New effects in spintronics derived from the symmetry of response functions

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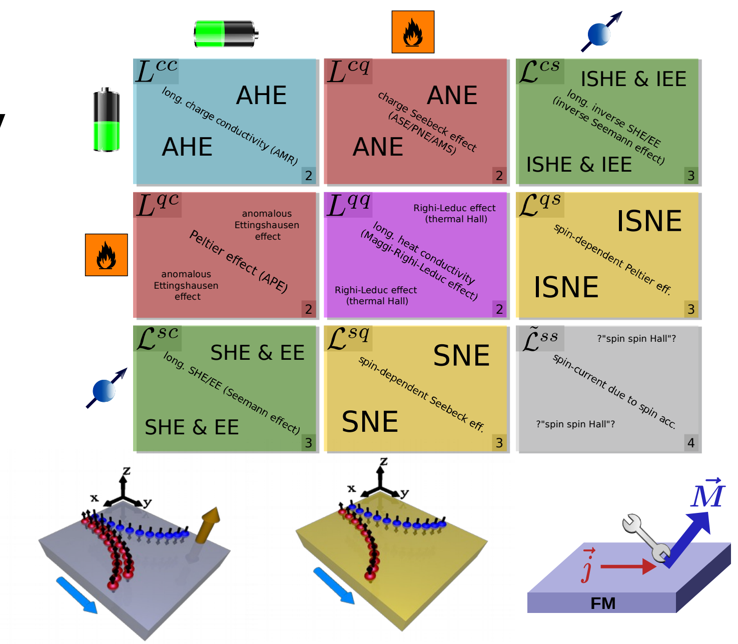
Outline

- Linear response formalism & group theory
- Application to electrical conductivity
- Application to spin-polarized conductivity
- Application to spin-orbit torques
- Summary and outlook



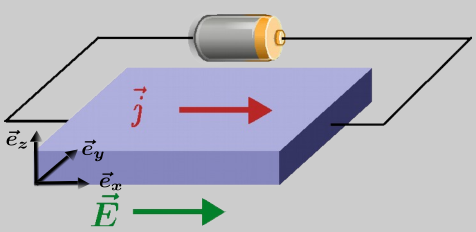
Linear response – Kubo formalism

Response in ***B*** caused by coupling to perturbation in ***A***

$$\tau_{\hat{B}_i \hat{A}_j}(\omega, \vec{H}) = \int_0^\infty dt\, e^{-i\omega t} \int_0^\beta d\lambda \left\langle \rho(\vec{H})\, \hat{A}_j\, \hat{B}_i(t + i\hbar\lambda; \vec{H}) \right\rangle$$

Example – electric current

$$\tau_{\hat{j}\hat{j}}(\omega) = \int_0^\infty dt\, e^{-i\omega t} \int_0^\beta d\lambda\, \mathrm{Tr}\left(\rho_0\, \hat{j}\, \hat{j}(t + i\hbar\lambda)\right)$$



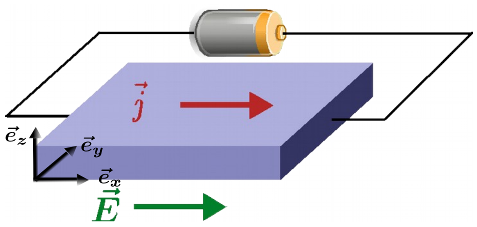
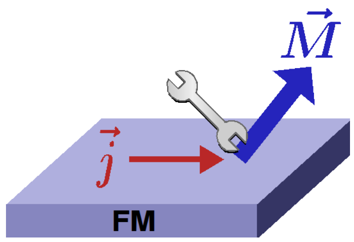
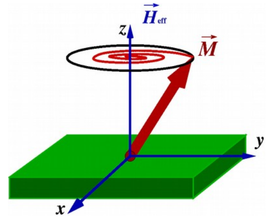
$$\tau_{\hat{j}\hat{j}} = \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

- symmetric part:
  longitudinal charge conductivity
  Anisotropic Magnetoresistance (AMR)
- antisymmetric part:
  transverse, anomalous Hall effect (AHE)

Linear response – Kubo formalism

Response in ***B*** caused by coupling to perturbation in ***A***

$$\tau_{\hat{B}_i \hat{A}_j}(\omega, \vec{H}) = \int_0^{\infty} dt\, e^{-i\omega t} \int_0^{\beta} d\lambda \left\langle \rho(\vec{H}) \hat{A}_j \hat{B}_i(t + i\hbar\lambda; \vec{H}) \right\rangle$$

	observable		perturbation	
Charge, heat or spin current	$\vec{j}^{\,c}$ $\vec{j}^{\,q}$ J^s		$\vec{j}^{\,c}$ $\vec{j}^{\,q}$ J^s	
Torque	$\vec{T}$	spin-orbit torque	$\vec{j}^{\,c}$	
	$\vec{T}$	Gilbert damping	$\vec{T}$	

Symmetry of response tensors

- Make use of transformation properties of involved operators
- Account for symmetry operations in space as well as space and time

space	**unitary** operations	u
space + time	**anti-unitary** operations with time reversal T	$\mathrm{a} = \mathrm{T}\,\mathrm{v}$

	non-magnetic	magnetic
space groups	230	1651
point groups	32	122
Laue groups	12	37

symmetry determination: ***FINDSYM***
ISOTROPY Software Suite, iso.byu.edu.
H. T. Stokes & D. M. Hatch, J. Appl. Cryst. 38, 237 (2005)

Application to electrical conductivity

$$\sigma_{ij} = \tau_{\hat{j}_i \hat{j}_j}(\omega, \vec{H}) = \int_0^\infty dt\, e^{-i\omega t} \int_0^\beta d\lambda \left(\rho(\vec{H}) \hat{j}_j \hat{j}_i (t + i\hbar\lambda; \vec{H}) \right)$$

for unitary operators *u*:

$$\sigma_{ij} = \sum_{kl} \sigma_{kl}\, D(P_R)_{ki} D(P_R)_{lj}$$

for anti-unitary operators *a*:

$$\sigma_{ij} = \sum_{kl} \sigma_{lk}\, D(P_R)^*_{ki} D(P_R)^*_{lj}$$

Pseudoalgorithm

- determine symmetry of system
- loop over symmetry operations
  - set up system of linear eqs. in elements $\{\sigma_{ij}\}$
- solution gives restrictions
  - element is linear combination of other elements
  - element is its negative → element is zero

Only the magnetic Laue group has to be considered

same transformation behavior for thermal transport ⇒ same tensor shapes

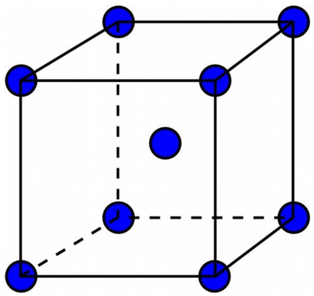
W. H. Kleiner, Phys. Rev. **142**, 318 (1966)

Application to electrical conductivity

Only the magnetic Laue group has to be considered

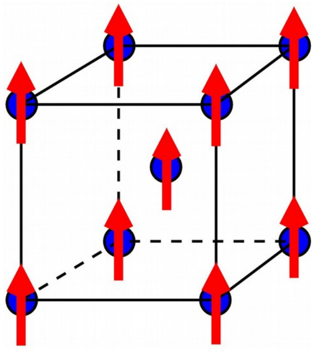
paramagnetic

$m\bar{3}m1'$



ferromagnetic

$4/mm'm'$

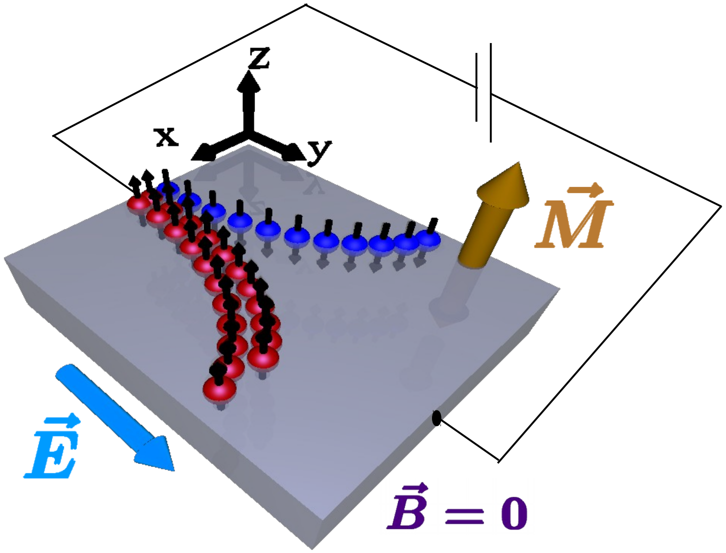


$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{xx} \end{pmatrix}$$

Isotropic conductivity

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

Anomalous Hall effect



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Electrical conductivity - tensor shapes

Results obtained by analytic computation using computer algebra system (CAS)

Non-magnetic materials

magnetic Laue group	$\boldsymbol{\tau}'$	$\boldsymbol{\sigma}$
$\bar{1}1'$	$\begin{pmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$
$2/m1'$	$\begin{pmatrix} \tau_{xx} & 0 & \tau_{zx} \\ 0 & \tau_{yy} & 0 \\ \tau_{xz} & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{xz} & 0 & \sigma_{zz} \end{pmatrix}$
$mmm1'$	$\begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$\bar{3}1'$, $4/m1'$, $6/m1'$	$\begin{pmatrix} \tau_{xx} & -\tau_{xy} & 0 \\ \tau_{xy} & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$\bar{3}1m1'$, $\bar{3}m11'$, $4/mmm1'$, $6/mmm1'$	$\begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$m\bar{3}1'$, $m\bar{3}m1'$	$\begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{xx} & 0 \\ 0 & 0 & \tau_{xx} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{xx} \end{pmatrix}$

