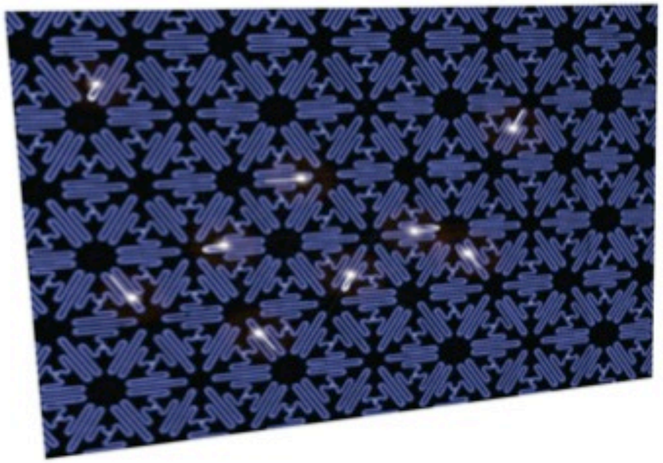
Flat bands with Strongly correlated photons

Sebastian Schmidt (ETH Zurich)

In collaboration with: M. Biondi, E. Niewenburg, S. Huber, G. Blatter (ETH)



Review: S. Schmidt & J. Koch,
Ann. Phys. (Berlin) 525, 395-412 (2013)

ETH
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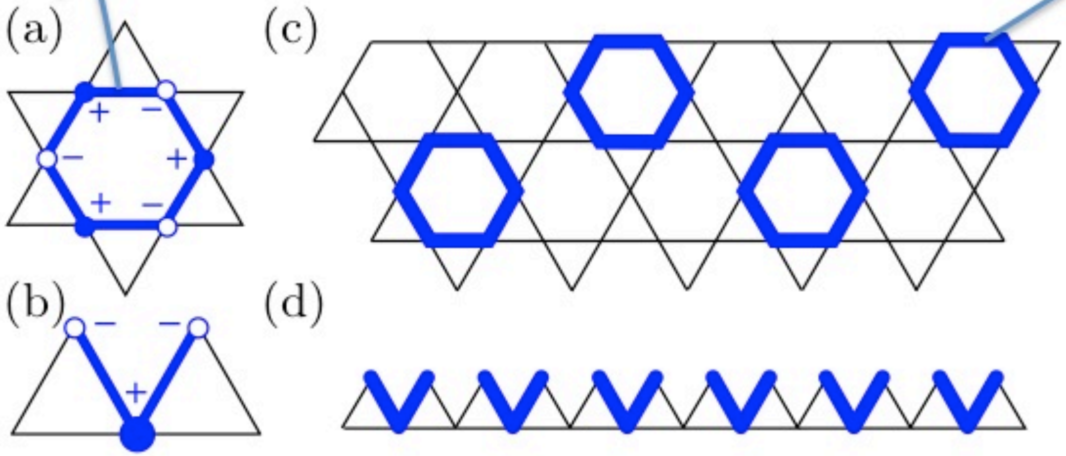
Flat bands

localized, degenerate
s.-p. states

CDW at fractional fillings
for repulsive interactions



enhanced interactions

➡ highly correlated, topological and exotic states of matter

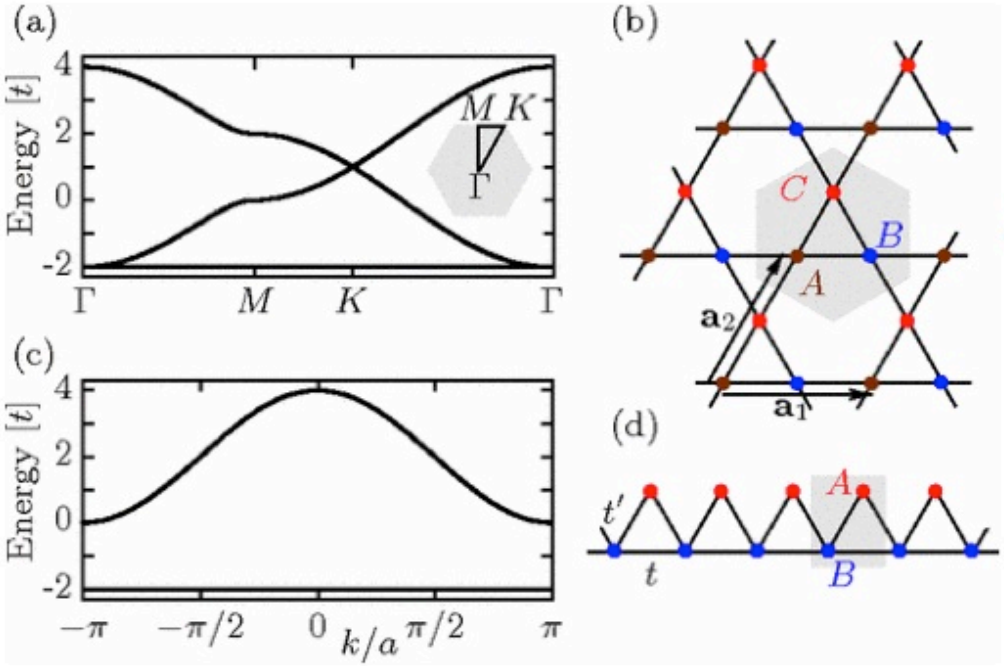
Zhitomirsky and Tsunetsugu, PRB 2004
Zhitomirsky and Tsunetsugu, Prog.Theor.Phys. 2005
Wu et. al. PRL 2007
Bergman et. al. PRB 2008
Huber et. al. PRB 2008 ....

Flat Bands

$$H = \sum_j h_j^{\mathrm{local}} + \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j$$

- electrons
- spins
- atoms

Geometric frustration → dispersionless bands

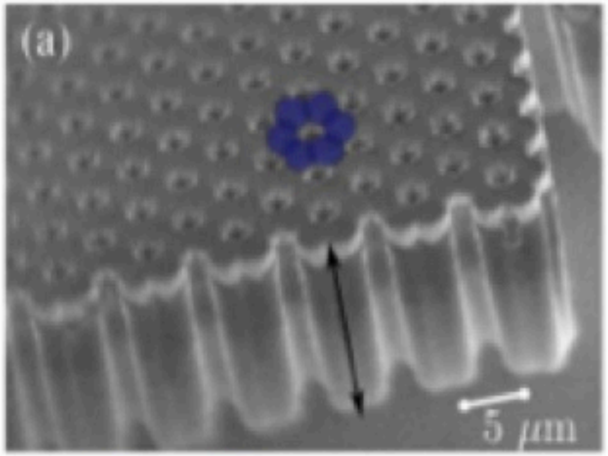


Kagome lattice

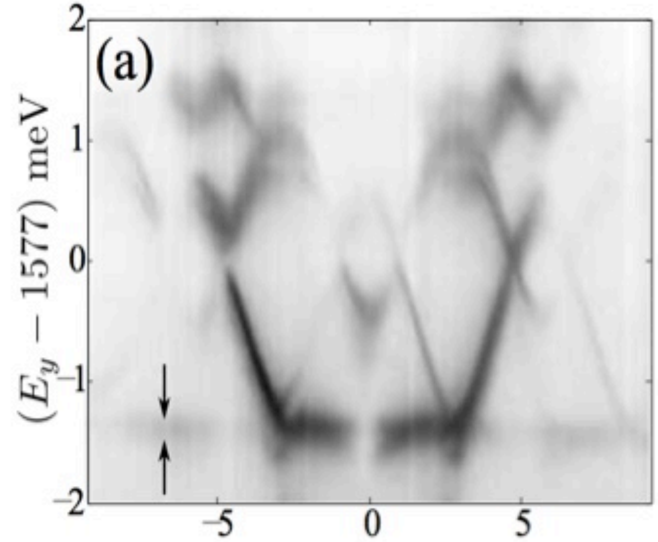
Sawtooth chain

Photonic lattices

Micropillar cavity lattice



Jacqmin et al., PRL 2014



Tight-binding model for photons

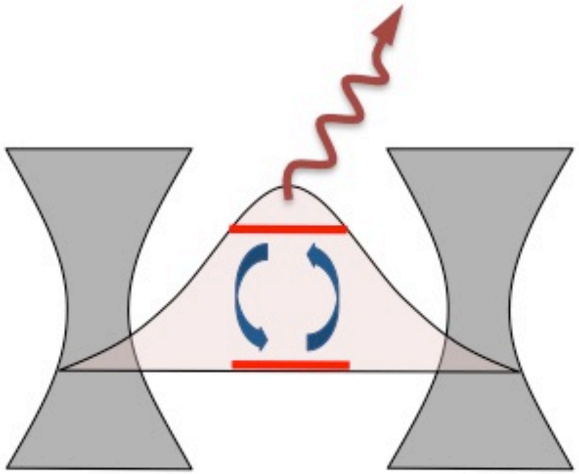
$$H = \sum_j h_j^{\mathrm{local}} + \sum_{\langle ij \rangle} J_{ij} a_i^\dagger a_j$$

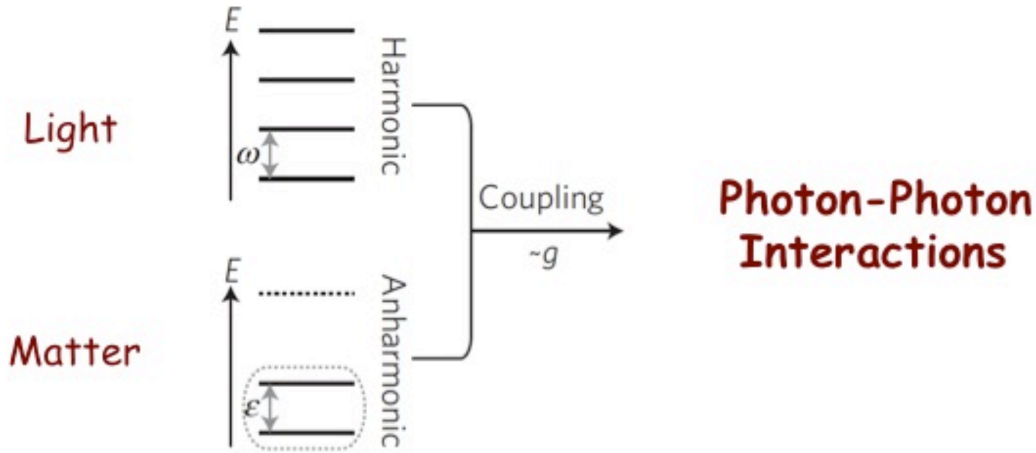
- full coherent control
- arbitrary geometries
- tunable parameters

Application of flat bands: slow light polaritons

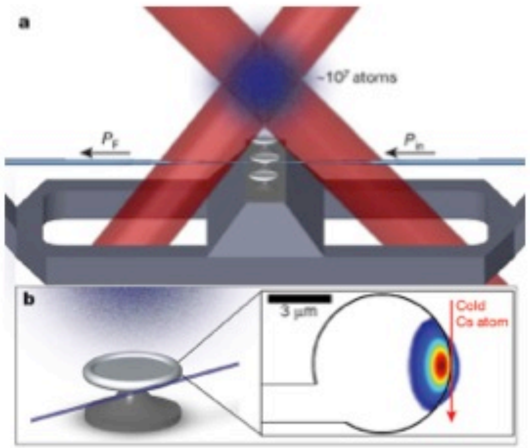
Interactions ?

