

# Van der Waals

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Future: Chemistry Department, Basel University

# Overview

- Tutorial (first half)
  - Why do we care?
  - What is it?
  - What's the problem with vdW?
  
- Recent contributions (second half)
  - Many-body dispersion in DFT
  - 3-body dispersion effects
  - Many-body dispersion effects



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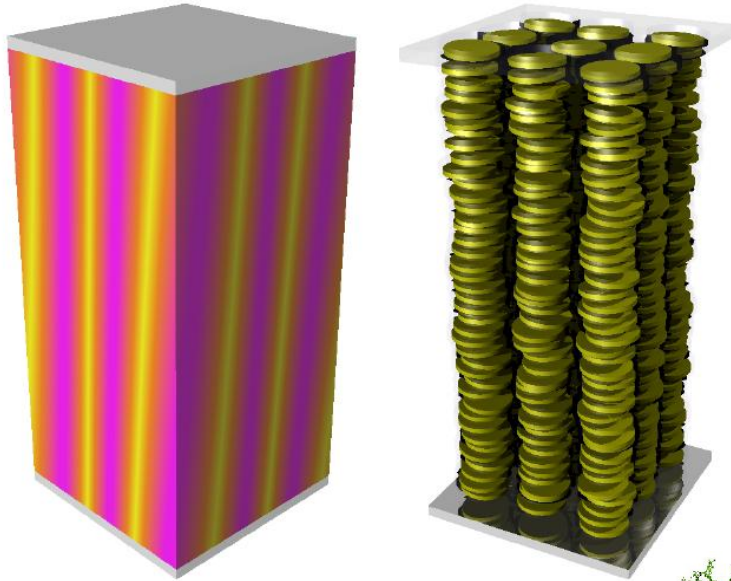
# Why do we care?

- "There is plenty of room at the bottom" (Feynman)
- Intermolecular bonds are weaker than chemical bond, yet crucial for many highly relevant processes
  - Self-assembly
  - Supra-molecular
  - Adhesion, adsorption, physisorption
  - Liquid phase (Solid covalent, Gas ideal)
  - Nano- and meso-scale objects
  - Soft matter
  - Biological systems



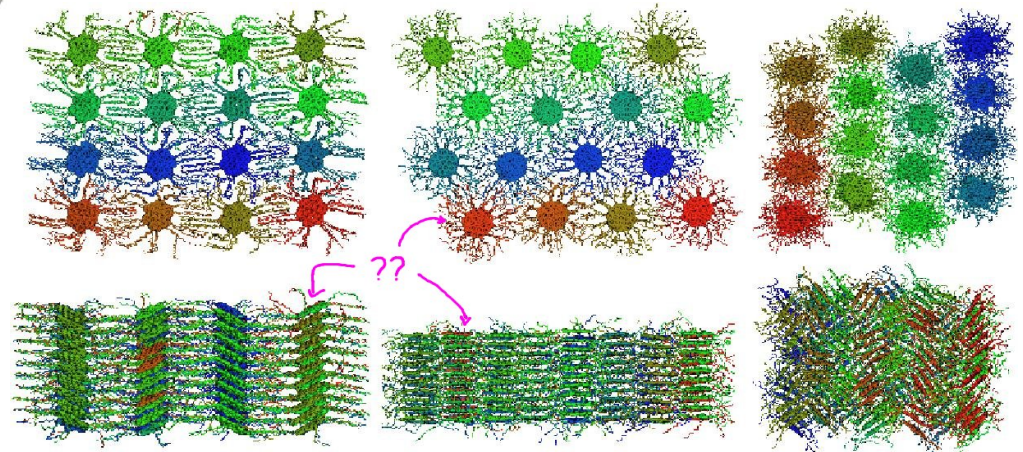


# Why do we care?



$$k = \frac{J^2}{\hbar} \sqrt{\frac{\pi}{\lambda k_B T}} \exp \left[ -\frac{\lambda}{4k_B T} \right]$$

Papers by D. Andrienko et al,  
OAvL & Andrienko, JCP (2007)



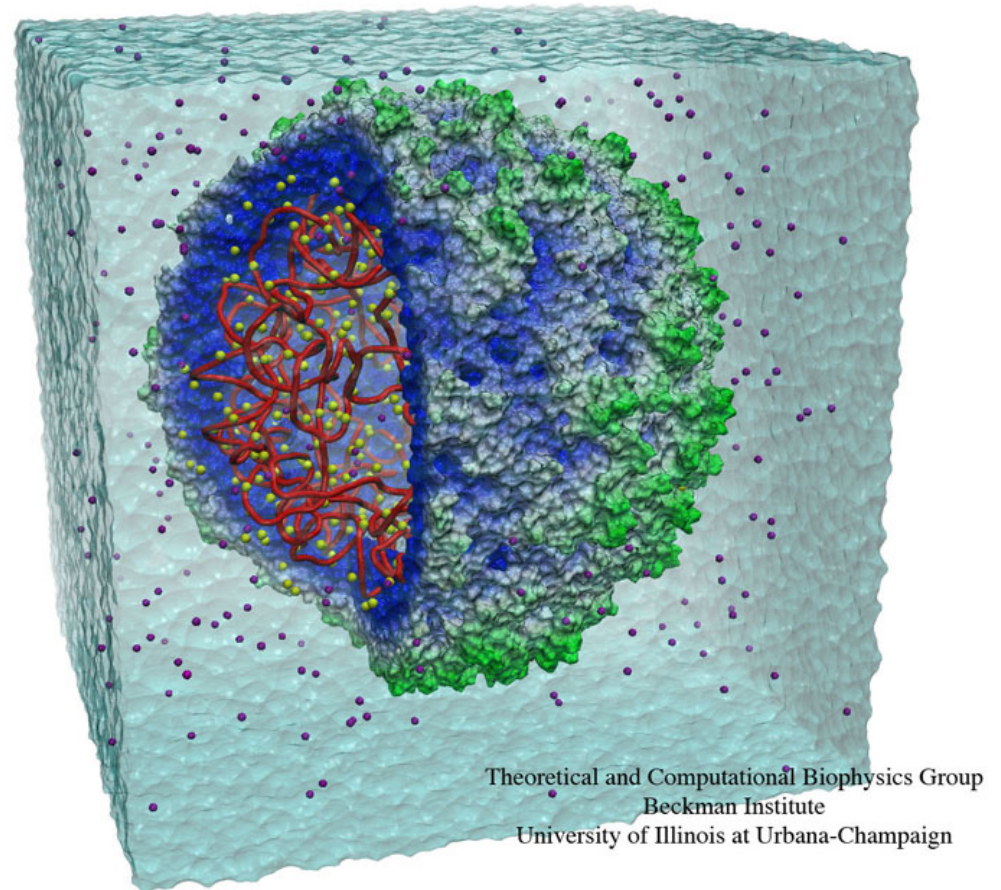
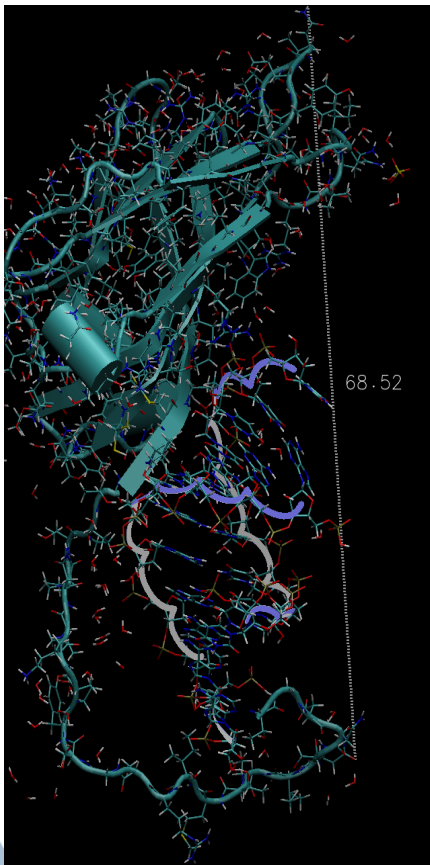
Herringbone (300 K) and hexagonal (400 K) configurations of  $C_{12}$ ; columnar disordered (400 K) of  $C_{10-6}$



# Why do we care?

Molecular dynamics of satellite tobacco mosaic virus (STMV), complete with protein, RNA, ions, and water

K. Schulten and coworkers (2006-)



Icosahedral structure with 3 units/face

→ 3k atom x 3 x 20 ~ 180k atom ~ 1m atom in solution  
for tens of nanoseconds

→ Coarse-graining to 5k particle for microseconds

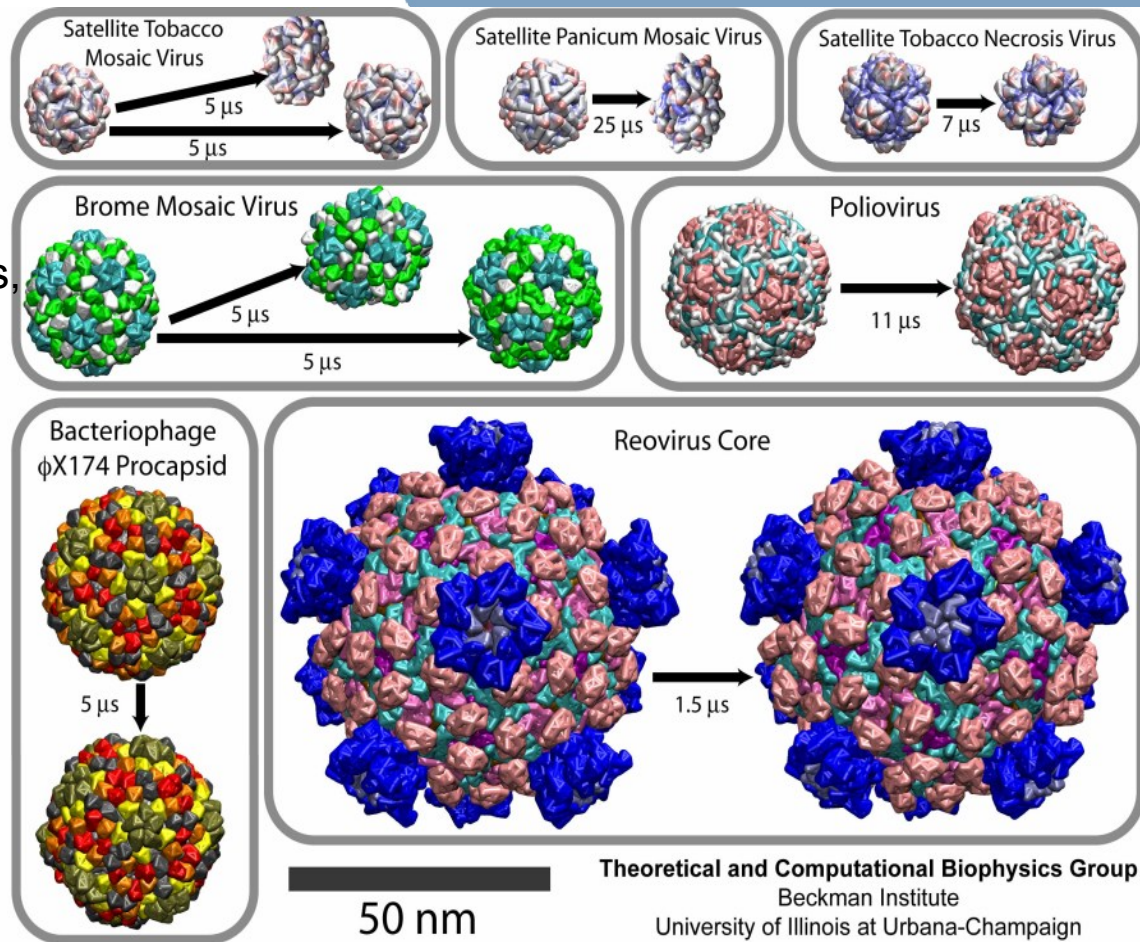
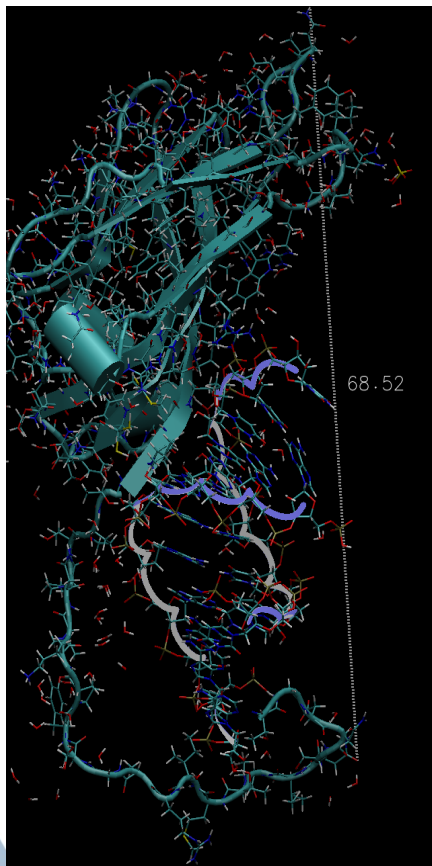
Parallelization across atoms →  $N_{\text{atoms}} < N_{\text{cores}}$



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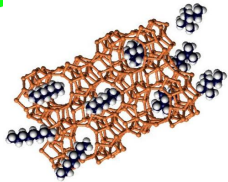
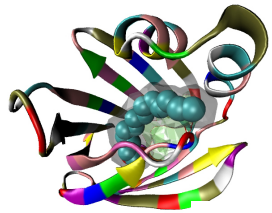
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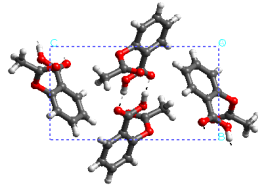
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# Why do we care?