Thermo-electrical response tensor

Electrical conductivity tensor

Magnetic materials

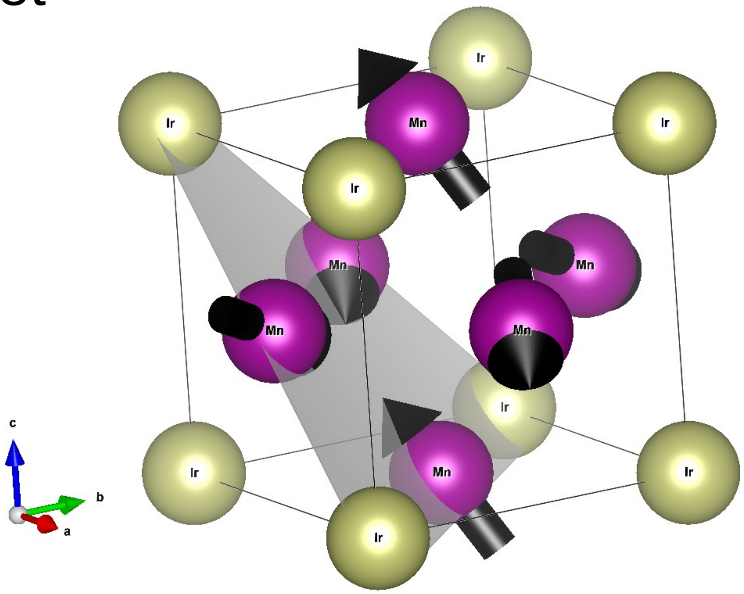
magnetic Laue group	$\boldsymbol{\tau}'$	$\boldsymbol{\sigma}$
$2'/m'$	$\begin{pmatrix} \tau_{xx} & -\tau_{yx} & \tau_{zx} \\ -\tau_{xy} & \tau_{yy} & -\tau_{zy} \\ \tau_{xz} & -\tau_{yz} & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ -\sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & -\sigma_{yz} & \sigma_{zz} \end{pmatrix}$
$m'm'm$	$\begin{pmatrix} \tau_{xx} & -\tau_{yx} & 0 \\ -\tau_{xy} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$4'/m$	$\begin{pmatrix} \tau_{yy} & -\tau_{xy} & 0 \\ -\tau_{yx} & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$4'/mm'm$	$\begin{pmatrix} \tau_{xx} & -\tau_{xy} & 0 \\ -\tau_{xy} & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$4'/mmm'$	$\begin{pmatrix} \tau_{yy} & 0 & 0 \\ 0 & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$\bar{3}1m'$, $\bar{3}m'1$, $4/mm'm'$, $6/mm'm'$	$\begin{pmatrix} \tau_{xx} & \tau_{xy} & 0 \\ -\tau_{xy} & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$6'/m'$	$\begin{pmatrix} \tau_{xx} & -\tau_{xy} & 0 \\ \tau_{xy} & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$6'/m'm'm$, $6'/m'mm'$	$\begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{xx} & 0 \\ 0 & 0 & \tau_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$m\bar{3}m'$	$\begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{xx} & 0 \\ 0 & 0 & \tau_{xx} \end{pmatrix}$	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{xx} \end{pmatrix}$

class a) contains time reversal *T*

class c) contains combined operations a=v *T*

Transverse conductivity and magneto-optics in non-collinear antiferromagnets

Mn_3Ir – a prototype non-collinear antiferromagnet



- Cu_3Au structure
- moments in (111) plane (Kagome lattice)
- magnetic space group: $R\bar{3}m'$

Prediction of **anomalous Hall effect (AHE)**

and **magneto-optical Kerr effect (MOKE)**

- based on analysis of electronic structure

  Chen, Niu, and MacDonald, PRL **112**, 017205 (2014)

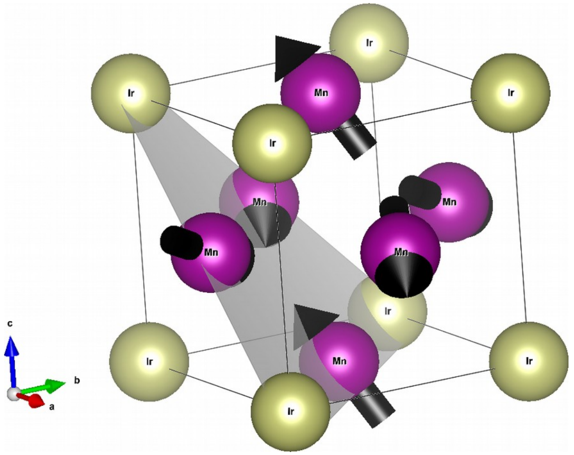
- Natural consequence of Kleiner's tables
  for the shape of the conductivity tensor

  Kleiner, PR **142**, 318 (1966)

Mn_3Ir – electrical conductivity

Electrical conductivity tensor

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$



z-direction along [111]

numerical work based on Kubo-Středa equation

$$\sigma_{\mu\nu} = \frac{\hbar}{4\pi V}\mathrm{Tr}\left\langle \hat{J}_\mu (G^+ - G^-)\hat{j}_\nu G^- - \hat{J}_\mu G^+ \hat{j}_\nu (G^+ - G^-)\right\rangle_c + \frac{e}{4\pi i V}\mathrm{Tr}\left\langle (G^+ - G^-)(\hat{r}_\mu \hat{J}_\nu - \hat{r}_\nu \hat{J}_\mu)\right\rangle_c$$

Smrčka and Středa, JPC 10, 2153 (1977)
Lowitzer *et al.*, PRL **105**, 266604 (2010)

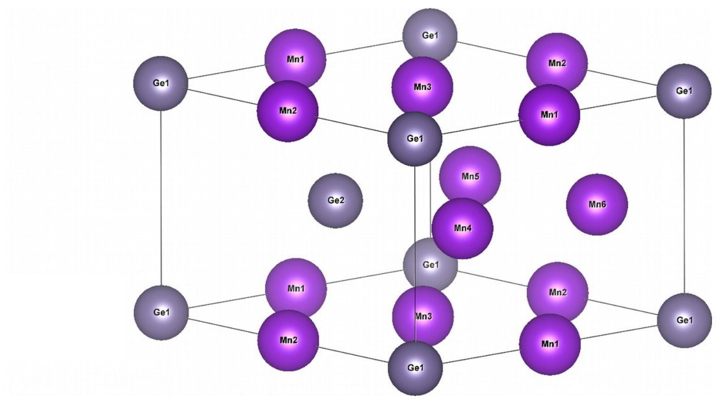
- confirms tensor shape
- Anomalous Hall conductivity

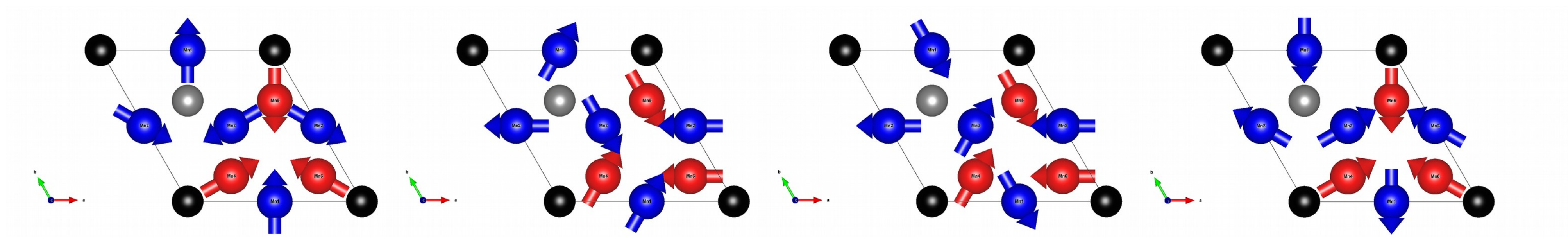
275 $(\Omega\ \mathrm{cm})^{-1}$ this work
218 $(\Omega\ \mathrm{cm})^{-1}$ Chen *et al.* (2014)

comparable in size to Fe, Co, and Ni

Mn_3Ge – non-collinear spin structures

- Space group $P6_3/mmc$ (194)
- Mn atoms couple antiferromagnetically on Kagome lattices (stacked along [0001])
  - magnetic moments in {0001}
  - various triangular configurations discussed
    Zhang *et al.* JPCM **25**, 206006 (2013), Kübler and Felser, EPL **108**, 67001 (2014)





AFM1 ***AFM2*** ***AFM3*** ***AFM4***

- AHC predicted by Kübler and Felser (for ***AFM3*** & ***AFM4***)
- Excluded for ***AFM1***

Mn_3Ge – electrical conductivity

- Electrical conductivity
  - predicted tensor shapes (MSGs determined using ***FINDSYM***)

$$\begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ -\sigma_{xz} & 0 & \sigma_{zz} \end{pmatrix} \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ -\sigma_{xz} & -\sigma_{yz} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ -\sigma_{xz} & 0 & \sigma_{zz} \end{pmatrix}$$

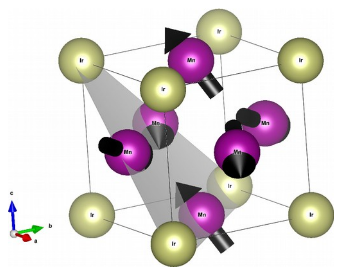
	AFM1	***AFM2***	***AFM3***	***AFM4***
MSG	Am'm'2	P-6'2c'	Pm'	Am'm'2
MLG	**m'm'm**	**6'/m'mm'**	**2'/m'**	**m'm'm**