Strong light-matter coupling



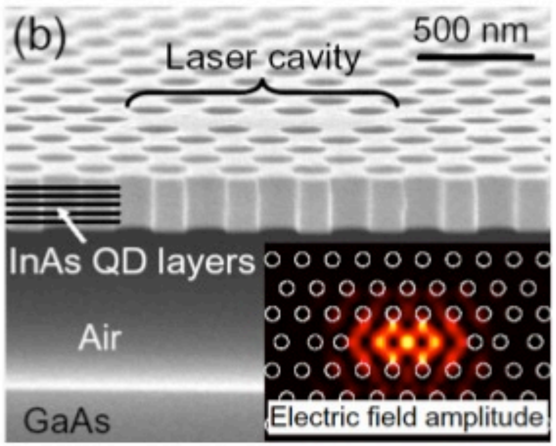


AMO



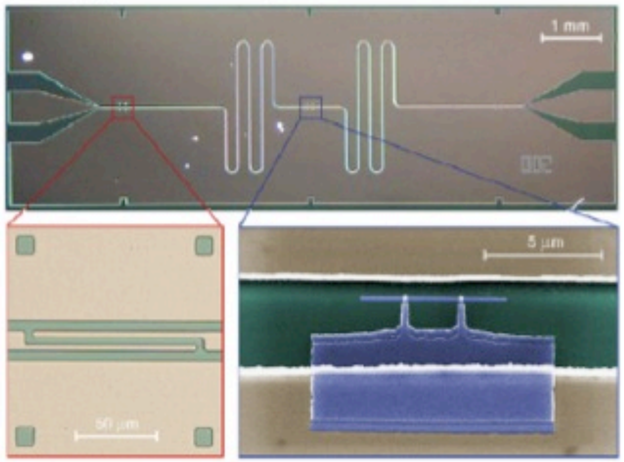
Caesium atoms
Nature **443**, 671 (2006)

Solid State



Semiconductor quantum dots
Nature **432**, 197 (2004)

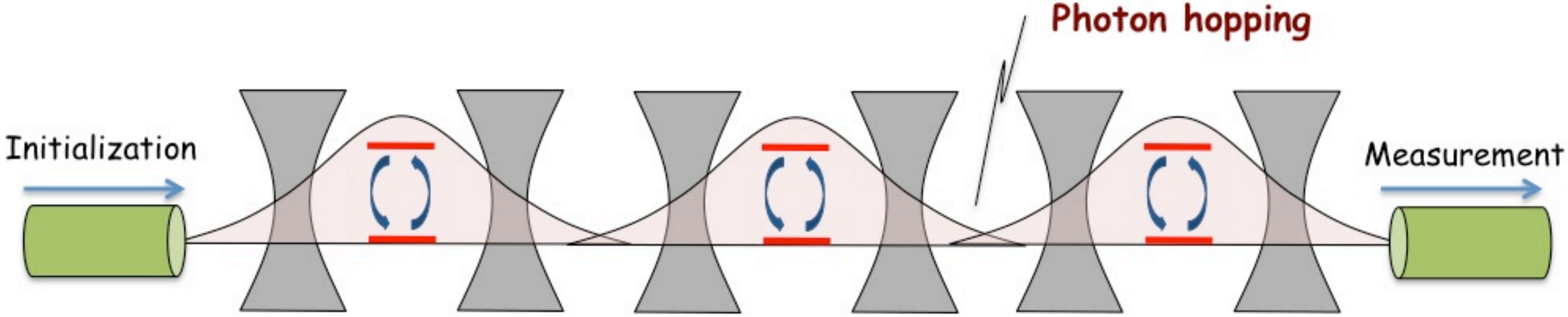
Quantum Electronics



Superconducting qubits
Nature **431**, 159 (2004)

Quo Vadis?

Scaling up & understanding **coupled systems**



Greentree et.al. Nat. Phys. **466**, 856 (2006)

Quantum Devices

Single-photon source

entangled photon pair source

quantum-limited amplifiers

...

Quantum Simulation

Strong correlations

Frustration

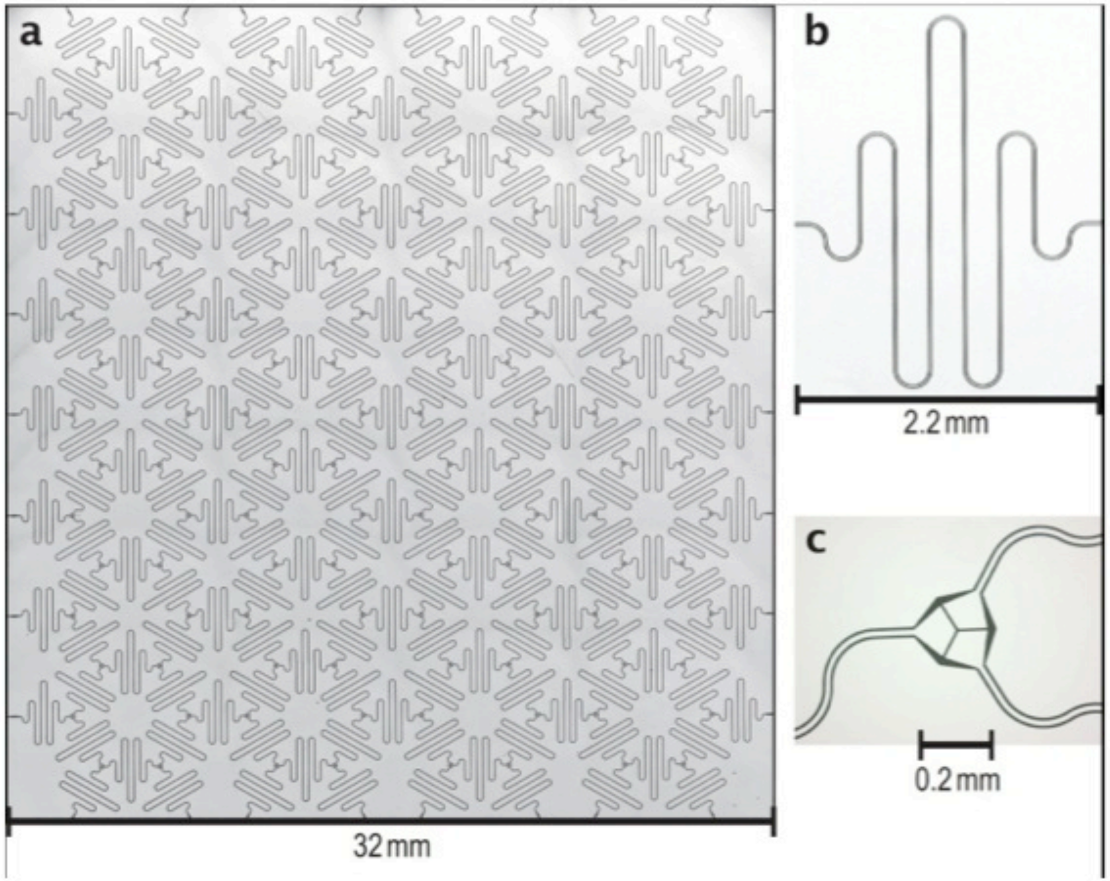
Topological states

...

New aspects: Drive and Dissipation !

Circuit QED Kagome lattice

On-going experiment: Qdevice Lab @ Princeton (A. Houck)



A. Houck, H. Tureci, J. Koch, Nature Physics 2012

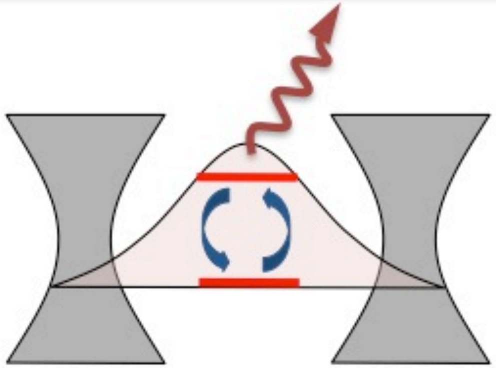
Outline

Cavity QED primer

Jaynes-Cummings model

Photon Blockade

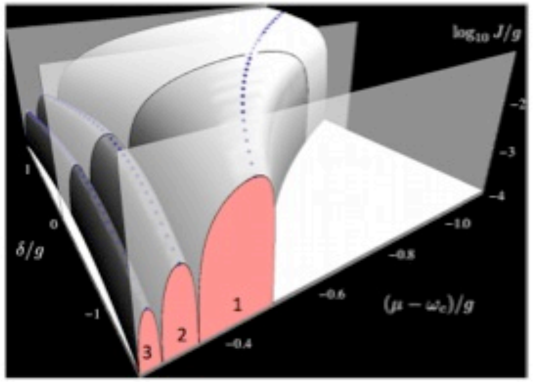
Circuit QED



Strong correlations

Self-trapping of photons

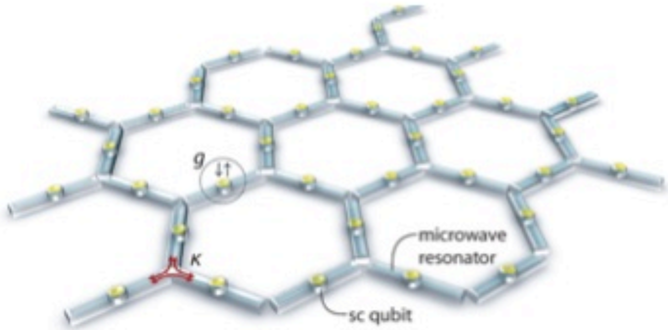
Non-equilibrium SF-MI transition of polaritons



Flat bands

Lieb chain

Crystalline order of photons

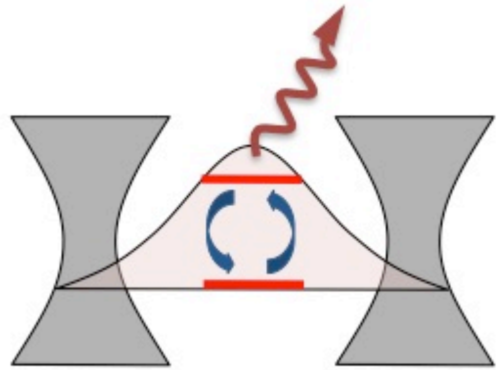


Cavity QED primer

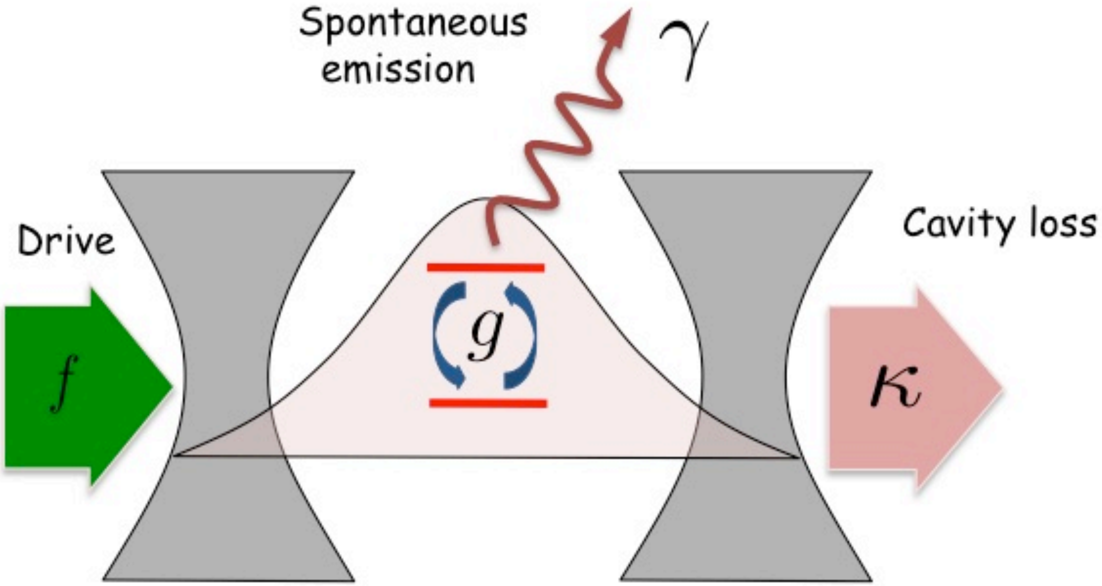
Jaynes-Cummings model

Photon Blockade

Circuit QED



Light-matter interactions



$$H = H_{\rm JC} + H_{\rm dr} + H_\kappa + H_\gamma$$

Jaynes-Cummings model

$$H_{\rm JC} = \omega_r a^\dagger a + \omega_q \sigma^+\sigma^- + g(a^\dagger\sigma^- + a\sigma^+).$$

