Coherence by elevated temperature

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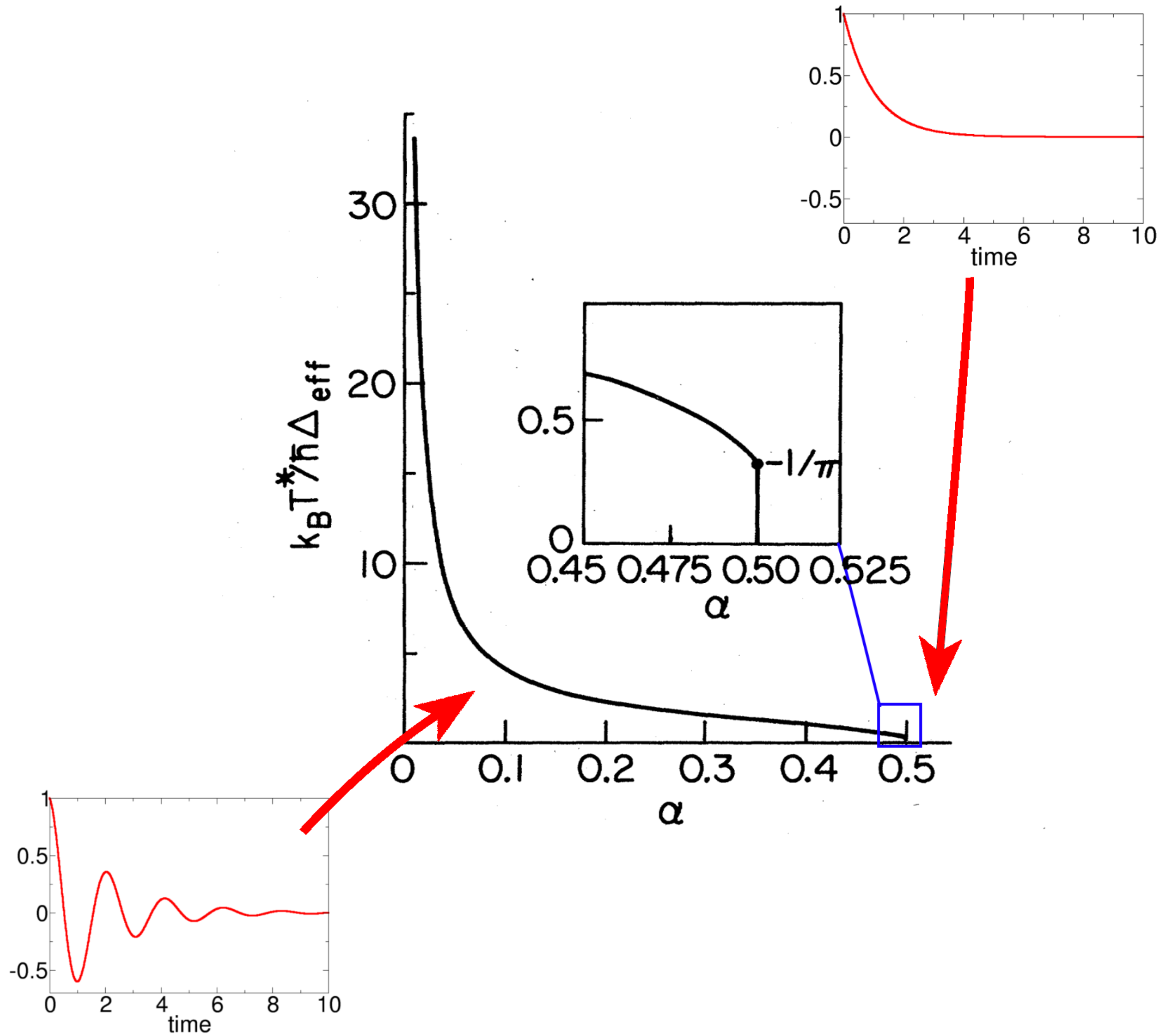
Goal

dynamics of dissipative quantum systems in ‘dissipation’-T-plane?

here: unbiased, ohmic spin boson model in scaling limit



(Garg ’85; Weiss, Grabert ’86; Leggett et al. ’87)

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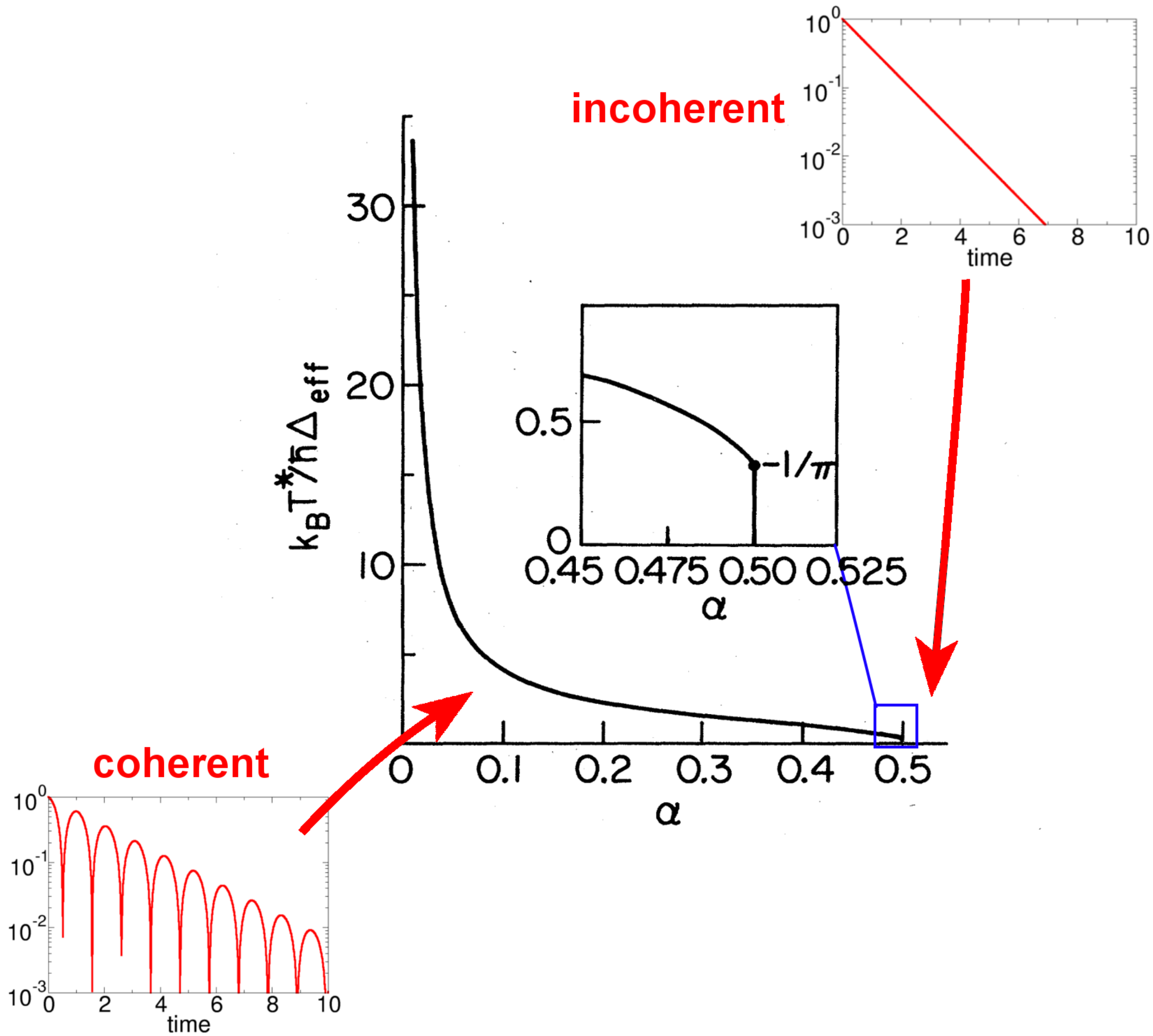


(Garg ’85; Weiss, Grabert ’86; Leggett et al. ’87)

The unbiased, ohmic spin-boson model

Hamiltonian

$$H = -\frac{\Delta}{2}\sigma_x + \sum_k \omega_k b_k^\dagger b_k - \sum_k \frac{\lambda_k}{2}\sigma_z \left(b_k^\dagger + b_k\right),$$

(ohmic) spectral density

$$J(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k) = 2\alpha\omega\Theta(\omega_{\mathrm{c}} - \omega)$$

scaling limit—'universal' physics

ω_{c} larger than any other energy scale

emergent (Kondo) scale

$$T_{\mathrm{K}} = \Delta \left(\Delta/\omega_{\mathrm{c}}\right)^{\alpha/(1-\alpha)}$$

to be specific: initial condition and observable

$$\rho(t=0) \sim |\uparrow\rangle \otimes e^{-\beta H_{\mathrm{bath}}}, \quad P(t) = \langle \sigma_z(t) \rangle$$

(Leggett et al. '87, Weiss '12)

The interacting resonant level model

mapping at low energies (by bosonization)

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \sqrt{\frac{\Gamma_0}{2\pi\nu}} \sum_k (d^\dagger a_k + d a_k^\dagger) + \frac{U}{2\nu}(d^\dagger d - d d^\dagger) \sum_{kk'} : a_k^\dagger a_{k'} :$$

lead density of states

$$D(\omega) = \sum_k \delta(\varepsilon_k - \omega) = \nu \quad \text{for} \quad \omega \ll \omega_c$$

mapping of parameters

$$U = 1 - \sqrt{2\alpha}\ , \quad \Gamma_0 = \Delta^2/\omega_c\ , \quad g = 2U - U^2 = 1 - 2\alpha$$

advantage

$\alpha = 1/2$ ($g = 0$) corresponds to $U = 0$ (Toulouse limit)

(Leggett et al. '87, Weiss '12)

Two complementary RG mehods: FRG and RTRG

FRG

(Metzner,. . . ,VM,. . . '12)

- aims at time-dependent irreducible (Keldysh) vertex functions
- auxiliary lead cutoff
- all orders in $\Delta^2/\omega_c = \Gamma_0$
- truncation: keep only lowest order in g on rhs of flow equations
- analytical results for $T_K t \ll 1$ and $(T_K t)^{|g|} \gg 1$

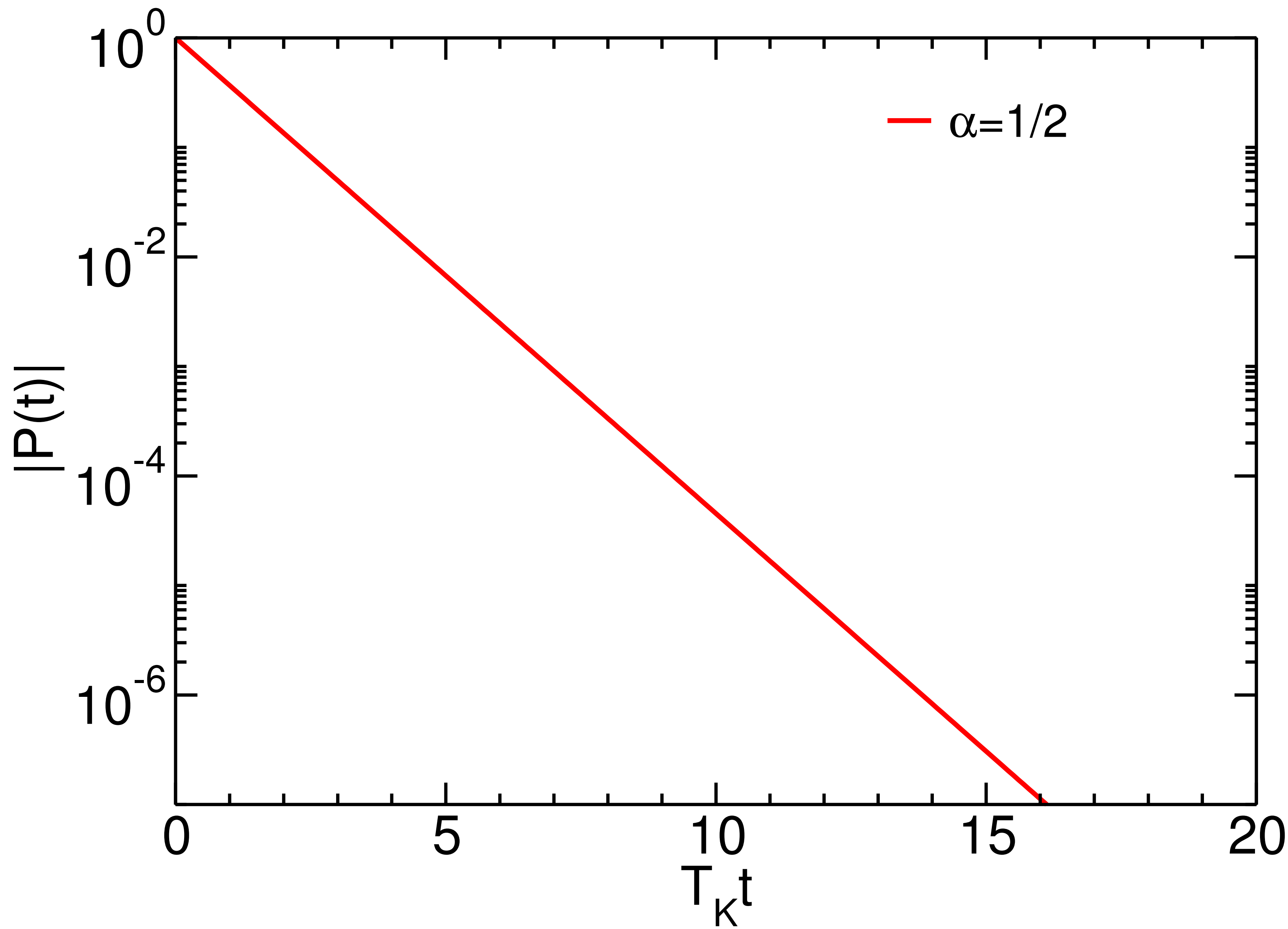
RTRG

(Schoeller '09)