  - numerical results confirm prediction **in all cases**
  - Comparison to results obtained by Kübler & Felser [1]
    - Discrepancies for ***AFM1*** (no AHE) & ***AFM4*** (2 different AHCs)
    - ***AFM2*** not considered in [1]
    - Agreement for ***AFM3***

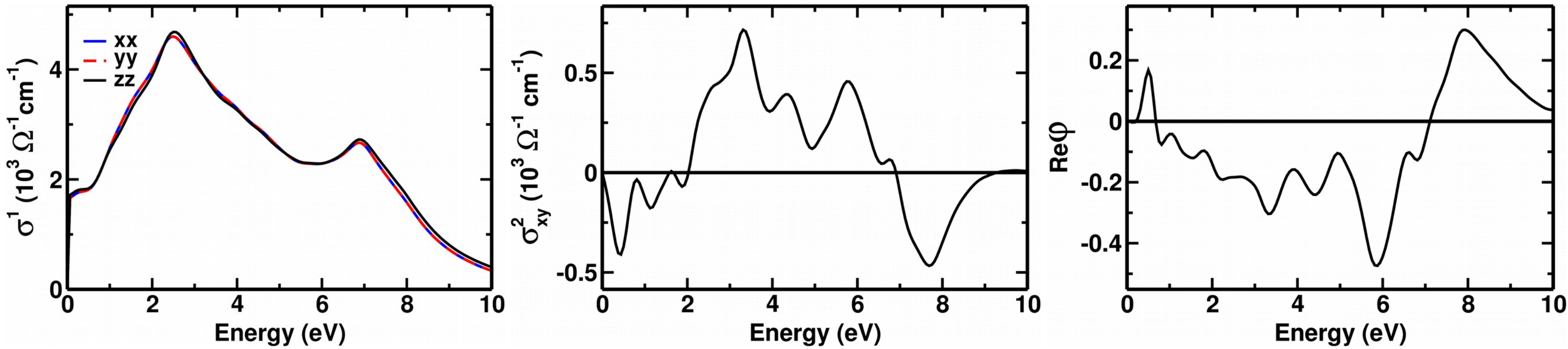
[1] Kübler & Felser, EPL **108**, 67001 (2014)

Mn_3Ir – optical conductivity and Kerr angle

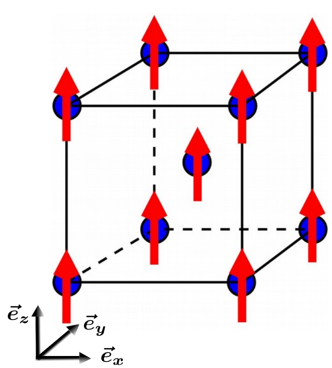
Mn_3Ir

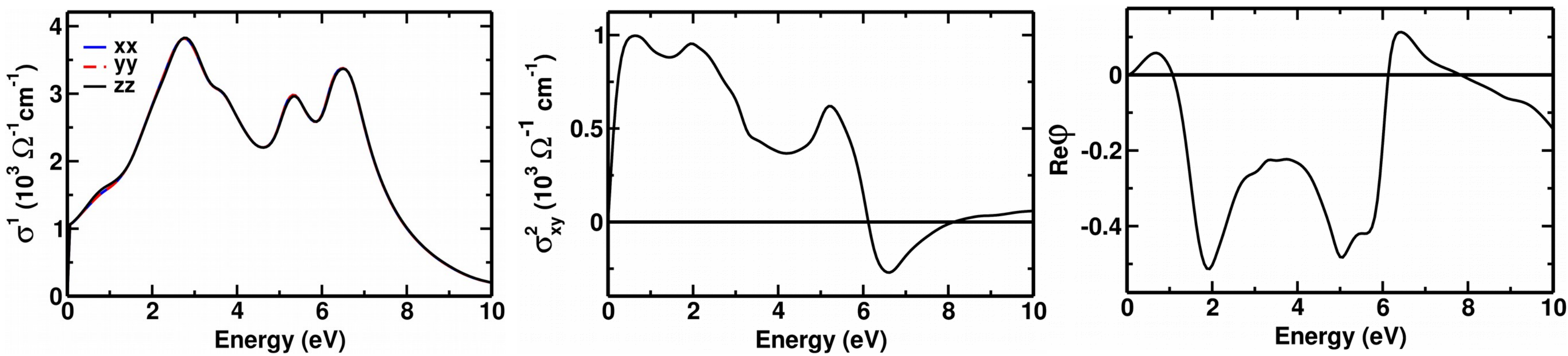


z-direction along [111]



bcc Fe





- results in line with symmetry prediction

$$\sigma^1_{xx}(\omega) = \sigma^1_{yy}(\omega) \neq \sigma^1_{zz}(\omega) \quad \sigma^2_{xy}(\omega) = -\sigma^2_{yx}(\omega)$$

- same tensor shape as for bcc-Fe, comparable in magnitude

Wimmer, *et al.*, unpublished (2015)

X-ray absorption

- One-electron approximation to *Fermi's Golden Rule*

$$\mu^{i}_{\mathbf{q}\lambda}(\omega) \propto \sum_f |\langle \Psi_f \,|\, X_{\mathbf{q}\lambda} \,|\, \Psi_i \rangle|^2 \; \delta(E_f - E_i - \hbar\omega)$$

- Electron-photon interaction operator

$$X_{\mathbf{q}\lambda}(\mathbf{r}) = -\frac{1}{c} \underbrace{\mathbf{j}_{el}}_{\text{electric current density operator}} \cdot \mathbf{A}_{\mathbf{q}\lambda}(\mathbf{r}) = e\,\alpha \cdot \hat{\varepsilon}_\lambda \, \mathbf{A} \, e^{i\mathbf{q}\mathbf{r}}$$

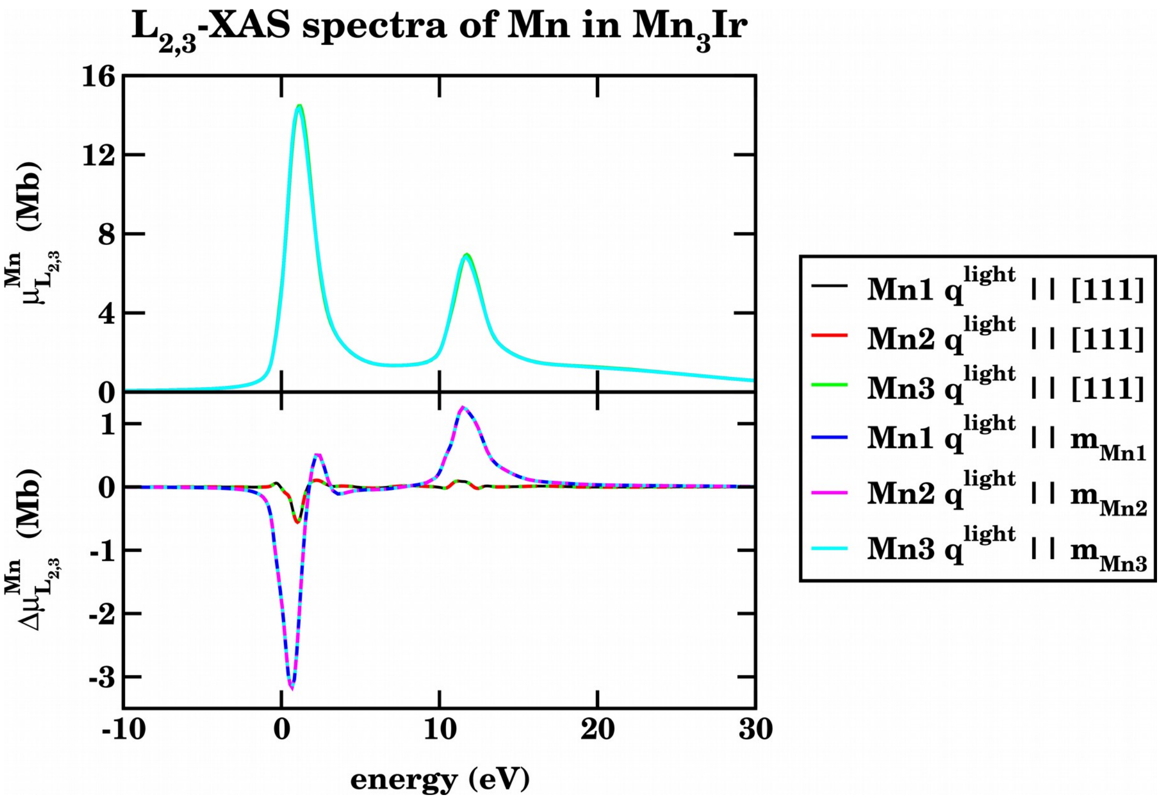
- Absorption coefficient in terms of Green function