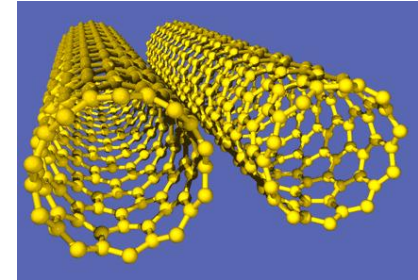


*Finite-gap  
molecules  
and solids*



*Multipole effects;  
Short-range interaction*

*Low-dimensional systems*

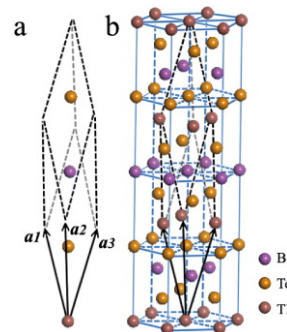


*Hybrid organic/inorganic  
interfaces*

*Metallic systems*



*Ionic  
systems*



# Why do we care?

## Approaches for Optimizing the First Electronic Hyperpolarizability of Conjugated Organic Molecules

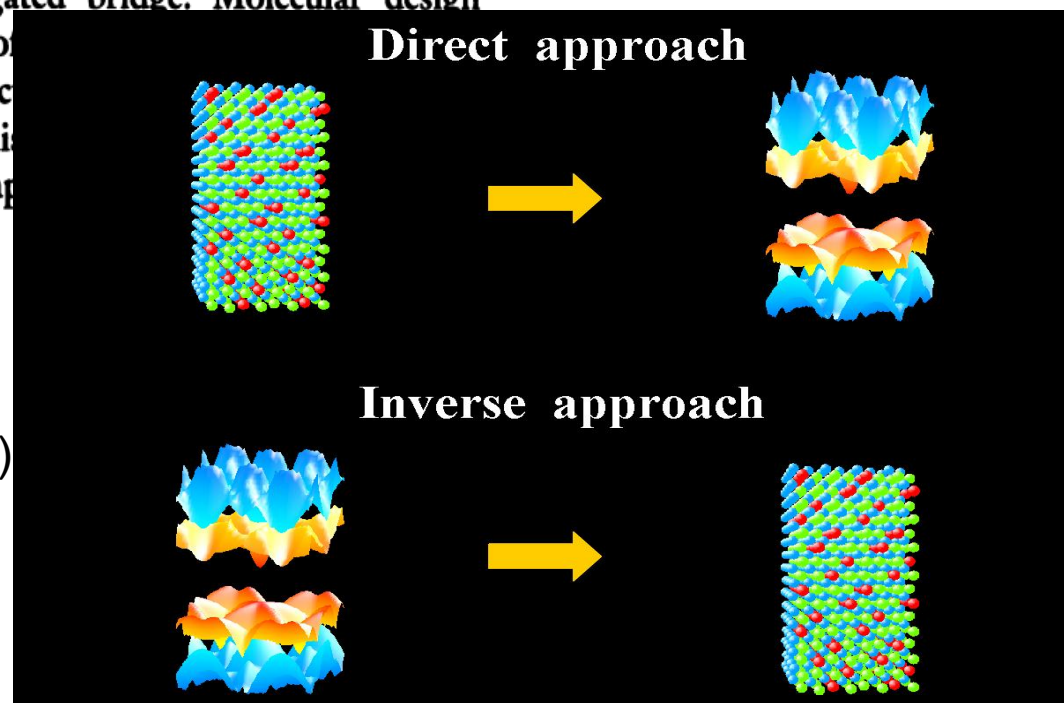
Science (1991)

S. R. MARDER, D. N. BERATAN, L.-T. CHENG

A two-state, four-orbital, independent electron analysis of the first optical molecular hyperpolarizability,  $\beta$ , leads to the prediction that  $|\beta|$  maximizes at a combination of donor and acceptor strengths for a given conjugated bridge. Molecular design strategies that focus on the energetic manipulations of the bridge for the optimization of  $\beta$ . The limitations of molecular bridge structures are highlighted and more promising structures are suggested. Experimental results supporting the validity of this approach are presented.

Many more

- Nørskov et al (Stanford/DTU)
- Curtarolo&Beratan&Yang (Duke)
- Ceder (MIT)
- Wolverton (Northwestern)
- Zunger (NREL)
- Yamashita (UT)



A Franceschetti & A Zunger, Nature (1999)

# Overview

## → Tutorial (first half)

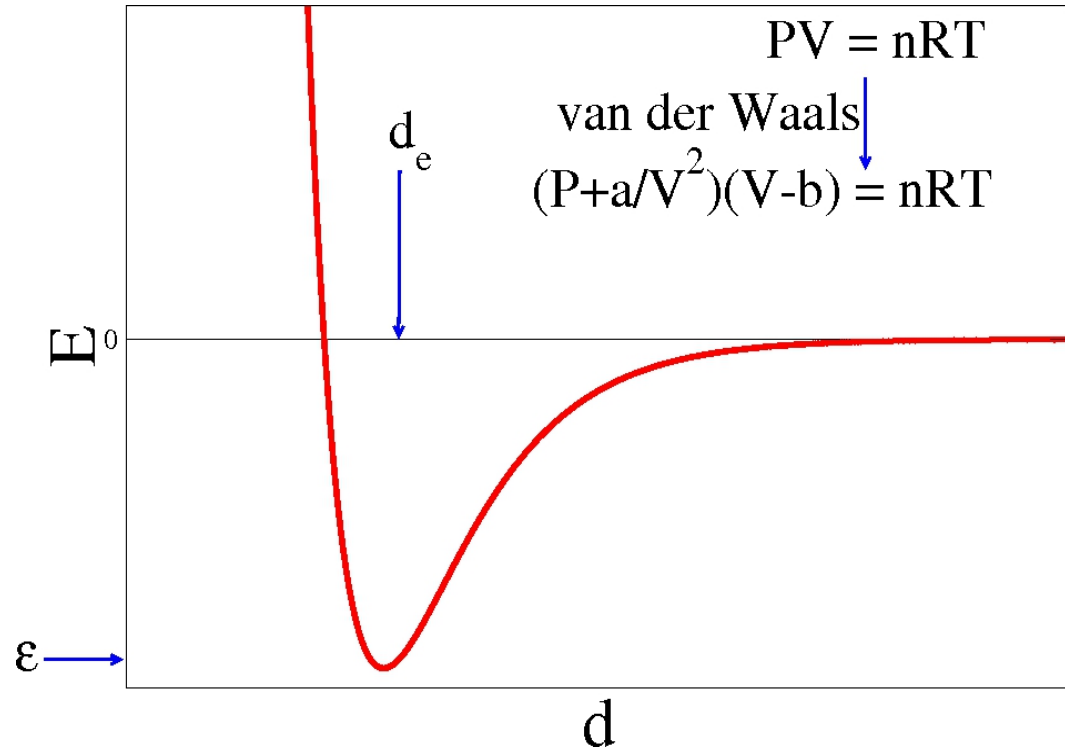
- Why do we care?
- What is it?
- What's the problem with vdW?

## → Recent contributions (second half)

- Many-body dispersion in DFT
- 3-body dispersion effects
- Many-body dispersion effects



# What is it?



## Good books

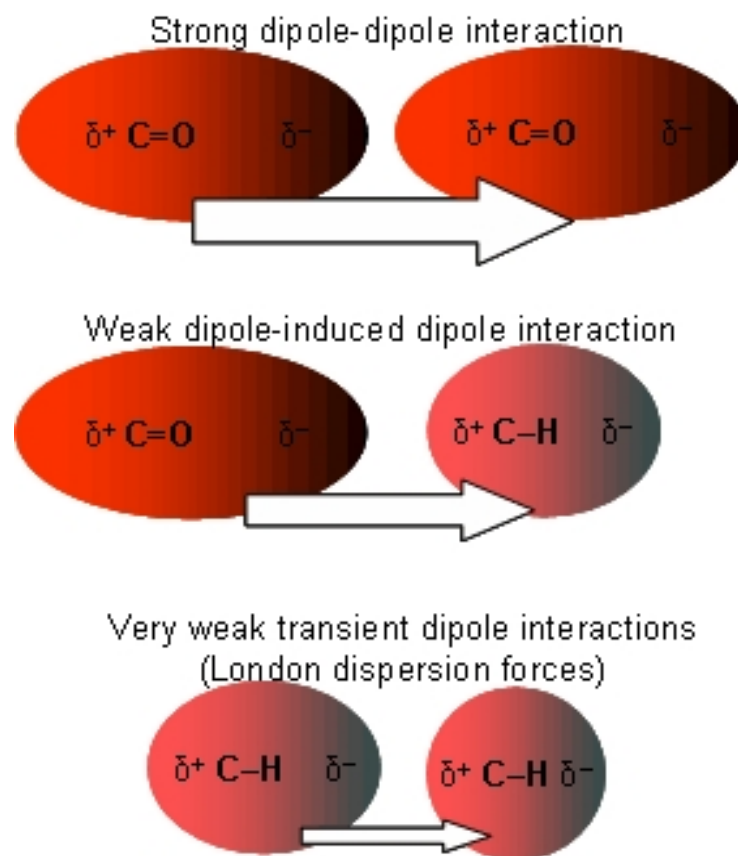
- "Theory of intermolecular forces" by A. J. Stone
- "Intermolecular and surface forces" by J. N. Israelachvili
- "Intermolecular interactions" by I. G. Kaplan
- "Van der Waals forces: A Handbook for biologists, engineers and physicists" by V. A. Parsegian





# What is it?

Origin	Sign	Additive?
Electrostatic	+/-	yes
Induction	-	no
Dispersion	-	no but small
Resonance	+/-	no
Magnetic	+/- (small)	yes
Exchange	-	no
Repulsion	+	no
Charge Transfer	-	no
Penetration	-	yes





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# What's the problem with vdW?

Accuracy/  
Transferability

$$H\Psi = E\Psi$$

Full CI, Quantum Monte Carlo

Correlated wavefunctions, MBPT  
(MP2, RPA, CCSD(T), GW, ...)

**Density-functional theory  
(LDA/GGA, meta, hybrids, ...)**

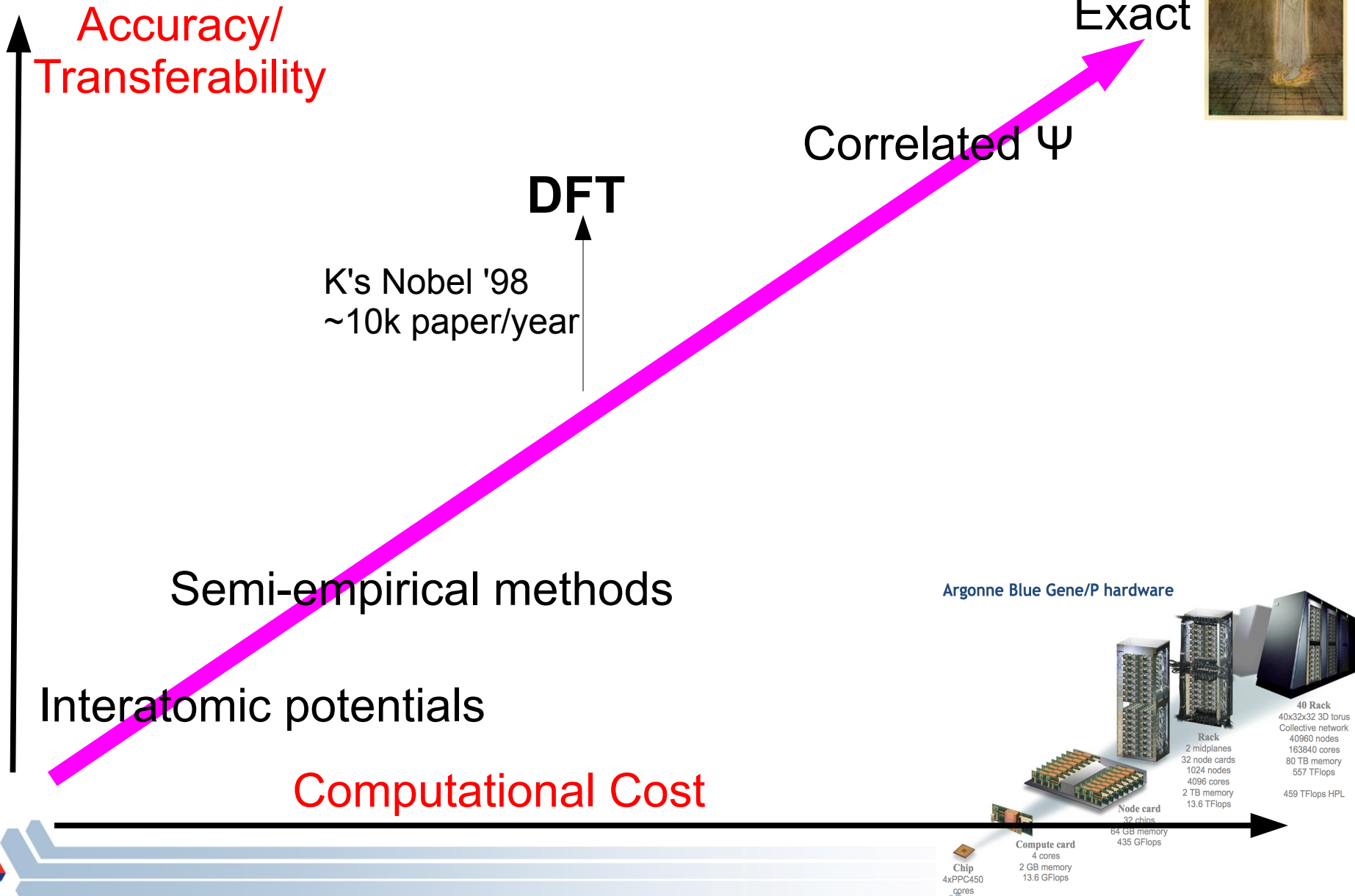
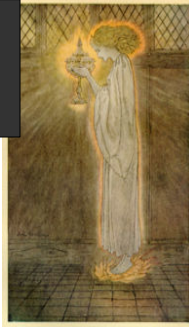
Semi-empirical methods  
(AM1, PM6, ZINDO, TB-DFT)

Interatomic potentials (“force fields”)  
( Tersoff, Brenner, Foiles, Pettifor, Karplus etc)

