Stability of skyrmion lattices and symmetries of Dzyaloshinskii-Moriya magnets

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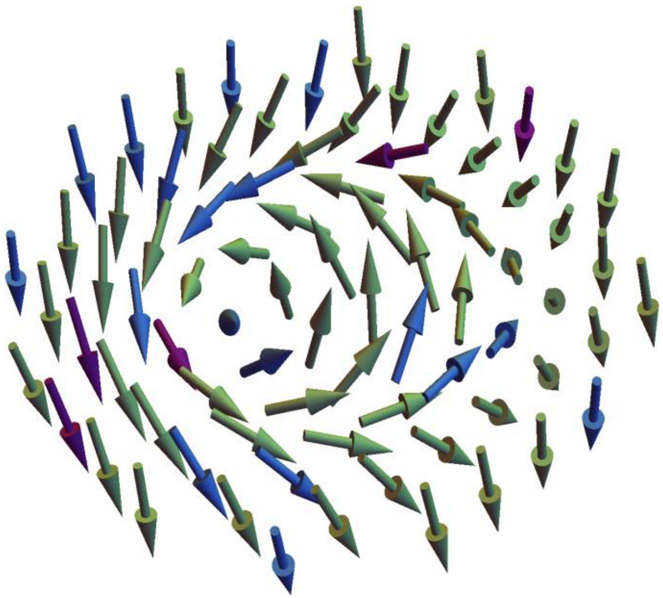
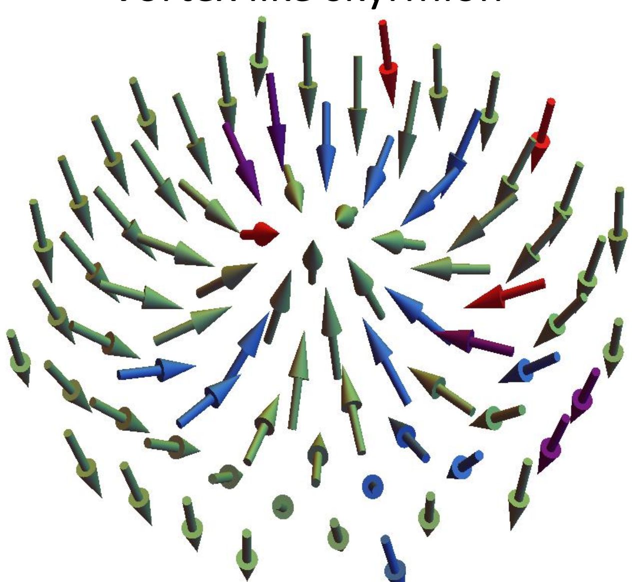
Outline

- Discuss possible 2D Dzyaloshinskii-Moriya magnets
- Study phase diagram and peculiarities of skyrmions for various DMI
- Discuss a theory describing interplay between magnons and skyrmions
- Discuss manipulation of skyrmions and merons by temperature gradients

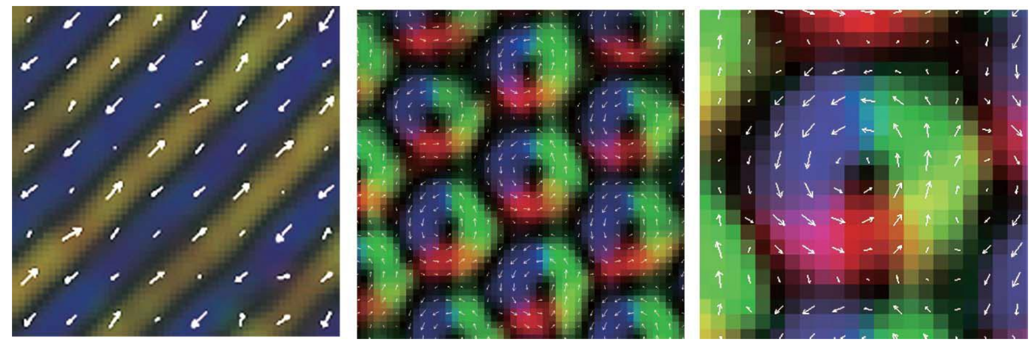
Skyrmions in Dzyaloshinskii-Moriya magnets



Vortex like skyrmion



Hedgehog like skyrmion



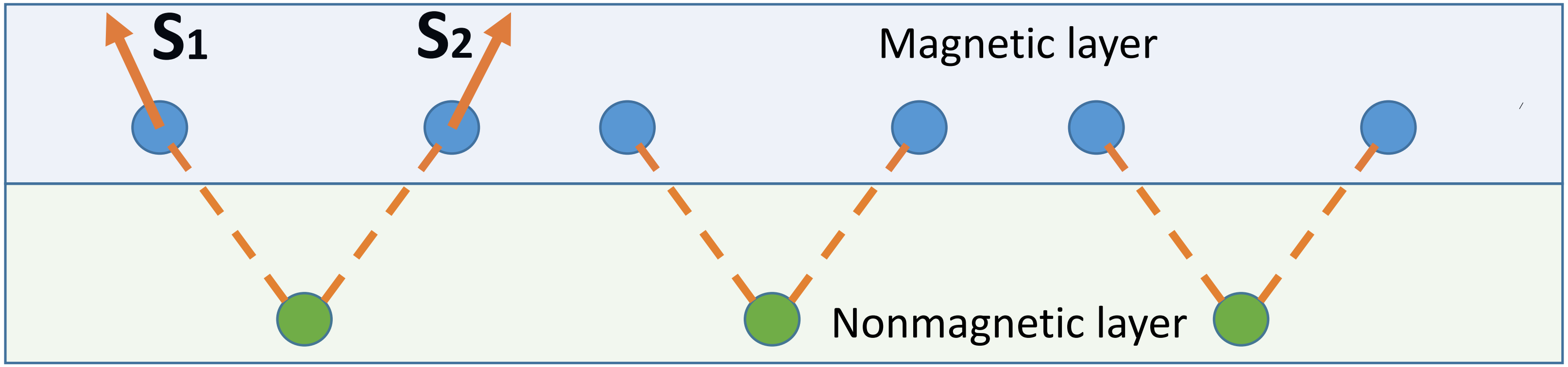
Tokura group, Nature 465, 901–904 (2010)
Muhlbauer et al. Science 323, 915 (2009)