EM field (cavity mode) — Two-level system — Interaction

Coherent Drive

$$H_{\rm drive} = f(ae^{i\omega_d t} + a^\dagger e^{-i\omega_d t})$$

Strong coupling: $g \gg \kappa, \gamma, \gamma_\phi$

Weak drive: $g \gg f$

Dissipation via Master equation

$$\partial_t\rho = -i[H_{\rm JC} + H_{\rm drive}, \rho] + \kappa\mathcal{D}[a]\rho + \gamma\mathcal{D}[\sigma^-]\rho + \gamma_\phi\mathcal{D}[\sigma^z]\rho$$

Lindblad operator

$$\mathcal{D}[A]\rho = A\rho A^\dagger - \{A^\dagger A, \rho\}/2$$

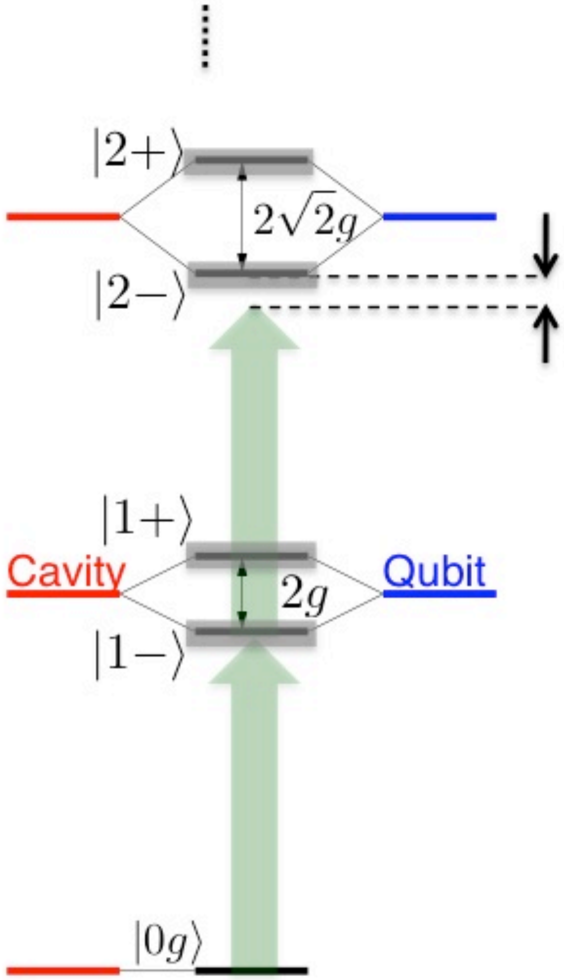
(Born-Markov)

$$\dot\rho = \mathcal{L}\rho \qquad \rho(t) = e^{\mathcal{L}t}\rho(0)$$

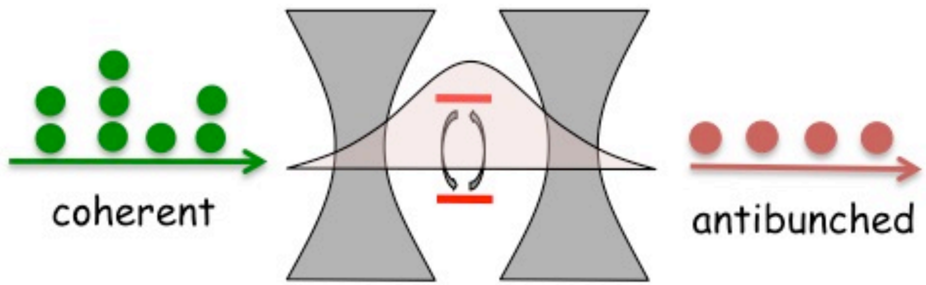
Photon blockade

Lets consider: $\delta = \omega_r - \omega_q = 0$

$|n, \pm\rangle \sim |g, n\rangle \pm | e, n-1\rangle$ JC ladder: dressed states (polaritons)



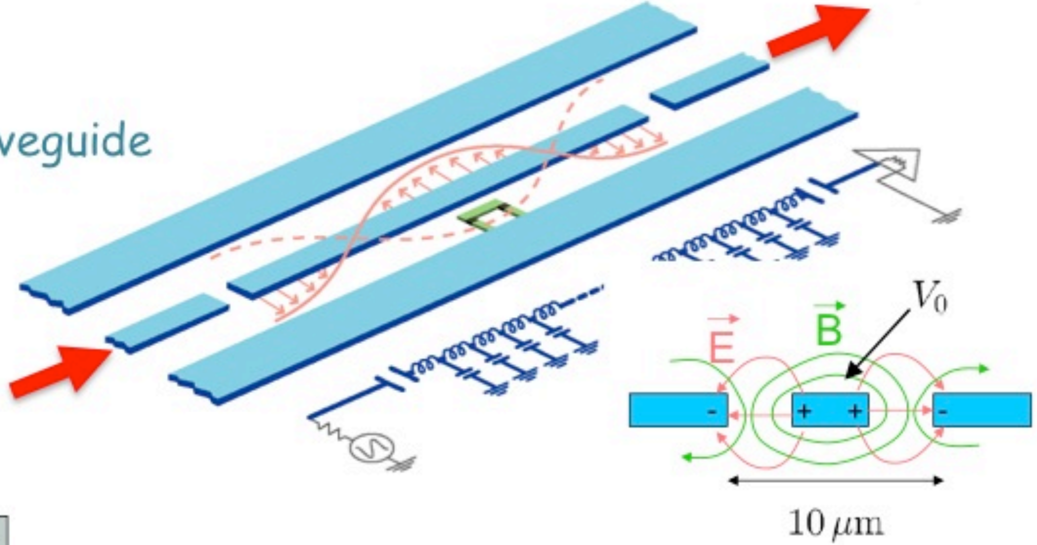
Nonlinearity $U = g(2-\sqrt{2})$ → **Photon blockade**



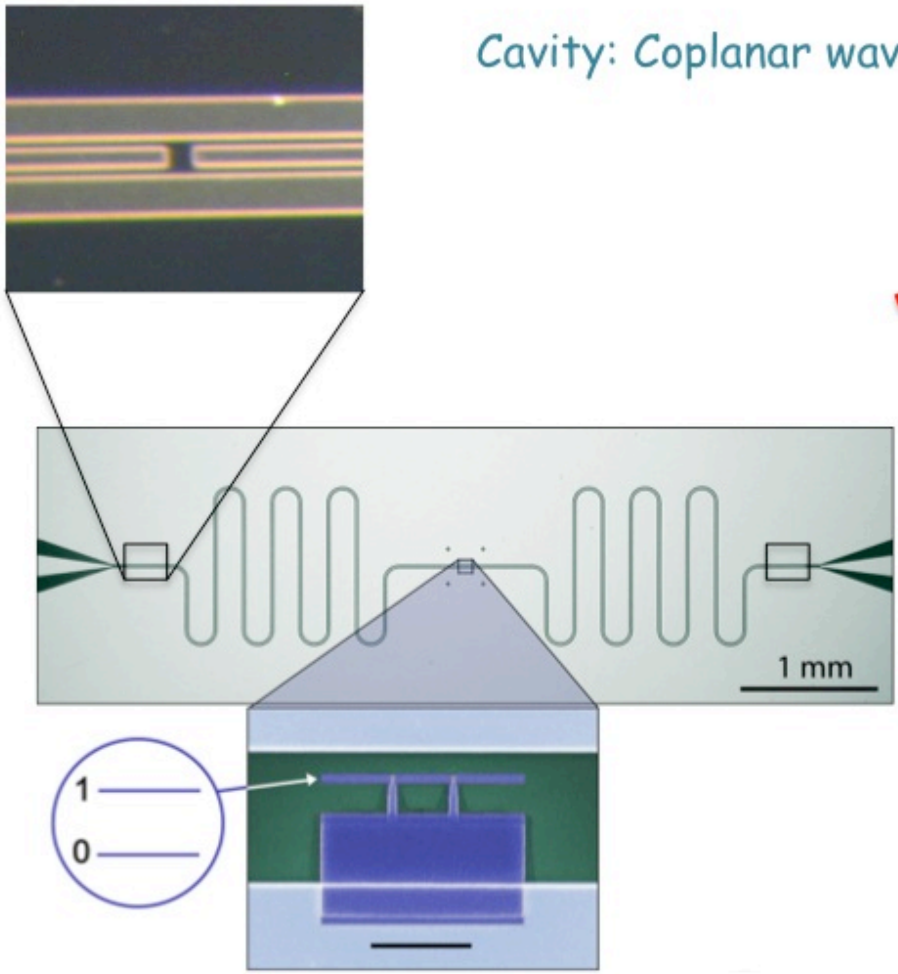
Single Photon Source

Circuit QED realization

Cavity: Coplanar waveguide

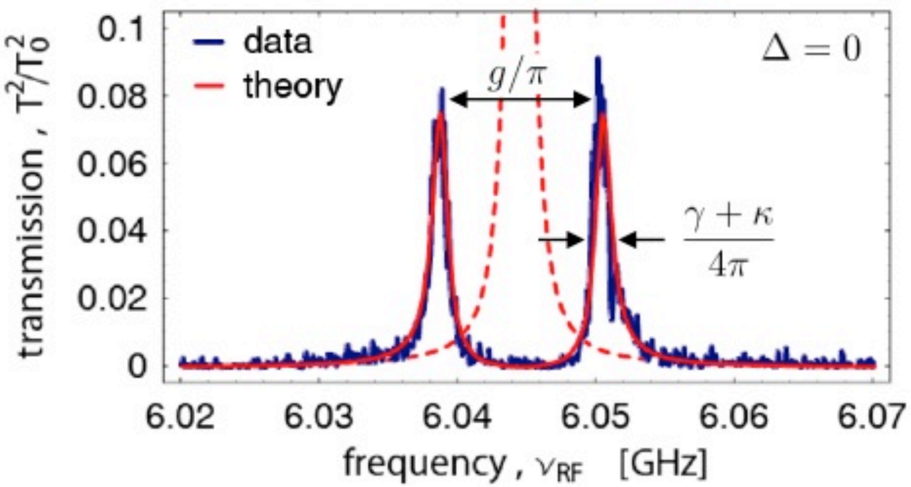


Wallraff et. al., Nature (2004)



Artificial atom: Cooper pair box

Vacuum Rabi spliting



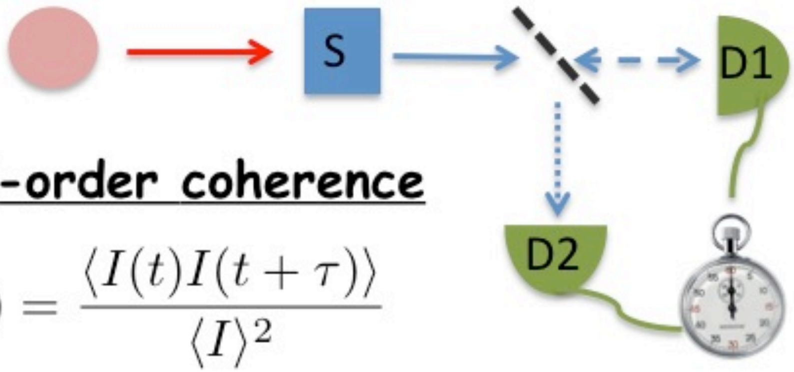
Parameter regime

Table 1 Circuit QED parameters and typical values.