- aims at reduced density matrix (of spin)
- setup in Laplace-Liouville space: Laplace variable E as cutoff
- approximation: all terms $\mathcal{O}\left(g\frac{\Delta^2}{\omega_c E}\right)$ but not $\mathcal{O}\left(g\left[\frac{\Delta^2}{\omega_c E}\right]^2\right)$
- analytical results for all t

$$T = 0$$

Spin expectation value for $T = 0$



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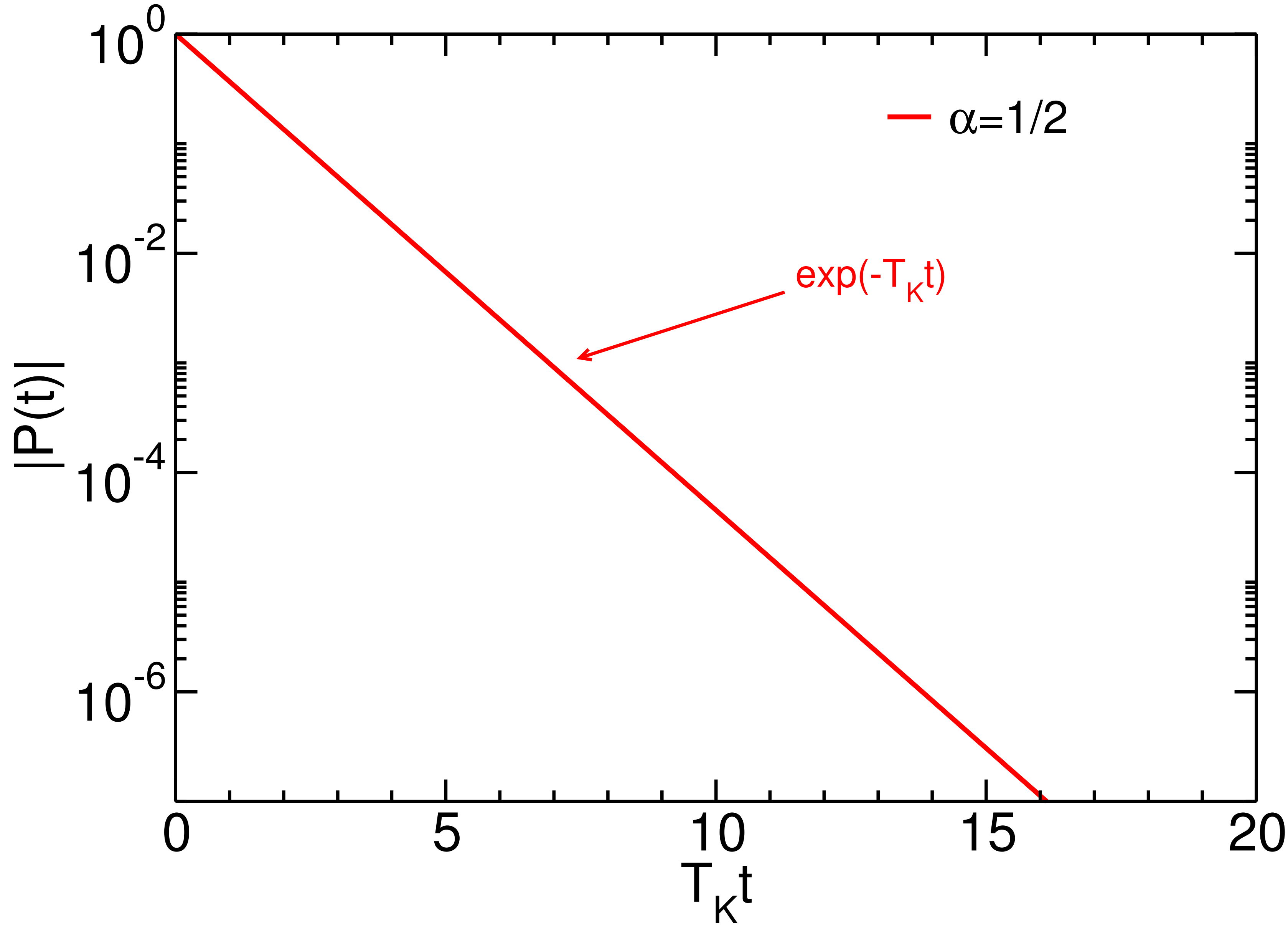

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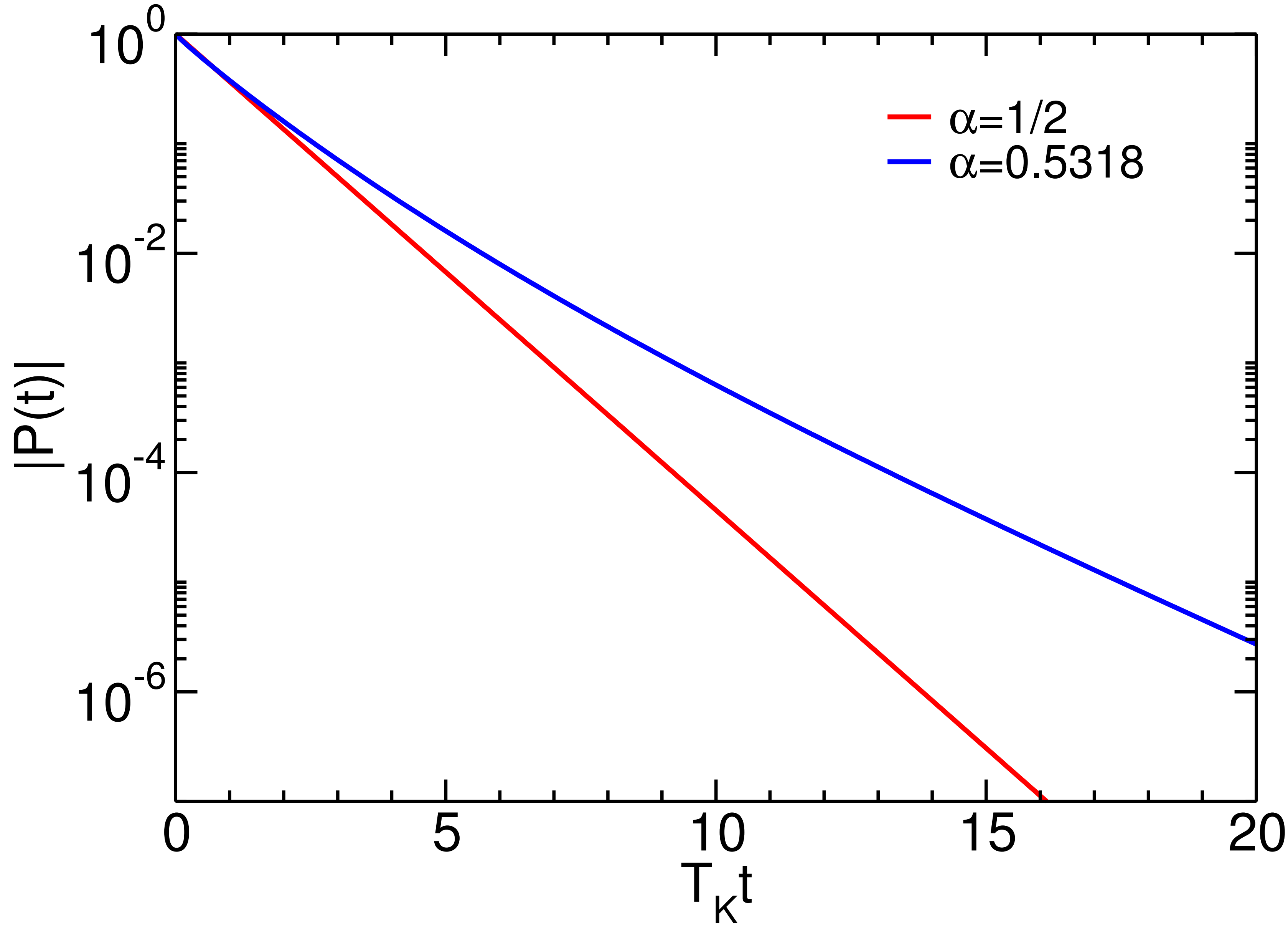

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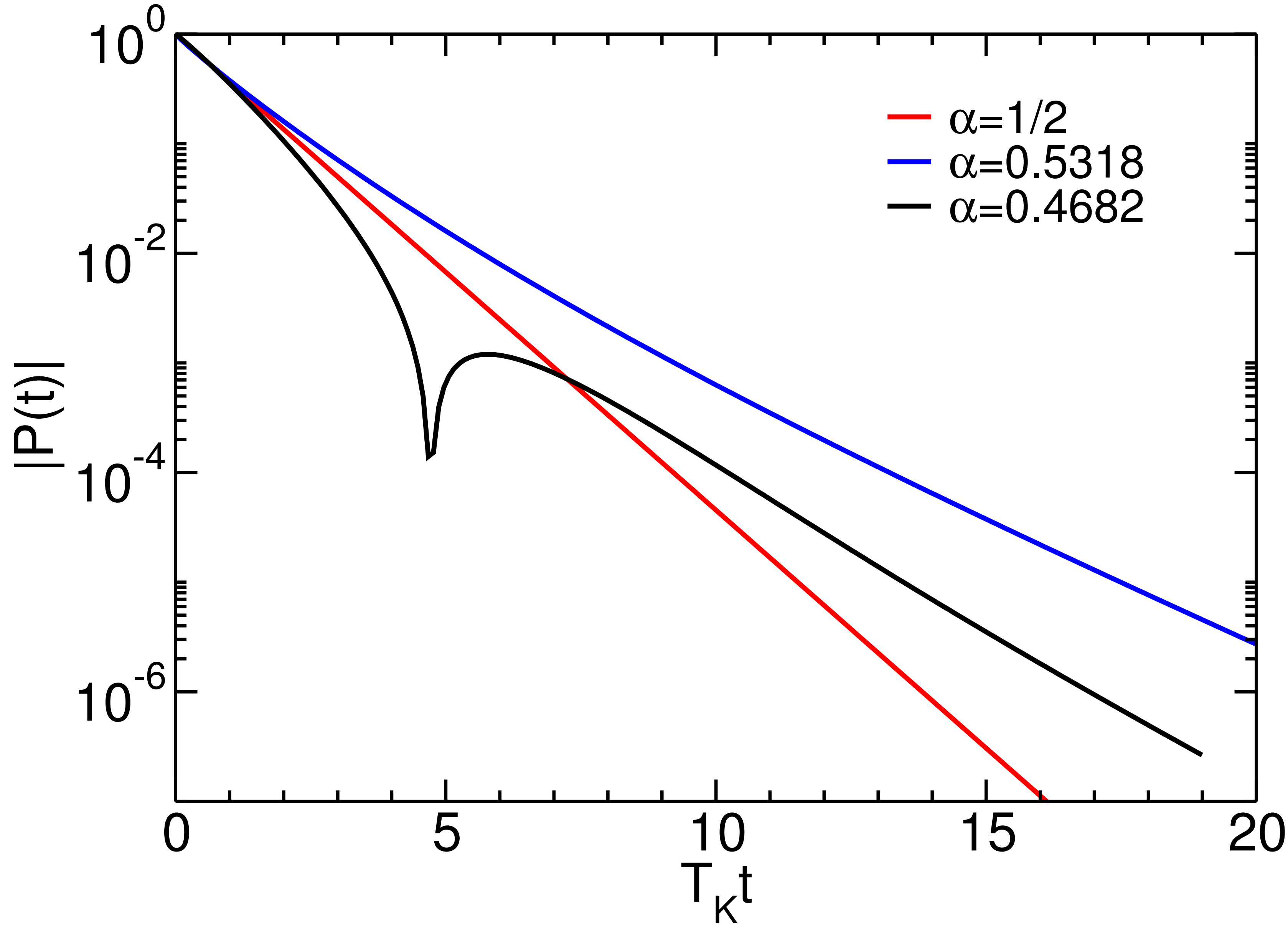