$$\mathcal{G} = \int d^2\mathbf{r}(\partial_x \mathbf{m} \times \partial_y \mathbf{m}) \cdot \mathbf{m}/(4\pi)$$

Topological charge associated with skyrmion.

Bogdanov & Yablonskii, JETP (1988)
Bodanov & Hubert, JMMM (1994, 1999)
Bogdanov & Roessler, PRL (2001)
Roessler, Bogdanov & Pfleiderer, Nature (2006)

Microscopic origin of skyrmions



$$H_{\mathrm{DMI}} = \sum \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

1. Dzyaloshinskii-Moriya interaction arises in the presence of inversion asymmetry and spin-orbit interactions.

2. This leads to canting of the neighboring magnetic moments.

A. Fert, V. Cros& J. Sampaio
Nature Nanotechnology 8, 152–156 (2013)

Continuous form of Dzyaloshinskii-Moriya interaction

$$F = (A/2)(\partial_\alpha \mathbf{m})^2 + \mathbf{D}_\alpha(\mathbf{m} \times \partial_\alpha \mathbf{m}) + \mathbf{m}\cdot\mathbf{H} + K m_z^2$$

1. DMI is characterized by a second rank tensor $D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$

2. We separate Dzyaloshinskii-Moriya tensor into symmetric and antisymmetric parts:

$$D_{\alpha\beta} = D_{\alpha\beta}^{sym} + \varepsilon_{\alpha\beta\gamma} D_\gamma^{ant}$$

3. High symmetry cases:

$\mathbf{D}^{ant} = D\mathbf{n}$ --- Structural asymmetry

$D_{\alpha\beta}^{sym} = D\delta_{\alpha\beta}$ --- Noncentrosymmetric systems

Spontaneous formation of magnetic texture

1. Free energy can be lowered by formation of magnetic texture (spiral):

$$F = \frac{A}{2}Q^2 - DQ + \frac{K}{2} = \frac{A}{2}(Q - \frac{D}{A})^2 - \frac{D^2}{2A} + \frac{K}{2}$$

2. Typical wavevector of the magnetic texture becomes:

$$Q = \frac{D}{A}$$

3. Magnetic texture wins over ferromagnetic state when:

$$\frac{D^2}{KA} > 1$$

4. This inequality has to be satisfied if we wish to form skyrmions.

Skyrmions in ultra-thin magnetic films

1. In a quasi-2D system we can write a general form of spin-orbit interaction, e.g. corresponding to C2v symmetry:

$$H = \frac{p^2}{2m} + (\boldsymbol{\alpha}_i p_i) \cdot \hat{\boldsymbol{\sigma}}$$

2. Coulomb interactions are spin independent, thus we gauge out the spin-orbit interaction:

$$U^\dagger H U = \frac{p^2}{2m} + \mathcal{O}(\alpha^2)\,; \quad U = e^{-i(2m/\hbar)\hat{\boldsymbol{\sigma}}\cdot(\boldsymbol{\alpha}_i r_i)/2}$$

3. By rotating magnetization back to account for the gauge transformation, we obtain the Dzyaloshinskii-Moriya interaction (second order terms are not important):

$$F = \frac{A}{2}(\partial_i \tilde{m})^2 = \frac{A}{2}(\mathcal{D}_i m)^2\,; \quad m = \hat{R}\tilde{m}$$

$$\mathcal{D}_i = \partial_i + \hat{R}\partial_i \hat{R}^{-1} = \partial_i + (2m/\hbar)\boldsymbol{\alpha}_i \times$$

Skyrmions in ultra-thin magnetic films

1. DMI is first order in spin-orbit interaction and the typical wavelength of the texture is defined by the spin precession length:

$$2\pi A/D = h/(2m\alpha)$$

2. Textured state should also win over the ferromagnetic state as K is second order in spin orbit interaction and can be tuned such that:

$$\frac{D^2}{KA} > 1$$

3. This should apply to ultrathin magnetic films, e.g., Pt/Co(0.6 nm)/AlOx, where by changing the composition of Pt and Co we can change the perpendicular anisotropy.