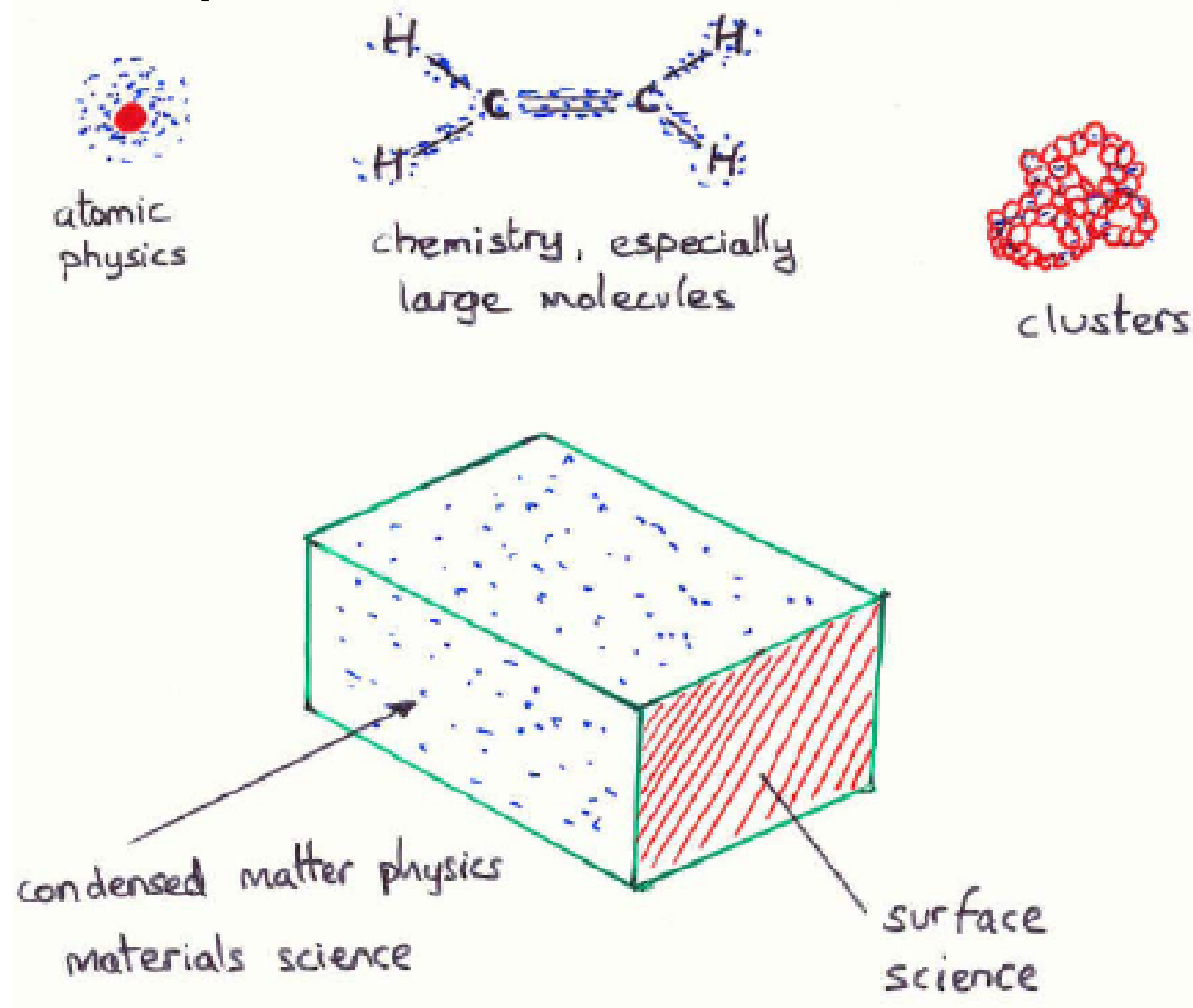


# What's the problem with vdW?

$$H\Psi = E\Psi$$



# What's the problem with vdW?



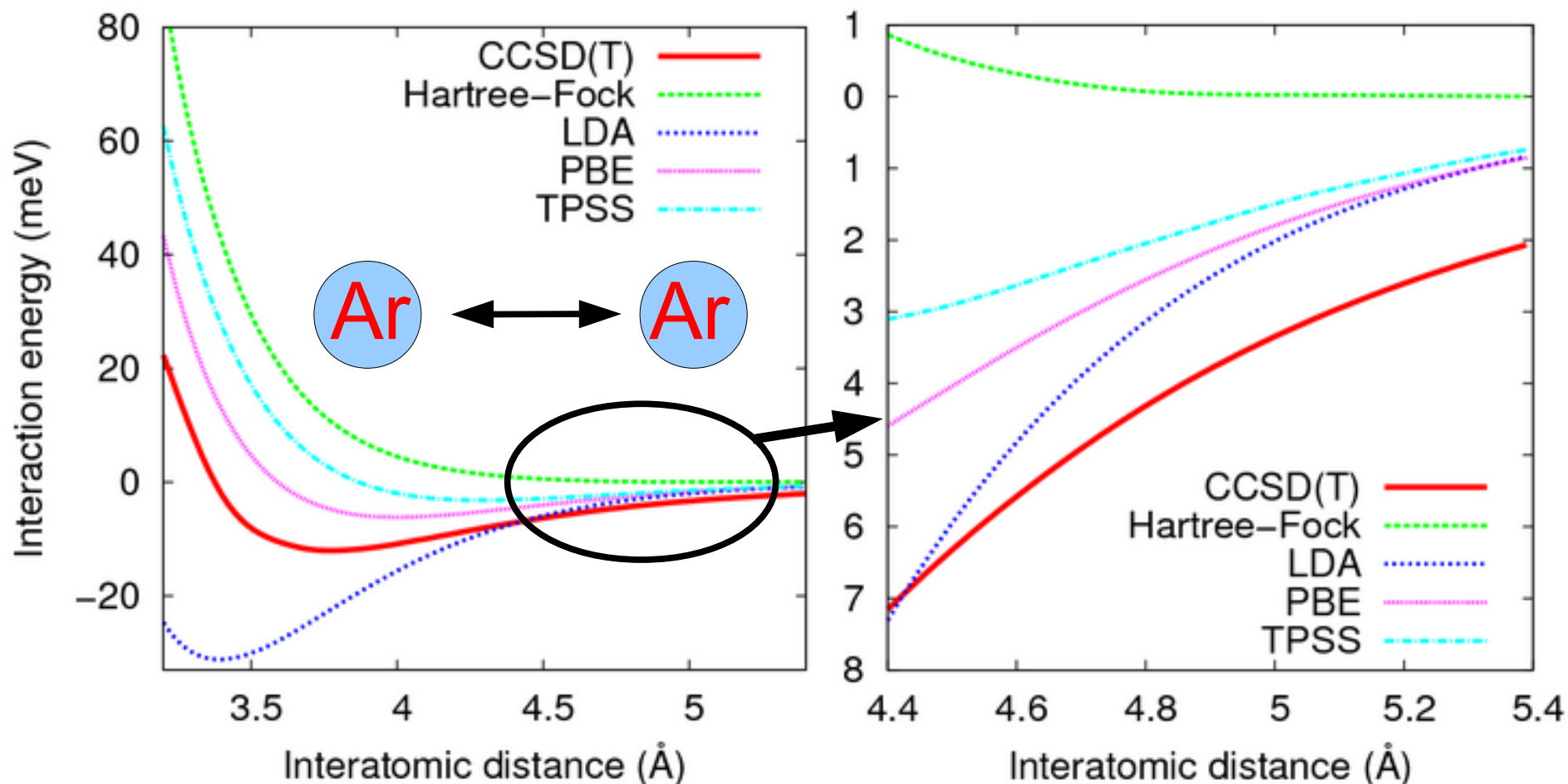
## Density-functional theory (DFT)

Kieron Burke, UC Irvine



# What's the problem with vdW?

$$E_{xc} = E_{ex}^{GGA \text{ or } EX} + E_{corr}^{LDA, GGA} + E_{corr}^{non-local}$$



# What's the problem with vdW?

$$E_{xc} = E_{ex}^{\text{GGA or EX}} + E_{\text{corr}}^{\text{LDA,GGA}} + E_{\text{corr}}^{\text{non-local}}$$

- › vdW-DFT: Non-local functionals (depend explicitly on  $r$  and  $r'$ ) (*Dion, Langreth, Lundqvist et al.* PRL (2004)).
- › Empirical atom centered potentials (*von Lilienfeld, Tavernelli, Roethlisberger, Sebastiani* PRL (2004), *DiLabio* CPL (2008))
- › Empirical density functionals (*Truhlar et al.* JCP (2006), *Goddard* PNAS (2004))
- › Interatomic (pairwise or beyond) dispersion corrections (Many ways)

*Ahlich, Scoles et al* (70's and 80's); *Elstner, Hobza, Frauenheim, Kaxiras et al* (2001); *Wu and Yang JCP* (2002); *Grimme J. Comp. Chem.* (2004,2006); *Johnson and Becke JCP* (2005-2007); *Silvestrelli* PRL (2008); *Tkatchenko and Scheffler PRL* (2009); and others ...

What's the problem with vdW?

Langreth-Lundqvist vdW-DFT



# What's the problem with vdW?

Langreth-Lundqvist functional

$$E_{xc} = E_{ex}^{GGA}[n(\mathbf{r})] + E_{corr}^{LDA}[n(\mathbf{r})] + E_{corr}^{non-local}[n(\mathbf{r})]$$

$$E_{corr}^{non-local}[n(\mathbf{r})] = \frac{1}{2} \int d^3r d^3r' n(\mathbf{r}) K(\mathbf{r}, \mathbf{r}') n(\mathbf{r}')$$

Dion, Rydberg, Schroeder, Langreth, Lundqvist, **PRL** (2004).

Lee, Murray, Kong, Lundqvist, Langreth, **PRB** (2010).



# What's the problem with vdW?

Langreth-Lundqvist functional (vdW-DF-04 and vdW-DF-10)

$$E_{xc} = E_{ex}^{GGA}[n(\mathbf{r})] + E_{corr}^{LDA}[n(\mathbf{r})] + E_{corr}^{non-local}[n(\mathbf{r})]$$

$$E_{corr}^{non-local}[n(\mathbf{r})] = \frac{1}{2} \int d^3r d^3r' n(\mathbf{r}) K(\mathbf{r}, \mathbf{r}') n(\mathbf{r}')$$

vdW-DF-04

Exchange: revPBE

Local corr.: LDA

No free parameters

$C_6$  error: ~ 20%

vdW-DF-10

Exchange: PW86

Local corr.: LDA

2 parameters

$C_6$  error: ~ 60%<sup>(\*)</sup>

<sup>(\*)</sup> Vydrov and van Voorhis, **PRA** (2010).

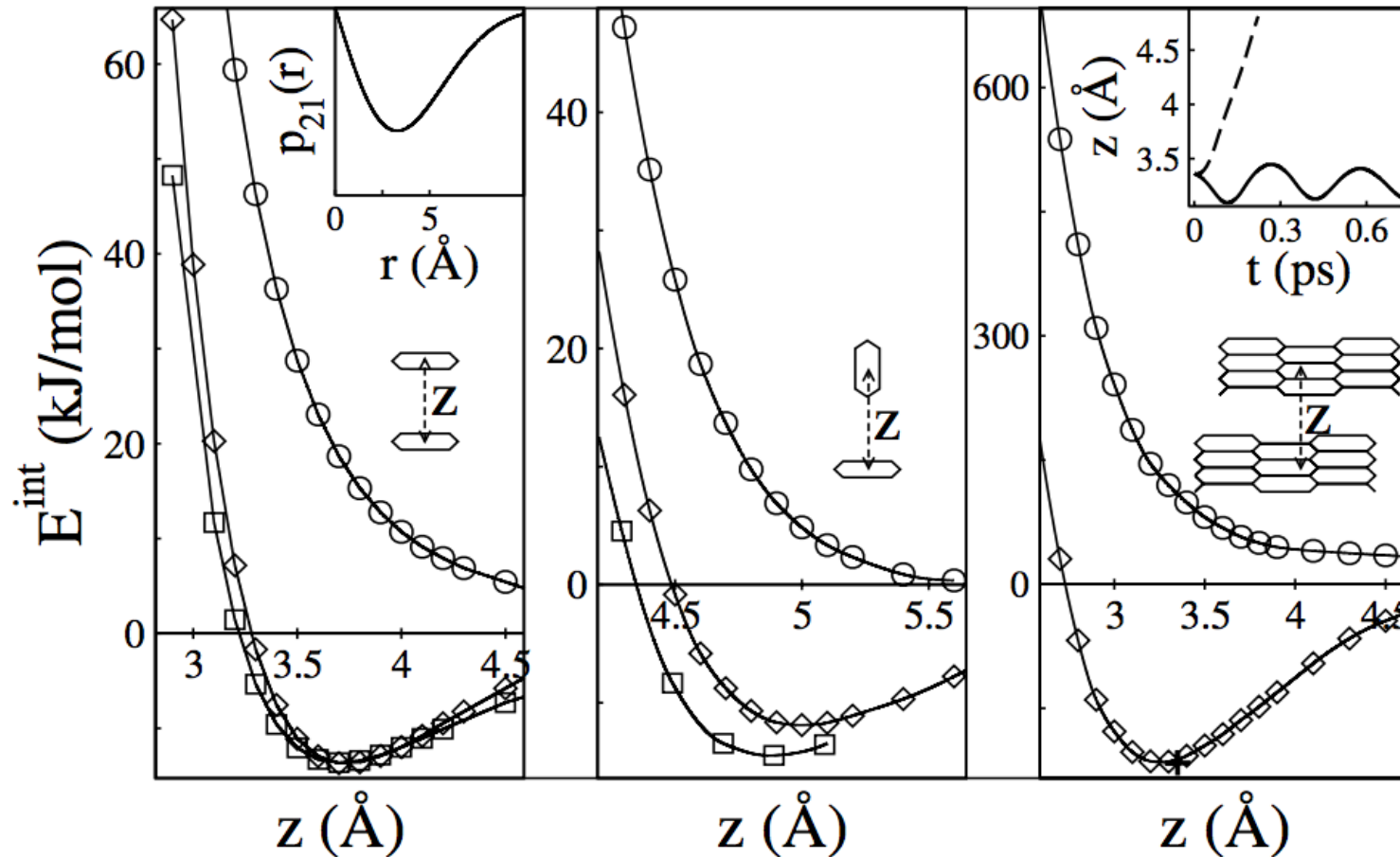


What's the problem with vdW?

Dispersion corrected atom  
centered potentials (DCACP)



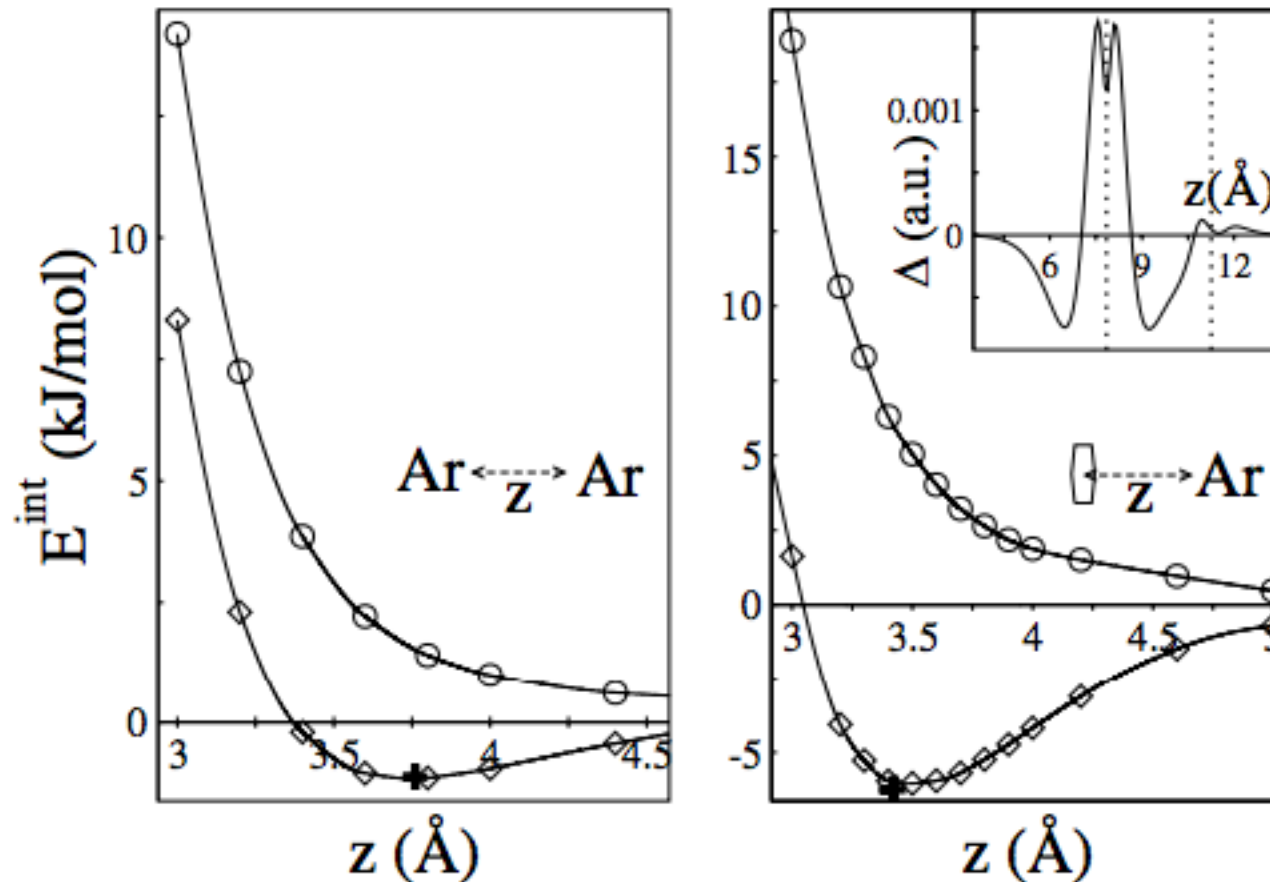
# What's the problem with vdW?