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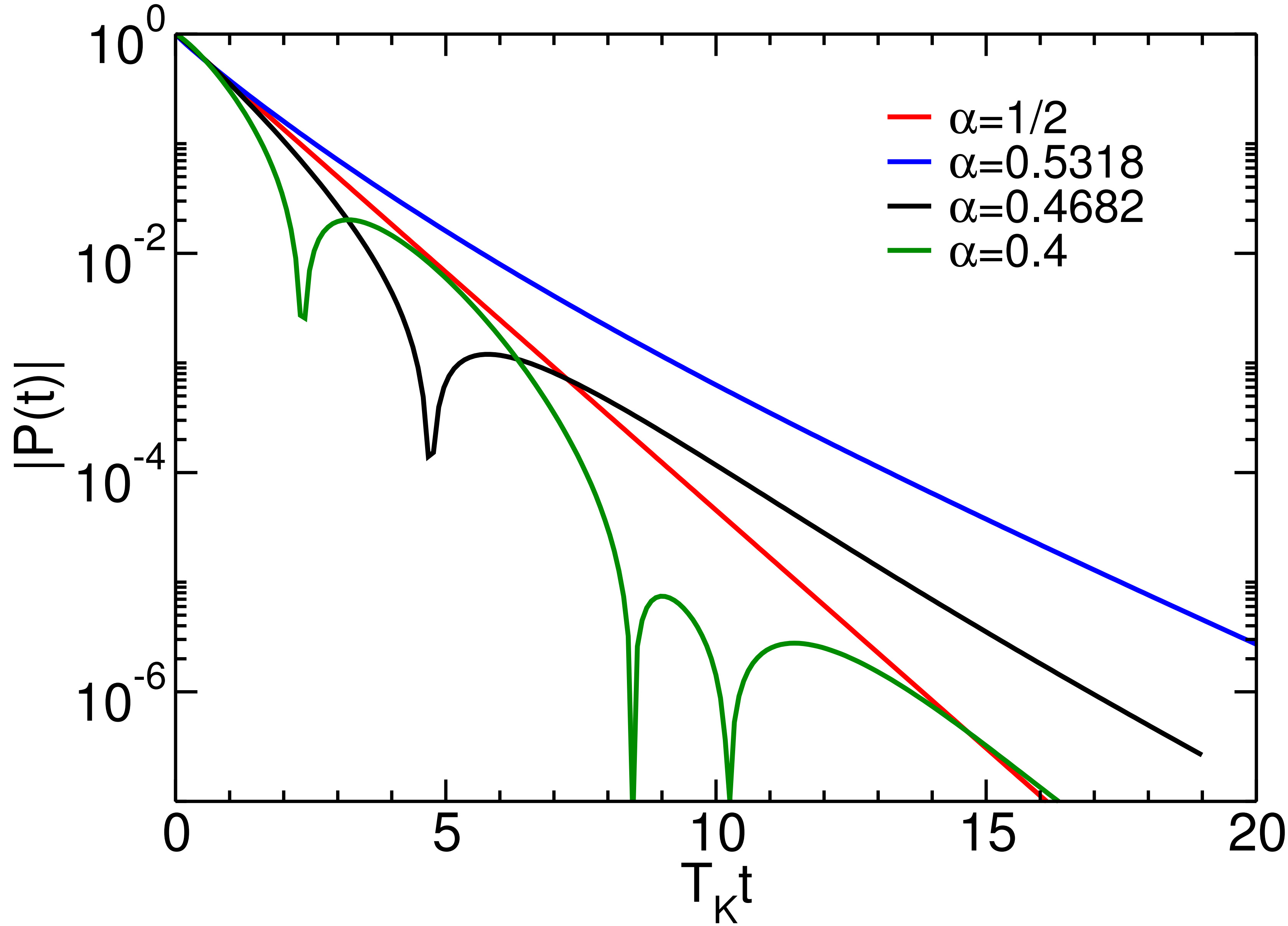

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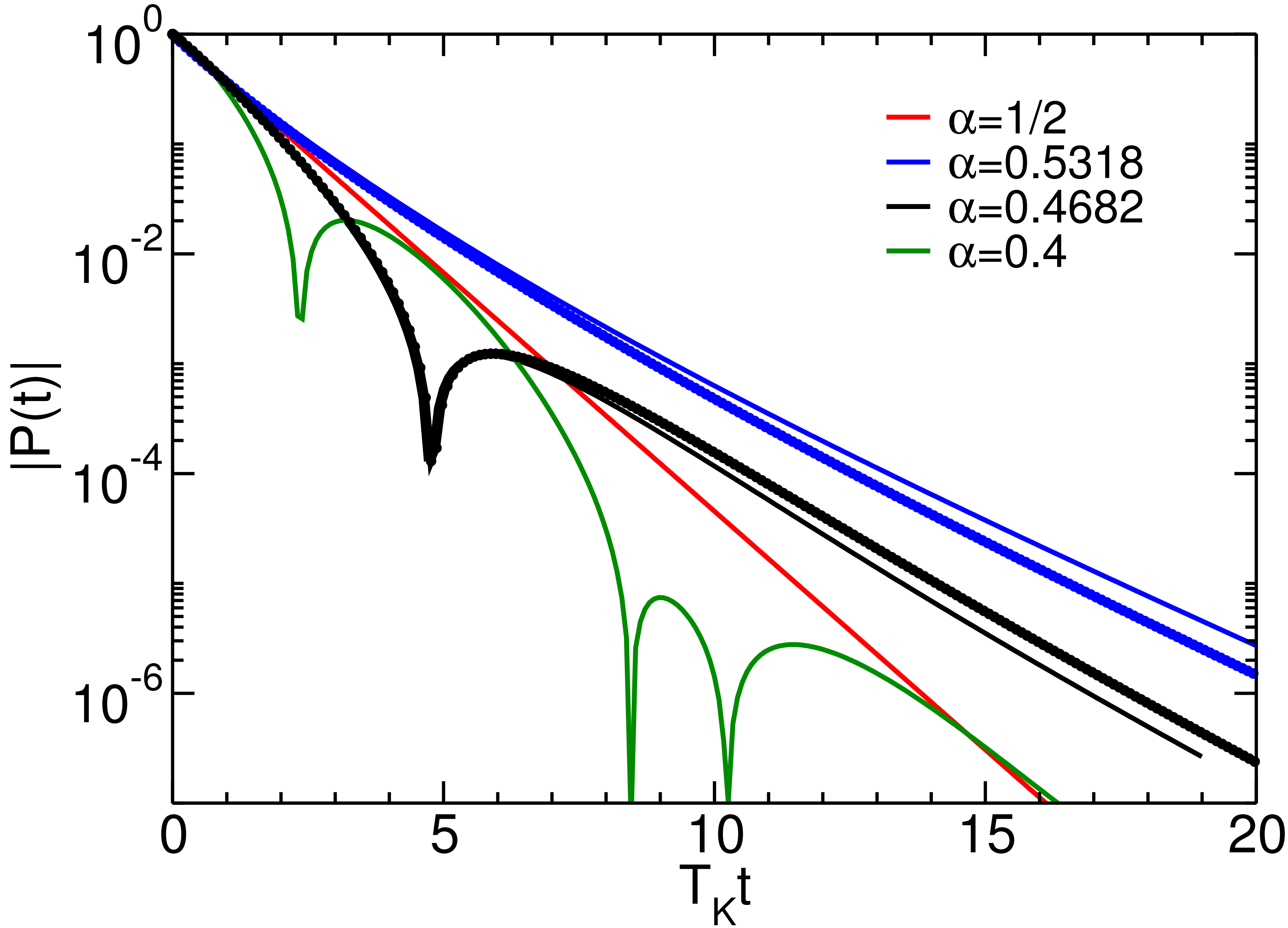


Take a closer look at RTRG flow equations

Laplace transform

$$
\begin{aligned}
P(t) &= \frac{i}{2\pi}\int_{-\infty+i0^+}^{\infty+i0^+} dE e^{-iEt}\Pi_1(E) \\
\Pi_1(E) &= [E + i\Gamma_1(E)]^{-1}
\end{aligned}
$$

IRLM: charge relaxation rate Γ_1

flow equations

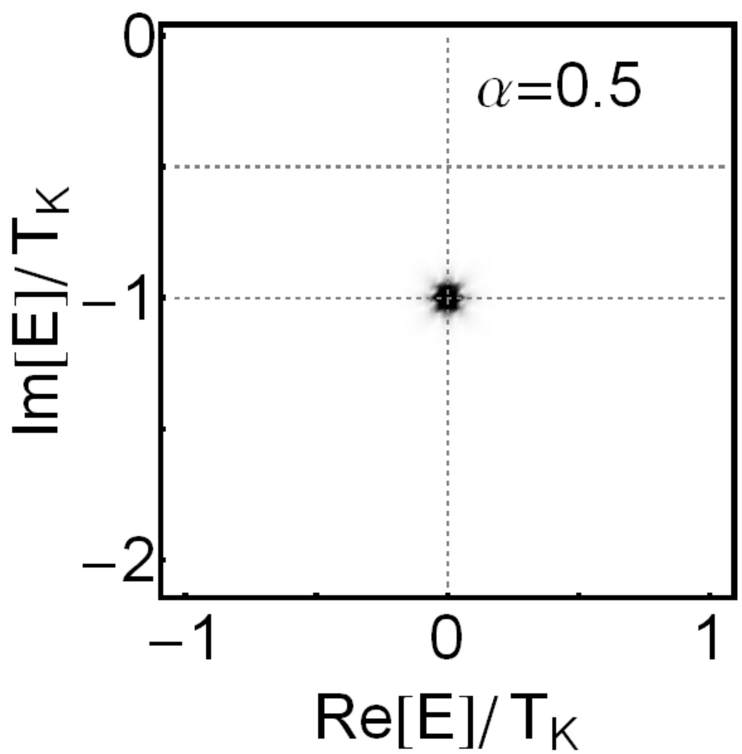
$$
\begin{aligned}
\frac{d\Gamma_{1/2}(E)}{dE} &= -g\Gamma_1(E)\Pi_{2/1}(E) \\
\Pi_2(E) &= [E + i\Gamma_2(E)/2]^{-1}
\end{aligned}
$$

IRLM: level broadening $\Gamma_2/2$

initial conditions

$$
\Gamma_n(i\omega_{\rm c}) = \Delta^2/\omega_{\rm c}\ , \quad n = 1, 2
$$

Numerical solution in complex E-plane



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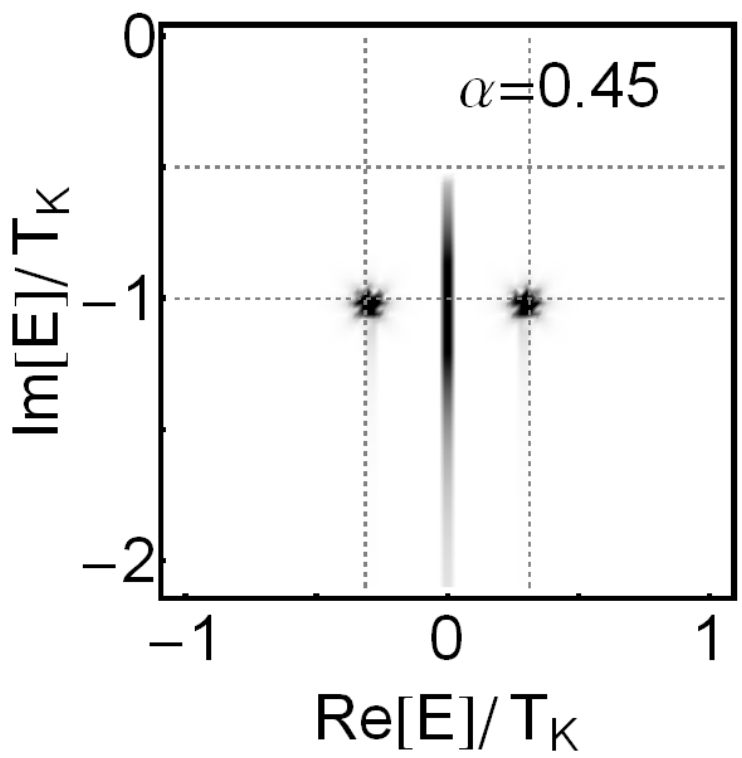

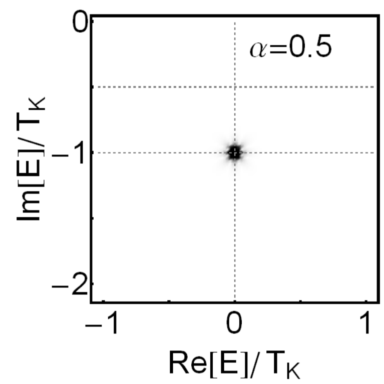

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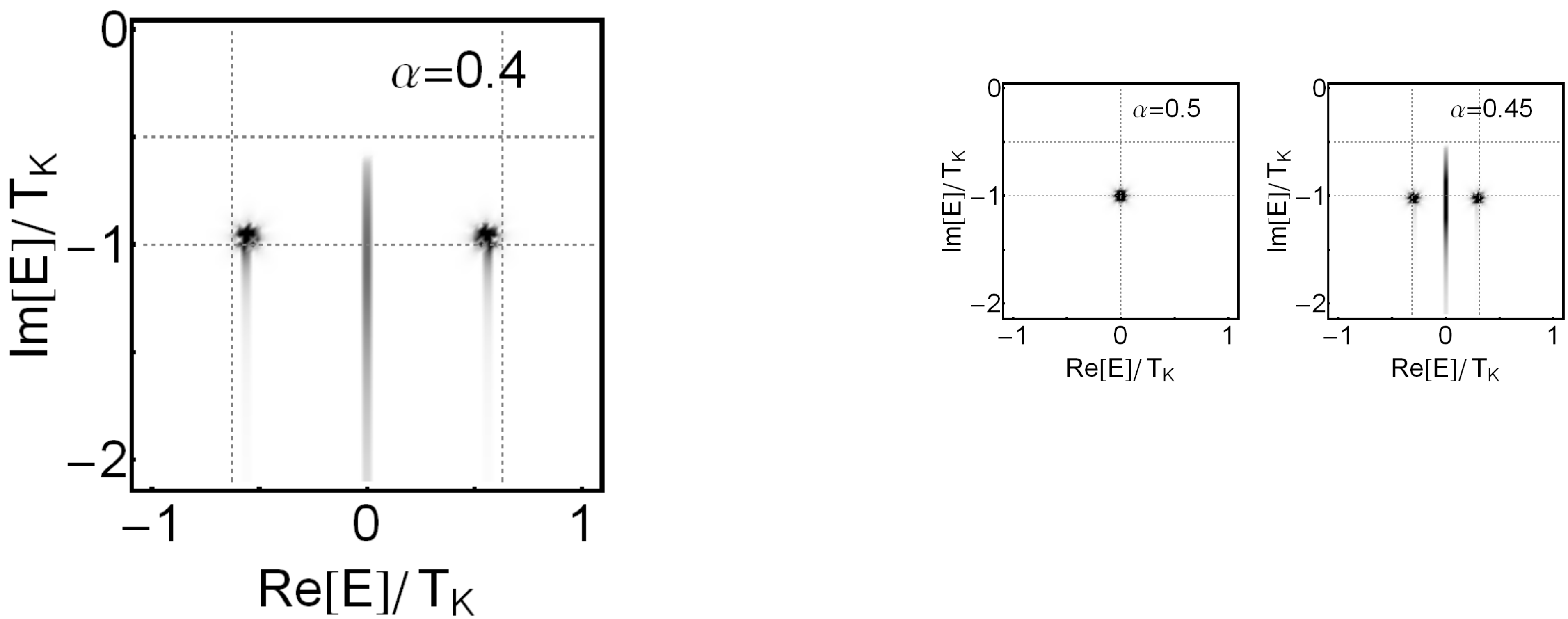