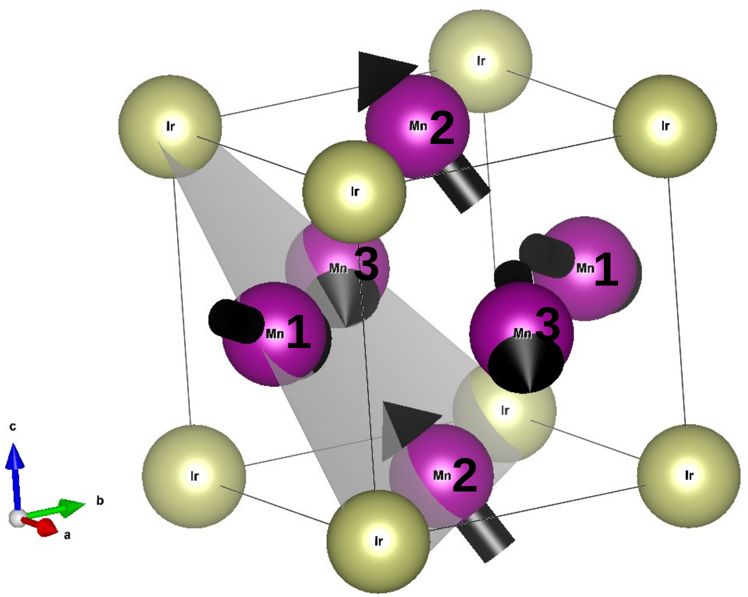
$$\mu_{\mathbf{q}\lambda}(\omega) \propto \sum_i \langle \, \Psi_i \,|\, X^{\dagger}_{\mathbf{q}\lambda} \; \Im G^{+}(E_i + \hbar\omega) \; X_{\mathbf{q}\lambda} \,|\, \Psi_i \, \rangle \, \theta(E_i + \hbar\omega - E_F)$$

- Relation to transport effects
  - XMCD ($\mu_{q+} - \mu_{q-}$) **circular dichroism** ⟺ **AHE**
  - XMLD ($\mu_{q\overline{\lambda}} - \mu_{q'\overline{\lambda}}$) **linear dichroism** ⟺ **AMR**

- Construction of spectra as sum of site-resolved absorption coefficients
  - wavevectors $\mathbf{q}$ along high symmetry directions → connection to $\underline{\boldsymbol{\sigma}}$

Mn_3Ir – Mn $L_{2,3}$ X-ray absorption spectra



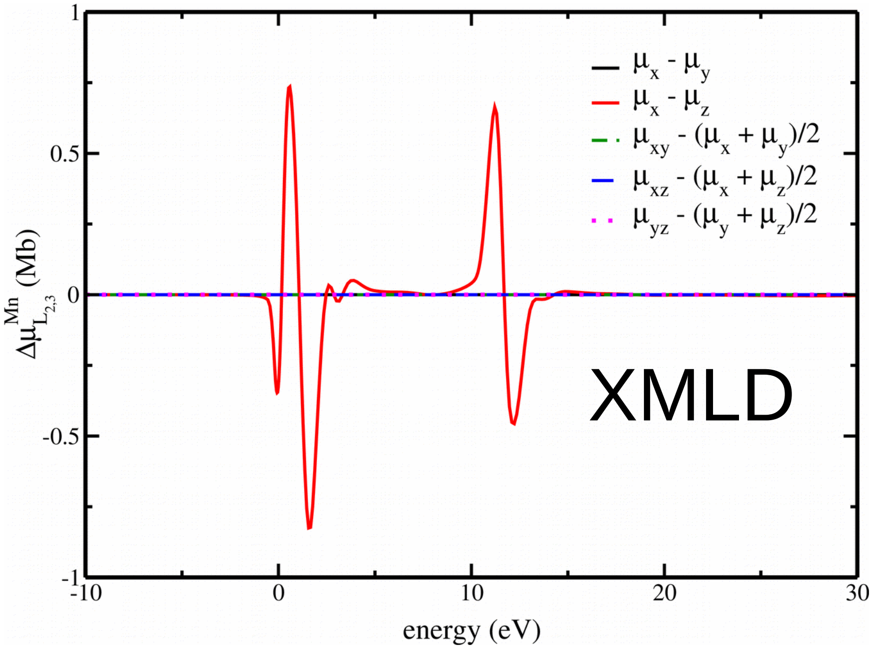


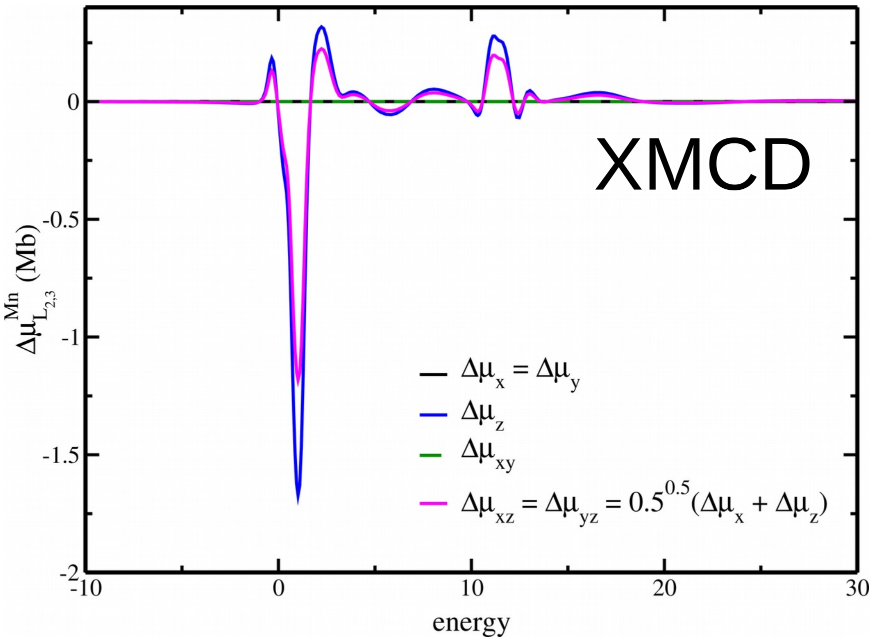
z-direction along [111]

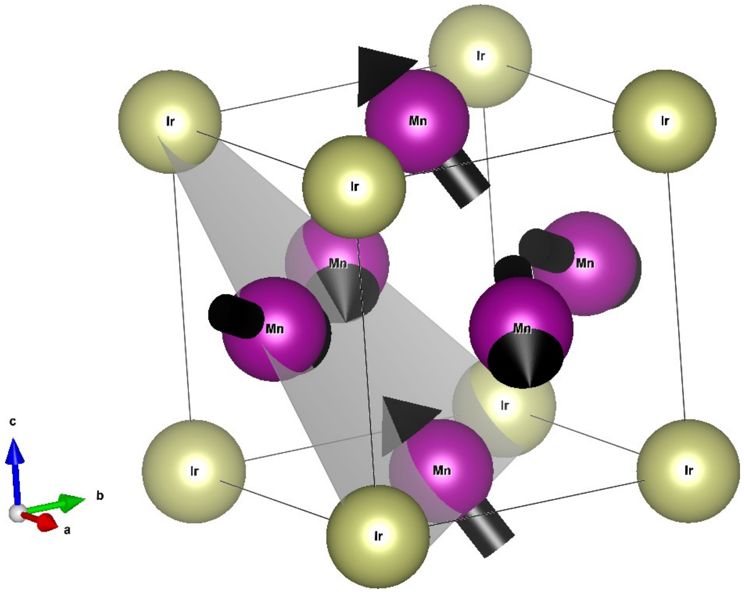
- spectra by superposition of site-resolved abs. coeffs. $\mu^{n}_{\vec{q}\lambda}(\omega)$
- incidence $\vec{q}$: [111] vs. direction of $\vec{m}_n$ (polar geometry)
  - same total absorption
  - in **both** cases XMCD (larger for polar geometry)

Wimmer, *et al.*, unpublished (2015)

Mn_3Ir – magnetic linear and circular dichroism







z-direction along [111]

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

Wimmer, *et al.*, unpublished (2015)

Spin-polarized conductivity

$$\tau_{(\mathcal{T}_k\hat{j}_i)\hat{j}_j}(\omega,\vec{H}) = \int_0^\infty dt\, e^{-i\omega t} \int_0^\beta d\lambda \left(\rho(\vec{H})\hat{j}_j \mathcal{T}_k \hat{j}_i(t+i\hbar\lambda;\vec{H})\right)$$

$$\hat{j}_j = -|e|c\,\alpha_j$$ ← Dirac matrix

Using a relativistic spin polarization operator [1,2,3]: $$\mathcal{T}_k = \beta\Sigma_k - \frac{\gamma_5\Pi_k}{mc}$$

for unitary operators *u*: $$\sigma^k_{ij} = \sum_{lmn} D(P_R)_{li}\, D(P_R)_{mj}\, D(P_R)_{nk}\, \sigma^n_{lm}$$

for anti-unitary operators *a*: $$\sigma^k_{ij} = -\sum_{lmn} D(P_R)_{li}\, D(P_R)_{mj}\, D(P_R)_{nk}\, \sigma'^n_{lm}$$

Only the magnetic Laue group has to be considered

⇒ Spin polarized conductivity tensor

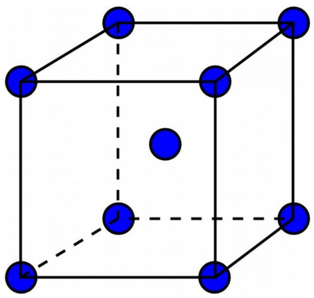
[1] V. Bargmann, E. P. Wigner, Proc. Natl. Acad. Sci. U.S.A. **34**, 211 (1948)
[2] A. Vernes, B.L. Györffy, P. Weinberger, Phys. Rev. B **76**, 012408 (2007)
[3] S. Lowitzer, Ködderitzsch, H. Ebert, Phys. Rev. B **82**, 140402 (2010)