von Lilienfeld, Tavernelli, Sebastiani, Roethlisberger, PRL (2004)



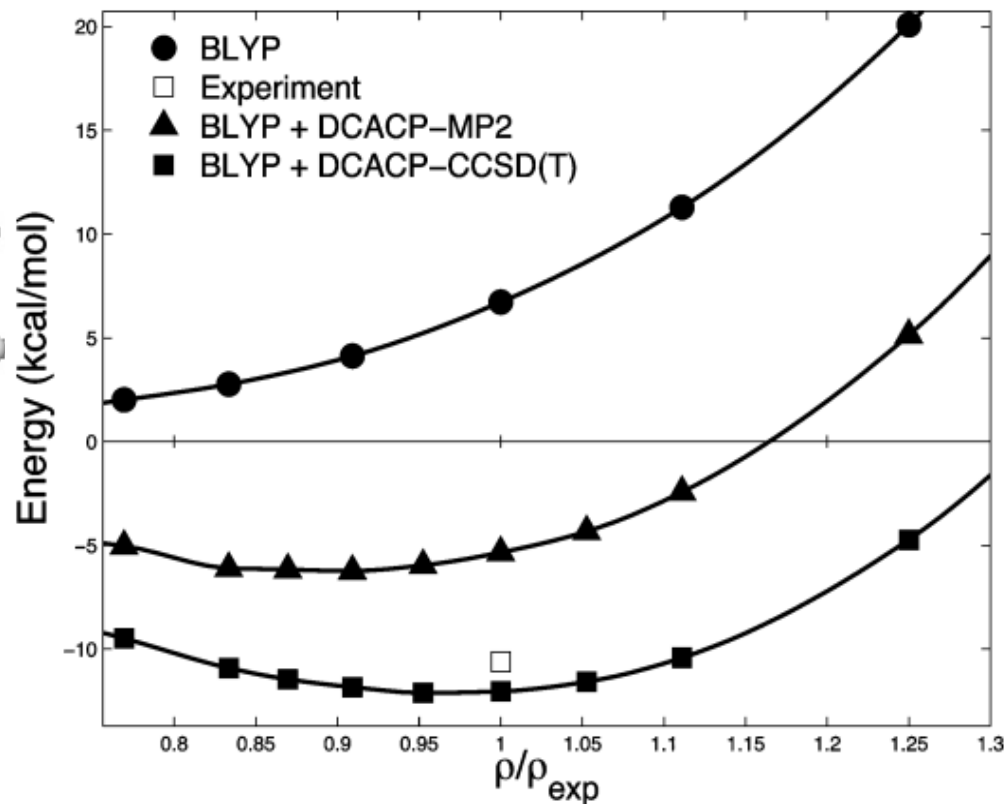
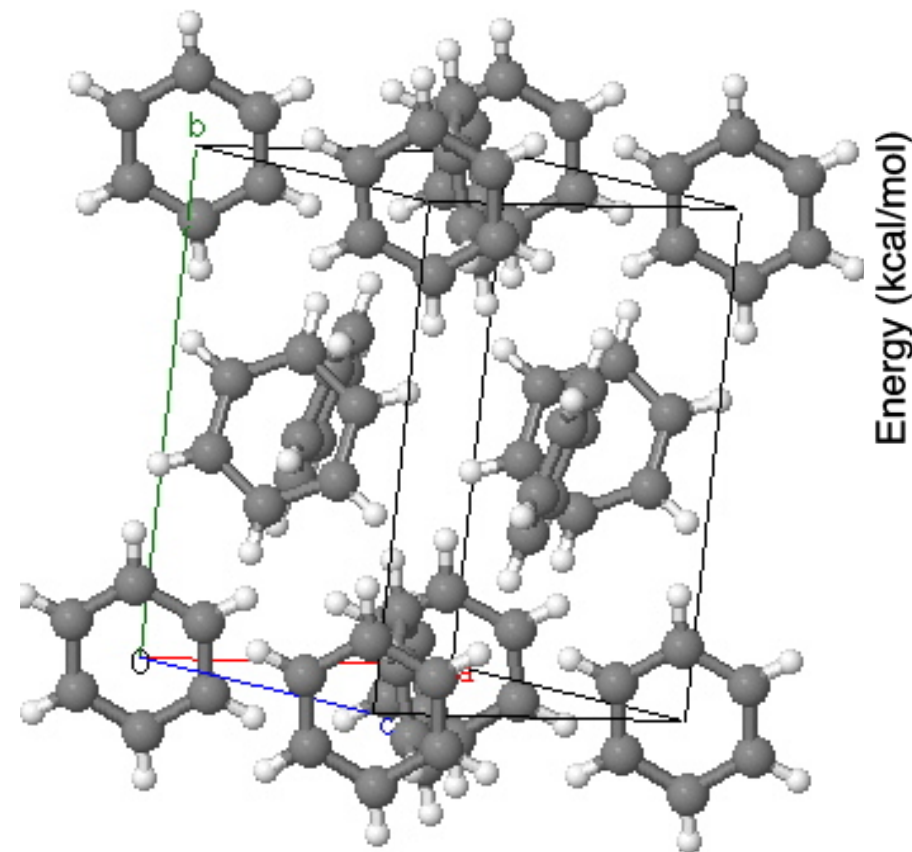
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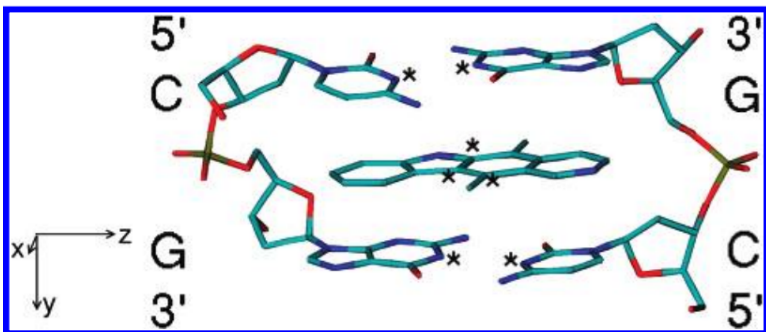
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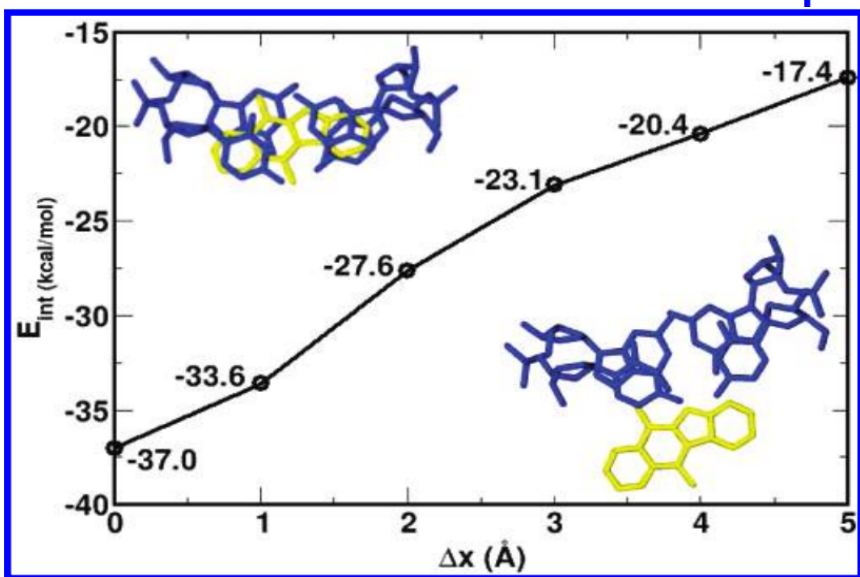
Tapavizca, Lin, von Lilienfeld, Tavernelli, Coutinho-Neto, Roethlisberger, JCTC (2007)



# What's the problem with vdW?



B-DNA	$\Delta x = 0 \text{ \AA}$	$\Delta x = 5 \text{ \AA}$	Ellipticine
 LUMO 2.28	 LUMO 2.19	 LUMO 1.55	 LUMO -2.09
 HOMO-1 -0.19	 HOMO 0.01	 HOMO-1 -0.66	 HOMO -4.65
 HOMO-4 -0.43	 HOMO-2 -0.48	 HOMO-4 -0.84	



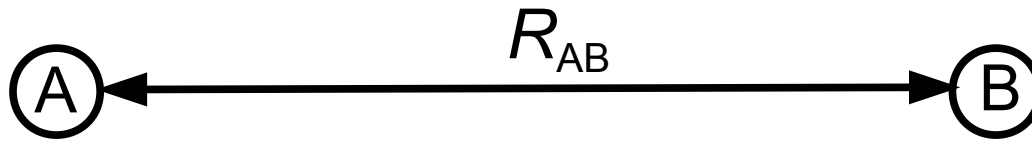
Lin, von Lilienfeld, Coutinho-Neto, Tavernelli, Roethlisberger, JCPA (2007)





What's the problem with vdW?

## Pair-wise corrected DFT



$$E_{xc} = E_{ex}^{\text{GGA or EX}} + E_{\text{corr}}^{\text{LDA,GGA}} + E_{\text{corr}}^{\text{non-local}}$$

$$E^{vdW}(R) = - \left( f_6(R) \frac{C_6}{R^6} + f_8(R) \frac{C_8}{R^8} + f_{10}(R) \frac{C_{10}}{R^{10}} + \dots \right)$$

$$E_{\text{vdW}}^{(2)} = - \frac{C_6^{AB}}{R_{AB}^6}$$

$$C_6^{AB} = \frac{3}{\pi} \int \alpha_A(i\omega) \alpha_B(i\omega) d\omega$$



# What's the problem with vdW?

## Pair-wise corrected DFT

Grimme's D1,D2 (2004-2006): Parameterization for many elements in the periodic table

- Highly empirical, some very *ad hoc* approximations

Jurečka, Hobza *et al.* (2007): Accurate parameterization for organic molecules

- Better theoretical ground, but still very empirical

Johnson and Becke (2005-2008), Silvestrelli (2008): On-the-fly  $C_6$  and vdW radii from HF or DFT orbitals

- Reduced empiricism, errors of  $\sim 20\%$ - $40\%$  in  $C_6$  coefficients

Tkatchenko and Scheffler (2009): On-the-fly  $C_6$  coefficients and vdW radii from ground-state electron density

- First-principles  $C_6$  accurate to **5%**



# What's the problem with vdW?

## TS-vdW method: Atomic vdW parameters from first principles

Dissociative asymptotics with an accuracy of 5%

Transparent partitioning of vdW coefficients over atoms

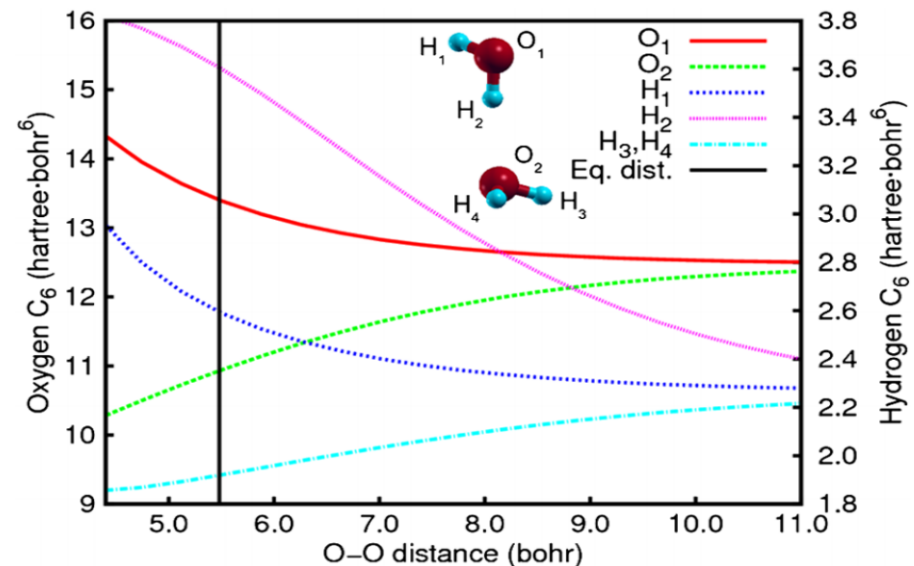
On-the-fly atomic polarizability ratio proportional to atomic volume ratio

No computational overhead

Valid for entire periodic table

$$C_{6AA}[n(\mathbf{r})] = \left( \frac{V_A[n(\mathbf{r})]}{V_A^{free}[n^{free}(\mathbf{r})]} \right)^2 C_{6AA}^{free}$$

$$C_{6AB} = \frac{2C_{6AA}C_{6BB}}{\left[ \frac{\alpha_B^0}{\alpha_A^0} C_{6AA} + \frac{\alpha_A^0}{\alpha_B^0} C_{6BB} \right]}$$