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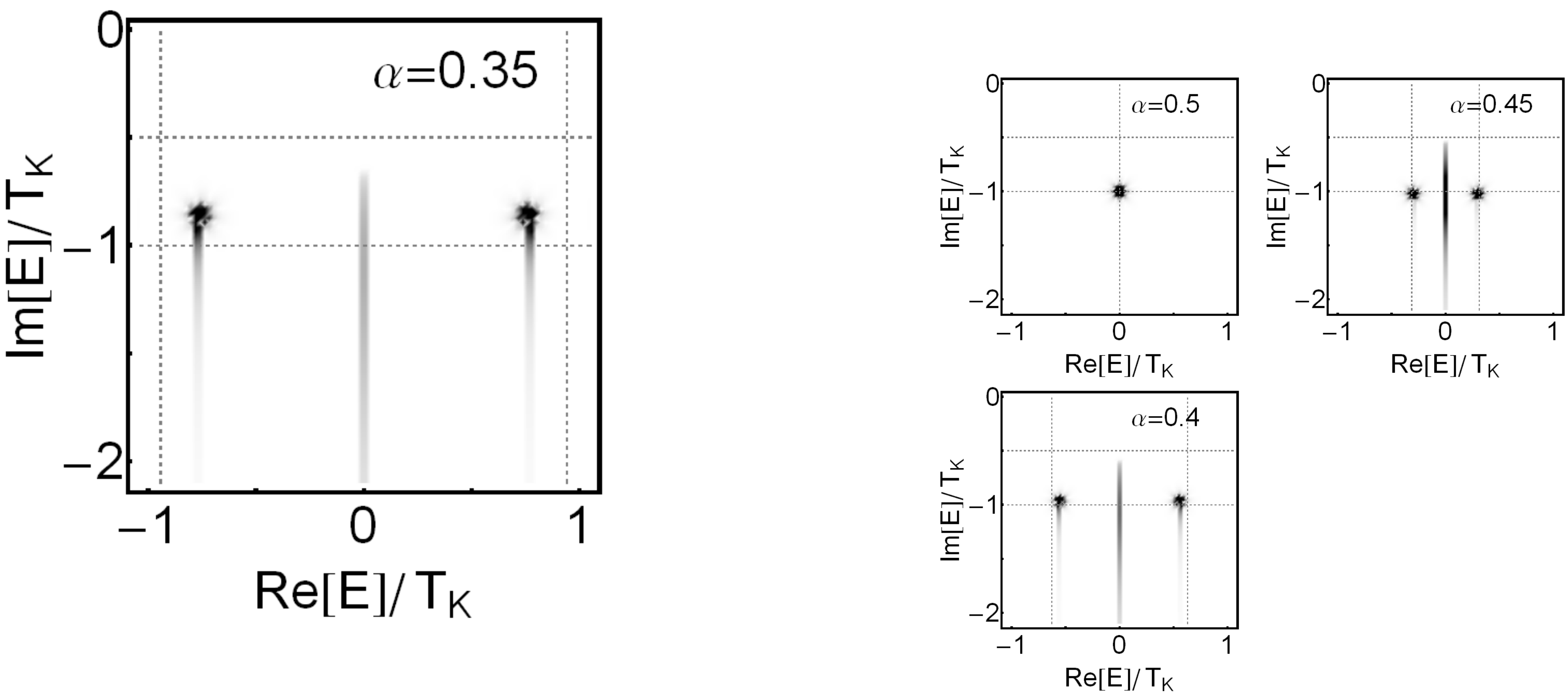

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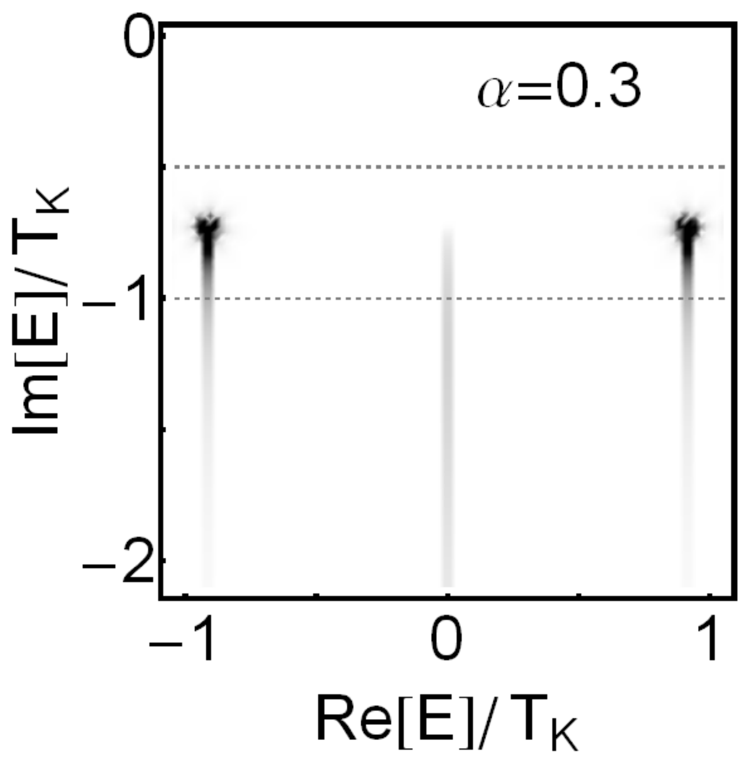
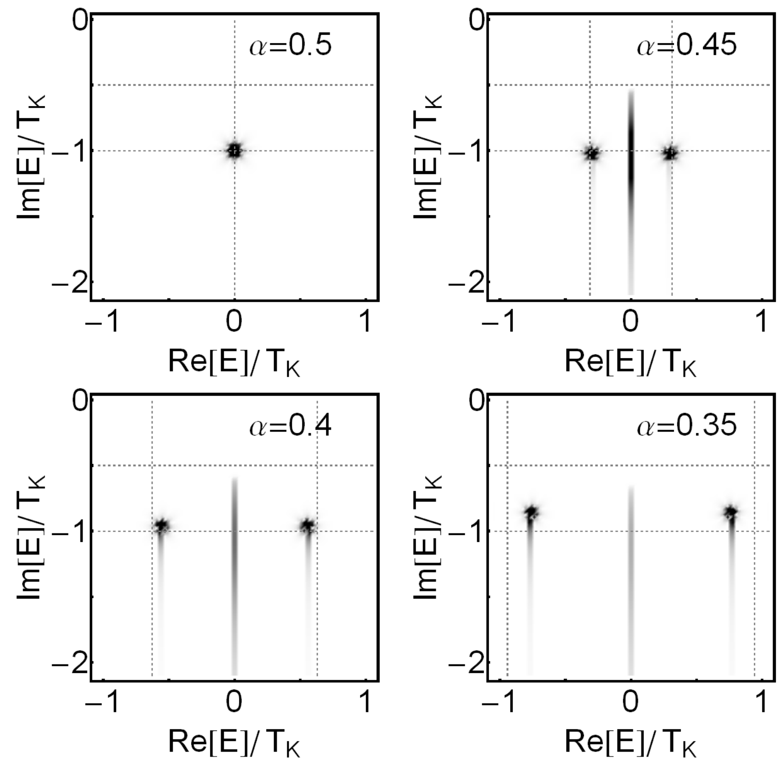
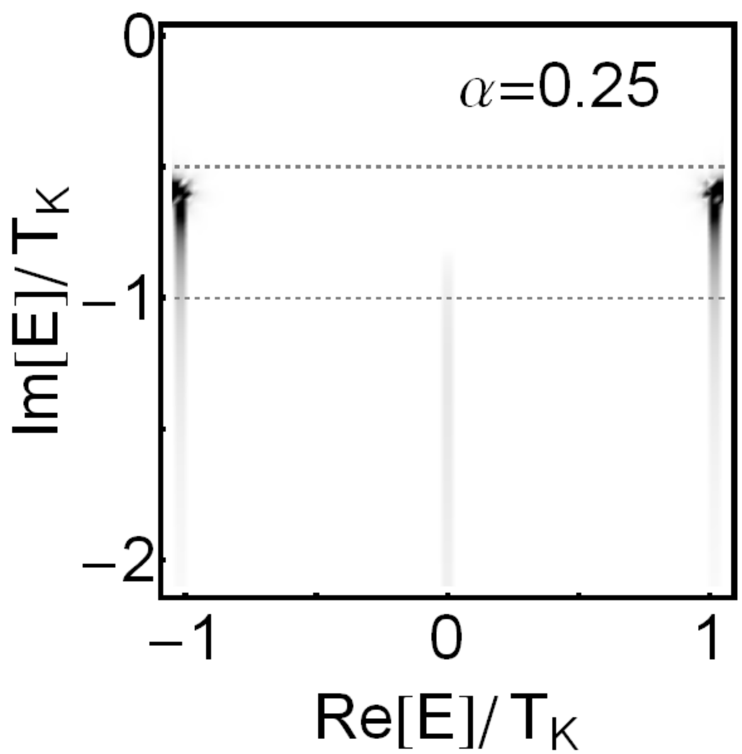


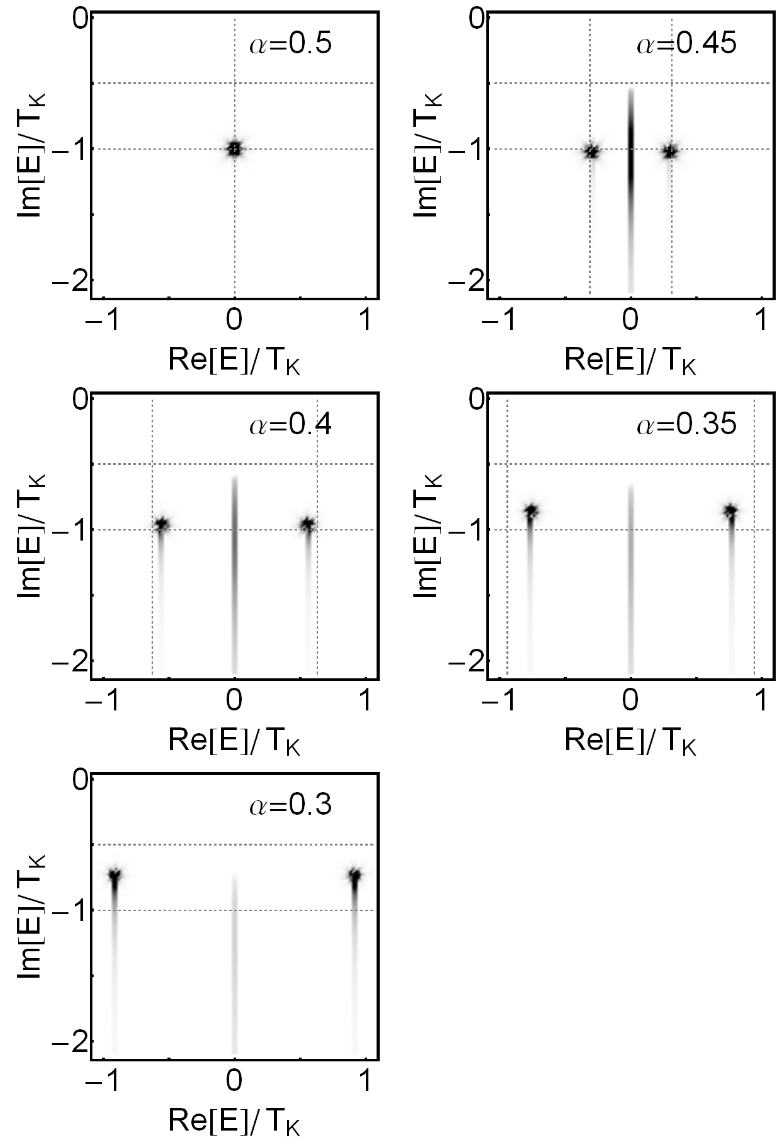


$$\alpha_c \approx 0.3$$

Numerical solution in complex E-plane



$$\left|\partial_{\mathrm{Im}\,E}\mathrm{Re}\,\Pi_1 + \partial_{\mathrm{Re}\,E}\mathrm{Im}\,\Pi_1\right|$$



Conclusion

three distinct regimes

- $1/2 < \alpha < 1$: incoherent
- $\alpha_c < \alpha < 1/2$: partially coherent
- $0 < \alpha < \alpha_c$: asymptotically coherent

still controlled?

