Pairing Symmetry of Superfluid in Three-Component Repulsive Fermionic Atoms in Optical Lattices

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Introduction

Cold atoms in optical lattices: high controallability

→ Quantum simulator

Repulsively interacting three-component fermionic atoms in optical lattices

Three-component fermionic gases realized in $^6$Li atoms.

Ottenstein et al, PRL 101, 203202 (2008); Huckans et al, PRL 102, 165302 (2009)

Three kinds of internal degrees of freedom:

Three kinds of repulsions:
Model

\[ H = -t \sum_{\langle i, j \rangle} \sum_{\alpha=1,2,3} c_{i\alpha}^+ c_{j\alpha} + \frac{1}{2} \sum_{i=1} \sum_{\alpha \neq \beta} U_{\alpha\beta} n_{i\alpha} n_{i\beta} - \sum_{i,\alpha} \mu_{\alpha} n_{i\alpha}, \quad U_{\alpha\beta} > 0 \]

According to the experiments for $^6$Li atoms

balanced population: $n_1 = n_2 = n_3 = n$,

Half filling: $n = 1/2$, total atom number $N = 3/2$

$U'', U', U > 0$
Properties at half Filling: N=3/2

Inaba, Miyatake, and Suga, PRA 80, 051602(R) (2010)
Miyatake, Inaba, and Suga, PRA 81, 051602(R) (2010)

DMFT at T=0

\( U'' \gg U \) For \( U'' \gg U' \), paired Mott insulator (PMI)

For \( U'' \gg U' \)

\( U'' \ll U \) For \( U'' \ll U \)

color-selective Mott state (CSM)

\( \cdot \) Two Mott states in ‘color paramagnetic’ sector

\( \cdot \) the ground state

color-density wave (CDW)

color selective AF (CSAF)
Pair fluctuations are enhanced

- in the FL close to PMI transition point,
- close to HF.

Superfluid is expected to appear there.
Inaba and Suga, PRL 108, 255301 (2012)

**Half filling in ‘color paramagnetic’ sector**

**Self-energy functional approach**

\[ n = \frac{1}{2}, \quad \frac{U}{U'} = 0.1 \]

![Graph showing the phase diagram with regions labeled FL (Fermi liquid), SF (superfluid), and PMI (paired Mott insulator).]

SF at HF has been obtained with DMFT + CTQMC.

Okanami, Takemori, Koga, arXiv: 1401.5610

Prof. Koga’s talk at 10:30 in June 23 (Mon.)
**Close to HF: DMFT + modified iterated-perturbation theory**

Inaba and Suga, PRL 108, 255301 (2012)

Superfluid appears for

- $U/U' < 0.11$: large difference in repulsions,
- $0.42 < n \leq 0.5$: close to HF (HF: $n=0.5$).

Effective attractive interaction is caused by density fluctuations of unpaired color-3 atoms.
Aim

Pairing symmetry of superfluid state in repulsively interacting three-component fermionic atoms in optical lattices.

Model

\[
H = -t \sum_{\langle i,j \rangle} \sum_{\alpha=1,2,3} c_{i\alpha}^+ c_{j\alpha} + \frac{1}{2} \sum_i \sum_{\alpha \neq \beta} U_{\alpha\beta} n_{i\alpha} n_{i\beta} - \sum_{i,\alpha} \mu_{\alpha} n_{i\alpha}, \quad U_{\alpha\beta} > 0
\]

balanced population: \( n_1 = n_2 = n_3 = n \), HF: \( n = 1/2 \)

\( U' \gg U > 0 \)

Cooper pairs: color-1 and 2 atoms
Effective interaction $\tilde{U}_{12}(\mathbf{q})$ between color-1 & 2 atoms

RPA diagrams and ladder diagrams

$$\tilde{U}_{12}(\mathbf{q}) = U + \frac{3}{2} U^2 \chi_s(\mathbf{q}) - \frac{1}{2} U^2 \chi_c(\mathbf{q})$$

$$\chi_s(\mathbf{q}) = \frac{\chi_{11}^{(0)}(\mathbf{q})}{1 - U \chi_{11}^{(0)}(\mathbf{q})}$$

$$\chi_c(\mathbf{q}) = \frac{R^2 \chi_{11}^{(0)}(\mathbf{q}) + 2 \chi_{33}^{(0)}(\mathbf{q}) - 2 U \chi_{11}^{(0)}(\mathbf{q}) \chi_{33}^{(0)}(\mathbf{q})}{1 + U \chi_{11}^{(0)}(\mathbf{q}) - 2 U' \chi_{11}^{(0)}(\mathbf{q}) \chi_{33}^{(0)}(\mathbf{q})}$$

$\chi_s(\mathbf{q})$ and $\chi_c(\mathbf{q})$ have a possibility of divergence. → competition
\[ \lambda \Delta(k) = -\frac{1}{N} \sum_{k'} \bar{U}_{12}(k - k') \frac{\tanh(\beta \xi_{k'}/2)}{2 \xi_{k'}} \Delta(k') \]

\( \Delta(k) \) : SF order parameter
\( \lambda \) : eigenvalue    SF transition occurs at \( \lambda=1 \).