Model Parameter m		**Typical values** m/2π	**In-situ tunability?**
ω_r	resonator frequency	1…15 GHz	Yes – with SQUID terminated resonators [57]
ω_q	qubit frequency	100 MHz … 15 GHz [64]	Yes – e.g., via global or local magnetic flux [58]
g	photon-qubit coupling strength	0…400 MHz	Yes [55, 59]
J	photon hopping strength	1…100 MHz [60]	Yes – with additional coupler circuits [51, 53]
κ	photon escape rate at port	10 kHz … 80 MHz [60]	Yes – with additional coupler circuits [51, 61]
γ_κ	intrinsic photon escape/dissipation rate	$\geq$ 5 kHz [62]	No
γ_1	qubit relaxation rate	$\geq$ 10 kHz [24]	Yes – e.g., via Purcell e ect [63]
γ_{ϕ}	qubit dephasing rate	$\geq$ 50 kHz [24]	Yes – by inducing extra noise

Review: S. Schmidt & J. Koch, Ann. Phys. (Berlin) 525, 395-412 (2013)

Measurement



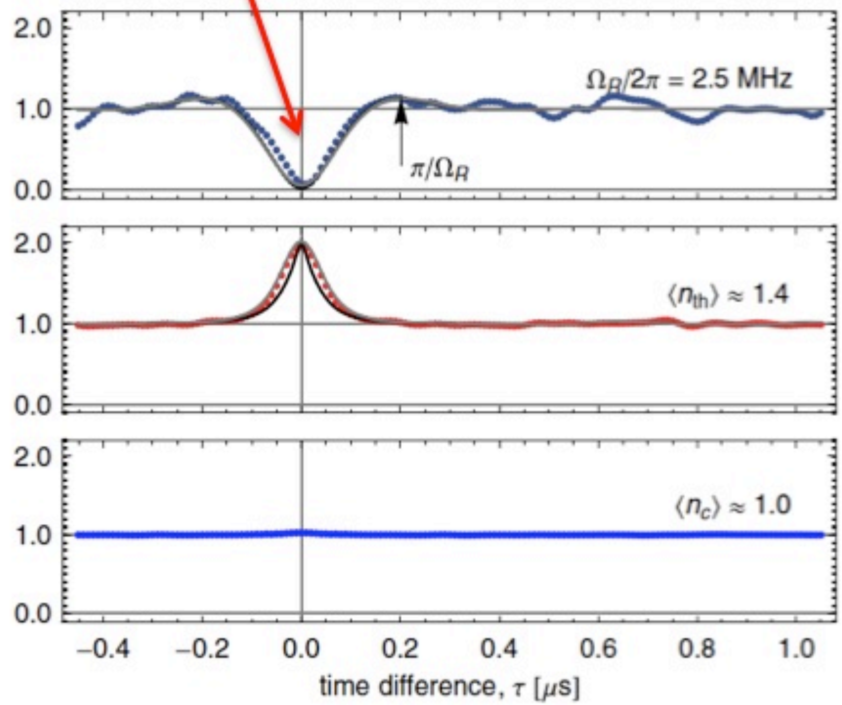
Second-order coherence

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I \rangle^2}$$

Statistics

$$g^{(2)}(0) = \frac{\langle n(n-1) \rangle}{\langle n^2 \rangle}$$

Photon blockade dip



Lang et al, PRL 2011

↔ Antibunching (Cavity field)

Sub-poissonian $g^{(2)}(0) < 1$

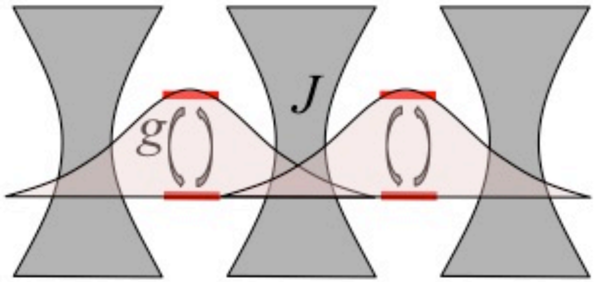
↔ Bunching (Thermal field)

Super-poissonian $g^{(2)}(0) > 1$

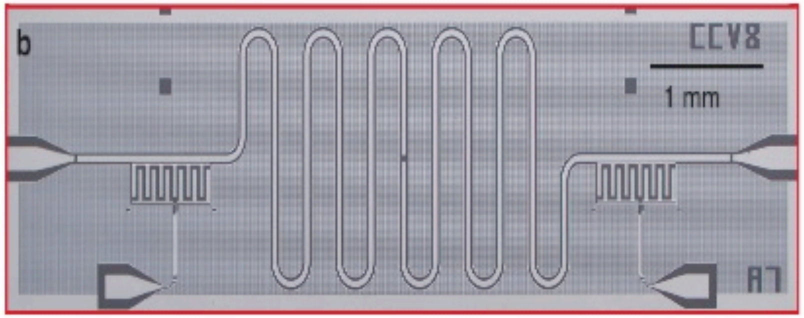
↔ Coherent (Random field)

Poissonian $g^{(2)}(0) = 1$

Self-trapping of photons



↔



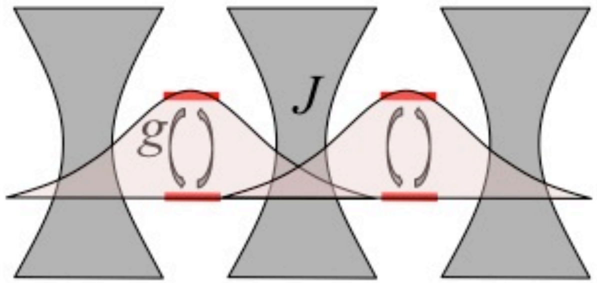
Theory:
Schmidt et al, PRB 82, 100507(R) (2010)

Experiment:
Raftery et al., arXiv:1312.2963 (2013)

Interactions vs. Hopping ➡ Dynamical (phase) transition

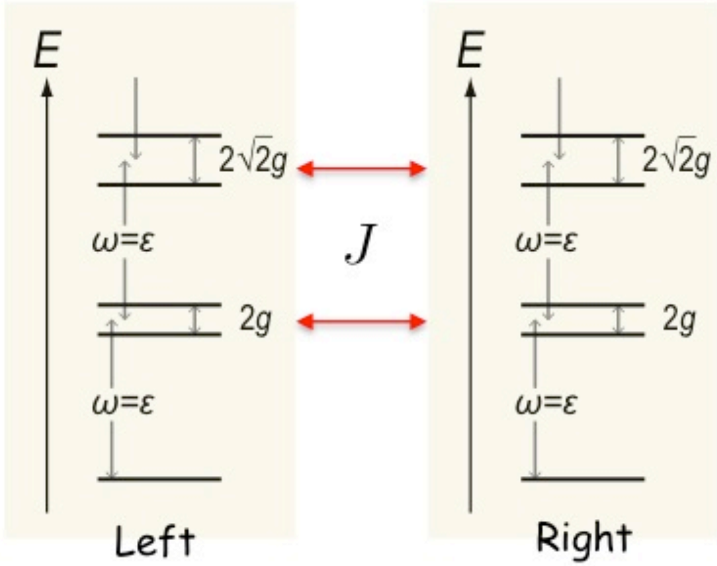
Two cavities

Interactions vs. Hopping ➡ Dynamical (phase) transition



Jaynes-Cummings dimer (JCD)

$$H = H_{\mathrm{JC,L}} + H_{\mathrm{JC,R}} + J(a_L^\dagger a_R + \mathrm{h.c.})$$



for large number of photons $N \gg 1$

➡ sharp localization-delocalization transition at $g_c = \alpha\sqrt{N}J$

Schmidt et al, PRB 82, 100507(R) (2010)

JCD Eigenstates

Total number of excitations conserved $N = n_L + n_R + \sigma_L^z + \sigma_R^z$

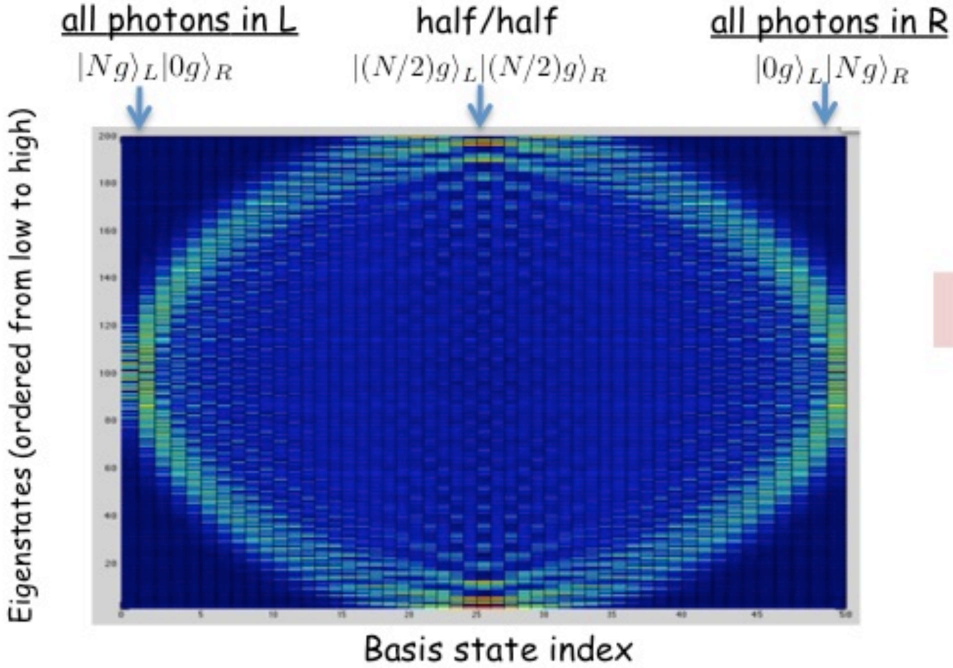
$N = 50$

(sector with both qubits in the ground state ~g)

Imbalance

$$Z = \frac{n_L - n_R}{n_L + n_R}$$

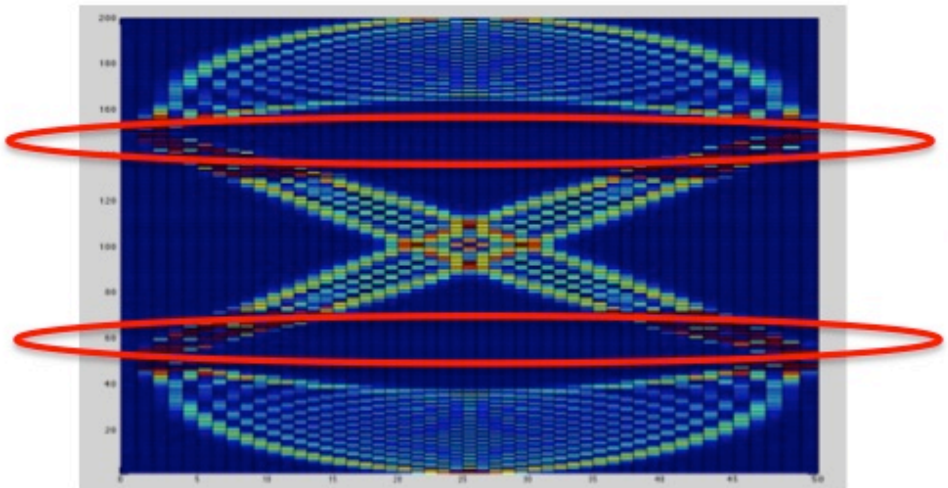
$g \ll J$



Delocalized states

$\langle\psi_i|Z|\psi_i\rangle = 0$

$g \gg J$



Localized states

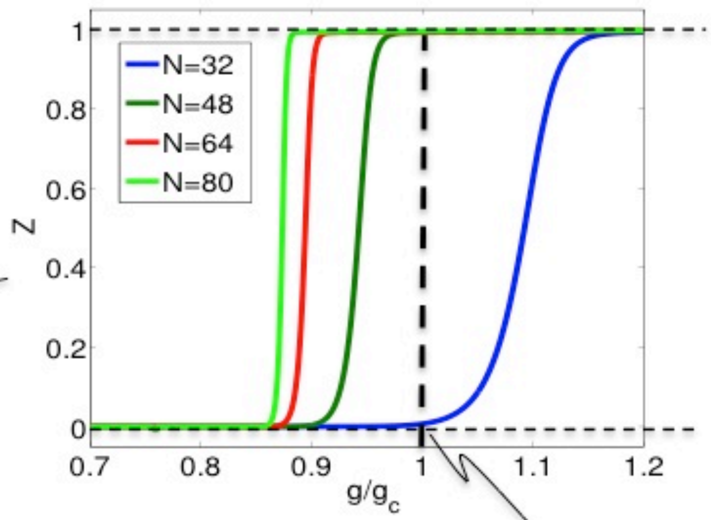
$\langle\psi_i|Z|\psi_i\rangle \neq 0$

Dynamical (phase) transition

Imbalance of an eigenstate

$$Z = \langle\psi_i|\hat{Z}|\psi_i\rangle$$

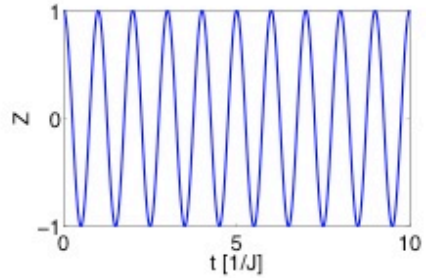
i=50



all photons localized

all photons delocalized

probe dynamics!



semiclassical approximation $\langle a_i\sigma_i^-\rangle \approx \langle a_i\rangle\langle\sigma_i^-\rangle$

→ $g_c^{\mathrm{cl}} \sim 2.8\sqrt{N}J$

sharp delocalization-localization transition !