RTRG for SBM (small α!) gives $\alpha_c \approx 0.36$ (Kashuba, Schoeller '13)

Analytical results

$T_K t \ll 1$ from RTRG and FRG (compare to NIBA: Leggett et al' 87)

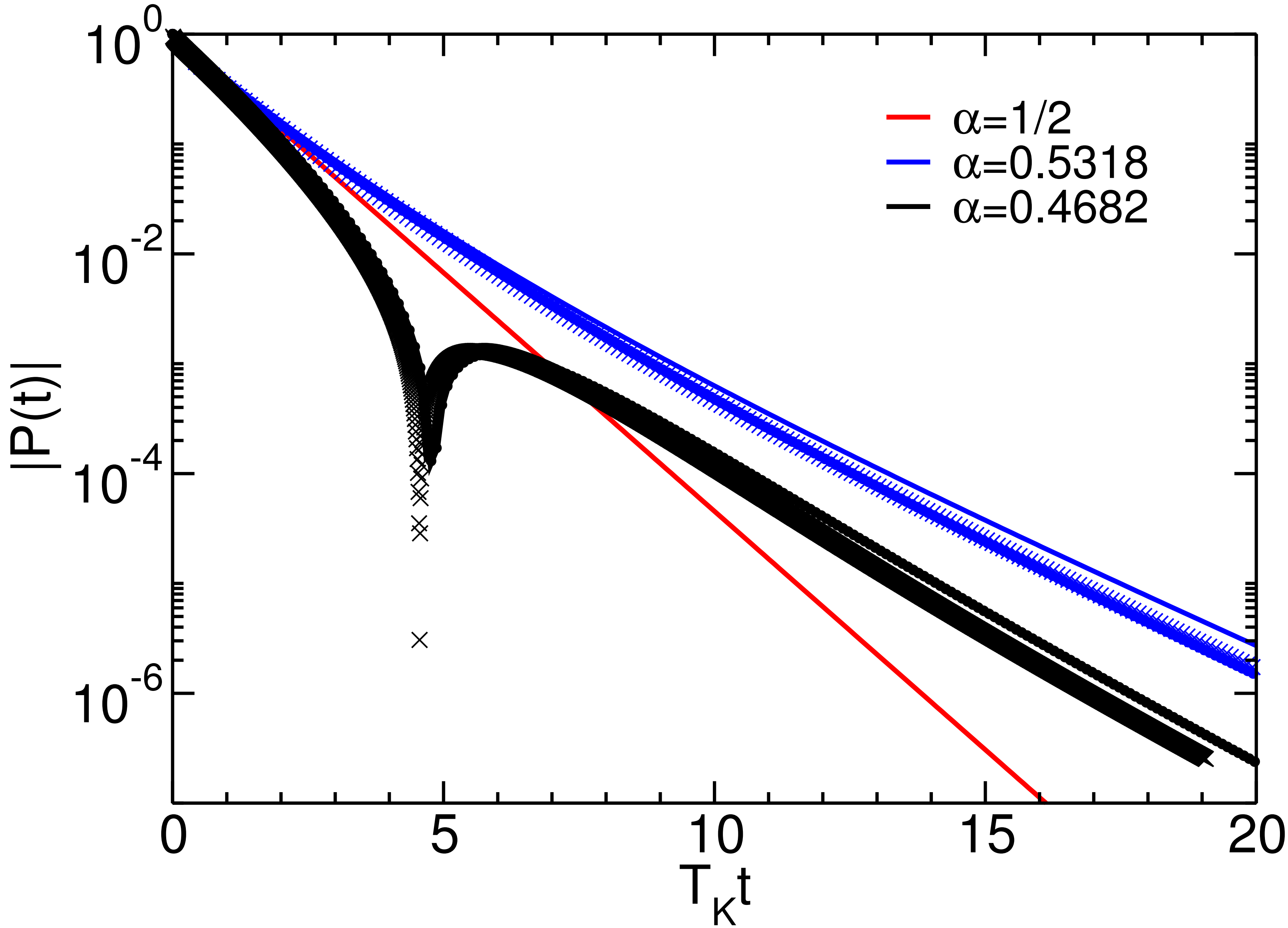
$$1 - P(t) = (T_K t)^{1+g}/\Gamma(2+g) + \mathcal{O}\left([T_K t]^{2+2g}\right)$$

$T_K t \gtrsim 1$ from RTRG

$$\begin{aligned}
P(t) &= P_{bc}(t) + P_p(t) \\
P_{bc}(t) &\approx \frac{1}{\pi}\mathrm{Im}\left\{e^{-\gamma t}\mathrm{E}_1\left(\left[\tfrac{1}{2}\Gamma_2^* - \gamma\right] t\right)\right\} \\
\gamma &= e^{-i\pi g}(T_K t)^g \\
P_p(t) &\approx 2\frac{1-g}{1+g}\cos(\Omega t)\, e^{-\Gamma_1^* t}\Theta(g) \\
\frac{\Gamma_2^*}{T_K} &\approx 2\left[\frac{\pi g}{2\sin(\pi g)}\right]^{\frac{1}{1+g}}, \quad \frac{\Omega + i\Gamma_1^*}{T_K} \approx e^{(i\pi + \ln 2)\frac{g}{1+g}},
\end{aligned}$$

- frequency Ω as in NIBA and CFT (Lesage, Saleur '98)
- for $(T_K t)^{|g|} \gg 1$: $P_{bc}(t) = -g[1 + 3\Theta(-g)]\frac{e^{-T_K t/2}}{(T_K t)^{1+|g|}}$ (Egger et al. '97)

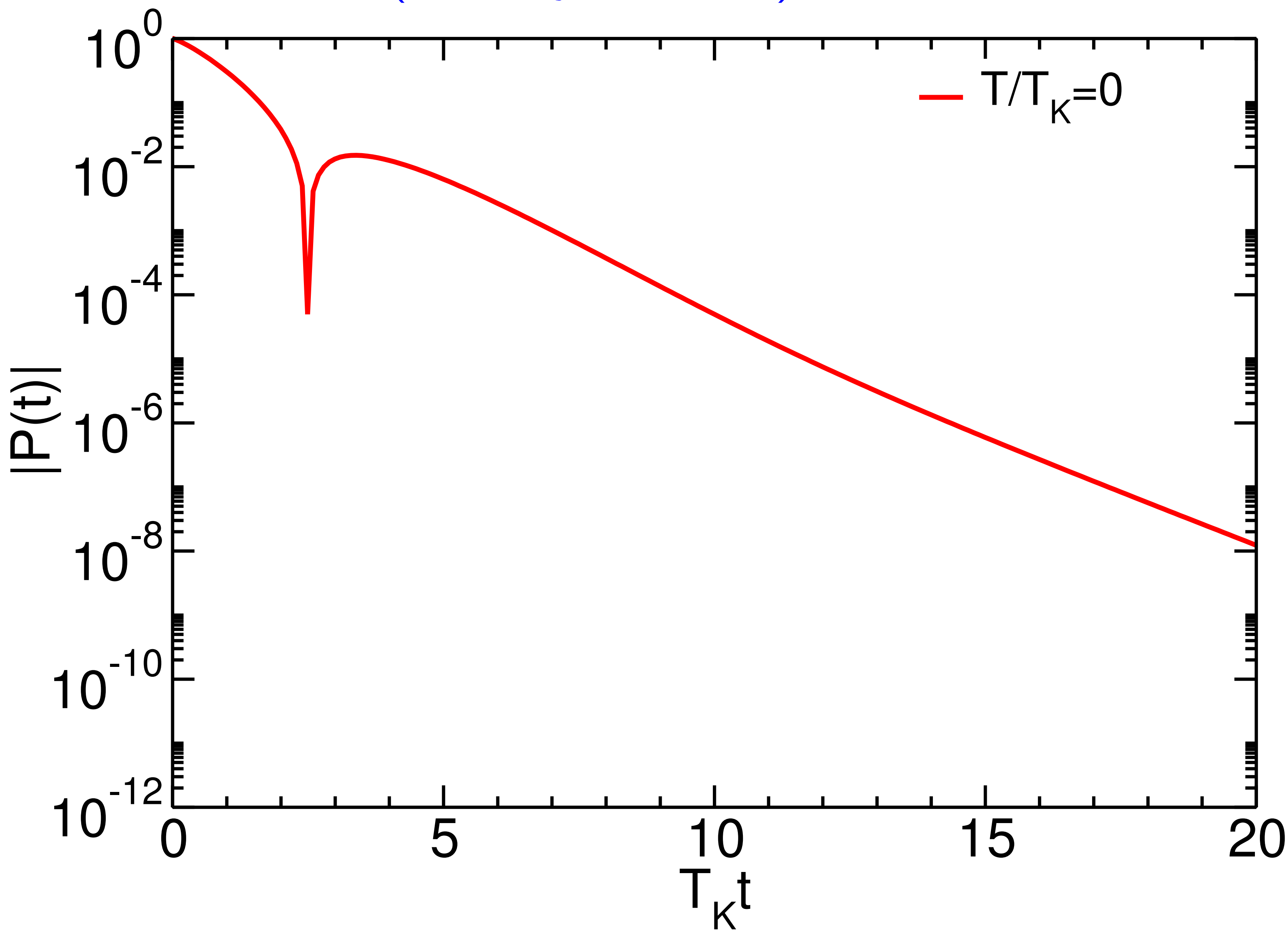
Comparison to numerics



$$T > 0$$

Spin expectation value for $T \geq 0$

FRG for $\alpha = 0.4045$ (partially coherent)



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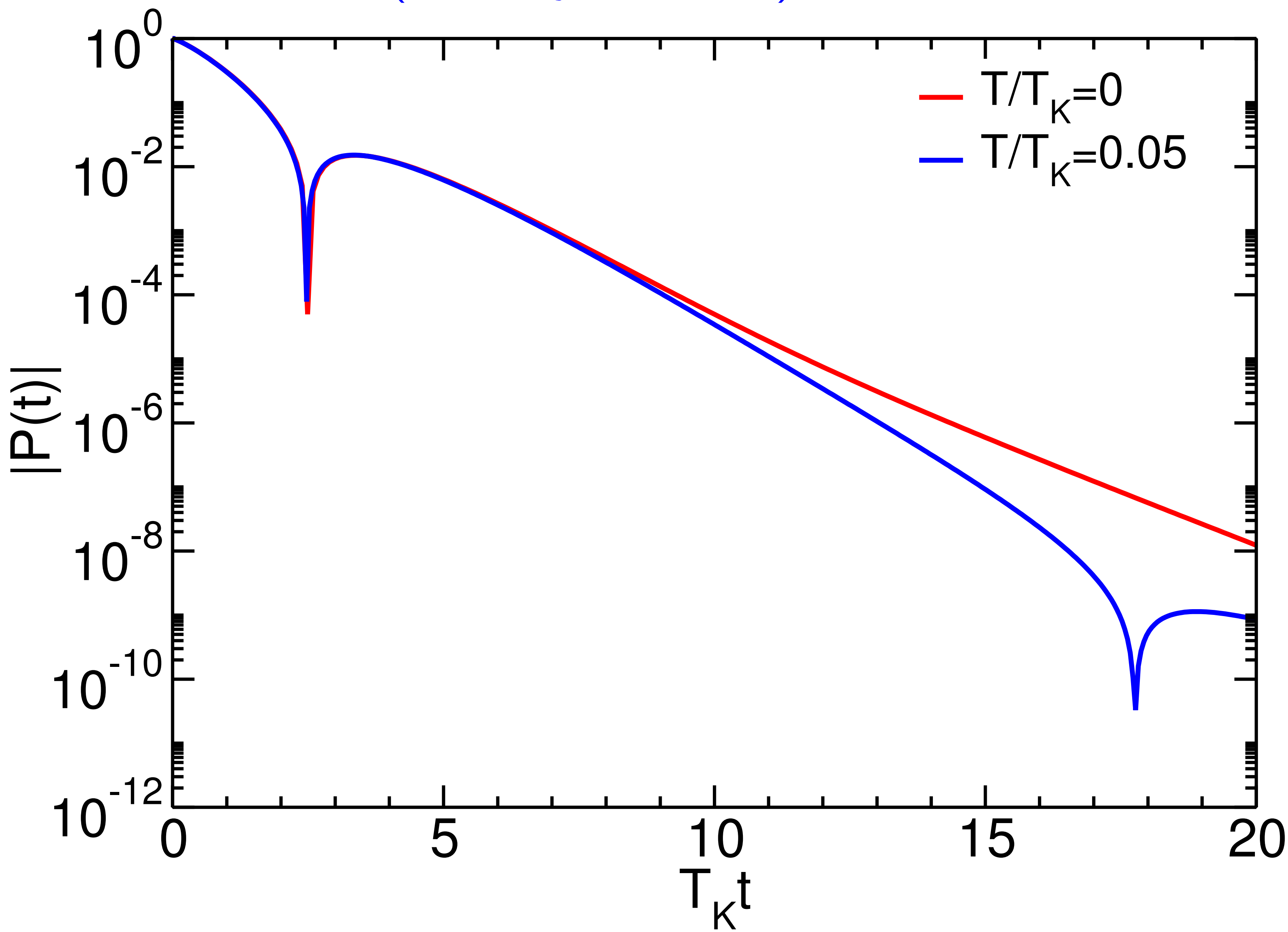