A. Tkatchenko and M. Scheffler, **PRL** (2009)



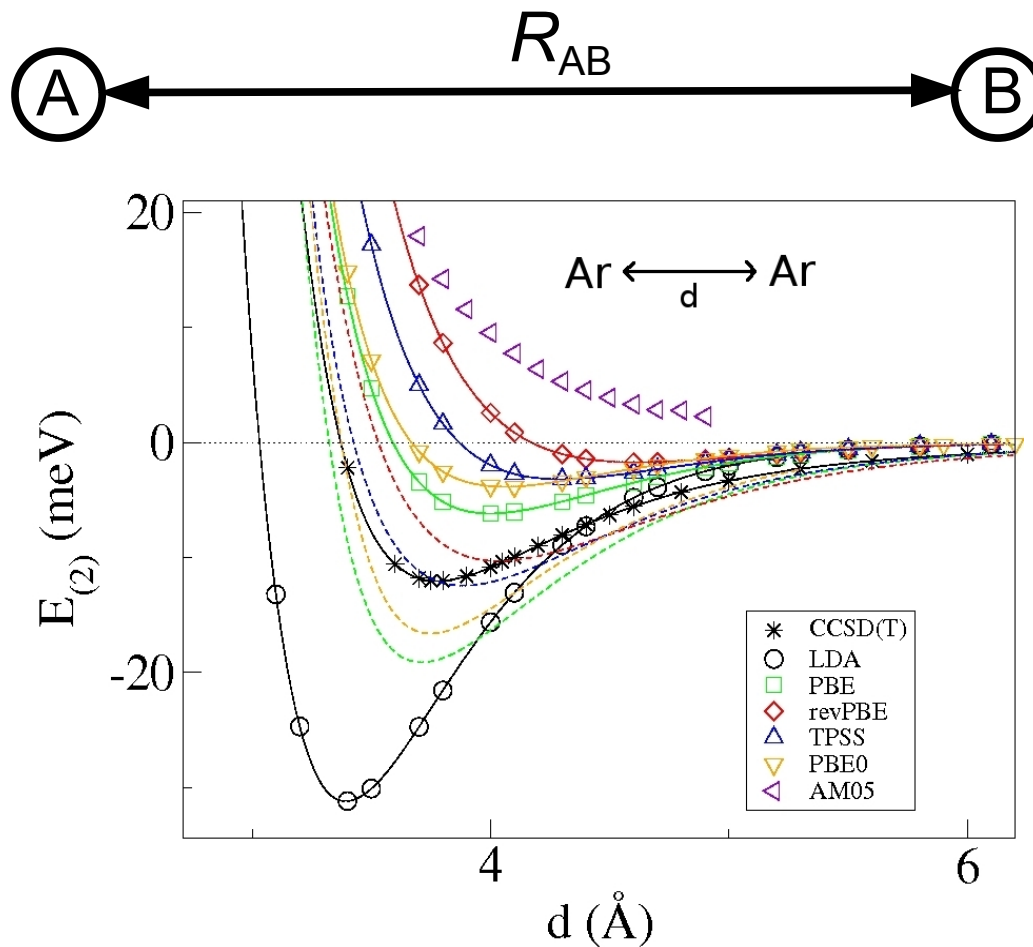
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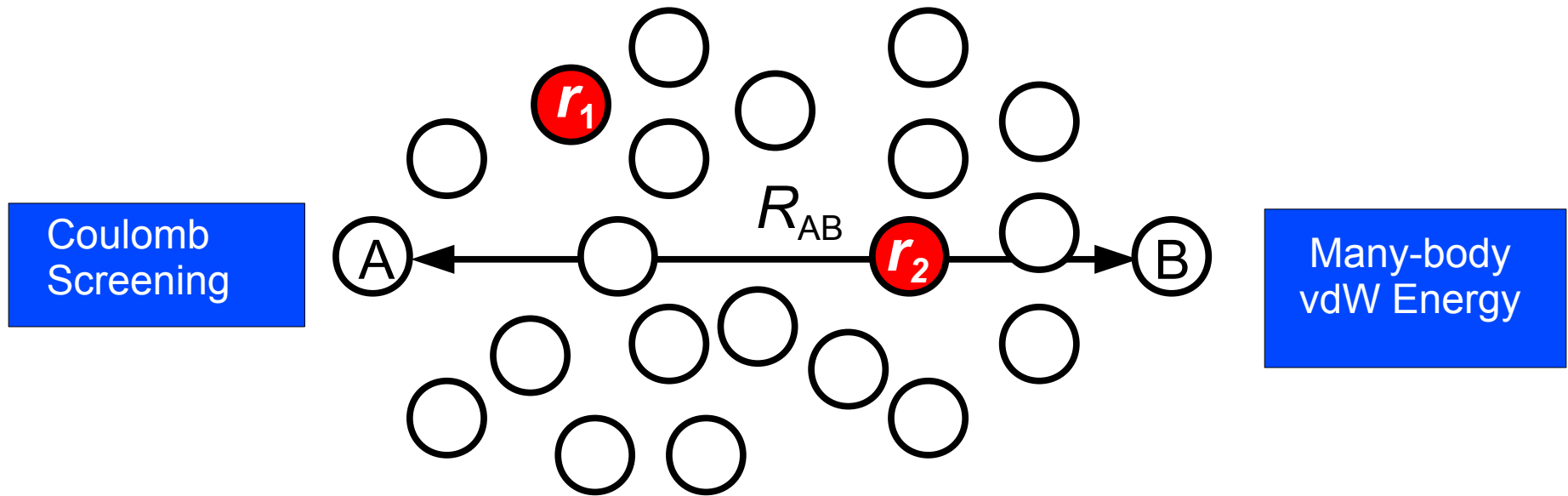


# Many-body dispersion in DFT

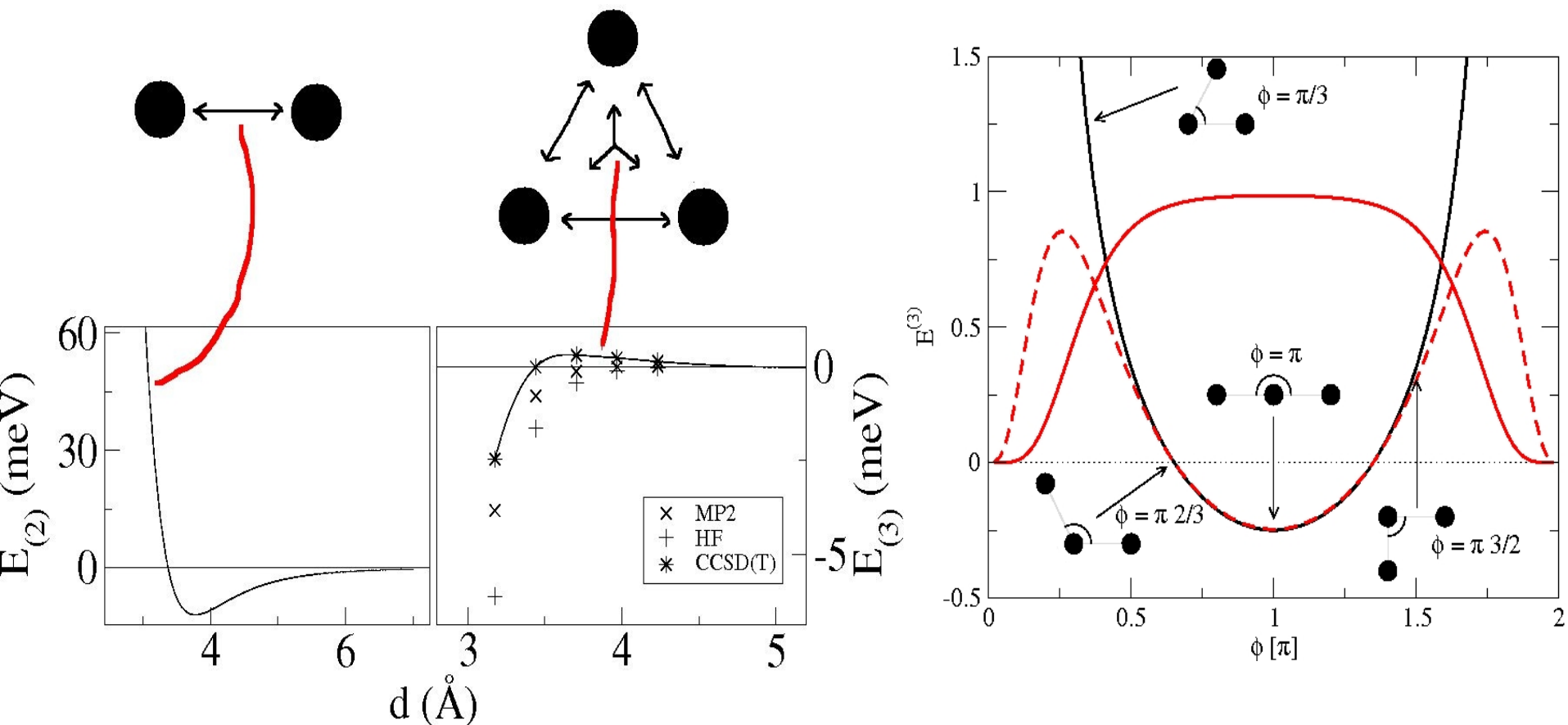
## Pair-wise corrected DFT



# Many-body dispersion in DFT



# Many-body dispersion in DFT

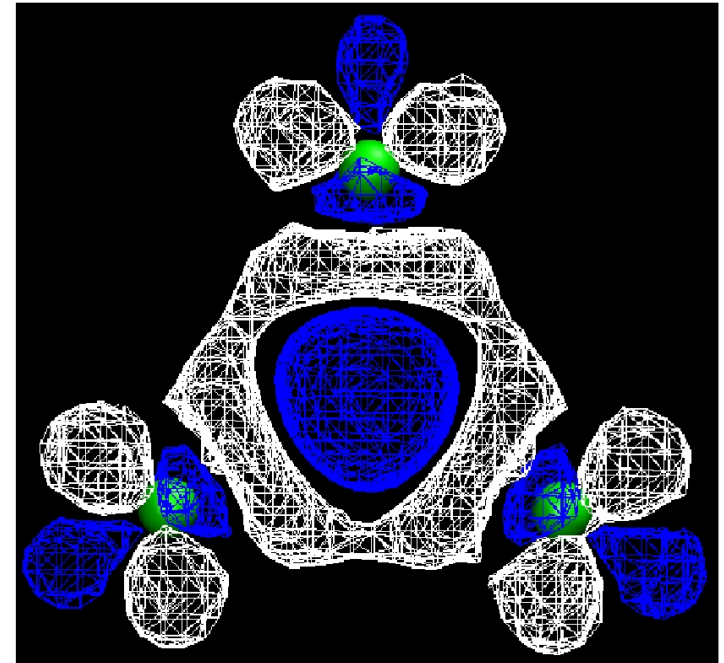
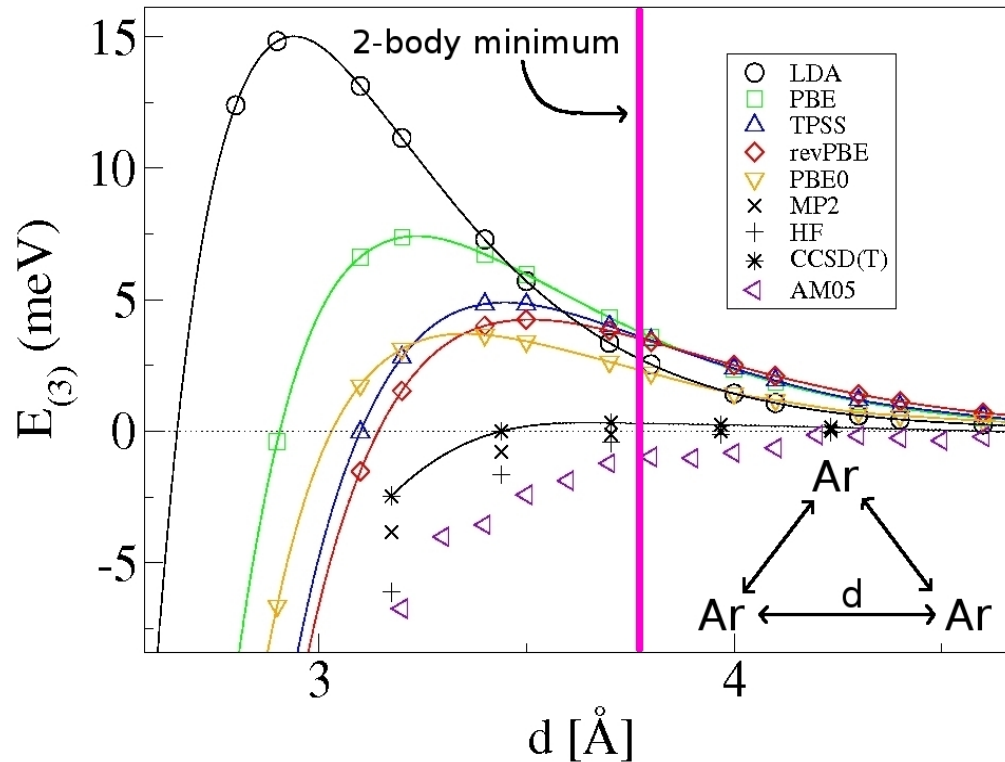


Axilrod & Teller *J Chem Phys* (1943); Muto *J Phys-Math Soc Japan* (1943)

$$E_{(3)}[A, B, C] = C_{9_{ABC}} \frac{3 \cos[\gamma_{AB}] \cos[\gamma_{AC}] \cos[\gamma_{BC}] + 1}{d_{AB}^3 d_{AC}^3 d_{BC}^3} + \dots$$



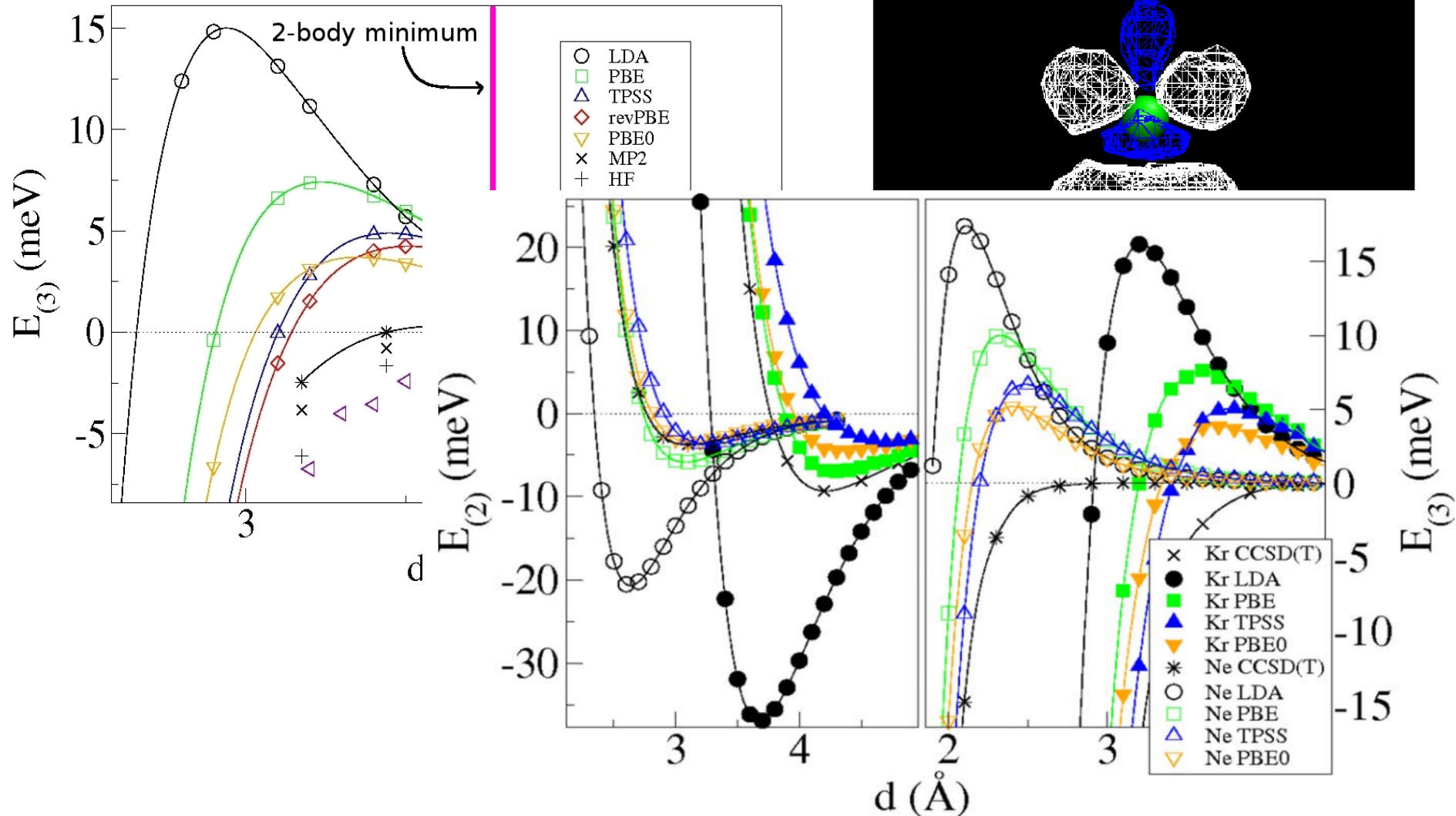
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Tkatchenko & von Lilienfeld, PRB (2008)