J. Sampaio, V. Cros, S. Rohart, A. Thiaville, A. Fert, Nature Nanotech. 8, 839-844, (2013);
C. Moreau-Luchaire, C. Moutafis, N. Reyren, J. Sampaio, N. Van Horne, C.A.F. Vaz, K. Bouzehouane, K. Garcia, C. Deranlot, P. Warnicke, P. Wohlhüter, J.M. George, J. Raabe, V. Cros, A. Fert, arXiv:1502.07853

4. This should also apply to possible magnetic interfaces in complex oxides, e.g., LaAlO3/SrTiO3.

S. Banerjee, O. Erten, and M. Randeria, Nat. Phys. 9, 626 (2013); X. Li,W. Liu, and L. Balents, Phys. Rev. Lett. 112, 067202 (2014); S. Banerjee, J. Rowland, O. Erten, M. Randeria, PRX 4, 031045 (2014)

Symmetries of 2D Dzyaloshinskii-Moriya magnet

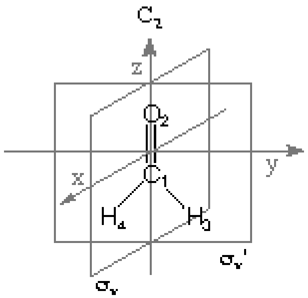
$$F = (A/2)(\partial_\alpha \mathbf{m})^2 + \mathbf{D}_\alpha(\mathbf{m} \times \partial_\alpha \mathbf{m}) + \mathbf{m} \cdot \mathbf{H} + K m_z^2\,;$$

$$D_{\alpha\beta} = D_{\alpha\beta}^{sym} + \varepsilon_{\alpha\beta\gamma} D_\gamma^{ant}\,;\ D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

1. Rashba-like symmetry: $\mathbf{D}^{ant} = D\mathbf{n}$

2. Dresselhaus-like symmetry: $D_{\alpha\beta}^{sym} = D\delta_{\alpha\beta}$

3. Combination of Rashba and Dresselhaus: corresponds to interface with C2v symmetry: $E,\ C_2,\ \sigma_v(xz),\ \sigma_v(yz)$.



4. Only one mirror symmetry corresponds to R+D plus tilted vector **n**.

5. Only C2 corresponds to R+D plus $D_{\alpha\beta}^{sym}$ with non-equal diagonal elements.

These are all possible situations for the DMI tensor.

Minimization procedure

1. Free energy is minimized with respect to ansatz solutions for Skyrmion:

$$\mathbf{m}_{sk} = \{\sin m_\theta \cos m_\phi, \cos m_\theta \sin m_\phi, \cos m_\theta\}$$

$$m_\theta = \pi(1 - r/R)\,; m_\phi = \phi + \phi_0$$

Minimization is done with respect to ϕ_0 , R.

2. Free energy is minimized with respect to spiral solution:

$$\mathbf{m}_{sp} = \mathbf{A}\sin(\mathbf{Q}\cdot\mathbf{r}) + \mathbf{B}\cos(\mathbf{Q}\cdot\mathbf{r})$$

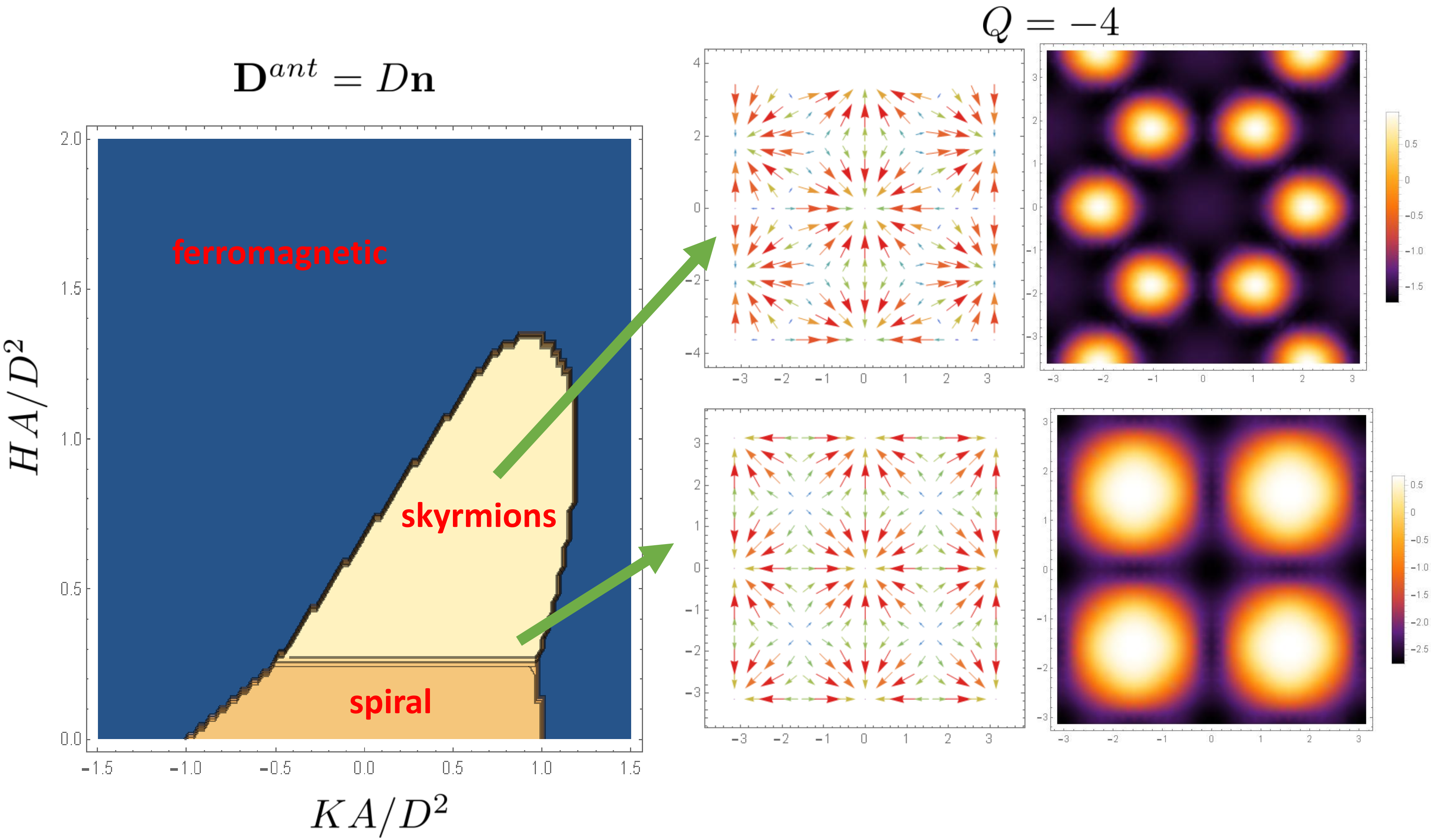
Minimization is done with respect to $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}$.