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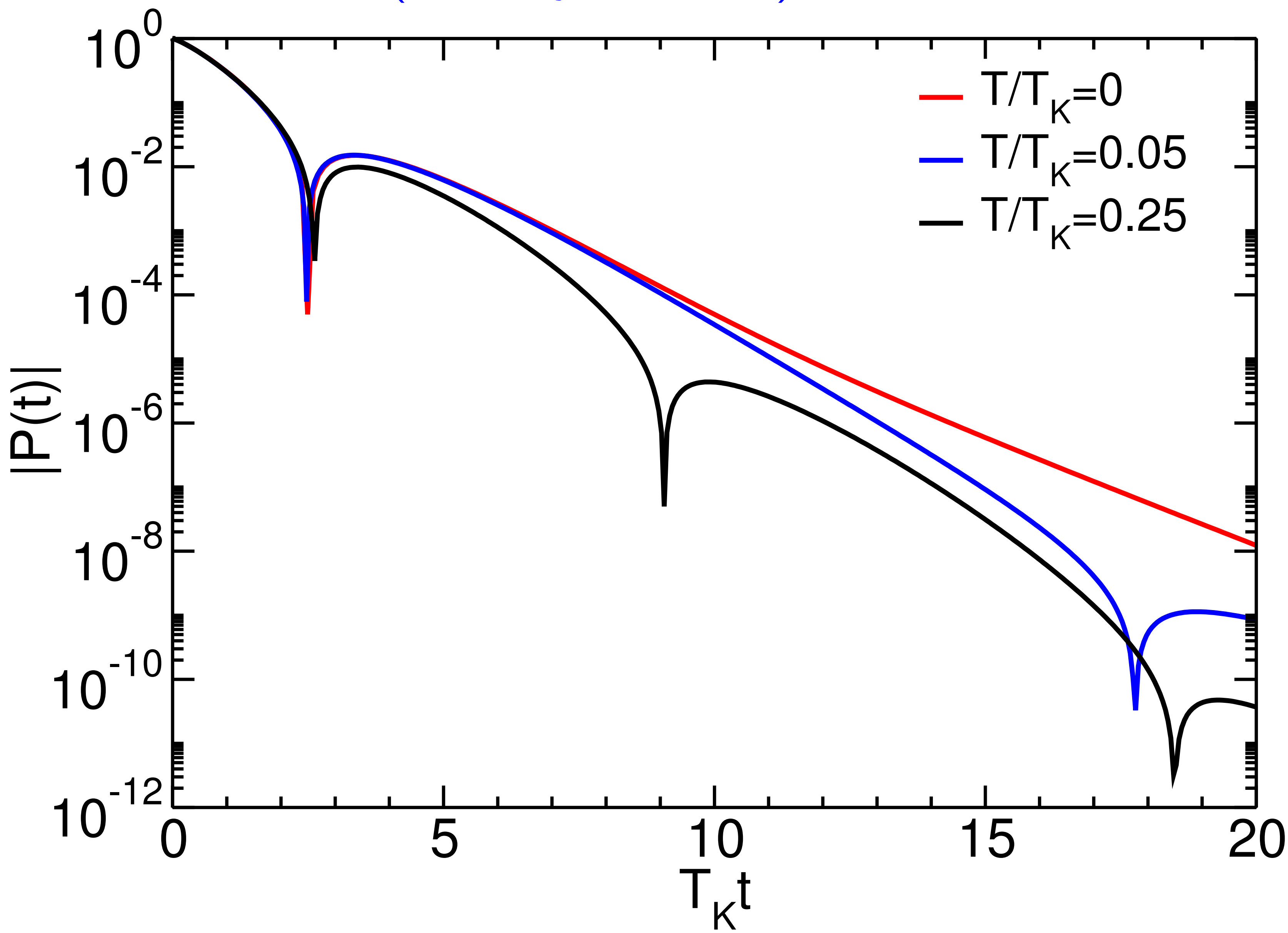

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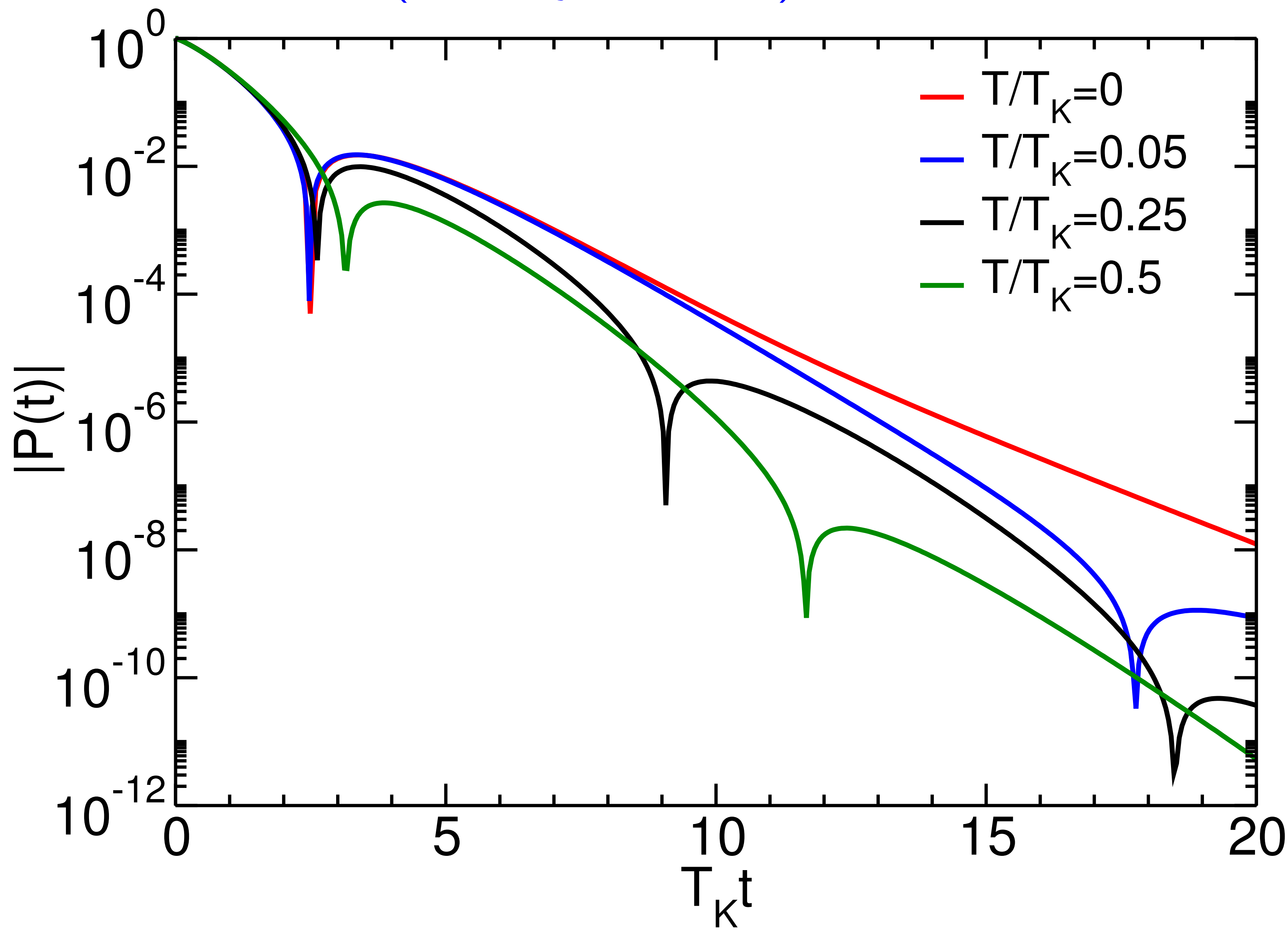

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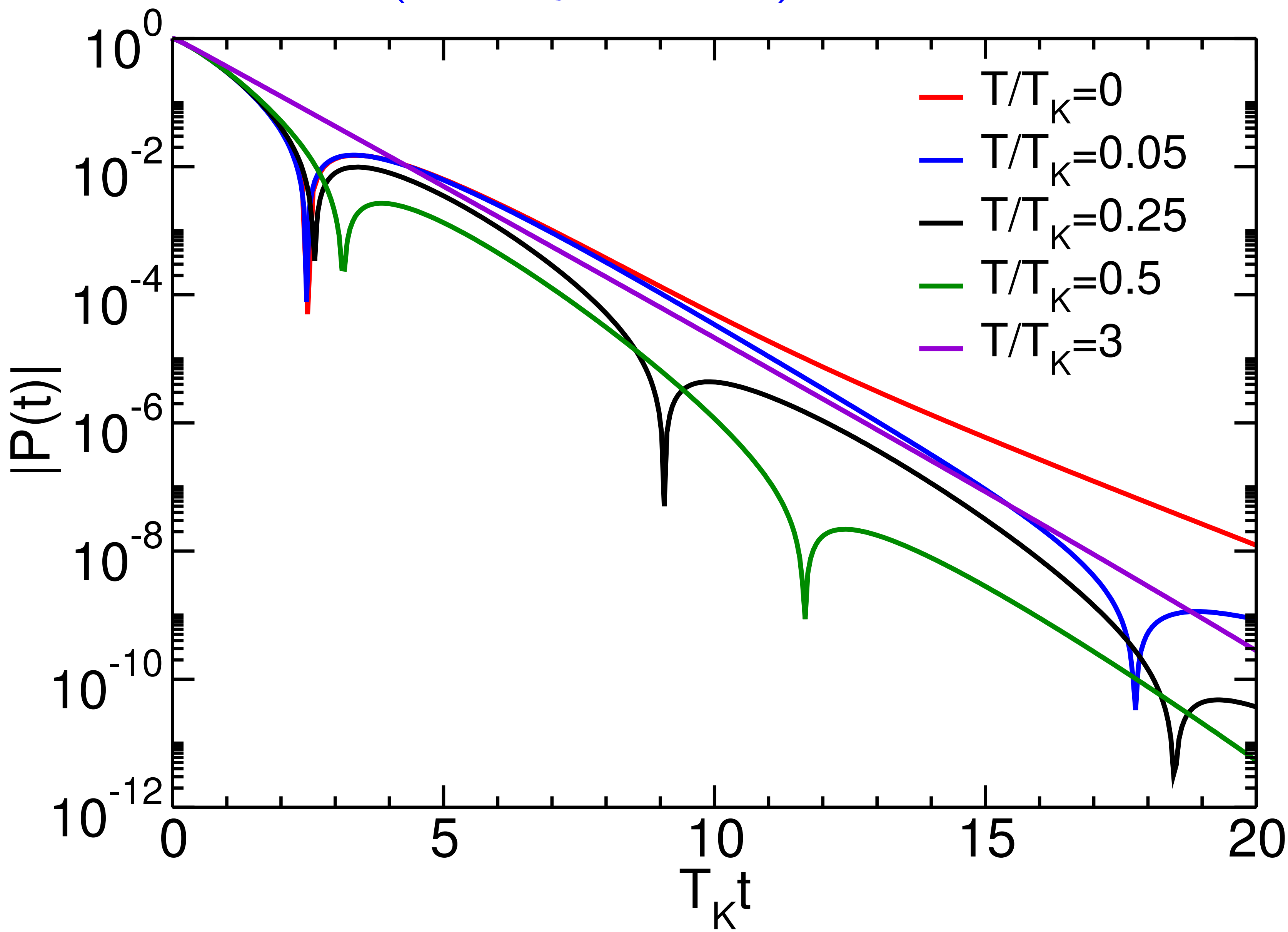
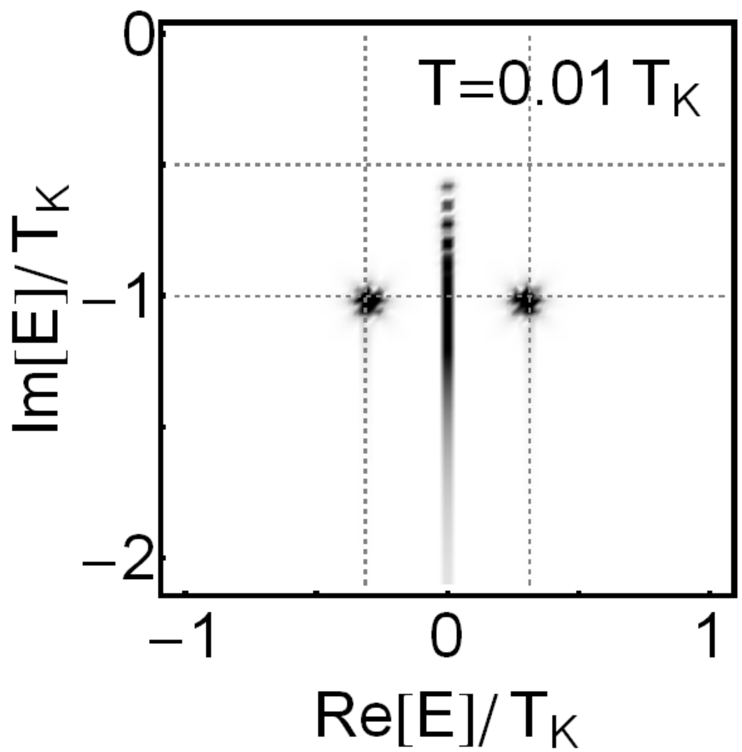


partially coherent → asymptotically coherent → incoherent?