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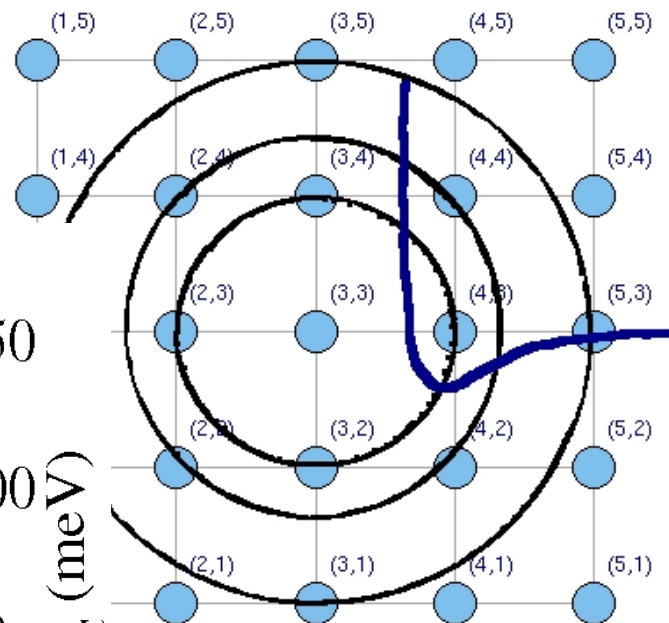
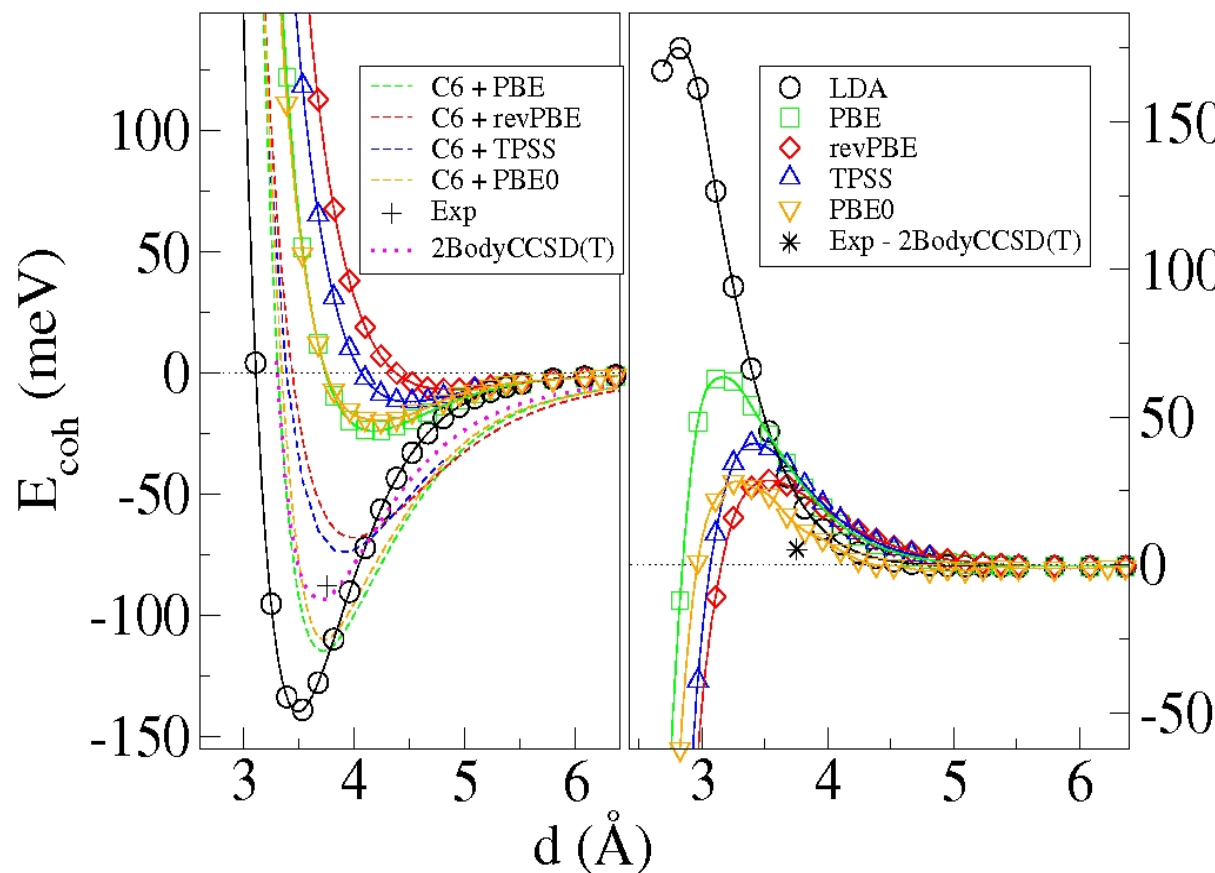
Tkatchenko & von Lilienfeld, PRB (2008)





# Many-body dispersion in DFT

For rare gas crystals, experimental  
MBC estimated to be up to 10% of cohesive energy



Tkatchenko & von Lilienfeld, PRB (2008)



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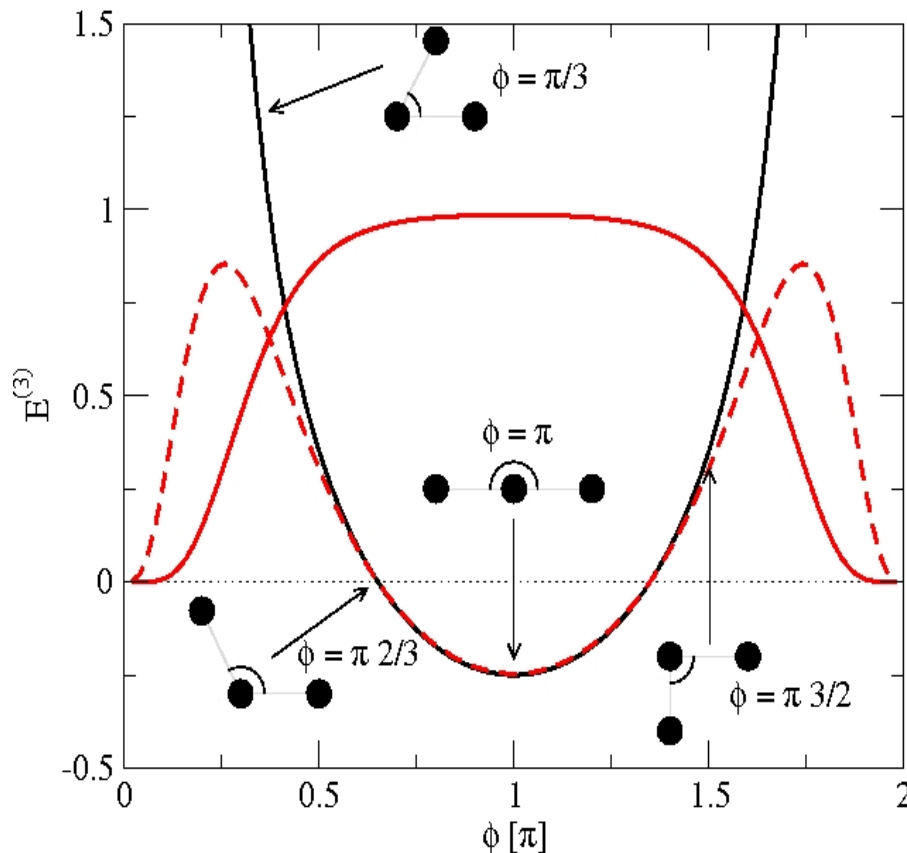
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# 3-body dispersion effects

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von Lilienfeld & Tkatchenko, JCP (2010)



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$$E_{(3)}[A, B, C] = C_{9_{ABC}} \frac{3 \cos[\gamma_{AB}] \cos[\gamma_{AC}] \cos[\gamma_{BC}] + 1}{d_{AB}^3 d_{AC}^3 d_{BC}^3} + \dots$$

$$C_{9_{IJK}} = \frac{3}{\pi} \int_0^\infty d\omega \alpha_I(i\omega) \alpha_J(i\omega) \alpha_K(i\omega)$$

$$C_{6_{IJ}} = \frac{3}{\pi} \int_0^\infty d\omega \alpha_I(i\omega) \alpha_J(i\omega)$$

$$C_{9_{IJK}} = \frac{8}{3} \frac{P_I P_J P_K (P_I + P_J + P_K)}{(P_I + P_J)(P_J + P_K)(P_K + P_I)},$$

$$P_I = C_{9_{III}} \frac{\alpha_{0,J} \alpha_{0,K}}{\alpha_{0,I}^2}$$

$$C_{9_{III}}[n(\mathbf{r})] \approx \left( \frac{V_I[n(\mathbf{r})]}{V_I^{\text{free}}[n^{\text{free}}(\mathbf{r})]} \right)^3 C_{9_{III}}^{\text{free}}$$

$$\frac{V_I[n(\mathbf{r})]}{V_I^{\text{free}}[n^{\text{free}}(\mathbf{r})]} = \left( \frac{\int d\mathbf{r} r^3 n_I(\mathbf{r})}{\int d\mathbf{r} r^3 n_I^{\text{free}}(\mathbf{r})} \right)$$



# 3-body dispersion effects

Atom	$\alpha$	$C_6$	$C_9$	$R_{vdW}$
H <sup>free</sup>	4.50	6.5	21.6	3.10
H <sup>s</sup>	2.75	2.42	4.91	2.63
He	1.38	1.46	1.47	2.65
C <sup>free</sup>	12.0	46.6	373	3.59
C <sup>sp</sup>	9.73	30.6	199	3.35
C <sup>sp<sup>2</sup></sup>	9.67	30.3	195	3.34
C <sup>sp<sup>3</sup></sup>	8.64	24.1	139	3.22
N <sup>free</sup>	7.40	24.2	117	3.34
N <sup>sp<sup>2</sup>,sp<sup>3</sup></sup>	6.36	17.9	74.4	3.18
O <sup>free</sup>	5.40	15.6	52.6	3.19
O <sup>sp<sup>2</sup></sup>	4.92	13.0	39.8	3.09
O <sup>sp<sup>3</sup></sup>	4.81	12.4	37.1	3.07
F <sup>free</sup>	3.80	9.52	24.2	3.04
F <sup>sp<sup>3</sup></sup>	3.46	7.89	18.3	2.95
Ne <sup>free</sup>	2.67	6.38	12.0	2.91
Si <sup>free</sup>	37.0	305	8550	4.20
Si <sup>sp<sup>3</sup></sup>	25.6	146	2846	3.72
P <sup>free</sup>	25.0	185	3561	4.01
S <sup>free</sup>	19.6	134	1925	3.86
S <sup>sp<sup>3</sup></sup>	18.2	115	1532	3.76
Cl <sup>free</sup>	15.0	94.6	1014	3.71
Cl <sup>sp<sup>3</sup></sup>	14.6	89.4	932	3.68
Ar	11.1	64.3	518	3.55
Br <sup>free</sup>	20.0	162	2511	3.93
Br <sup>sp<sup>3</sup></sup>	19.5	155	2340	3.90
Kr	16.8	130	1572	3.82

von Lilienfeld & Tkatchenko, JCP (2010)

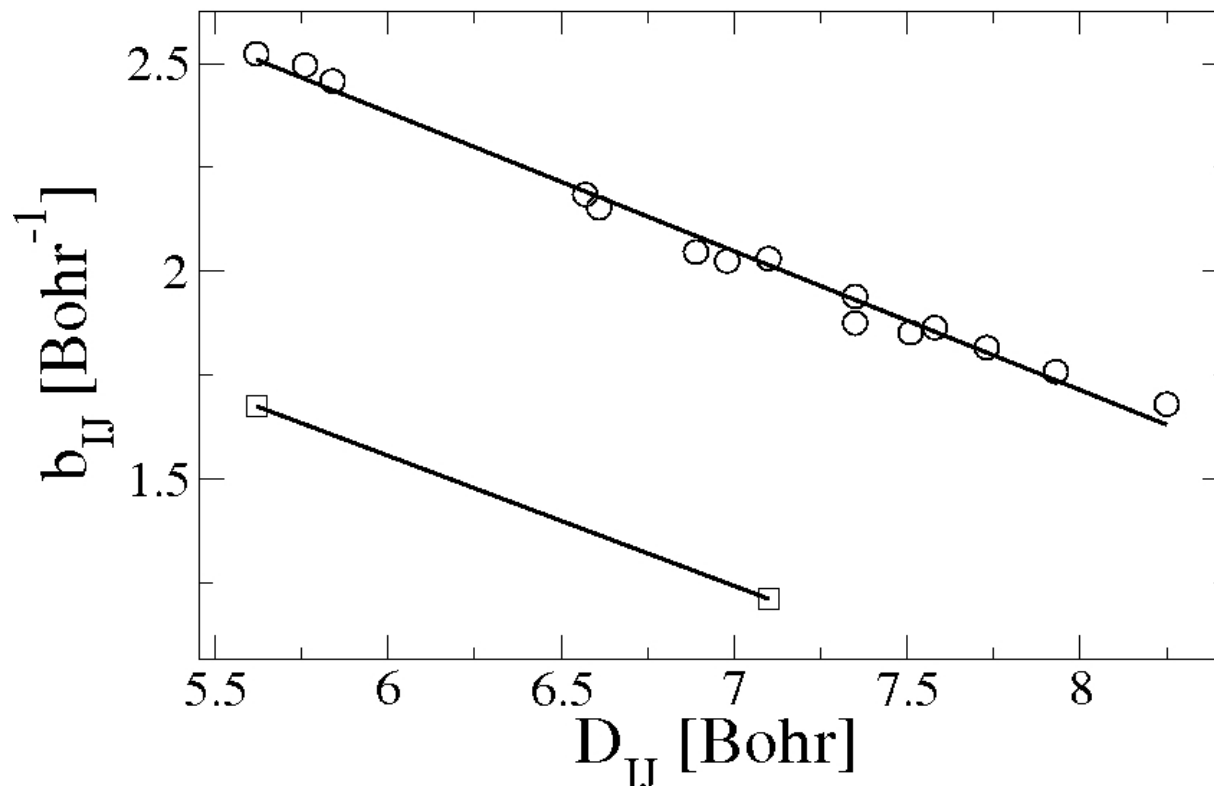


# 3-body dispersion effects

Damping according to Tang&Toennis

$$f_6^d(R_{IJ}) = 1 - e^{-b_{IJ}R_{IJ}} \sum_{k=0}^{n=6} \frac{(b_{IJ}R_{IJ})^k}{k!}$$

$$f_{\text{ATM}}^d(R_I, R_J, R_K) = f_6'(R_{IJ}) \times f_6'(R_{IK}) \times f_6'(R_{JK})$$



von Lilienfeld & Tkatchenko, JCP (2010)

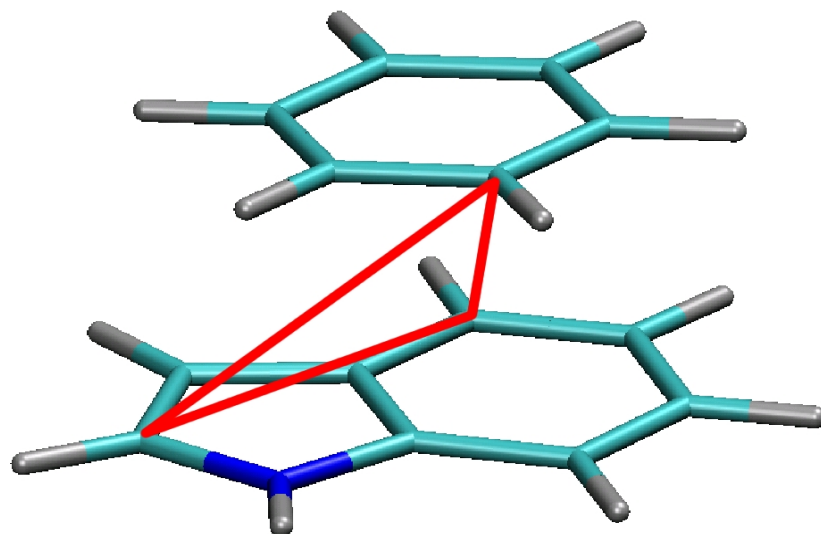
Tkatchenko&Scheffler, PRL (2009)



# 3-body dispersion effects

TABLE II. Interatomic  $E^{(2)}$  and  $E^{(3)}$  contributions to S22 benchmark data set results in kcal/mol. CCSD(T) results from S22 data set in Ref. 9.