3. In the second approach, we also solve over-damped LLG equation to find the local minimum.

4. In the third approach, we perform Monte Carlo simulations of the lattice model based on the Free energy.

S. Banerjee, J. Rowland, O. Erten, and M.Randeria, Phys. Rev. X 4, 031045 (2014)

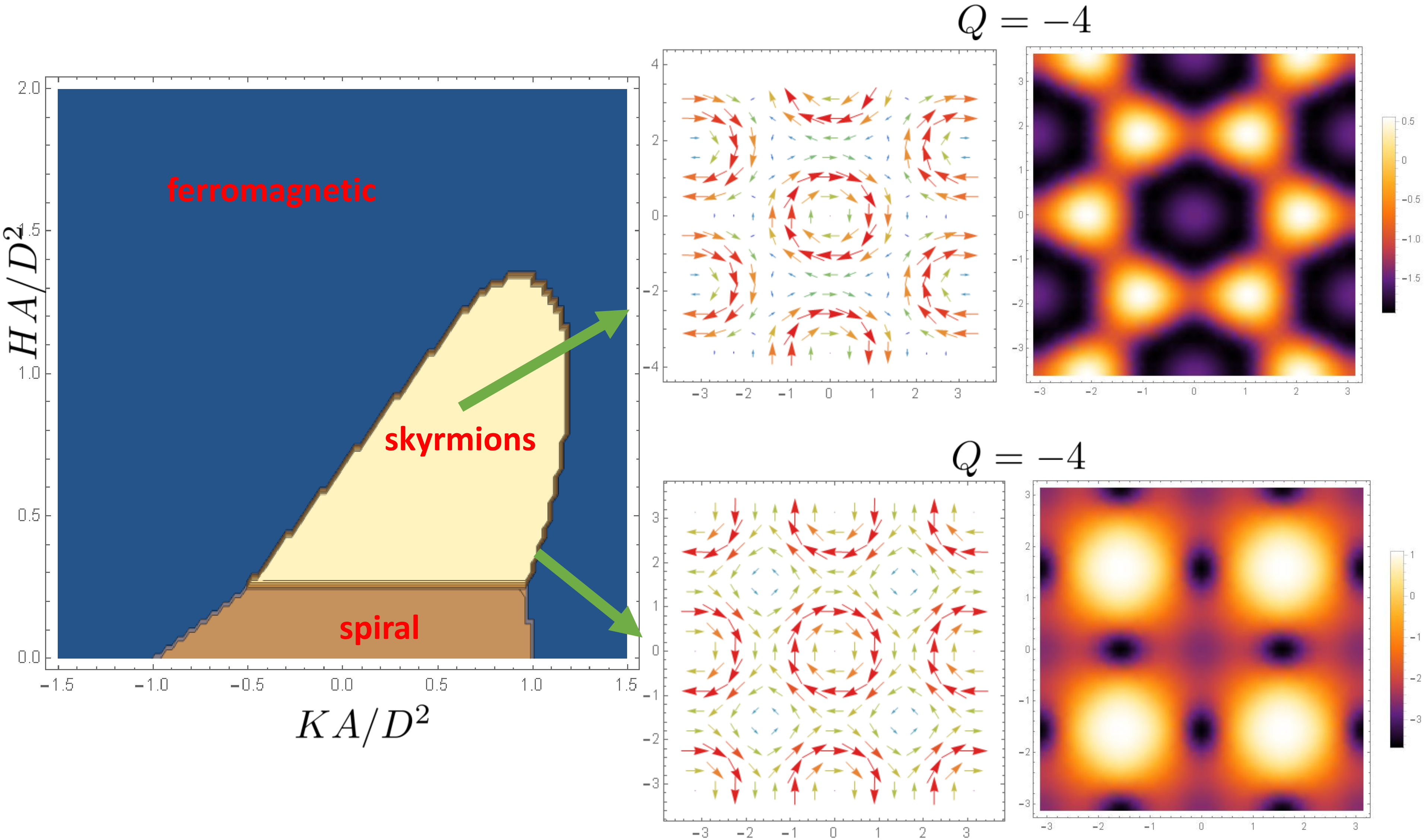
Rashba-like symmetry



$$F = (A/2)(\partial_\alpha \mathbf{m})^2 + D\boldsymbol{y}\cdot(\mathbf{m}\times\partial_x\mathbf{m}) - D\boldsymbol{x}\cdot(\mathbf{m}\times\partial_y\mathbf{m}) + \mathbf{m}\cdot\mathbf{H} + Km_z^2\,;$$

From the form of the free it is clear that the hedgehog skyrmion is favored.