Application to spin-polarized conductivity

Only the magnetic Laue group has to be considered

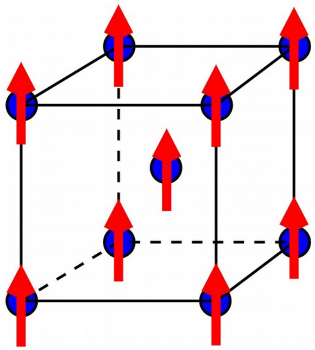
paramagnetic

$m\bar{3}m1'$



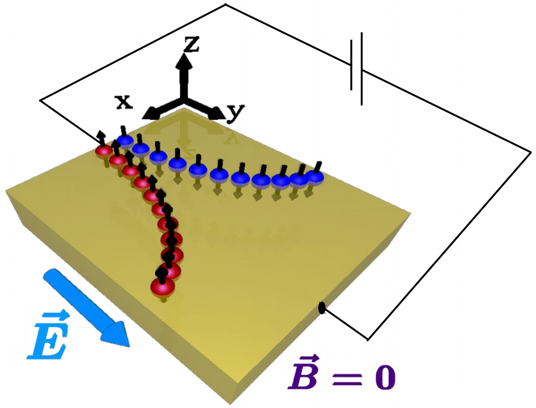
ferromagnetic

$4/mm'm'$



$$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$$

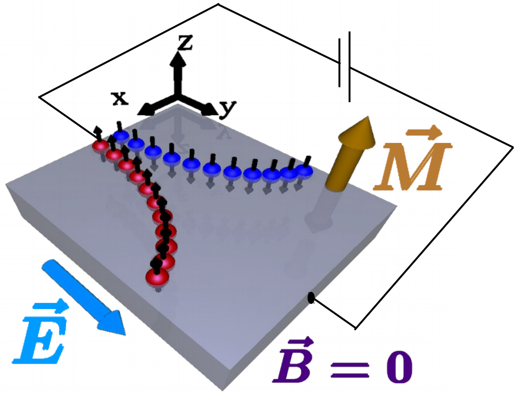


Spin-Hall effect in paramagnet

and

ferromagnet

(no need for two-current-model)



Spin-polarized conductivity - tensor shapes

Results obtained by analytic computation using computer algebra system (CAS)

Non-magnetic materials

magnetic Laue group	$\underline{\boldsymbol{\sigma}}^x$	$\underline{\boldsymbol{\sigma}}^y$	$\underline{\boldsymbol{\sigma}}^z$
$\bar{1}1'$	$\begin{pmatrix} \sigma^x_{xx} & \sigma^x_{xy} & \sigma^x_{xz} \\ \sigma^x_{yx} & \sigma^x_{yy} & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & \sigma^x_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & \sigma^y_{xy} & \sigma^y_{xz} \\ \sigma^y_{yx} & \sigma^y_{yy} & \sigma^y_{yz} \\ \sigma^y_{zx} & \sigma^y_{zy} & \sigma^y_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & \sigma^z_{xz} \\ \sigma^z_{yx} & \sigma^z_{yy} & \sigma^z_{yz} \\ \sigma^z_{zx} & \sigma^z_{zy} & \sigma^z_{zz} \end{pmatrix}$
$2/m1'$	$\begin{pmatrix} 0 & \sigma^x_{xy} & 0 \\ \sigma^x_{yx} & 0 & \sigma^x_{yz} \\ 0 & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & 0 & \sigma^y_{xz} \\ 0 & \sigma^y_{yy} & 0 \\ \sigma^y_{zx} & 0 & \sigma^y_{zz} \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ \sigma^z_{yx} & 0 & \sigma^z_{yz} \\ 0 & \sigma^z_{zy} & 0 \end{pmatrix}$
$mmm1'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^x_{yz} \\ 0 & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ \sigma^z_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$4/m1'$, $6/m1'$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & -\sigma^y_{xz} \\ \sigma^x_{zx} & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & \sigma^x_{xz} \\ \sigma^y_{zx} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$4/mmm1'$, $6/mmm1'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sigma^y_{xz} \\ 0 & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{3}1'$	$\begin{pmatrix} \sigma^x_{xx} & \sigma^y_{xx} & \sigma^x_{xz} \\ \sigma^y_{xx} & -\sigma^x_{xx} & -\sigma^y_{xz} \\ \sigma^x_{zx} & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & -\sigma^x_{xx} & \sigma^y_{xz} \\ -\sigma^x_{xx} & -\sigma^y_{xx} & \sigma^x_{xz} \\ \sigma^y_{zx} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$\bar{3}1m1'$	$\begin{pmatrix} \sigma^x_{xx} & 0 & 0 \\ 0 & -\sigma^x_{xx} & -\sigma^y_{xz} \\ 0 & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sigma^x_{xx} & \sigma^y_{xz} \\ -\sigma^x_{xx} & 0 & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{3}m11'$	$\begin{pmatrix} 0 & \sigma^y_{xx} & 0 \\ \sigma^y_{xx} & 0 & -\sigma^y_{xz} \\ 0 & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & 0 & \sigma^y_{xz} \\ 0 & -\sigma^y_{xx} & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$m\bar{3}1'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^z_{xy} \\ 0 & \sigma^y_{xz} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & 0 \\ \sigma^z_{xy} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ \sigma^y_{xz} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$m\bar{3}m1'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^z_{xy} \\ 0 & -\sigma^z_{xy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -\sigma^z_{xy} \\ 0 & 0 & 0 \\ \sigma^z_{xy} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Magnetic materials

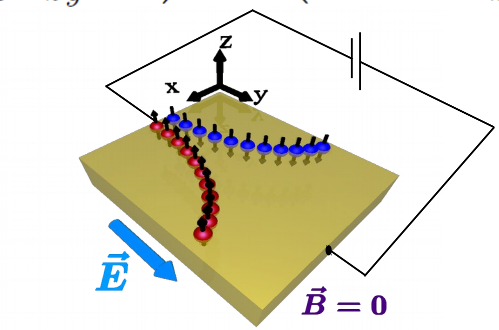
magnetic Laue group	$\underline{\boldsymbol{\sigma}}^x$	$\underline{\boldsymbol{\sigma}}^y$	$\underline{\boldsymbol{\sigma}}^z$
$2'/m'$	$\begin{pmatrix} \sigma^x_{xx} & \sigma^x_{xy} & \sigma^x_{xz} \\ \sigma^x_{yx} & \sigma^x_{yy} & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & \sigma^x_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & \sigma^y_{xy} & \sigma^y_{xz} \\ \sigma^y_{yx} & \sigma^y_{yy} & \sigma^y_{yz} \\ \sigma^y_{zx} & \sigma^y_{zy} & \sigma^y_{zz} \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & \sigma^z_{xz} \\ \sigma^z_{yx} & \sigma^z_{yy} & \sigma^z_{yz} \\ \sigma^z_{zx} & \sigma^z_{zy} & \sigma^z_{zz} \end{pmatrix}$
$4'/m$, $m'm'm$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & \sigma^y_{yz} \\ \sigma^y_{zx} & \sigma^y_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ \sigma^z_{yx} & \sigma^z_{yy} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$4'/mm'm$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & -\sigma^y_{xz} \\ \sigma^x_{zx} & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & -\sigma^x_{xz} \\ \sigma^y_{zx} & -\sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & -\sigma^z_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$4'/mmm'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^x_{yz} \\ 0 & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ \sigma^z_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$4/mm'm'$, $6/mm'm'$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & -\sigma^y_{xz} \\ \sigma^x_{zx} & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & \sigma^x_{xz} \\ \sigma^y_{zx} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$\bar{3}1m'$, $\bar{3}m'1$, $6'/m'$	$\begin{pmatrix} \sigma^x_{xx} & \sigma^y_{xx} & \sigma^x_{xz} \\ \sigma^y_{xx} & -\sigma^x_{xx} & -\sigma^y_{xz} \\ \sigma^x_{zx} & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & -\sigma^x_{xx} & \sigma^y_{xz} \\ -\sigma^x_{xx} & -\sigma^y_{xx} & \sigma^x_{xz} \\ \sigma^y_{zx} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$6'/m'm'm$	$\begin{pmatrix} \sigma^x_{xx} & 0 & 0 \\ 0 & -\sigma^x_{xx} & -\sigma^y_{xz} \\ 0 & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sigma^x_{xx} & \sigma^y_{xz} \\ -\sigma^x_{xx} & 0 & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$6'/m'mm'$	$\begin{pmatrix} 0 & \sigma^y_{xx} & 0 \\ \sigma^y_{xx} & 0 & -\sigma^y_{xz} \\ 0 & -\sigma^y_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^y_{xx} & 0 & \sigma^y_{xz} \\ 0 & -\sigma^y_{xx} & 0 \\ \sigma^y_{zx} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$m\bar{3}m'$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^z_{xy} \\ 0 & \sigma^y_{xz} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & 0 \\ \sigma^z_{xy} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^z_{xy} & 0 \\ \sigma^y_{xz} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

class a) contains time reversal *T*

class c) contains combined operations a=v *T*

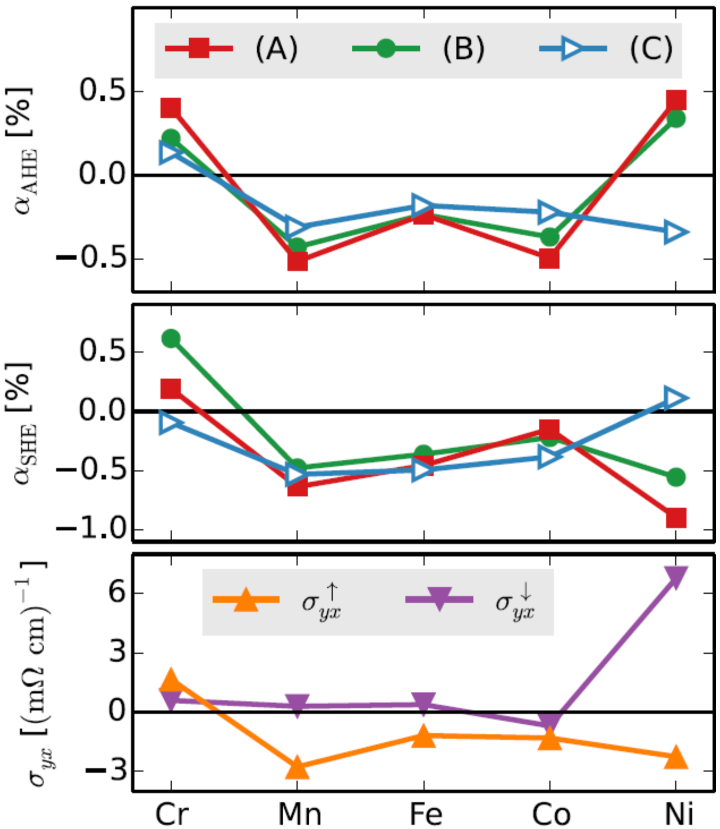
Electrical and spin-polarized conductivity tensor