Numerical solution of RTRG flow equations ($\alpha = 0.45$)



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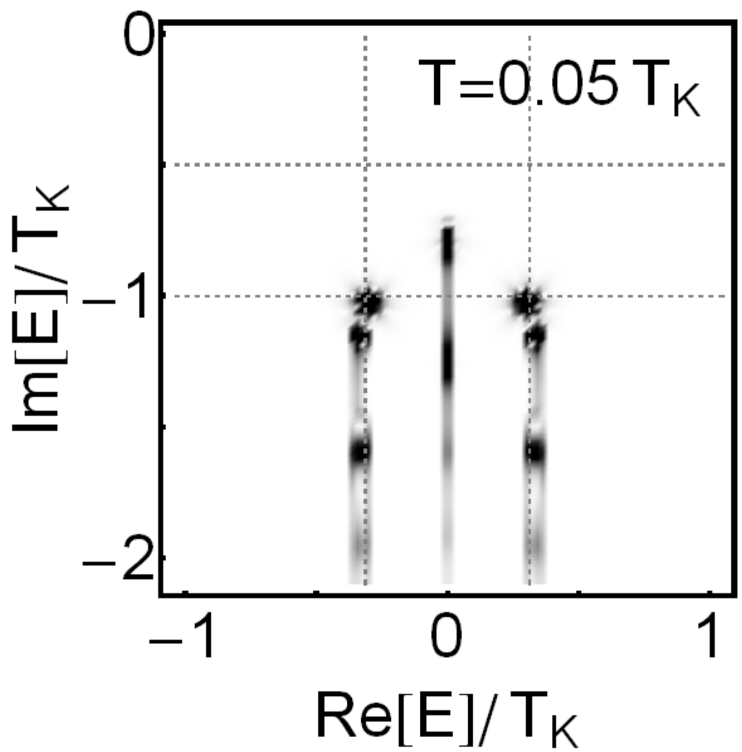

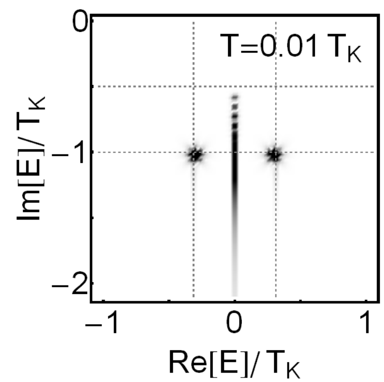

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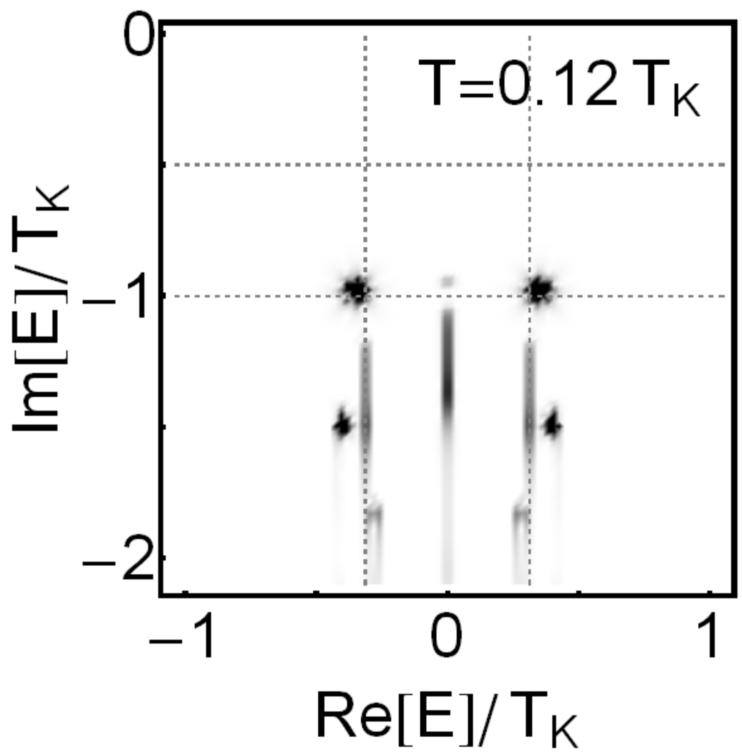
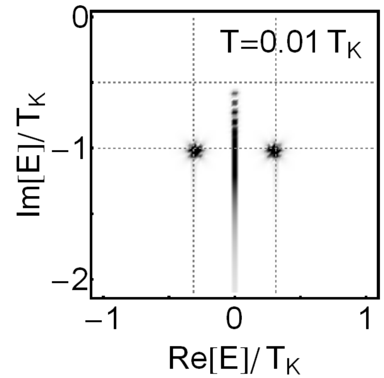
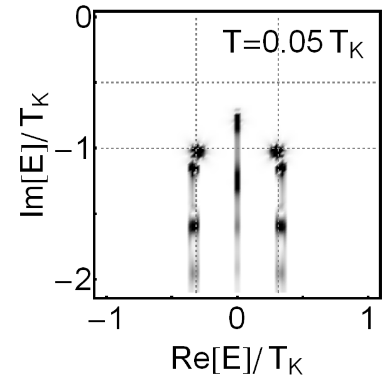
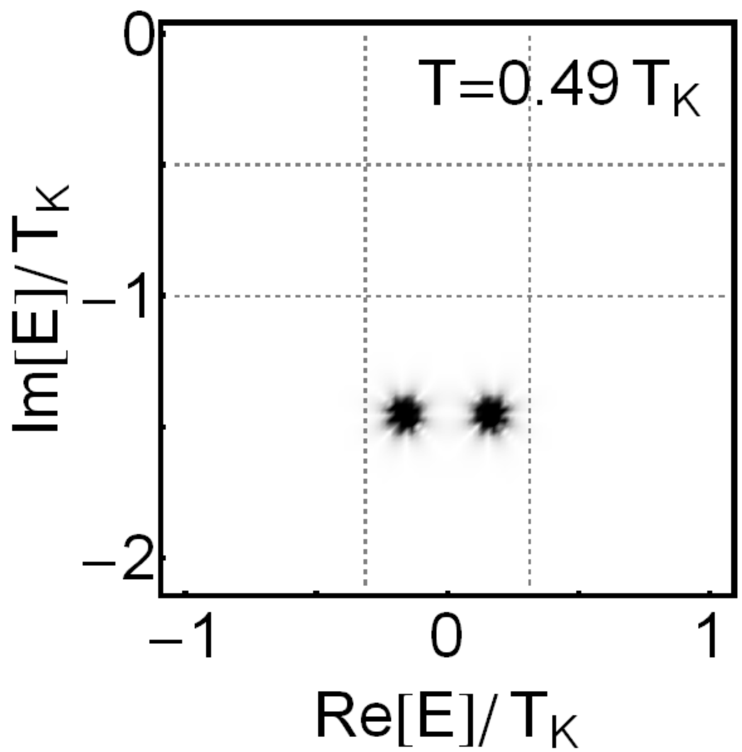


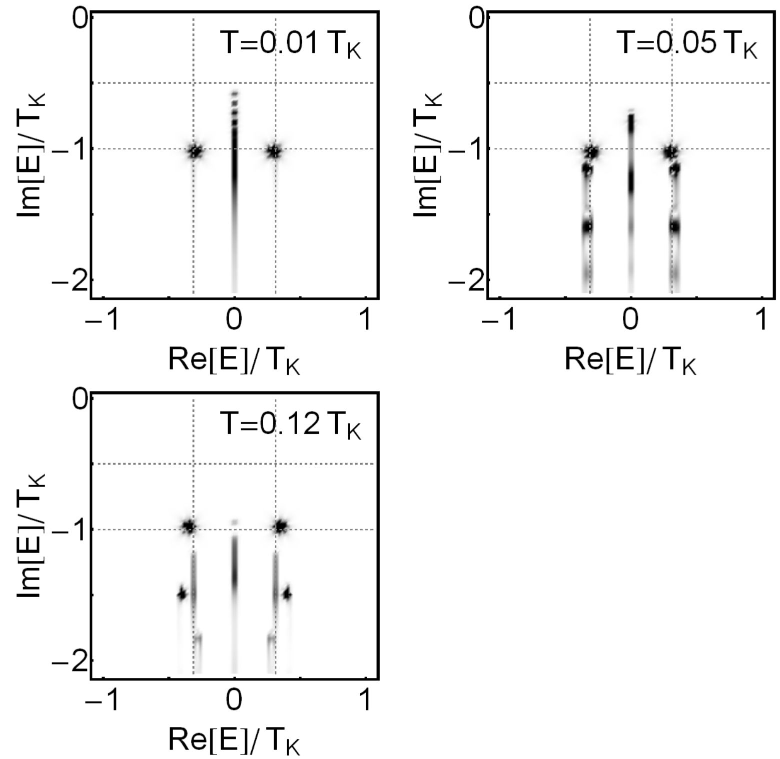




$T_{\text{c1}}(\alpha)$

Numerical solution of RTRG flow equations ($\alpha = 0.45$)