- Numerical diagonalization
- Iterative approximation
Results
$\Delta(k)$ in square optical lattices

$U/U'=0.1, \; U'/t=0.8, \; T/t=0.01$ at $n=0.49$

Extended s-wave pairing
	nodeless

Extended s-wave SF, although SF is adjacent to PMI at HF.
\( \tilde{U}_{12}(q) \) in square optical lattices

\[
\tilde{U}_{12}(q) = U + \frac{3}{2} U^2 \chi_s(q) - \frac{1}{2} U^2 \chi_c(q)
\]

\[ q = k - k' \]

\( \tilde{U}_{12}(q) < 0 \)

Peak in \( \chi_c \) at \( q = (\pi, \pi) \): strong CDW fluctuations close to HF

Large attractive peak in \( \tilde{U}_{12}(q) \)

\( U' \gg U \): CDW ground state at HF

strong CDW fluctuations
\( \Delta(r) \): Fourier component of \( \Delta(k) \)

- Local component
  \[ \Delta_0 \approx 3.00 \times 10^{-2}, \ r = (0,0) \]

- Nonlocal components
  \[ \Delta_1 \approx -3.86 \times 10^{-3}, \ r = (\pm 1, \pm 1), (\pm 1, \mp 1) \]
  \[ \Delta_2 \approx 1.40 \times 10^{-3}, \ r = (\pm 2, \pm 2), (\pm 2, \mp 2) \]

Local component \( \Delta_0 \) is dominant, although the strong attractive peak caused by CDW fluctuations appears in \( \tilde{U}_{12}(q) \).

Local correlation effects play an important role in this extended s-wave SF.
$\Delta(k)$ and $\tilde{U}_{12}(q)$ in triangular optical lattices

$U/U' = 0.1$, $U'/t = 0.8$, $T/t = 0.01$ at $n = 0.49$

Extended $s$-wave pairing

No large attractive peak in $\tilde{U}_{12}(q)$

CDW fluctuations are suppressed due to geometrical frustration.
\( \Delta(r) \): Fourier component of \( \Delta(k) \)

- Local component
  \( \Delta_0 \approx 3.12 \times 10^{-2}, \ r = (0,0) \)

- Nonlocal components
  \( \Delta_1 \approx -5.06 \times 10^{-4}, \ r = (\pm 2, \pm 1), (\pm 1, \mp 1), (\pm 1, \pm 2) \)
  \( \Delta_2 \approx -1.85 \times 10^{-4}, \ r = (\pm 1, \pm 1), (\pm 1, 0), (0, \pm 1) \)

Local component \( \Delta_0 \) is more dominant in triangular OL.

Local correlation effects in triangular OL are more dominant. 

\[ \text{DMFT picture} \] can be adequate in triangular OL.
Ext. s-wave pairing symmetry for $U/U' = 0.1$

In three-component $^6$Li fermionic gases, $U/U'$ can be (somewhat) controllable.

Next issue

What happens for SF pairing symmetry, when we change $U/U'$?
\( \Delta(k) \) in square optical lattices 2

\[ \frac{U}{U'} = 0.2, \frac{U'}{t} = 0.8, \frac{T}{t} = 0.01 \text{ at } n = 0.45 \]

**Nodal s-wave pairing**

\( \tilde{U}_{12}(q) \)

- Attractive peak around \( q = (\pi, \pi) \)
- Repulsive for other \( q \)

CDW fluctuations are reduced.
$U'/t=1.2, \ T/t=0.01$ at $n=0.45$

**U/U'=0.4**  \ **$d_{xy}$ pairing**

**U/U'=0.8**  \ **$d_{x^2-y^2}$ pairing**

**Dominant color AF fluctuations**
Largest $\lambda$ as a function of $U/U'$ and $T/t$

Extended $s$-wave pairing for $U/U' < 0.1$

$d_{x^2-y^2}$-wave pairing for $U/U' > 1.0$

Nodal $s$-wave
$U/U' \approx 0.2$

d$_{xy}$-wave
$U/U' \approx 0.3-0.6$

can be observed in experiments.
Relation to experiments

Three-component fermionic atoms in optical lattices

$^6$Li atoms in optical lattices, $^{173}$Yb-$^{171}$Yb mixture in optical lattices

Change in pairing symmetry can be probed with momentum-resolved one-particle excitation spectrums.

Stewart, Gaeber, and Jin, Nature 545, 744 (2008)

<table>
<thead>
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<th>$k = (0,0)$</th>
<th>$k = (\pi, 0)$</th>
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<td>$d_{x^2-y^2}$ wave</td>
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Summary

Pairing symmetry of superfluid state in repulsively interacting three-component fermionic atoms in optical lattices.

Extended s-wave SF appears for $U/U' < 0.2$ close to HF in square and triangular optical lattices.

Local correlation effects play an important role.

Pairing symmetry changes with increasing $U/U'$:

- ext. s-wave $\Rightarrow$ nodal s-wave $\Rightarrow d_{xy}$-wave $\Rightarrow d_{x^2-y^2}$-wave

This phenomena can be probed in experiments.

Conclusion

Three-component repulsive fermionic atoms in optical lattices can be a quantum simulator for controlling SF pairing symmetry.