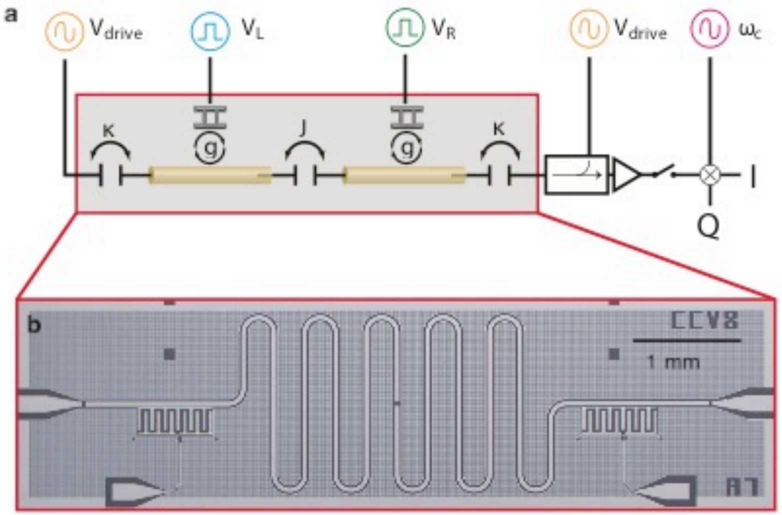
needs many photons: collective interaction effect → $N \gg 1$

critical coupling strength depends on quantum state → $g^q_{c,i} = \alpha_i\sqrt{N}J$

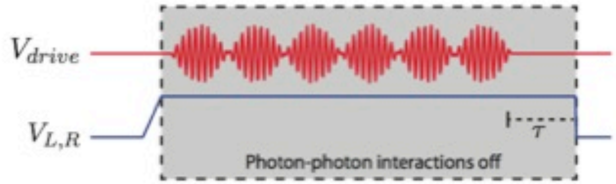
Experimental realization

Qdevice lab of A. Houck (Princeton)

J.Raftery, D. Sadri, S. Schmidt, H. Tureci, A. Houck, arxiv:1312.2963 (2013)



- Initialization: pulse + quench



$$V_{drive} = \sin(2\pi\nu_c t)\ \sin(2\pi J t)\ \Theta(-t)\ \Theta(t+\frac{m}{J})$$

- measures homdyne signal $\langle a_L \rangle$

- varies initial photon number

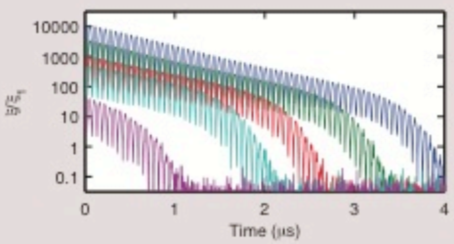
$$g_c = \alpha\sqrt{N}J \quad \longleftrightarrow \quad N_c = \left(\frac{g}{\alpha J}\right)^2$$

- weak dissipation

$$\nu_c = 6.34\ \text{GHz}$$
$$J = 8.7\ \text{MHz}$$
$$g = 190\ \text{MHz}$$
$$\kappa = 225\ \text{KHz}$$
$$g \gg \kappa$$

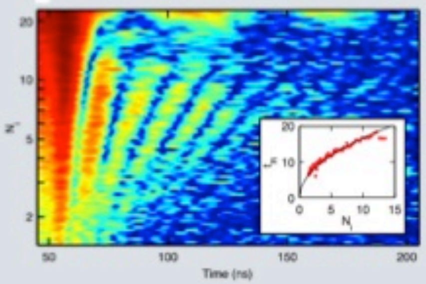
Phase diagram

Homodyne signal of the left cavity (initially localized in the left cavity)



Classical oscillations

$$t_J = \frac{\pi}{J}$$

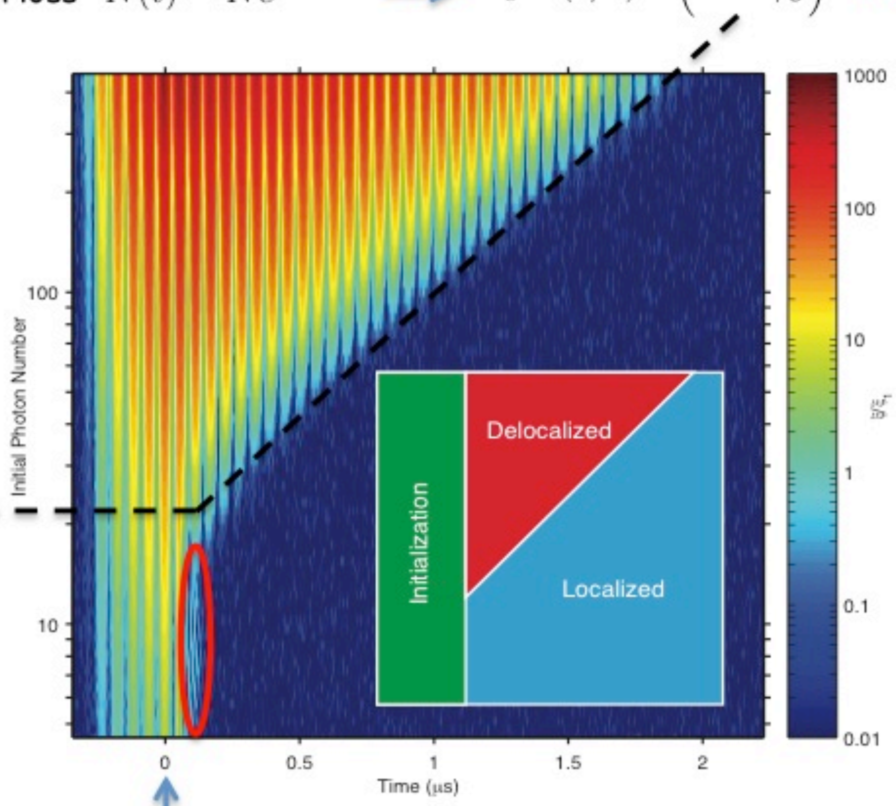


Quantum Revivals

$$t_{\rm rev} = \frac{2\pi\sqrt{N}}{g}$$

✓ Dissipation driven classical-to-quantum transition

Photon loss $N(t) = Ne^{-\kappa t}$ → $t_c \sim (1/\kappa)\ln\left(J\sqrt{N}/g\right)$ dissipation driven



quench

Crossover $t_{\rm J} \sim t_{\rm rev}$ ↔ $g \sim \sqrt{N}J$

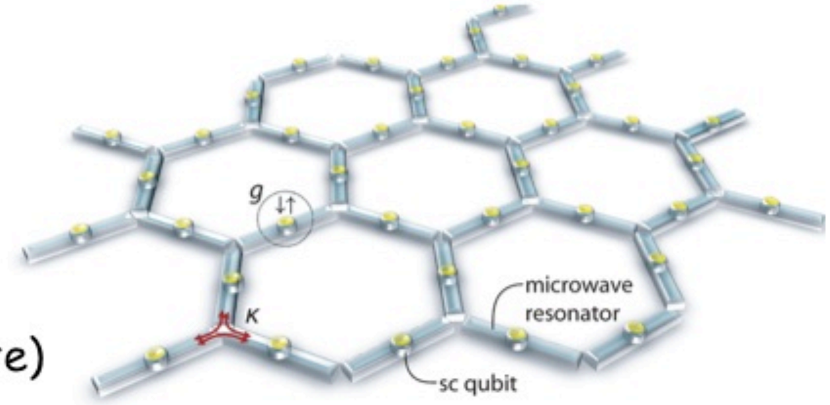
(consistent with semiclassical prediction)

Non-equilibrium SF-MI transition

Jaynes-Cummings-Hubbard model

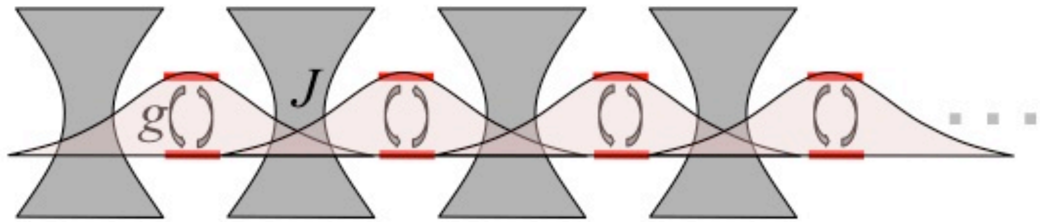
Equilibrium SF-MI transition (ground state)

Non-Equilibrium SF-MI transition (steady state)



Jaynes-Cummings-Hubbard model

Coupled cavity array (CCa)



Photons in a cavity interacting with a two-level system (g) and tunneling to neighbouring cavities (J)

$$H_{JCHM} = \sum_j H_{\mathrm{JC},j} + \sum_{\langle i,j\rangle} (J a_i^\dagger a_j + \mathrm{h.c.})$$

$$H_{\mathrm{JC}} = \omega_r a^\dagger a + \omega_q \sigma^+ \sigma^- + g(a^\dagger \sigma^- + a\sigma^+)$$

Greentree et.al. Nat. Phys. **466**, 856 (2006)
(for NV centers in diamond)

Schmidt and Blatter, PRL (2009)

Quantum phase diagram

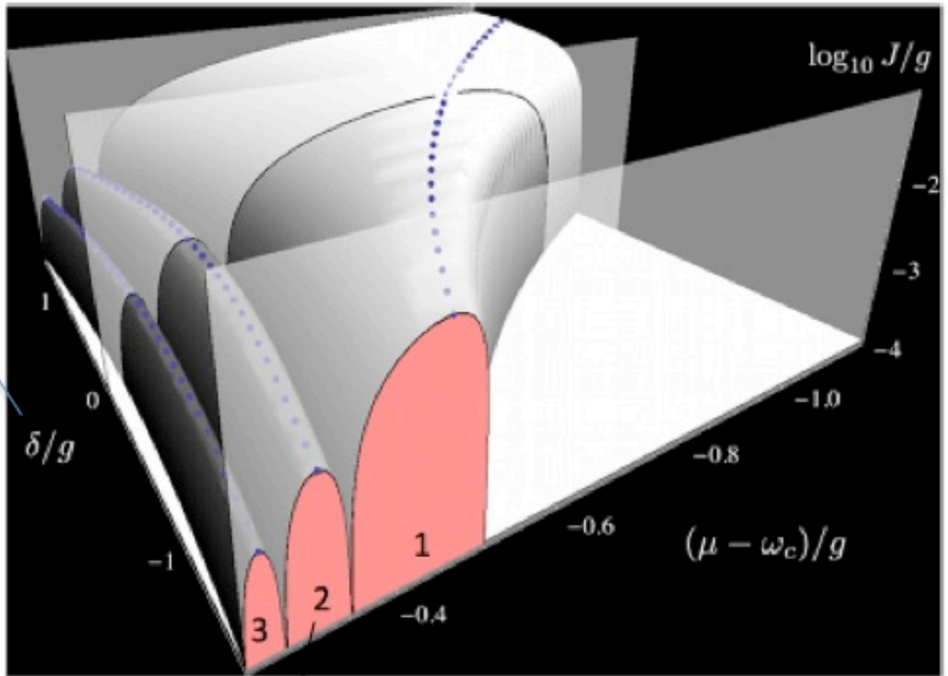
neglect drive and dissipation:

Introduce chemical potential

$$H = H_{\rm JCHM} + H_{\rm chem} \qquad H_{\rm chem} = \sum_j \mu \left(a_j^\dagger a_j + \sigma_j^+ \sigma_j^- \right)$$

our methods: Decoupling MFT, Linked-cluster, Slave-boson, Quantum Monte Carlo, ...

cavity-qubit detuning



Greentree et.al. Nat. Phys. (2006)
Angelakis et.al PRA (2007)
Aichhorn et.al PRL (2008)
Schmidt and Blatter, PRL (2009)
Koch and LeHur PRA (2009)
Schmidt and Blatter PRL (2010)
Hohenadler et al. PRA (2011)
.....