magnetic Laue group	$\underline{\boldsymbol{\sigma}}$	$\underline{\boldsymbol{\sigma}}^x$	$\underline{\boldsymbol{\sigma}}^y$	$\underline{\boldsymbol{\sigma}}^z$
$m\bar{3}m1'$ e.g.: Au	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{xx} \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^x_{yz} \\ 0 & -\sigma^x_{yz} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -\sigma^x_{yz} \\ 0 & 0 & 0 \\ \sigma^x_{yz} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \sigma^x_{yz} & 0 \\ -\sigma^x_{yz} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$4/mm'm'$ e.g.: FM bcc Fe	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -\sigma^x_{yz} \\ 0 & 0 & \sigma^x_{xz} \\ -\sigma^x_{zy} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$4/m1'$ e.g.: Au_4Sc	$\begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -\sigma^x_{yz} \\ 0 & 0 & \sigma^x_{xz} \\ -\sigma^x_{zy} & \sigma^x_{zx} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$
$2/m1'$ e.g.: Pt_3Ge	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sigma^y_{xz} \\ 0 & 0 & \sigma^y_{yz} \\ \sigma^y_{zx} & \sigma^y_{zy} & 0 \end{pmatrix}$	$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ \sigma^z_{yx} & \sigma^z_{yy} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$



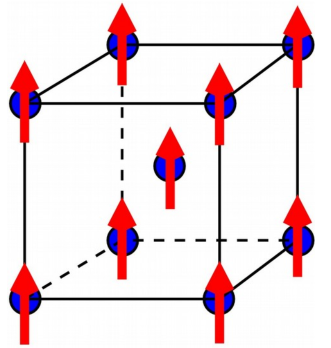
Wimmer, *et al.*, arXiv:1502.04947 (2015)

Anomalous and spin Hall effect in diluted ferromagnetic alloys M_1Pt_{99}



AHE
$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

SHE
$$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$$



ferromagnetic
4/mm'm'

skew scattering contributions to AHE, SHE
1 % magnetic impurities in Pt host

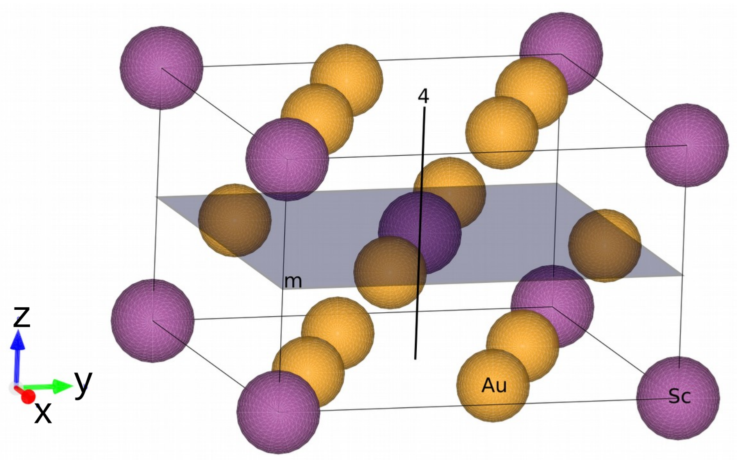
Approach	Transport description	Electronic structure	Spin-orbit coupling
Method A	BE	FP	Pauli
Method B	BE	ASA	Dirac
Method C	KSF	ASA	Dirac

- complex electronic structure at E_F
- skew scattering very sensitive to fine details

→ first-principles approach mandatory

Zimmermann, *et al.* PRB **90**, 220403(R) (2014)

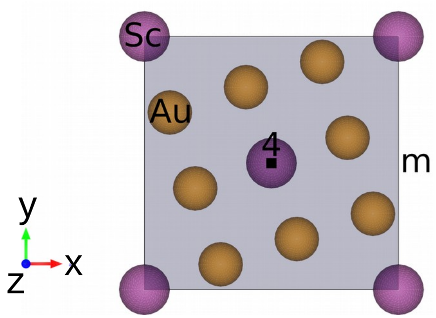
Additional *transverse* effects in 4/m1'

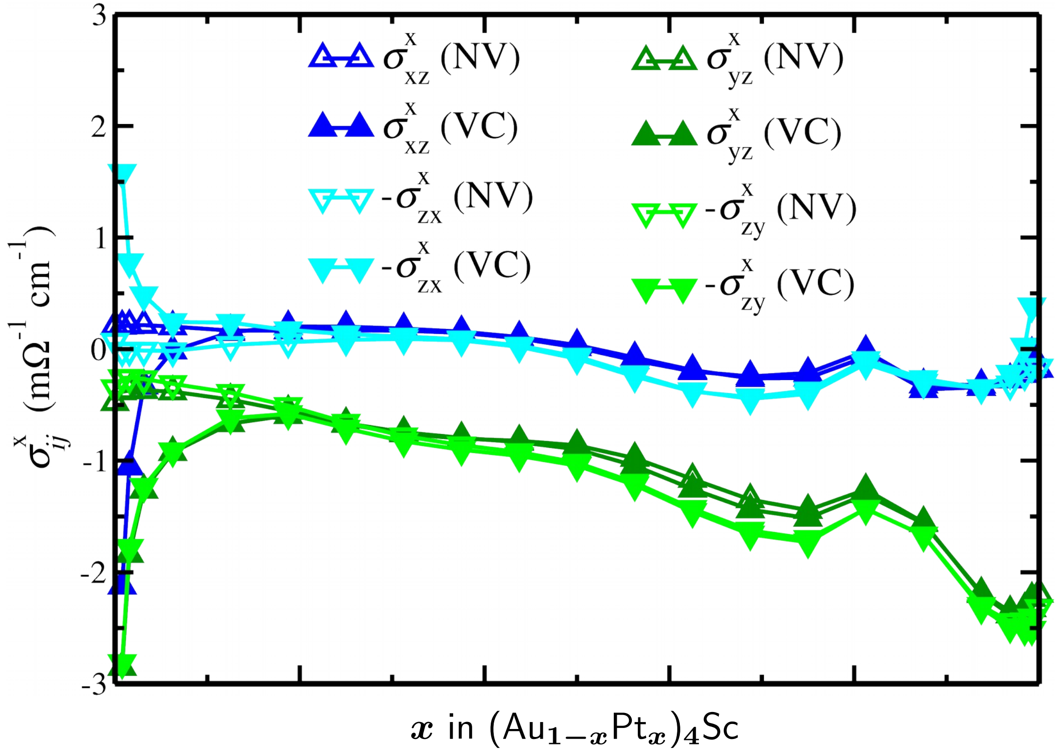


non-magnetic $(Au_{1-x}Pt_x)_4Sc$ - alloy

Au_4Mn - structure

4/m1'

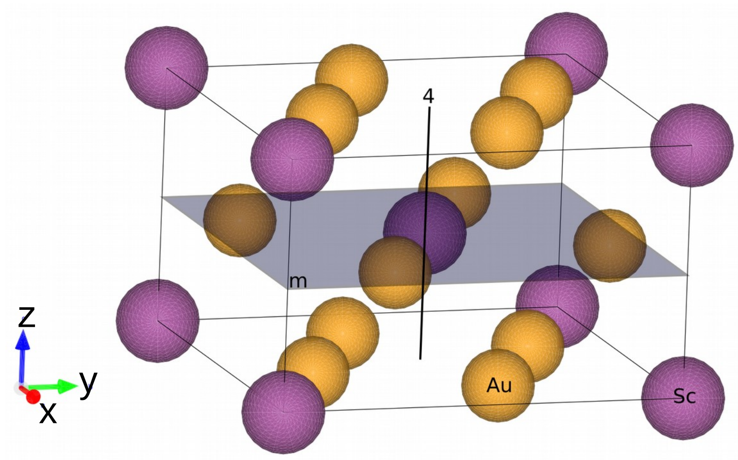




$$\begin{pmatrix} 0 & 0 & \sigma^x_{xz} \\ 0 & 0 & \sigma^x_{yz} \\ \sigma^x_{zx} & \sigma^x_{zy} & 0 \end{pmatrix}$$

Wimmer, *et al.*, arXiv:1502.04947 (2015)

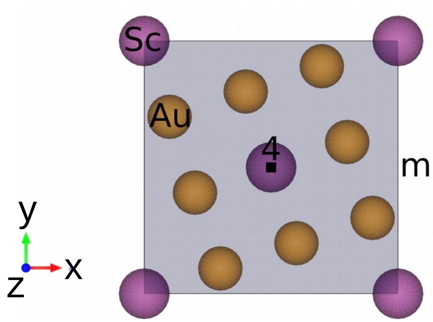
Longitudinal spin polarized currents in 4/m1'