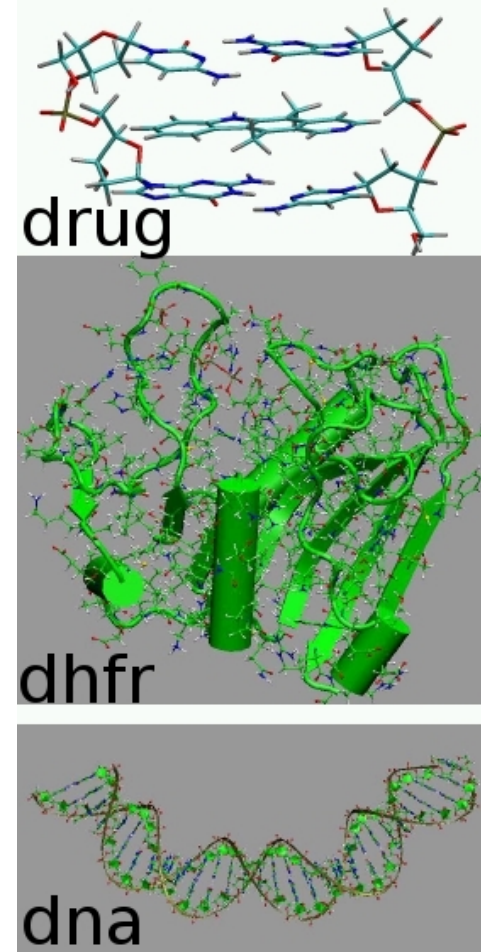
No.		$E^{(2)}$	$E^{(3)}$	CCSD(T)
1	(NH <sub>3</sub> ) dimer ( $C_{2h}$ )	-1.43	0.00	-3.17
2	(H <sub>2</sub> O) dimer ( $C_s$ )	-1.80	-0.01	-5.02
3	Formic acid dimer	-7.87	0.02	-18.61
4	Formamide dimer ( $C_{2h}$ )	-5.69	0.02	-15.96
5	Uracil dimer ( $C_{2h}$ )	-7.03	-0.07	-20.65
6	2-pyridoxine-2-aminopyridine ( $C_1$ )	-7.16	-0.02	-16.71
7	Adenine-thymine WC	-7.54	-0.04	-16.37
8	(CH <sub>4</sub> ) dimer ( $D_{3d}$ )	-0.68	0.02	-0.53
9	(C <sub>2</sub> H <sub>4</sub> ) dimer ( $D_{2d}$ )	-1.96	0.05	-1.51
10	Benzene-CH <sub>4</sub> ( $C_3$ )	-1.97	0.14	-1.50
11	Benzene dimer ( $C_{2h}$ )	-5.64	0.68	-2.73
12	Pyrazine dimer ( $C_s$ )	-5.92	0.60	-4.42
13	Uracil dimer ( $C_2$ )	-8.90	0.96	-10.12
14	Indole-benzene ( $C_1$ )	-8.48	1.10	-5.22
15	Adenine-thymine stack	-13.04	1.54	-12.23
16	Ethene-ethine ( $C_{2v}$ )	-0.93	0.01	-1.53
17	Benzene-H <sub>2</sub> O ( $C_s$ )	-2.35	0.15	-3.28
18	Benzene-NH <sub>3</sub> ( $C_s$ )	-2.15	0.14	-2.35
19	Benzene-HCN ( $C_s$ )	-2.84	0.15	-4.46
20	Benzene dimer ( $C_{2v}$ )	-3.61	0.21	-2.74
21	Indole-benzene T-shape	-5.24	0.30	-5.73
22	Phenol dimer ( $C_1$ )	-5.18	0.19	-7.05



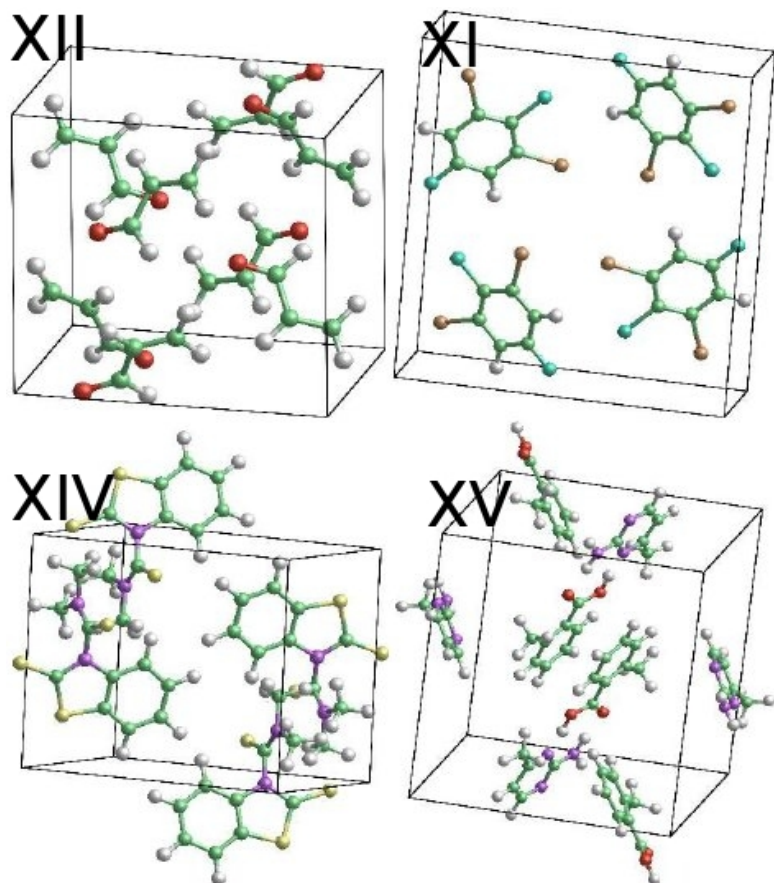


# 3-body dispersion effects

System	$E$	$E^{(3)}$	$E^{(2)}$
Ar crystal	$-2.03^a$	0.07	-2.17
benzene crystal	$-10.6^b$	1.67	-16.1
diamond	$-171.3^c$	2.03	-50.5
ice Ih	$-14.1^d$	0.04	-2.96
bi-graphene	$-1.20^e$	0.61	-2.13
$(C_{60})_2$	$-7.33^f$	1.02	-11.4
drug	$-37.0^g$	8.90	-57.0
Ala <sub>10</sub> ( $\alpha$ -FES)/residue	$-0.96^h$	0.44	-5.95
dhfr/amino acid <sup>i</sup>	-	1.60	-320.0
dna/base <sup>i</sup>	-	1.45	-241.67



# 3-body dispersion effects



# Polymorph	PW91+vdW <sup>a</sup>	PBE+vdW <sup>b</sup>	$E^{(2)}$	$E^{(3)}$
<b>XII</b>				
1	0.0	0.0	0.0	0.0
2	0.28	0.58	1.66	-0.07
3	0.47	0.40	-1.07	0.06
<b>XI</b>				
1	0.0	0.0	0.0	0.0
2	0.32	0.59	0.70	-0.09
3	0.34	0.88	1.77	-0.21
<b>XIV</b>				
1	0.0	0.0	0.0	0.0
2	0.47	1.05	1.94	-0.32
3	1.05	0.98	1.25	-0.13
<b>XV</b>				
1	0.0	0.0	0.0	0.0
2	0.50	0.35	1.54	0.02
3	0.59	0.75	2.72	-0.14

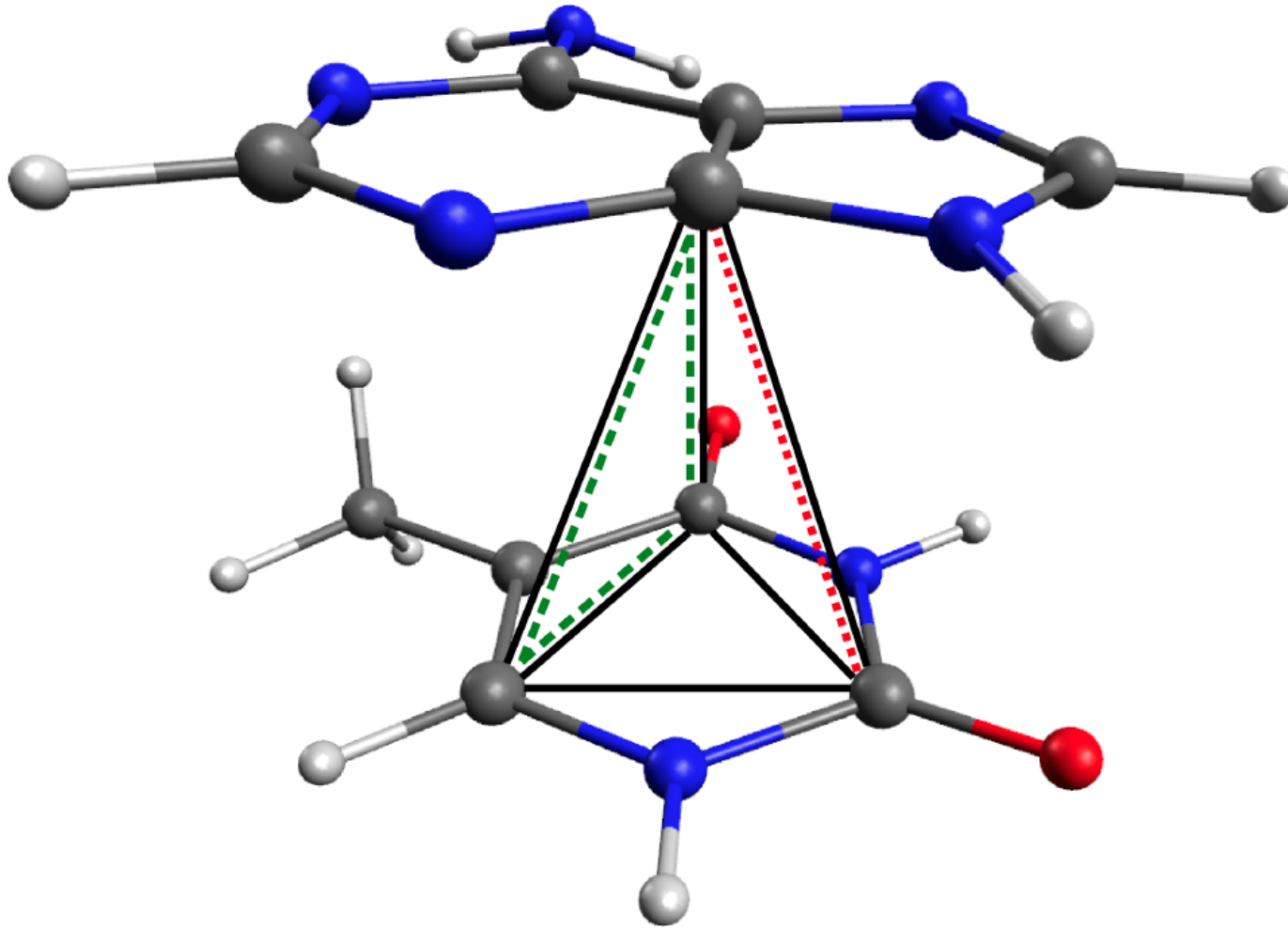


# Overview

- Tutorial (first half)
  - Why do we care?
  - What is it?
  - What's the problem with vdW?
- Recent contributions (second half)
  - Many-body dispersion in DFT
  - 3-body dispersion effects
  - Many-body dispersion effects



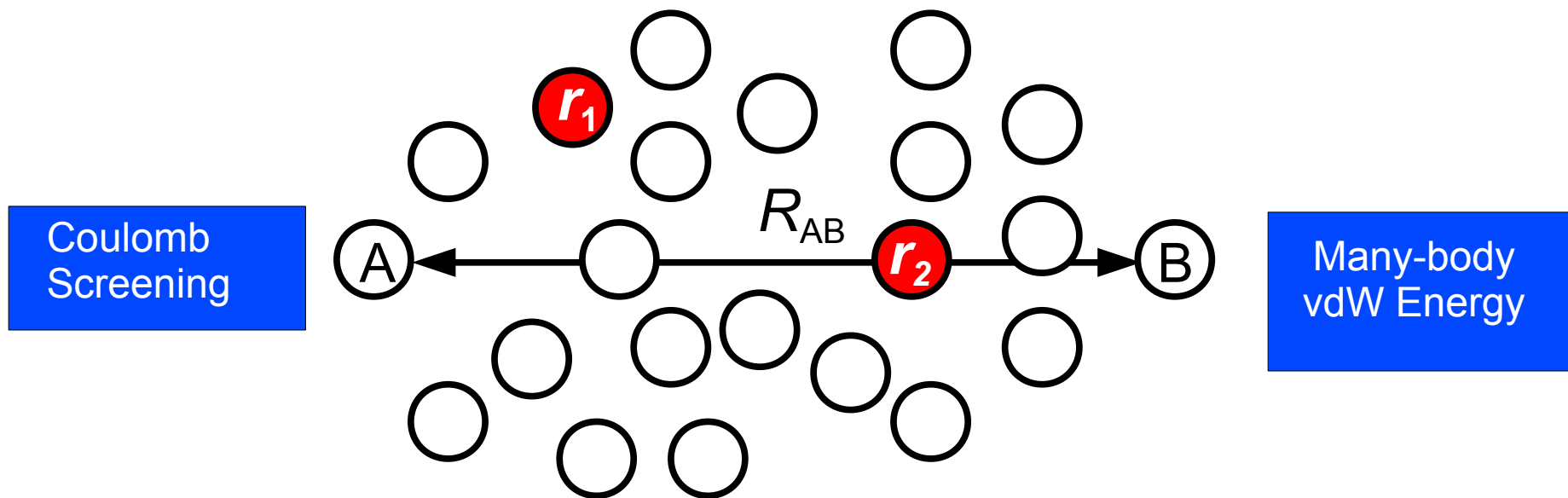
# Many-body dispersion effects



Tkatchenko, von Lilienfeld, DiStasio, submitted to PNAS (2012)