Photonic Mott insulator $\langle a_i \rangle = 0$

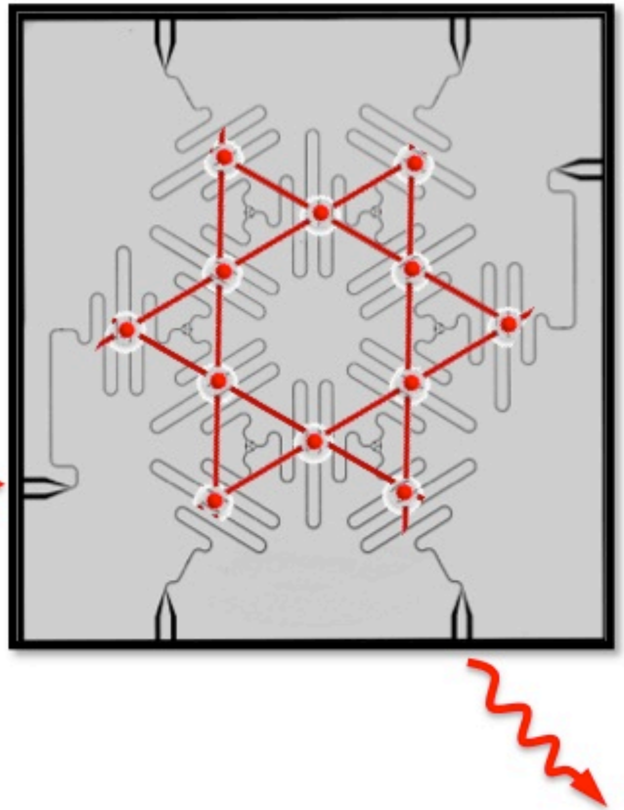
<u>Equilibrium Theory:</u>

- ✓phase diagram
- ✓elementary excitations
- ✓critical exponents
- ✓long-range hopping

main differences with Bose-Hubbard model:

- Sqrt{n} dependence of the particle-hole gap
- Mott lobes with flavor (detuning parameter)
- additional gapped conversion modes

New Challenges



- External Drive is needed
- Desired losses for detection
- Intrinsic (undesired) losses

→

Photon number is not conserved!

No grand-canonical ensemble

No chemical potential

→ Open (non-equilibrium) system

What is the fate of the SF-MI transition in non-equilibrium steady state?

JCHM with drive and dissipation

$$H = H_{\mathrm{JCHM}} + H_{\mathrm{drive}}$$

coherent pump with equal phases and drive strength

$$H_{\mathrm{drive}} = \sum_j f\left(a_j + a_j^\dagger\right)$$

<u>Master equation:</u>

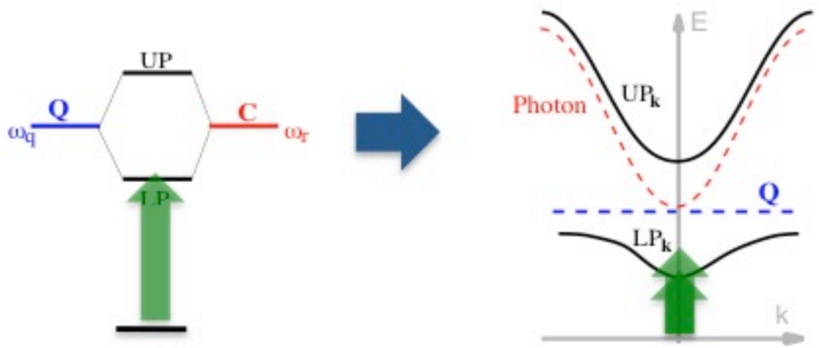
$$\partial_t \rho = -\, i[H_{\mathrm{JCHM}} + H_{\mathrm{drive}}, \rho] + \sum_i \left(\kappa \mathcal{D}[a_i]\rho + \gamma \mathcal{D}[\sigma_i^-]\rho + \gamma_\phi \mathcal{D}[\sigma_i^z]\rho\right)$$

Hopping-induced Bistability

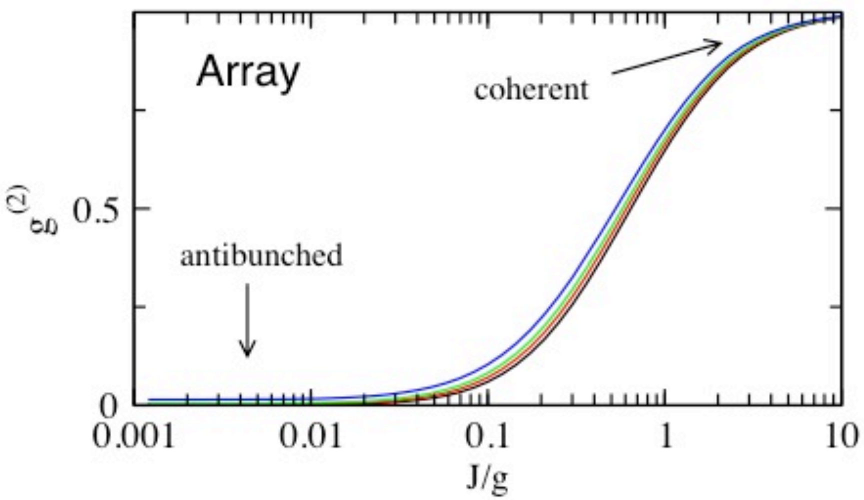
F. Nissen *et al.*, PRL 2012

Le Boite et al., PRL 2013

(a) Single cavity (b) Large array

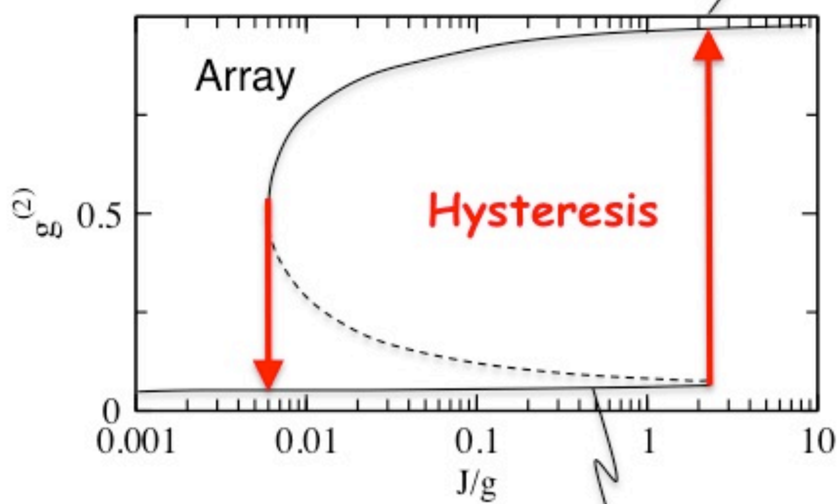


Case 1: Pump at the bottom of the band



Case 2: Pump into the band

delocalized branch



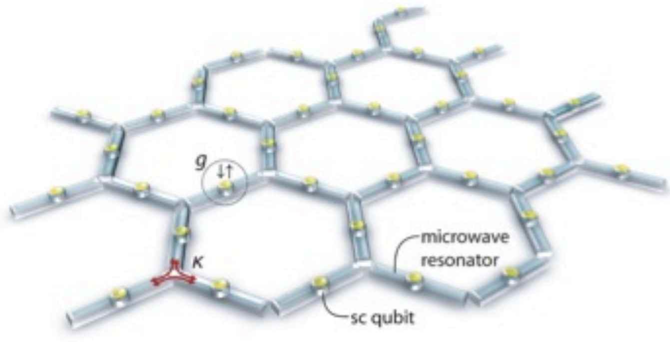
localized branch

Non-equilibrium SF-MI phase transition

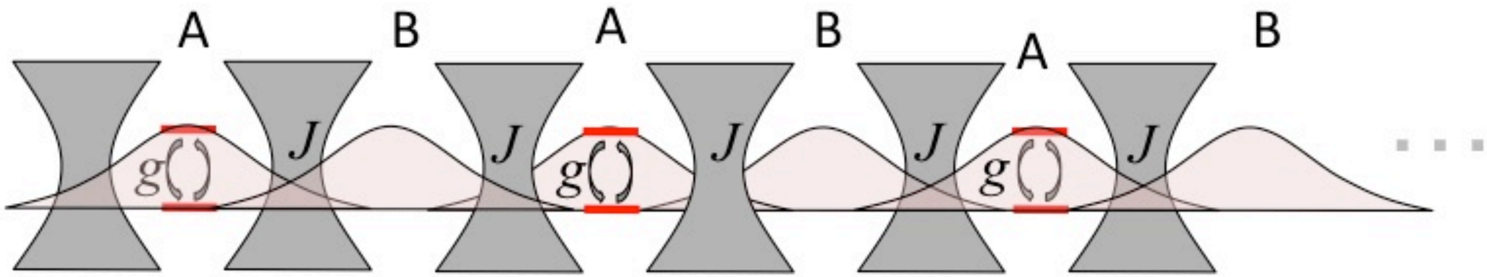
Flat bands

Lieb chain

Crystalline order of photons



Photonic lattices



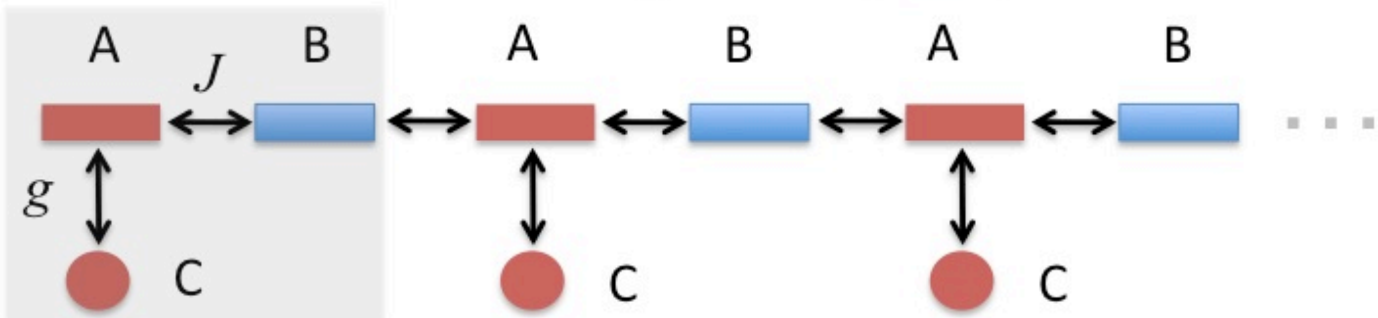
alternating quasi-1D JC lattice

$$H = \sum_j (h_{jA}^{\mathrm{JC}} + \omega_B\, b_j^\dagger b_j) + J \sum_j [b_j^\dagger (a_j + a_{j+1}) + \mathrm{H.c.}],$$

$$h_{jA}^{\mathrm{JC}} = \omega_A\, a_j^\dagger a_j + \omega_q \sigma_j^+ \sigma_j^- + g(a_j^\dagger \sigma_j^- + \mathrm{H.c.}).$$

on the single-particle level:

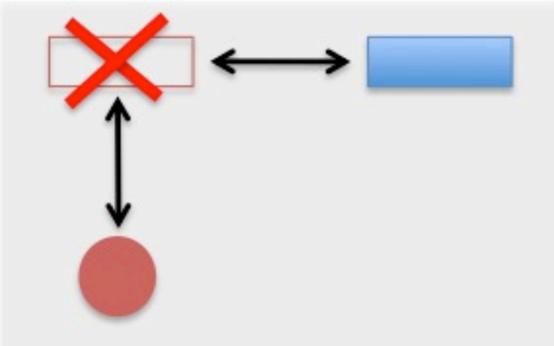
$$\sigma_j^+ \to c_j^\dagger$$



quasi-1D Lieb chain

Unit cell

Lieb chain unit cell



$$h = \begin{pmatrix} \omega_a & J & g \\ J & \omega_b & 0 \\ g & 0 & \omega_q \end{pmatrix}$$ (single excitation subspace)

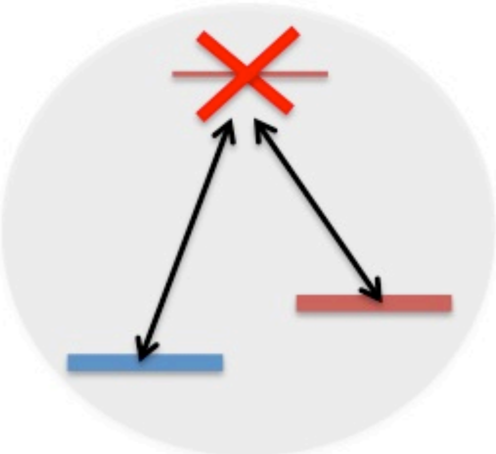
for $\omega_q = \omega_b$ → Quantum Interference

$\lambda = \omega_b$

$$|D\rangle = \frac{1}{\sqrt{g^2 + J^2}} \left(g b^\dagger - J \sigma^+ \right) |0\rangle$$

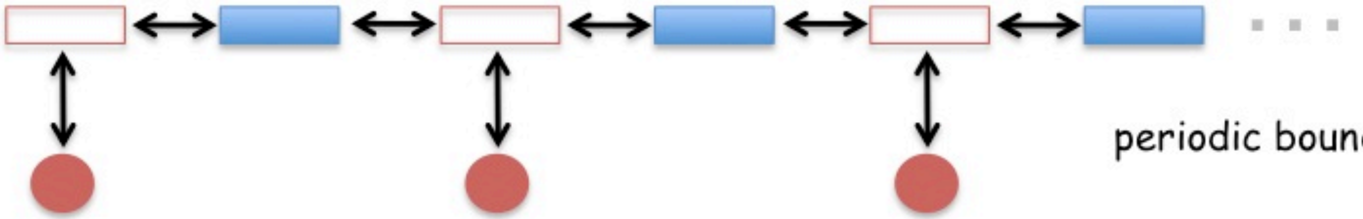
dark state polariton

3LS Lambda configuration



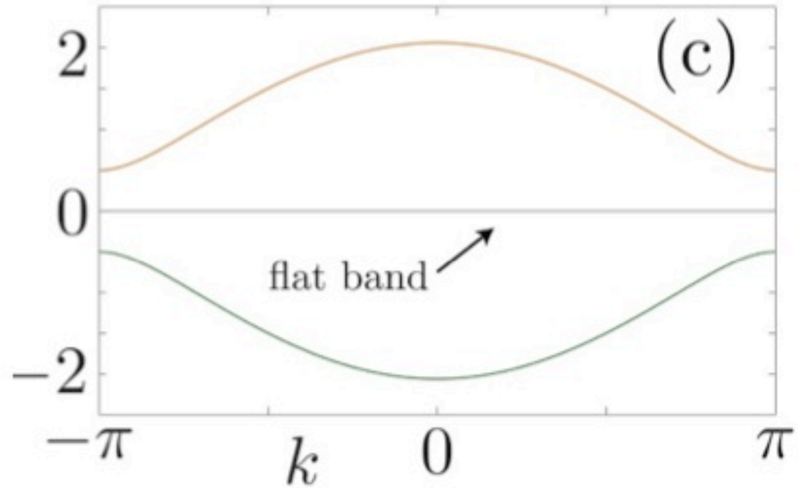
→ analogous to electromagnetically-induced transparancy (EIT effect)

Flat band



periodic boundary conditions

$$h_k = \begin{pmatrix} \omega_a & J(1+e^{-ik}) & g \\ J(1+e^{ik}) & \omega_b & 0 \\ g & 0 & \omega_q \end{pmatrix}$$



for $\omega_q = \omega_b$ ➡ Quantum Interference

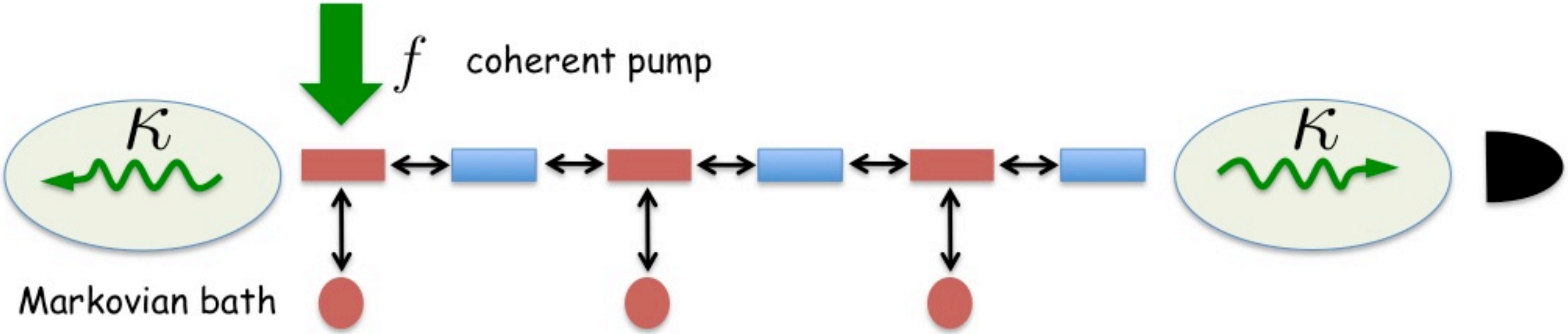
$$\lambda_F = \omega_b$$

Flat band energy

$$|\lambda_j\rangle = \frac{1}{\sqrt{g^2 + 2J^2}} \left[g b_j^\dagger - J(\sigma_j^+ + \sigma_{j+1}^+) \right] |0\rangle$$

lambda states = set of degenerate single particle states

Spectroscopy



$$\dot{\rho} = 0 = -i[\rho, H + H_{\mathrm{dr}}] + \kappa \mathcal{D}[a_1]\rho + \kappa \mathcal{D}[b_N]\rho$$

$$H_{\mathrm{dr}} = f\left(a_1 e^{i\omega_d t} + \mathrm{h.c.}\right)$$

Input/Output theory $\quad T = (\kappa/f)^2 |\langle b_N \rangle_{\mathrm{ss}}|^2$ $\quad$ Collet and Gardiner (1991)

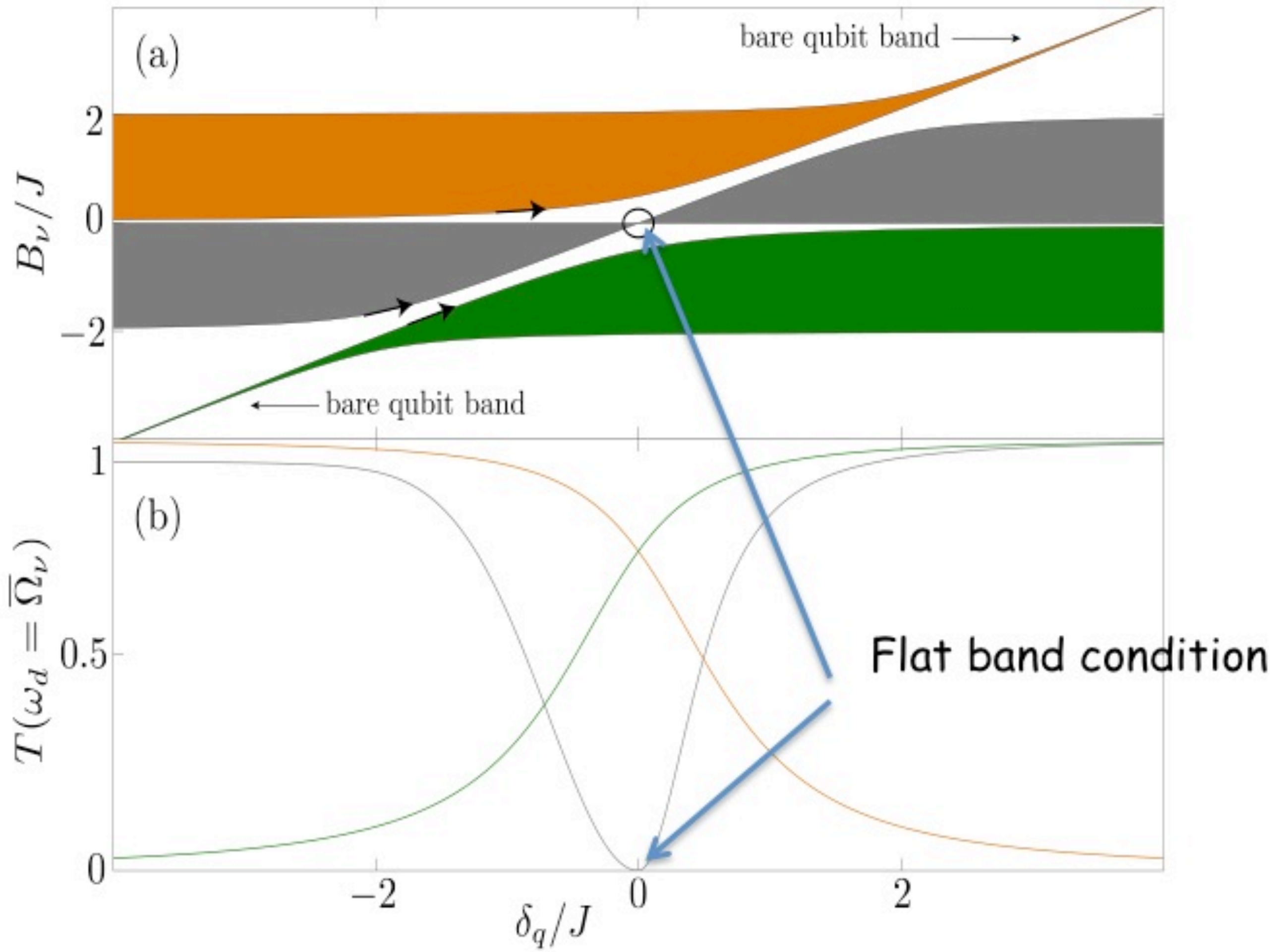
weak pump condition $\quad f \ll \kappa, g, J$

solve analytically in single excitation basis!

weak pump spectroscopy

$f \ll \kappa, g, J$

Open b.c.