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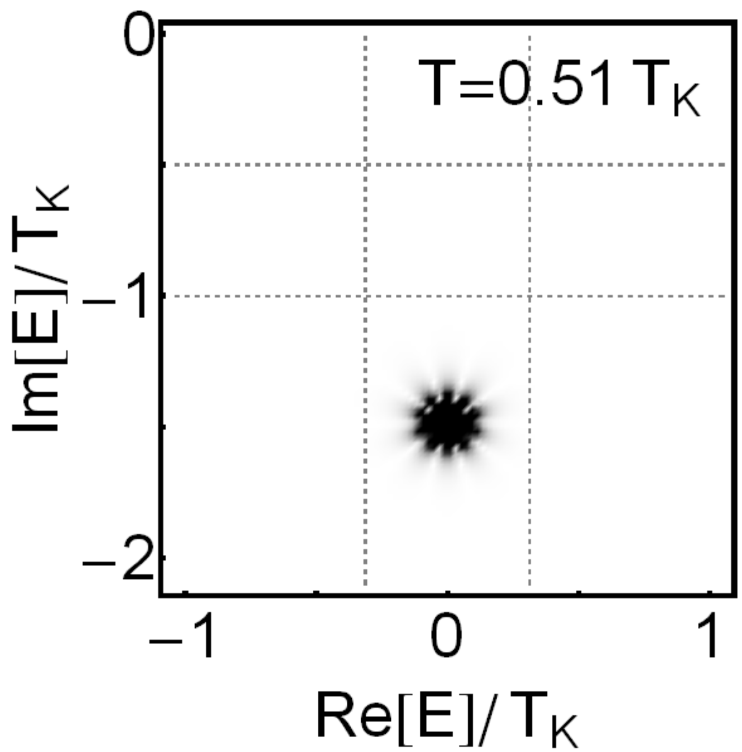
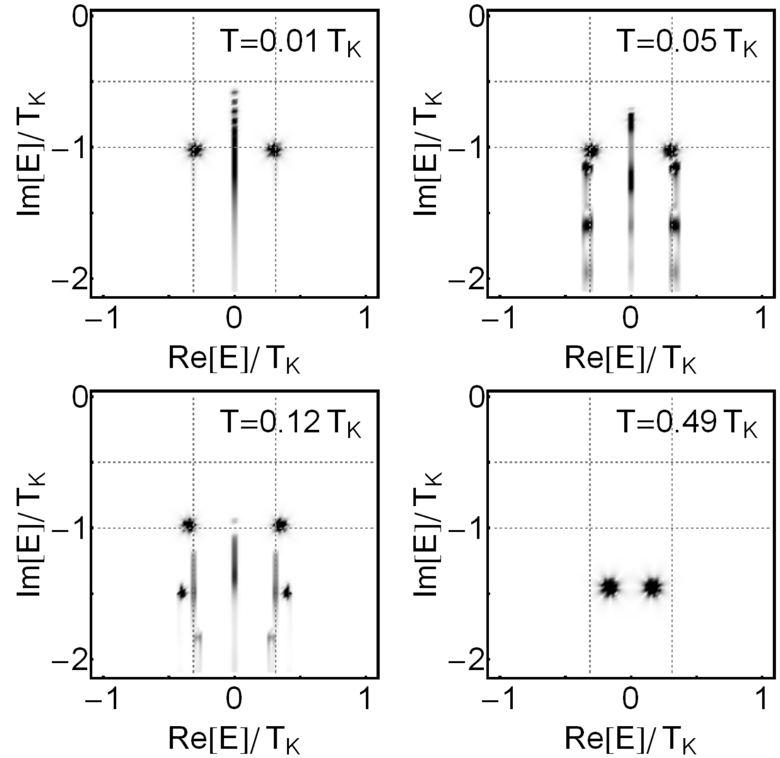
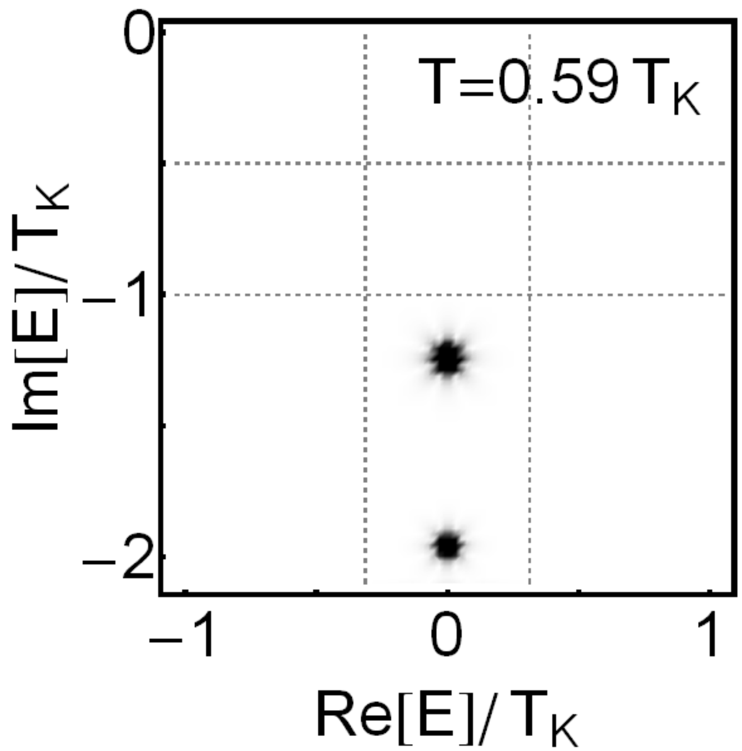
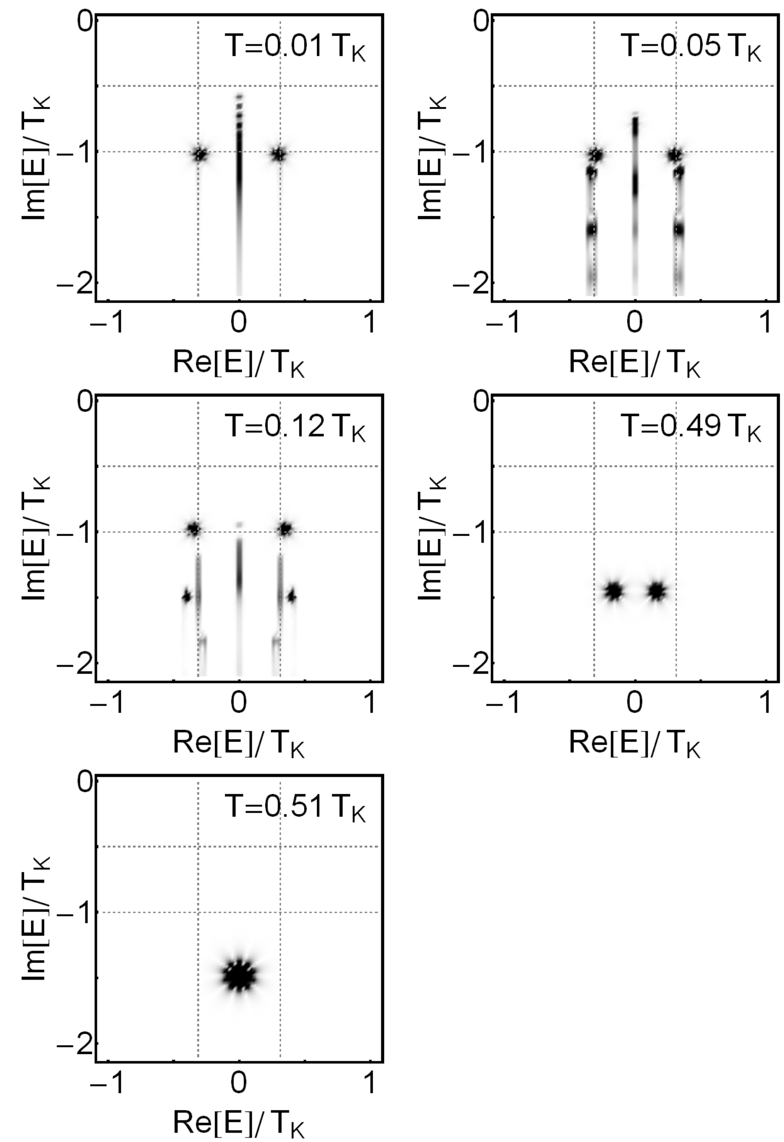




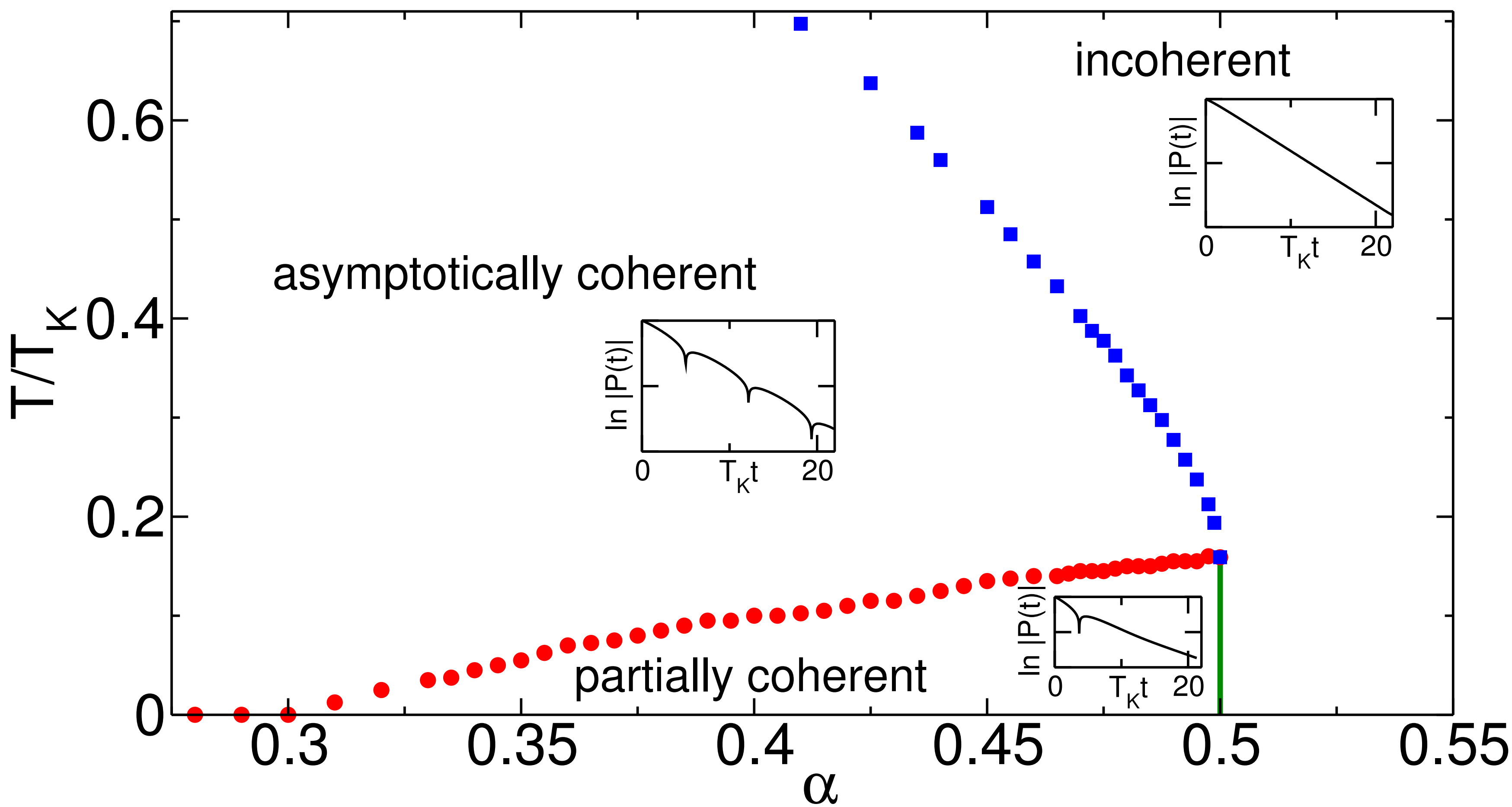
$T_{c2}(\alpha)$

Numerical solution of RTRG flow equations ($\alpha = 0.45$)





‘Phase’ diagram



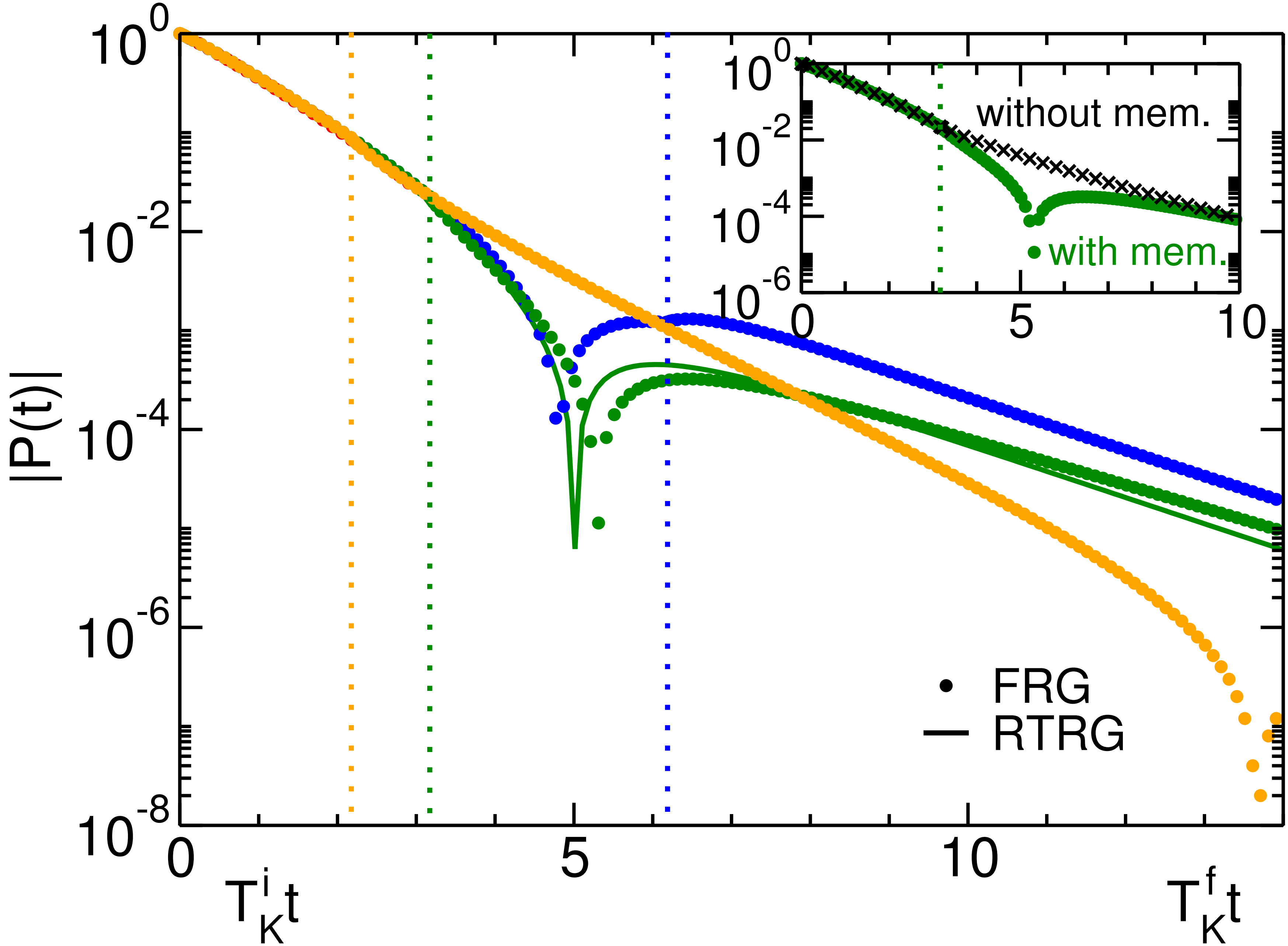
consider quench at $T = 0$

Does memory matter?

perform a (quantum) quench at time t_q

- time evolution up to $t = t_q$ as before out of spin up-state
- at t_q change α: $\alpha_i \longrightarrow \alpha_f$
- will the quantum system have a memory of the 'α_i-dynamics'
- quench from partially coherent to incoherent

Quench from partially coherent to incoherent



pronounced memory effects – non-Markovian dynamics

Summary

- three distinct dynamical regimes
- increase T: transition from partially to asymptotically coherent
- ‘coherence by elevated temperature’
- pronounced memory effects – non-Markovian dynamics

Refs.:

- $T = 0$ and quenches: Kennes, Kashuba, Pletyukhov, Schoeller, VM, Phys. Rev. Lett. **110**, 100405 (2013)
- mainly quenches: Kashuba, Kennes, Pletyukhov, VM, Schoeller, Phys. Rev. B **88**, 165133 (2013)
- $T > 0$: Kennes, Kashuba, VM, Phys. Rev. B **88**, 24110(R) (2013)