Dresselhaus-like symmetry



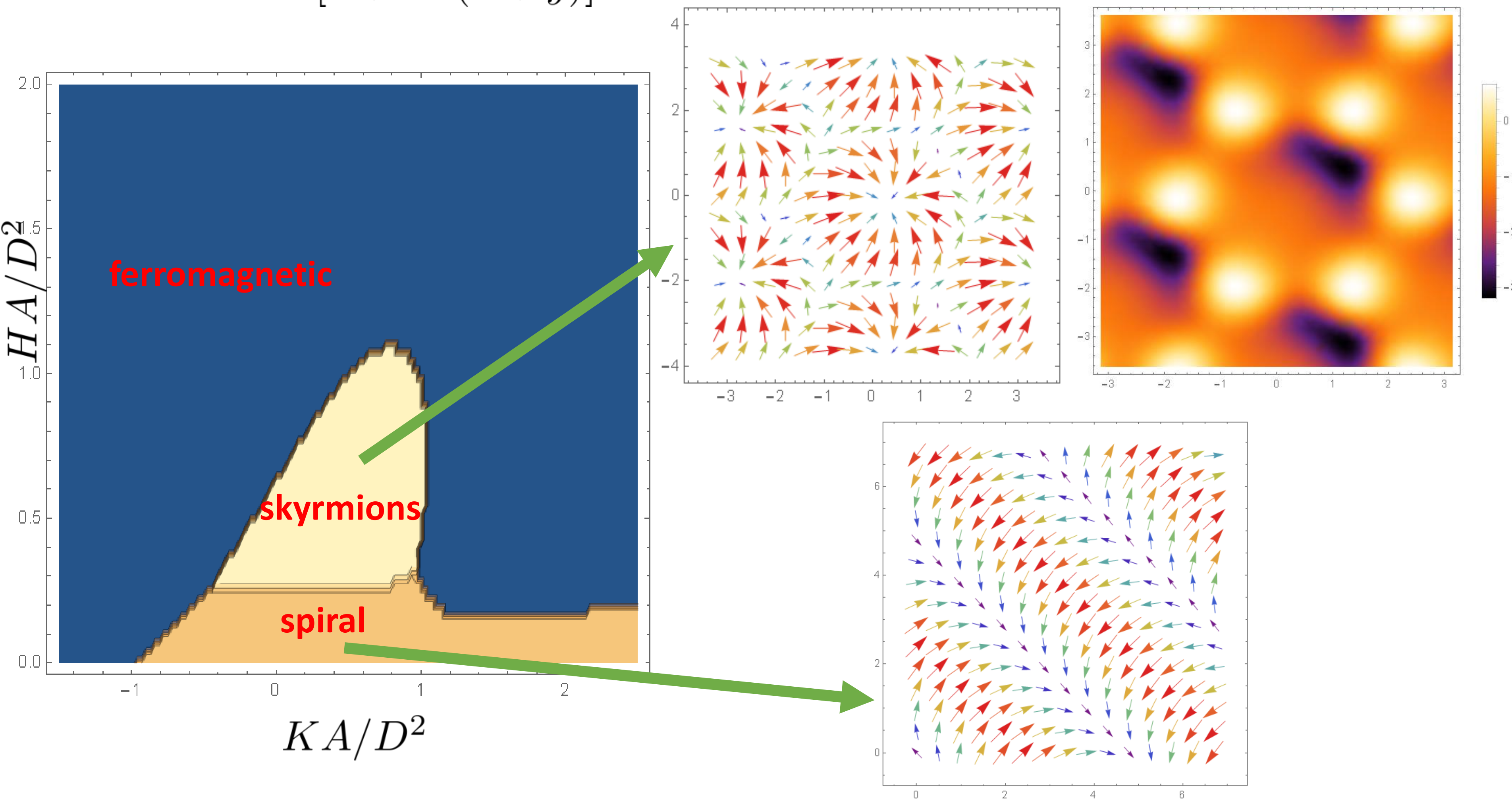
$$F = (A/2)(\partial_\alpha \mathbf{m})^2 + D\boldsymbol{x}\cdot(\mathbf{m}\times\partial_x\mathbf{m}) + D\boldsymbol{y}\cdot(\mathbf{m}\times\partial_y\mathbf{m}) + \mathbf{m}\cdot\mathbf{H} + Km_z^2\,;$$

From the form of the free it is clear that the vertex skyrmion is favored.