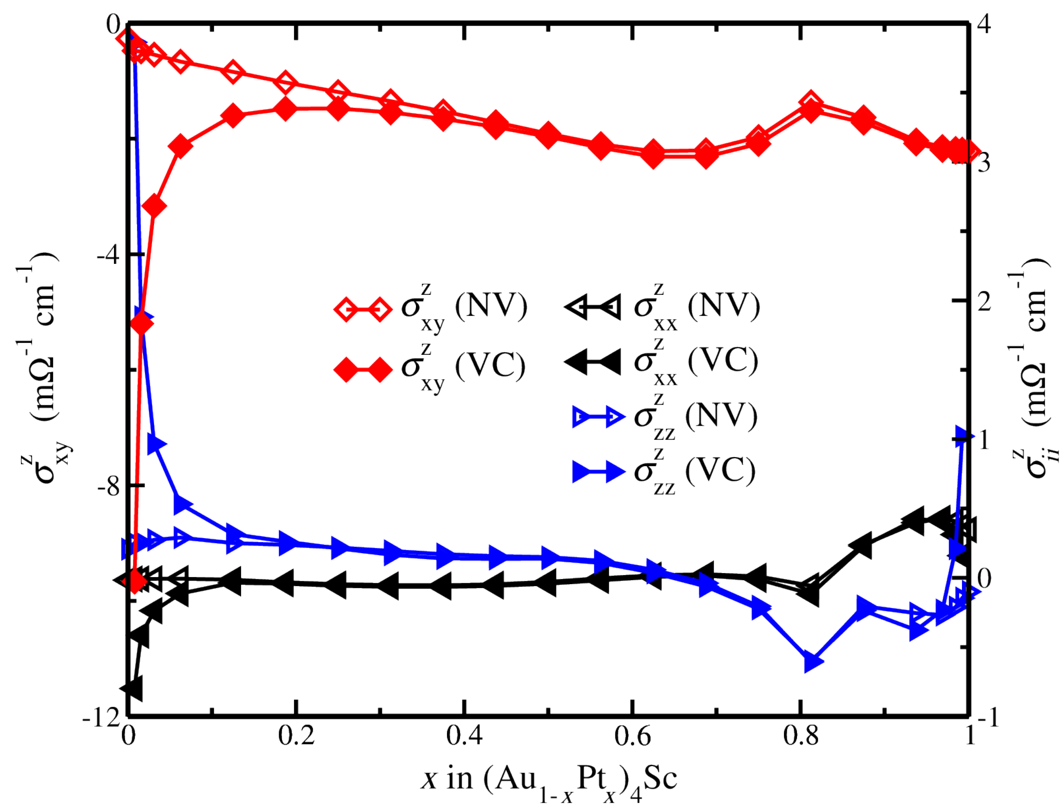


non-magnetic $(Au_{1-x}Pt_x)_4Sc$ - alloy

Au_4Mn - structure

4/m1'



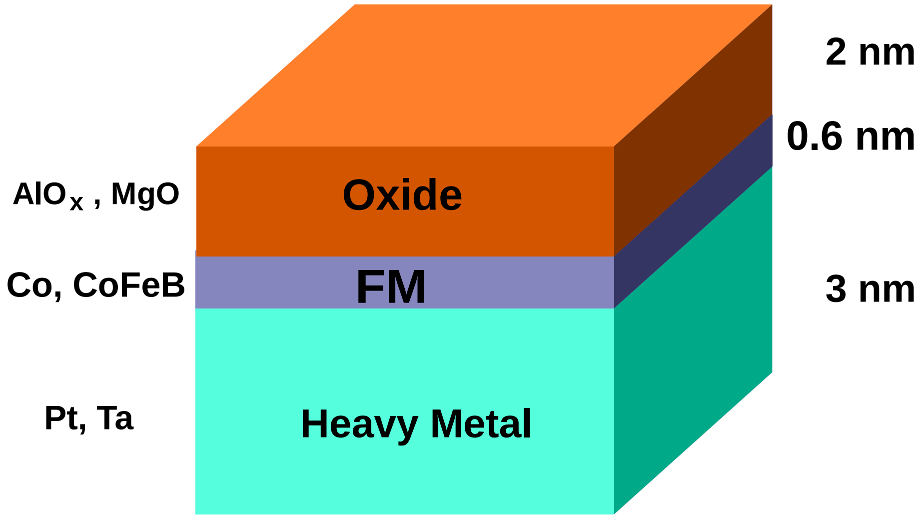


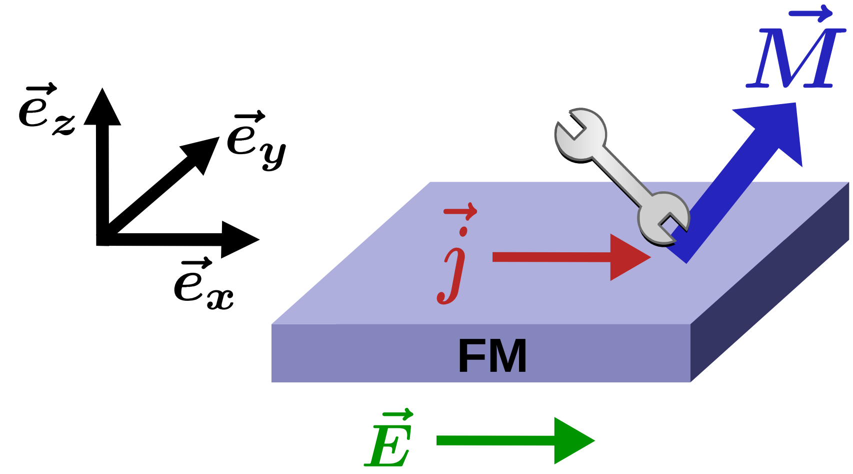
$$\begin{pmatrix} \sigma^z_{xx} & \sigma^z_{xy} & 0 \\ -\sigma^z_{xy} & \sigma^z_{xx} & 0 \\ 0 & 0 & \sigma^z_{zz} \end{pmatrix}$$

Wimmer, *et al.*, arXiv:1502.04947 (2015)

Spin-orbit torque – phenomenology

Systems lacking inversion symmetry





Torque $$\vec{T} = t\,\vec{E}$$

<u>Prediction:</u>

Manchon & Zhang, PRB **79**, 094422 (2009)
Garate & MacDonald, PRB **80**, 134403 (2009)

<u>Experiments:</u>

Miron *et al.*, Nat.Mat. **9**, 230 (2010)
Gambardella & Miron, Phil. Trans. R. Soc. A (2011) **369**, 3175
Garello *et al.*, Nat.Nanot. **8**, 587 (2013)

Possible avenue to avoid large switching currents (10^7 A / cm^2) that currently hamper progress in STT-RAM technology.

Linear response formulation for torkance

$$\tau_{\hat{B}_i \hat{A}_j}(\omega, \vec{H}) = \int_0^\infty dt\, e^{-i\omega t} \int_0^\beta d\lambda \left\langle \rho(\vec{H}) \hat{A}_j \hat{B}_i(t + i\hbar\lambda; \vec{H}) \right\rangle$$

Freimuth, *et al*, arXiv: 1305.4873

For torkance $\hat{B}_i = \hat{T}_i \qquad \hat{A}_j = \hat{j}_j$

Torque operator $T_i = \dfrac{\partial}{\partial u_i}\hat{H}$

Spin magentisation $\vec{m} = m\vec{u}$

Ebert & Mankovsky, PRB **79**, 045209 (2009)
Ebert, *et al*, PRL **107**, 066603 (2011)
Mankovsky, *et al*, PRB **87**, 014430 (2013)

Symmetry considerations for torkance t

$$\vec{T} = t\,\vec{E}$$

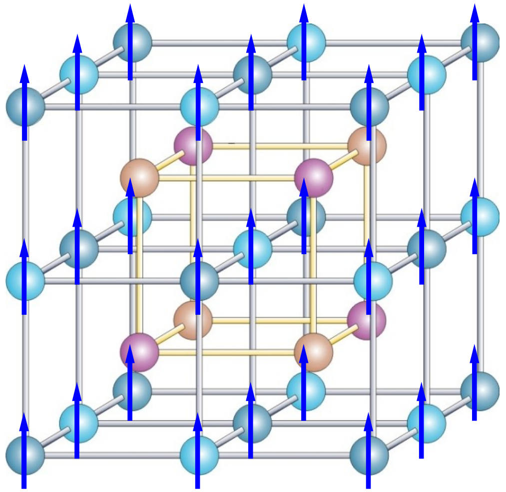
Transformation of torkance under symmetry operations of magnetic space group

$$t_{ij} = \sum_{kl} t_{kl} D(R)_{ki} D(R)_{lj} \det(R)$$ unitary operations

$$t_{ij} = \sum_{kl} t'_{lk} D(R)^*_{ki} D(R)^*_{lj} \det(R)$$ anti-unitary operations

all R → tensor shape

Example – inverse Heusler system



Cr_2VSb

http://en.wikipedia.org/wiki/File:Heusler_alloy_-_structure.png

$$t = \begin{pmatrix} t_{xx} & t_{xy} & 0 \\ -t_{xy} & -t_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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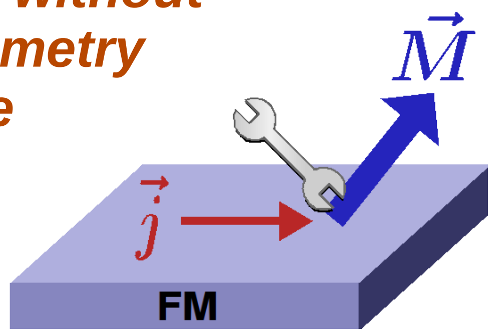
Application – spin-orbit torque