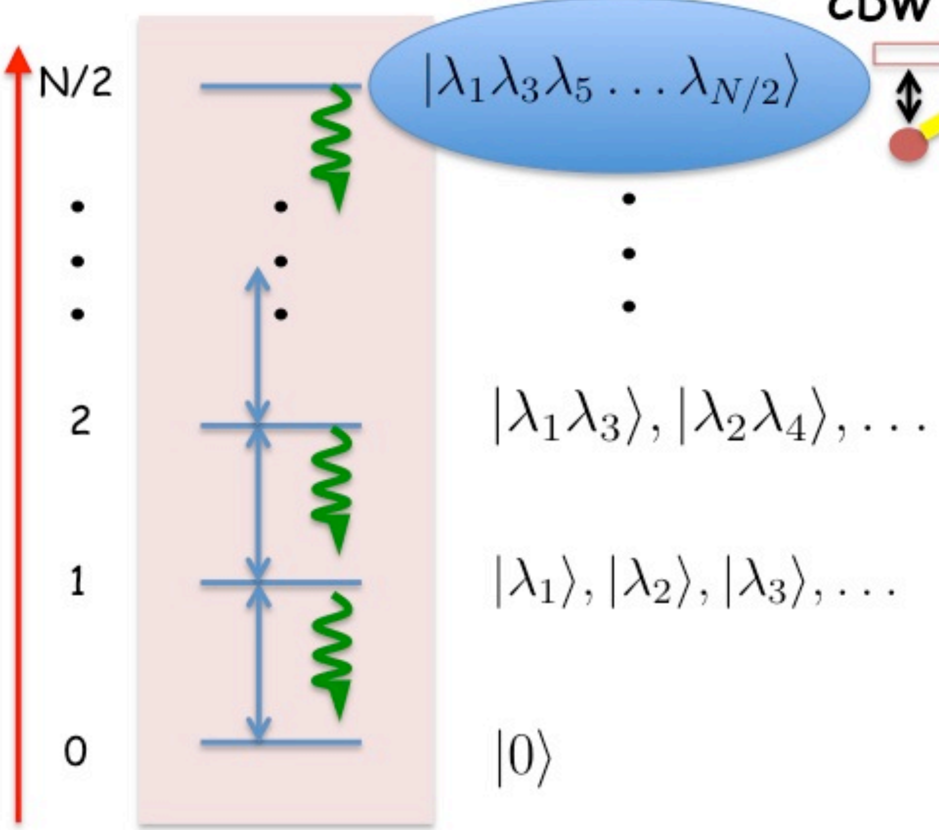


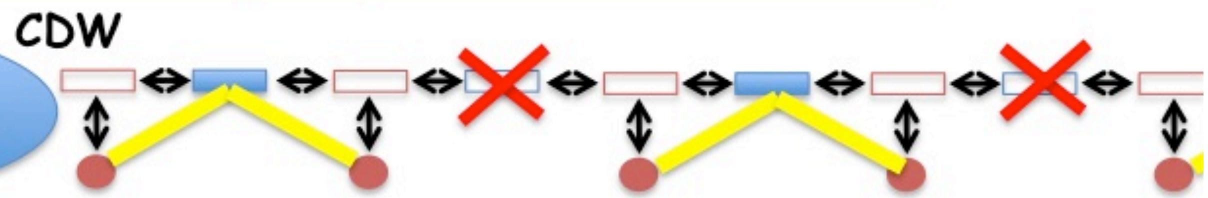
In-situ tunable parameter $\delta_q = \omega_q - \omega_b$

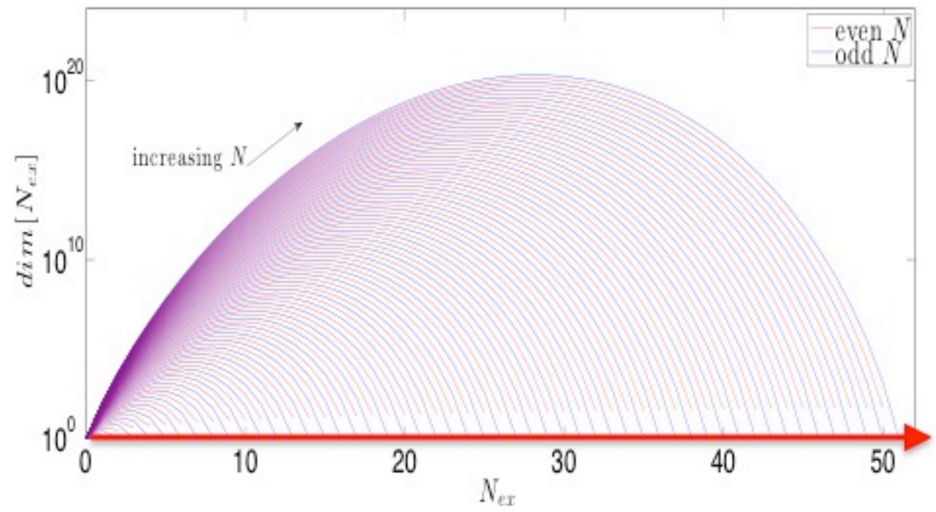
Flat band basis

What about interactions ?

Lambda states build exact many-body basis for the flat band!





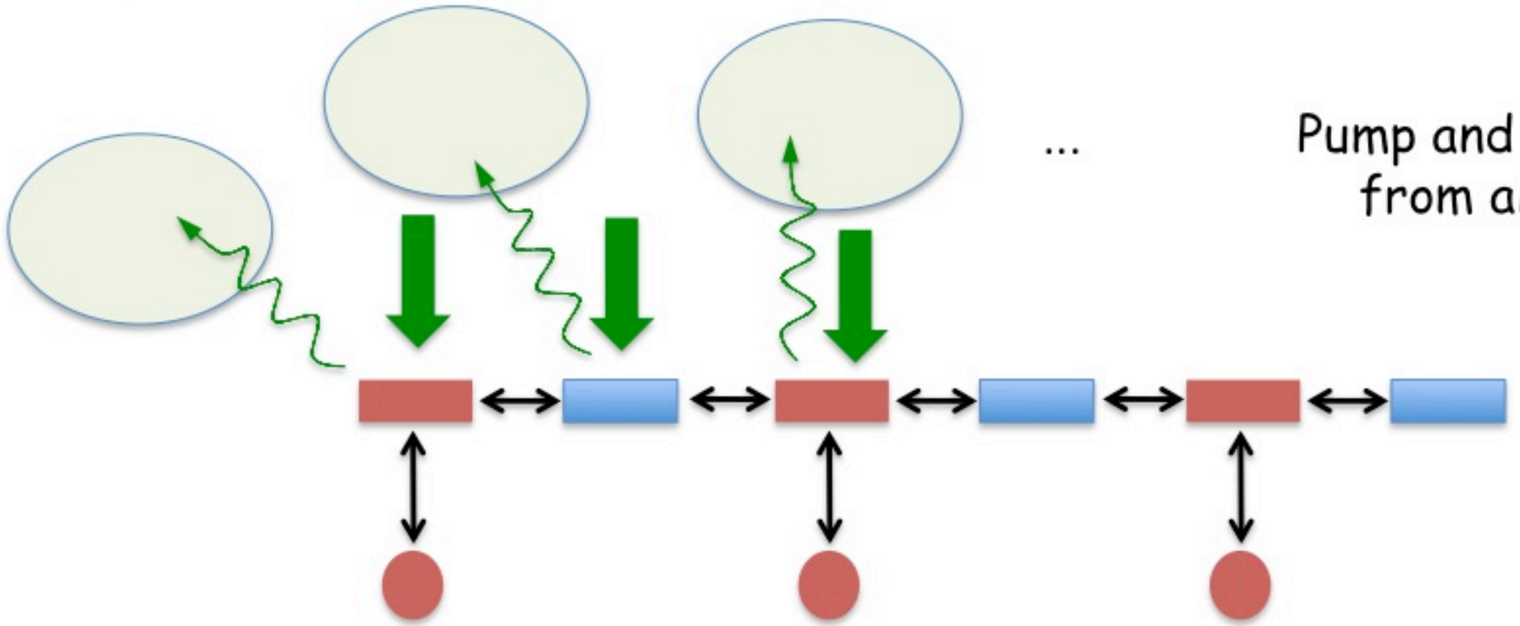


Flat band = driven, dissipative multi-level system

Size of the Liouvillian still increases exponentially

exact diagonalization up to 25 sites

Nonequilibrium steady state



Pump and dissipation from all cavities

$$\dot{\rho} = 0 = -i[\rho, H + H_{\mathrm{dr}}] + \kappa \sum_i \mathcal{D}[a_i]\rho + \kappa \sum_i \mathcal{D}[b_i]\rho$$

$$H_{\mathrm{dr}} = f \sum_i \left[(a_i + b_i) e^{i\omega_d t} + \mathrm{h.c.} \right]$$

steady state cavity occupation $\psi_i = \langle a_i \rangle_{\mathrm{ss}}, \langle b_i \rangle_{\mathrm{ss}}$

stronger pump $f \sim \kappa \ll g, J$

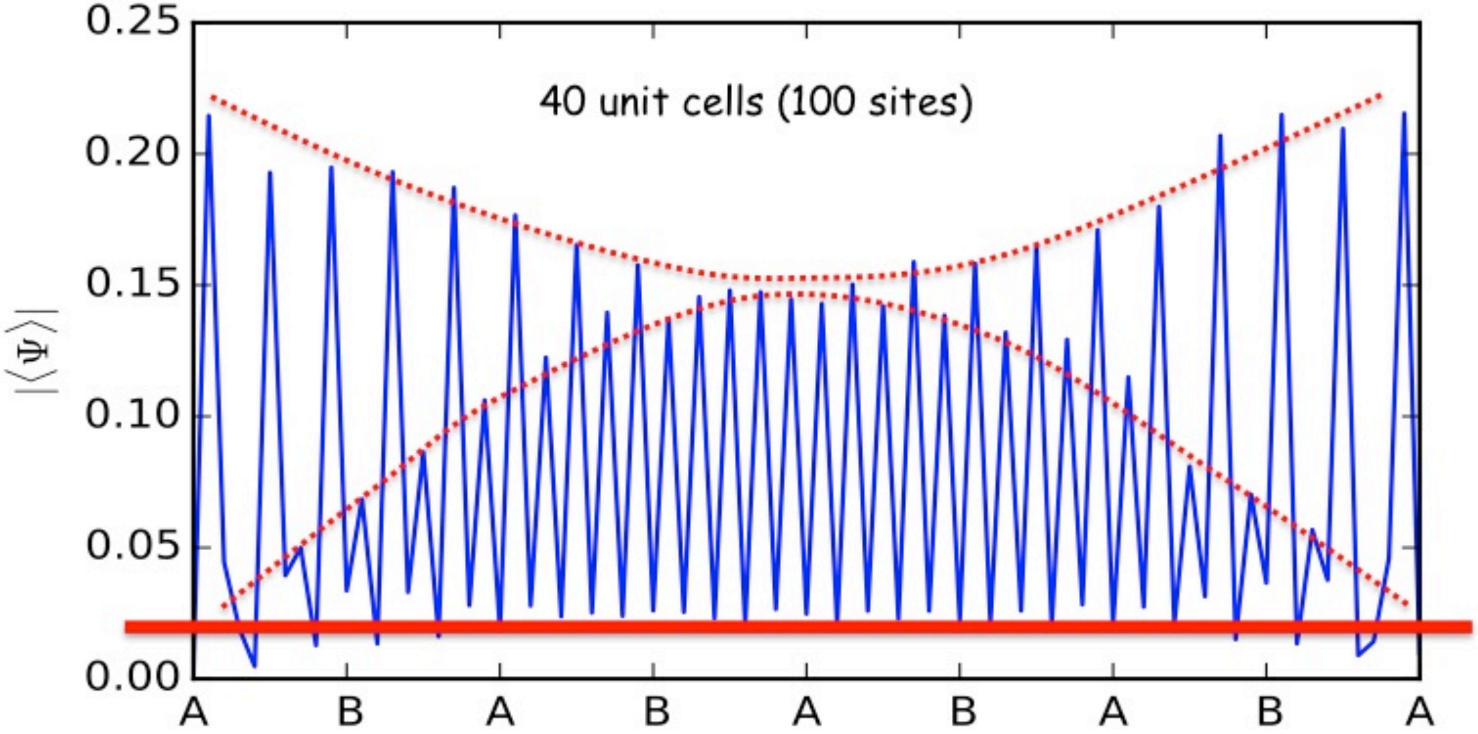
exact diagonalization of the Liouvillian in flat band subspace!

Open System TEBD/tDMRG

$$\dot{\rho} = \mathcal{L}\rho \quad \Rightarrow \quad \rho(t) = e^{\mathcal{L}t}\rho(0)$$

vectorized matrix product state (MPS)

Trotter decomposition + TEBD/DMRG



Thanks

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