Tilted Rashba-like symmetry

$$\mathbf{D}^{ant} = D[\hat{z} + 0.1(\hat{x} + \hat{y})]$$

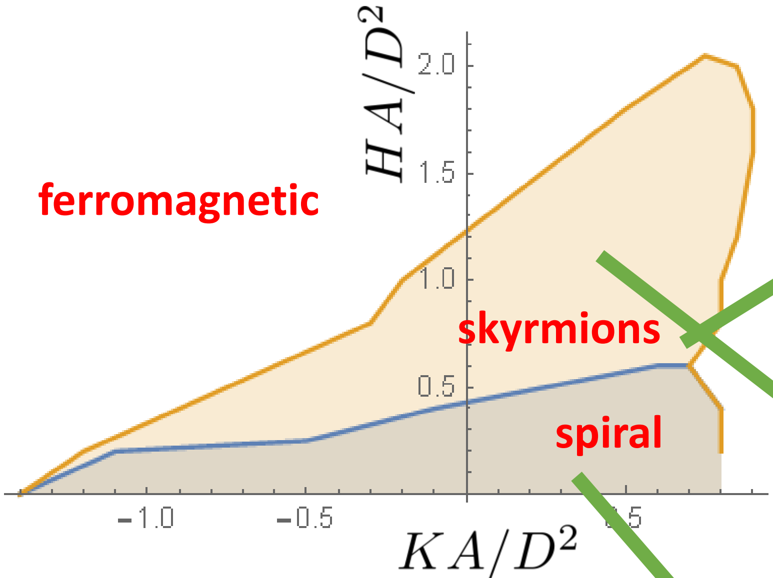
$$Q = -4$$



$$F = (A/2)(\partial_\alpha \mathbf{m})^2 + D\boldsymbol{z} \cdot (\mathbf{m} \times \partial_x \mathbf{m}) + \mathbf{m} \cdot \mathbf{H} + K m_z^2\,;$$

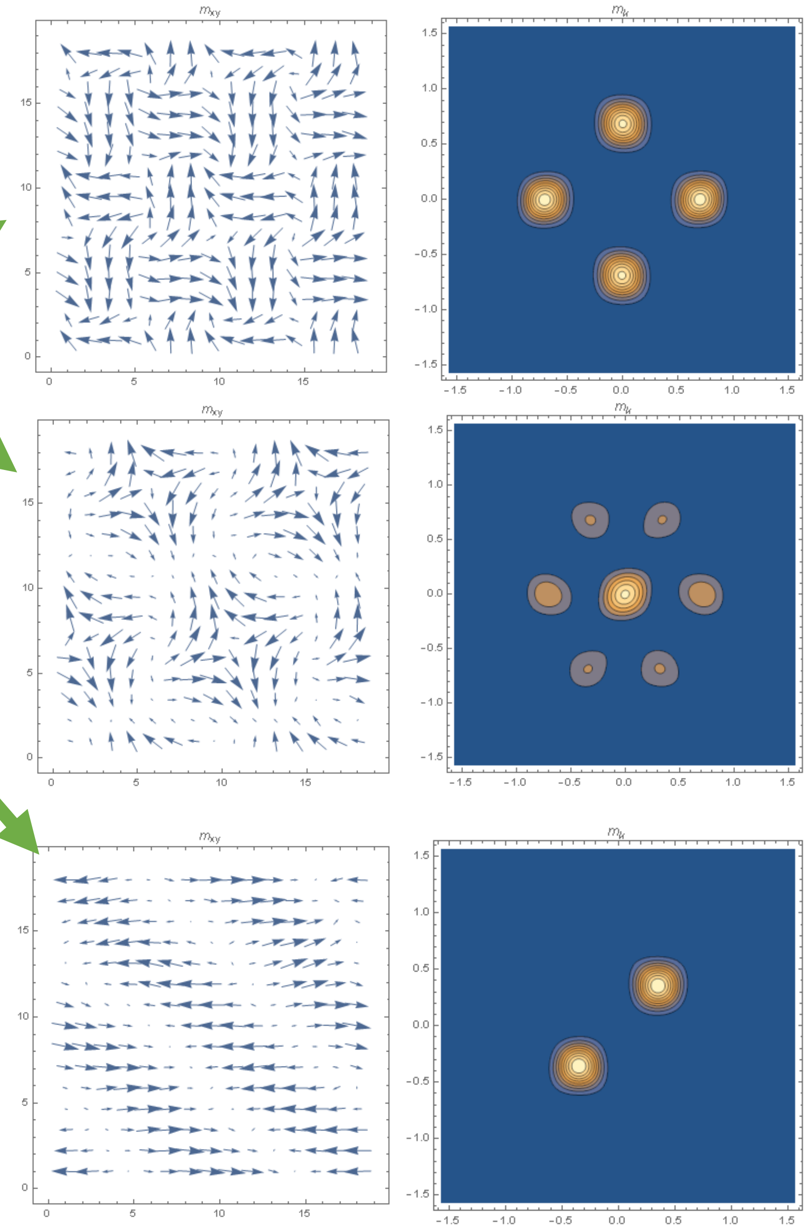
In-plane spiral is favored.

Combination of Rashba and Dresselhaus, MC



$$D_{\alpha\beta} = D^{sym}_{\alpha\beta} + \varepsilon_{\alpha\beta\gamma} D^{ant}_{\gamma}\,;$$

$$\mathbf{D}^{ant} = D\mathbf{n} \quad D^{sym}_{\alpha\beta} = D\delta_{\alpha\beta}$$



General trends:

1. Four-fold and six-fold symmetry.
2. Stability at lower fields for perpendicular Anisotropy.

Moving skyrmions by temperature gradient: stochastic LLG equation

1. Thermal effects are included via the stochastic LLG equation:

$$s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}}+\mathbf{m}\times(\mathbf{H}_{\text{eff}}+\mathbf{h})=0$$

$$\langle h_i(\mathbf{r},t)h_j(\mathbf{r}',t')\rangle = 2\alpha s k_B T(\mathbf{r})\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