Results obtained by analytic computation using computer algebra system (CAS)

magnetic point group	$\underline{t}$
$\bar{1}'$, 1, $2'$, m'	$\begin{pmatrix} t_{xx} & t_{xy} & t_{xz} \\ t_{yx} & t_{yy} & t_{yz} \\ t_{zx} & t_{zy} & t_{zz} \end{pmatrix}$
$\bar{1}$, $\bar{3}$, $\bar{3}1m$, $\bar{3}1m'$, $\bar{3}m'1$, $\bar{3}m1$, $\bar{4}3m$, $\bar{6}$, $\bar{6}2'm'$, $\bar{6}2m$, $\bar{6}m'2'$, $\bar{6}m2$, $2'/m'$, $2/m$, $4'/m$, $4'/mm'm$, $4'/mmm'$, $4/m$, $4/mm'm'$, $4/mmm$, $6'/m$, $6'/m'$, $6'/m'm'm$, $6'/m'mm'$, $6'/mm'm$, $6'/mmm'$, $6/m$, $6/mm'm'$, $6/mmm$, $m\bar{3}$, $m\bar{3}m$, $m\bar{3}m'$, $m'\bar{3}'m$, $m'm'm$, mmm	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
2, $2/m'$	$\begin{pmatrix} t_{xx} & 0 & t_{xz} \\ 0 & t_{yy} & 0 \\ t_{zx} & 0 & t_{zz} \end{pmatrix}$
$2'/m$, m, $m'm2'$	$\begin{pmatrix} 0 & t_{xy} & 0 \\ t_{yx} & 0 & t_{yz} \\ 0 & t_{zy} & 0 \end{pmatrix}$
$\bar{4}'2m'$, 222, $4'22'$, $m'm'm'$	$\begin{pmatrix} t_{xx} & 0 & 0 \\ 0 & t_{yy} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$
$\bar{4}'$, $2'2'2$, $4'$, $m'm'2$	$\begin{pmatrix} t_{xx} & t_{xy} & 0 \\ t_{yx} & t_{yy} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$
$\bar{4}'m2'$, $4'mm'$, $mm2$	$\begin{pmatrix} 0 & t_{xy} & 0 \\ t_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$m'mm$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & t_{yz} \\ 0 & t_{zy} & 0 \end{pmatrix}$
$\bar{3}'$, $\bar{6}'$, 3, $312'$, $31m'$, $32'1$, $3m'1$, 4, $4/m'$, $42'2'$, $4m'm'$, 6, $6'$, $6/m'$, $62'2'$, $6m'm'$	$\begin{pmatrix} t_{xx} & t_{xy} & 0 \\ -t_{xy} & t_{xx} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$
$\bar{4}$, $\bar{4}2'm'$, $\bar{4}m'2'$, $4'/m'$	$\begin{pmatrix} t_{xx} & t_{xy} & 0 \\ t_{xy} & -t_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

magnetic point group	$\underline{t}$
$\bar{4}2m$, $4'/m'm'm$	$\begin{pmatrix} t_{xx} & 0 & 0 \\ 0 & -t_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{4}m2$, $4'/m'mm'$	$\begin{pmatrix} 0 & t_{xy} & 0 \\ t_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{4}'3m'$, 23, $4'32'$, 432, $m'\bar{3}'$, $m'\bar{3}'m'$	$\begin{pmatrix} t_{xx} & 0 & 0 \\ 0 & t_{xx} & 0 \\ 0 & 0 & t_{xx} \end{pmatrix}$
$\bar{3}'1m'$, $\bar{3}'m'1$, $\bar{6}'2m'$, $\bar{6}'m'2$, 312, 321, $4/m'm'm'$, 422, $6'2'2$, $6'22'$, $6/m'm'm'$, 622	$\begin{pmatrix} t_{xx} & 0 & 0 \\ 0 & t_{xx} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$
$\bar{4}'m'2$, $4'2'2$	$\begin{pmatrix} t_{xx} & t_{xy} & 0 \\ t_{xy} & t_{xx} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$
$\bar{3}'1m$, $\bar{3}'m1$, $\bar{6}'2'm$, $\bar{6}'m2'$, $31m$, $3m1$, $4/m'mm$, $4mm$, $6'm'm$, $6'mm'$, $6/m'mm$, $6mm$	$\begin{pmatrix} 0 & t_{xy} & 0 \\ -t_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{4}'2'm$, $4'm'm$	$\begin{pmatrix} t_{xx} & t_{xy} & 0 \\ -t_{xy} & -t_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

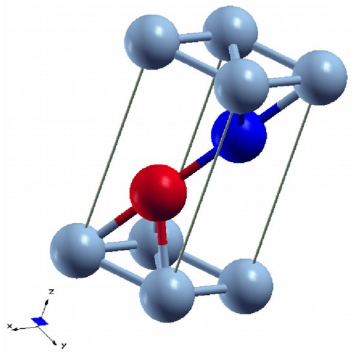
only materials without inversion symmetry show torkance



Results for multilayer system Cu/Fe_xCo_{1-x}/Pt

Trilayer test system

$\vec{e}_z$



$\vec{M} \parallel \vec{e}_z$

Unit cell

Pt

Fe_xCo_{1-x}

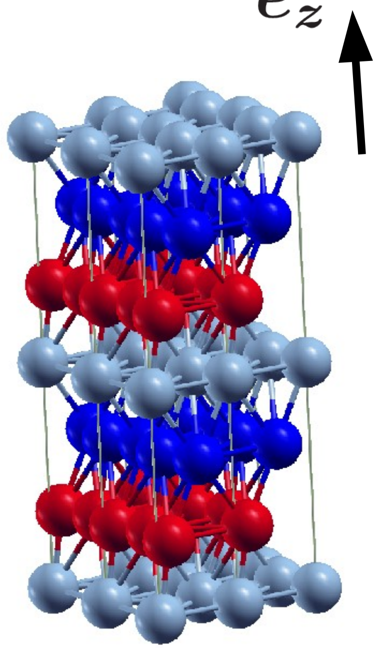
Cu

Pt

Fe_xCo_{1-x}

Cu

Pt

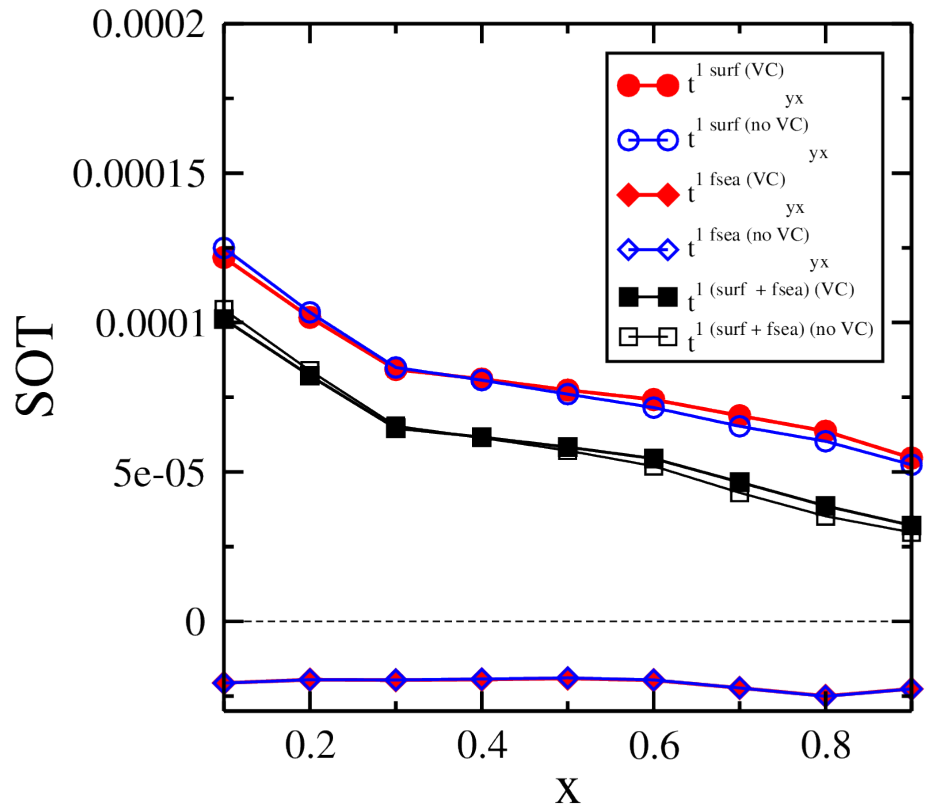


fcc (111) stacking

Symmetry

$$t = \begin{pmatrix} t_{xx} & -t_{yx} & 0 \\ t_{yx} & t_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

t_{yx} even w.r.t. $\vec{M} \rightarrow -\vec{M}$



- Fermi surface contribution dominant showing variation with concentration
- Fermi sea term almost constant

cf: similar observations as in
Freimuth, Blügel, Mokrousov, arXiv: 1305.4873

Summary and further applications

A universal scheme has been presented to

predict the shape of any linear response tensor

Spin-orbit induced effects

- **Electrical and spin conductivity**
  - Only magnetic Laue group relevant
  - AHE in non-collinear AFMs
  - Longitudinal Spin Conductivity
- **Torkance** (spin-orbit torque)
  - Only for systems without space inversion symmetry
- ***Gilbert damping***
- ***Edelstein effect***
- ***Thermoelectric effects***
- ***Layer-resolved conductivities***
- ***Non-linear response***

All results in full agreement with ab-initio calculations based on fully relativistic formalism

Further Applications not presented here

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Spin Caloric Transport