W. F. Brown, Phys. Rev. 130, 1677 (1963)

2. Transverse Fourier transform to reduce to one dimensional equation:

$$m_f(\mathbf{q},\omega,x)=\int\frac{d^{d-1}\boldsymbol{\rho}\,d\omega}{(2\pi)^d}e^{i(\omega t-\mathbf{q}\boldsymbol{\rho})}m_f(\mathbf{r},t)$$

2. Correlator is found after solving the Helmholtz equation:

$$A(\partial_x^2+k^2)m_f(x,\mathbf{q},\omega)=h(x,\mathbf{q},\omega)$$ --- Linearized LLG equation

3. In the rotated reference frame $S=S_x+iS_y$ for $\partial T(x)/\partial x=\text{constant}$

$$S=-\partial_x m_s\int\frac{d^{d-1}\mathbf{q}\,d\omega}{(2\pi)^d}\frac{\langle m_f(\mathbf{q},\omega,x)^*\partial_x m_f(\mathbf{q}',\omega',x)\rangle}{(2\pi)^d\delta(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega')}$$

LLG equation with magnonic torques

1. Amended LLG equation for slow component and magnonic torques

$$s(1+\alpha\mathbf{m}_s\times)\dot{\mathbf{m}}_\mathbf{s}+\mathbf{m}_s\times\mathbf{H}^s_{\text{eff}}=[1+\beta\mathbf{m}_s\times](\mathbf{j}\cdot\boldsymbol{\partial})\mathbf{m}_s$$

$$\mathbf{j}^s_m=-\hbar\mathbf{j}$$

A. Kovalev, Phys. Rev. B 89, 241101(R) (2014)
Se Kwon Kim, Yaroslav Tserkovnyak, arXiv:1505.00818

$$s(1+\alpha\mathbf{m}_s\times)\dot{\mathbf{m}}_\mathbf{s}+\mathbf{m}_s\times\mathbf{H}^s_{\text{eff}}=[1+\beta\mathbf{m}_s\times](\mathbf{j}_\alpha\mathcal{D}_\alpha)\mathbf{m}_s$$

Kovalev&Gungurdu, EPL (Europhys. Letters) 109, 67008 (2015)

Chiral derivative

1. By introducing a coordinate-dependent rotation at each point we can write the free energy with Dzyaloshinskii-Moriya interaction via rotated magnetizations, up to some added anisotropies.

$$F = (A/2)(\partial_\alpha \cdot \tilde{\mathbf{m}})^2 - \mathbf{m} \cdot \mathbf{H}$$

$$\mathbf{m} = \hat{R}(\mathbf{r})\tilde{\mathbf{m}}$$

2. To return to un-rotated magnetization we use the chiral derivative.

$$\mathcal{D}_\alpha = \partial_\alpha + \hat{R}\partial_\alpha \hat{R}^{-1}$$

Kim K.-W., Lee H.-W., Lee K.-J. and Stiles M. D., Phys. Rev. Lett., 111 (2013) 216601.

3. Chiral derivative for the most general form of Dzyaloshinskii-Moriya interaction:

$$F_{\mathrm{DMI}} = D_{\alpha\beta}\varepsilon_{\gamma\beta\nu} m_\nu \partial_\alpha m_\gamma$$

$$D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

$$\Rightarrow \quad \boxed{\mathcal{D}_\alpha = \partial_\alpha + (\mathbf{D}_\alpha / A)\times}$$

4. We separate Dzyaloshinskii-Moriya tensor into symmetric and antisymmetric parts:

$$D_{\alpha\beta} = D^{sym}_{\alpha\beta} + \varepsilon_{\alpha\beta\gamma} D^{ant}_\gamma$$

5. High symmetry cases:

$\mathbf{D}^{ant} = D\mathbf{n}$ --- Structural asymmetry

$D^{sym}_{\alpha\beta} = D\delta_{\alpha\beta}$ --- Noncentrosymmetric systems

Skyrmions under temperature gradient

1. Take LLG: $s(1+\alpha\mathbf{m}_s\times)\dot{\mathbf{m}}_\mathbf{s}+\mathbf{m}_s\times\mathbf{H}^s_{\text{eff}}=[1+\beta\mathbf{m}_s\times]\,(\mathbf{j}_\alpha\mathcal{D}_\alpha)\mathbf{m}_s$

2. We use Thiele approach by introducing generalized coordinates q, i.e. $\dot{\mathbf{m}}=\sum_i\dot{q}_i\partial_{q_i}\mathbf{m}$

3. The skyrmion velocity becomes:

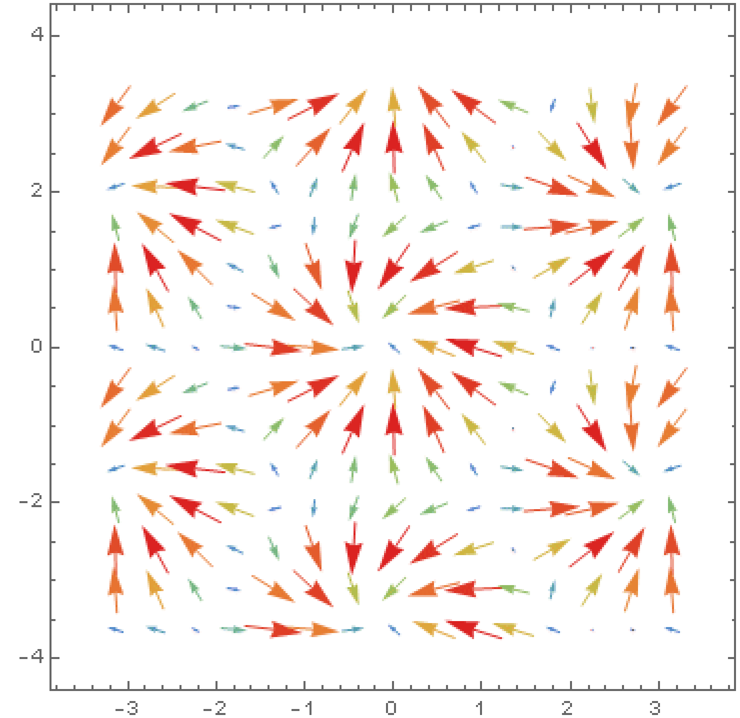
$$\boxed{\hat{\mathcal{G}}_1\mathbf{j}+s\hat{\mathcal{G}}\mathbf{v}+\beta\hat{\eta}_1\mathbf{j}+\alpha s\hat{\eta}\mathbf{v}=0}$$

$$\mathcal{G}_{ij}=\int d^2\mathbf{r}(\partial_i\mathbf{m}_s\times\partial_j\mathbf{m}_s)\cdot\mathbf{m}_s/(4\pi)$$

$$\mathcal{G}_{1\,ij}=\int d^2\mathbf{r}(\partial_i\mathbf{m}_s\times\mathcal{D}_j\mathbf{m}_s)\cdot\mathbf{m}_s/(4\pi)$$

$$\eta_{ij}=\int d^2\mathbf{r}(\partial_i\mathbf{m}_s\cdot\partial_j\mathbf{m}_s)/(4\pi)$$

$$\eta_{1\,ij}=\int d^2\mathbf{r}(\partial_i\mathbf{m}_s\cdot\mathcal{D}_j\mathbf{m}_s)/(4\pi)$$



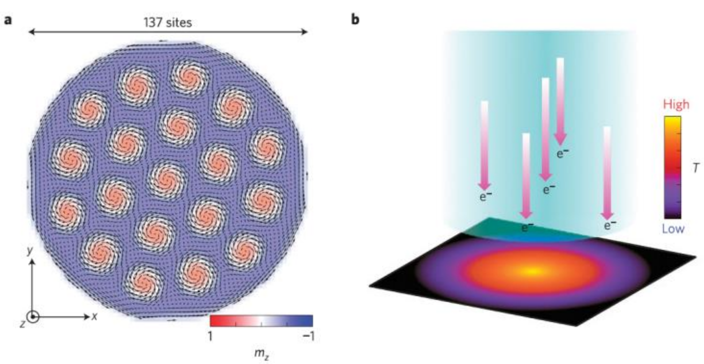
Skyrmionic spin Seebeck effect

1. Skyrmions will move in the direction of the hot region with additional side motion.

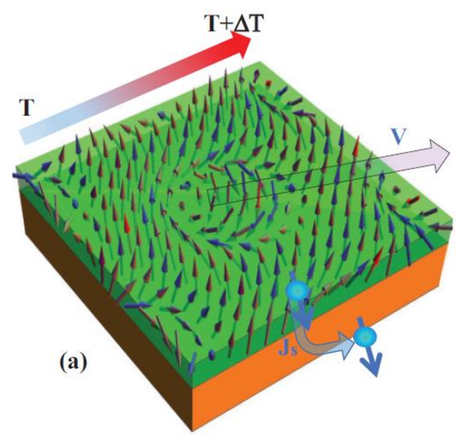
$$v_x = -\hbar j \frac{\mathcal{G}^2 + \alpha\eta\eta_1\beta}{s(\mathcal{G}^2 + \alpha^2\eta^2)}$$

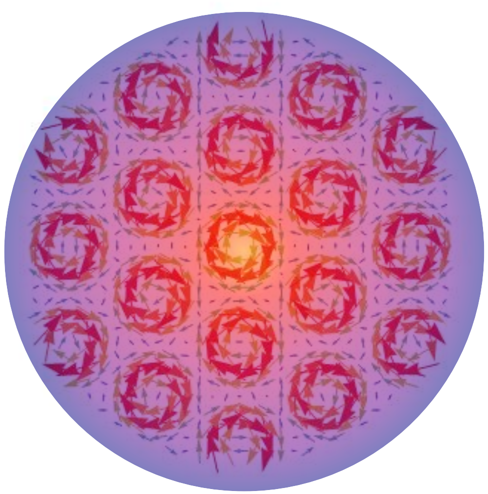
$$v_y = -\mathcal{G}\hbar j \frac{\alpha\eta - \beta\eta_1}{s(\mathcal{G}^2 + \alpha^2\eta^2)}$$

2. Two different regimes $\alpha\eta > \beta\eta_1$ and $\alpha\eta < \beta\eta_1$
3. Possible detection via spin pumping in the neighboring Pt layer.
4. For $\mathrm{Cu_2OSeO_3}$ the longitudinal velocity is estimated 0.1 m/s for 1K/µm



M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshibae, J. Zang, M. Mostovoy, Y. Tokura & N. Nagaosa, Nature Materials 13, 241–246 (2014)





Conclusions

- We analized skyrmions in different magnets with DMI
- We found rich phase diagram with various phases corresponding to skyrmions, merons and spirals
- We described motion of skyrmions in response to temperature gradient
- We found dissipative tensor renormalization due to DMI

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