# Many-body dispersion effects



Use DFT+MBD method

## TS-vdW method

$$\alpha_A^0 = \alpha_A^0[n(\mathbf{r})]; \quad \omega_A^0 = \omega_A^0[n(\mathbf{r})]$$

## Self-consistent electrostatic screening (SCS)

$$\alpha_p(i\omega) = \alpha_p^{\text{TS}}(i\omega) + \alpha_p^{\text{TS}}(i\omega) \sum_{q \neq p}^N \mathcal{T}_{pq} \alpha_q(i\omega)$$

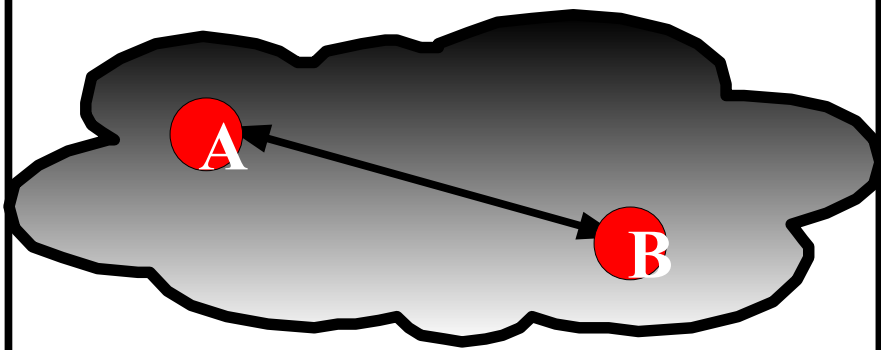
## Many-body vdW energy for a system of coupled oscillators (CFDM)

$$H = -\frac{1}{2} \sum_{i=1}^N \nabla_{\chi_i}^2 + \frac{1}{2} \sum_{i=1}^N \omega_i^2 \chi_i^2 + \sum_{i>j=1}^N \omega_i \omega_j \sqrt{\alpha_i \alpha_j} \chi_i \mathcal{T}_{ij} \chi_j.$$

$$E_{\text{vdW}} = \frac{1}{2} \sum_{p=1}^{3N} \sqrt{\lambda_p} - \frac{3}{2} \sum_{p=1}^N \omega_p^{\text{SCS}},$$

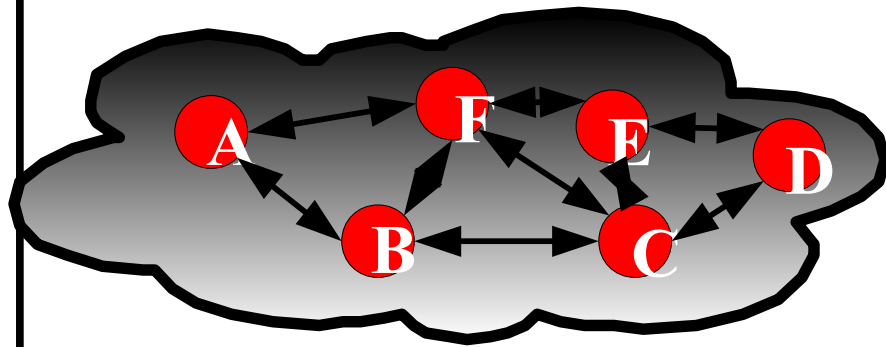
# Many-body dispersion effects

The current state-of-the-art  
(*Grimme, Becke, Tkatchenko/Scheffler, Langreth/Lundqvist, ...*):  
Effective screening and  
two-body energy



Valid for  
small molecules *or*  
homogeneous dielectrics

The new state-of-the-art:  
Full many-body screening  
and energy for a system of  
quantum oscillators



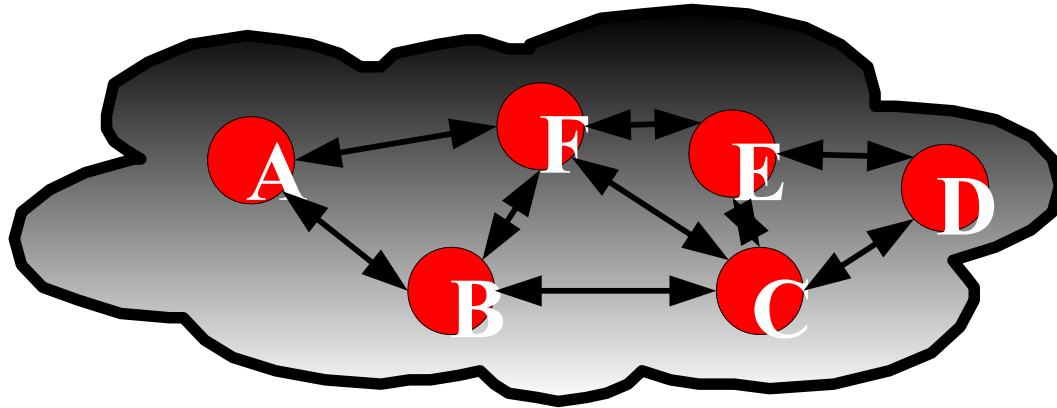
Valid for **small and large molecules, insulators**, metals, interfaces, ...





# Many-body dispersion effects

## Important features of DFT+MBD



Seamless treatment of short-range (quantum) and long-range (classical) screening

Models anisotropy, polarization, and depolarization for non-metallic molecules and solids

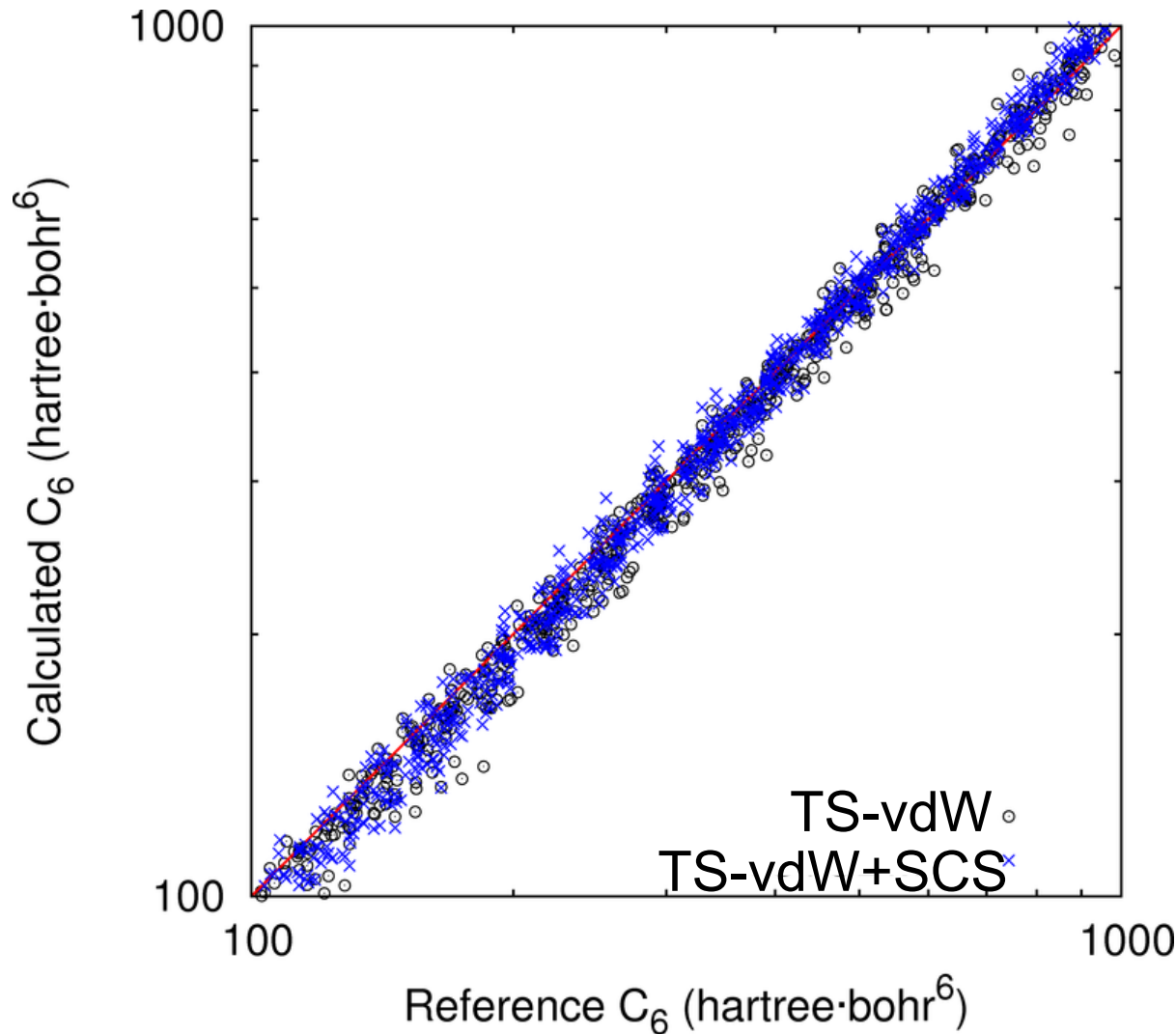
Computes many-body vdW energy to *infinite order* with a single parameter

Negligible computational cost compared to DFT (MBD calculation takes 1 min. for 1000 atoms on 1 processor)



# Many-body dispersion effects

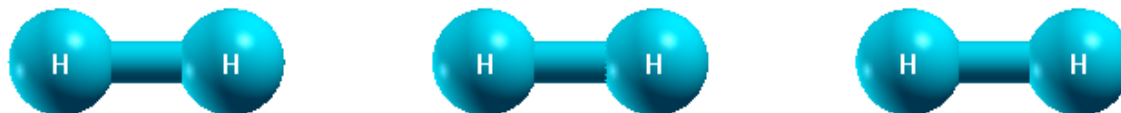
$C_6$  coefficients: **1225** atomic/molecular dimers



$$C_6^{AB} = \frac{3}{\pi} \int \alpha_A(i\omega) \alpha_B(i\omega) d\omega$$

# Many-body dispersion effects

## SCS: Polarization and anisotropy in $H_6$



	$C_{6\perp}$	$C_{6\parallel}$	$C_6^{iso}$	
TS-vdW:	166	161	165	
+SCS:	89	692	223	
<b>LR-CCSD:</b>	<b>115</b>	<b>638</b>	<b>238</b>	<b>(Reference)</b>

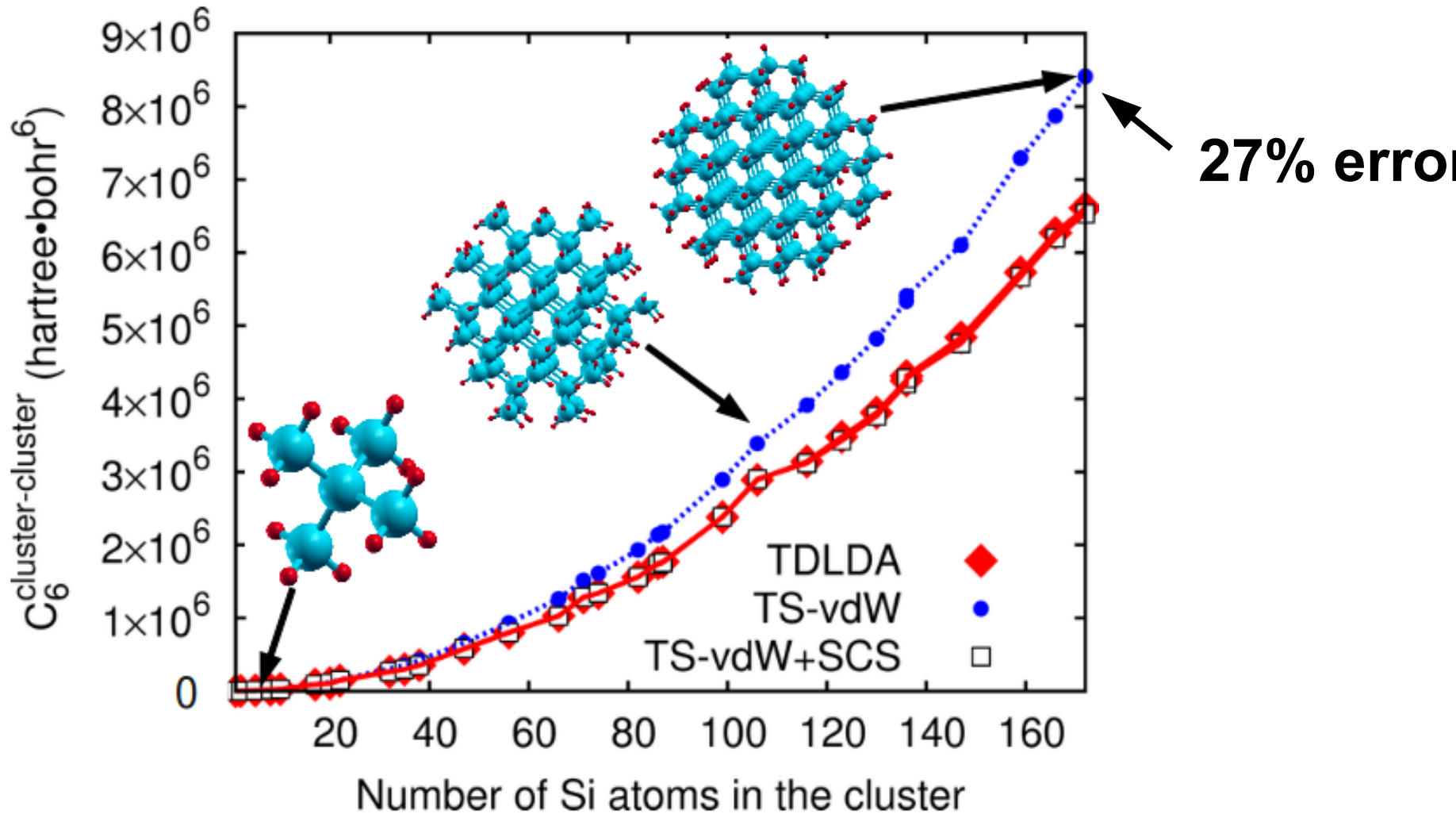
$$C_6^{AA} = \frac{3}{\pi} \int [\alpha_A(i\omega)]^2 d\omega$$

*All values in HartreeBohr<sup>6</sup>*



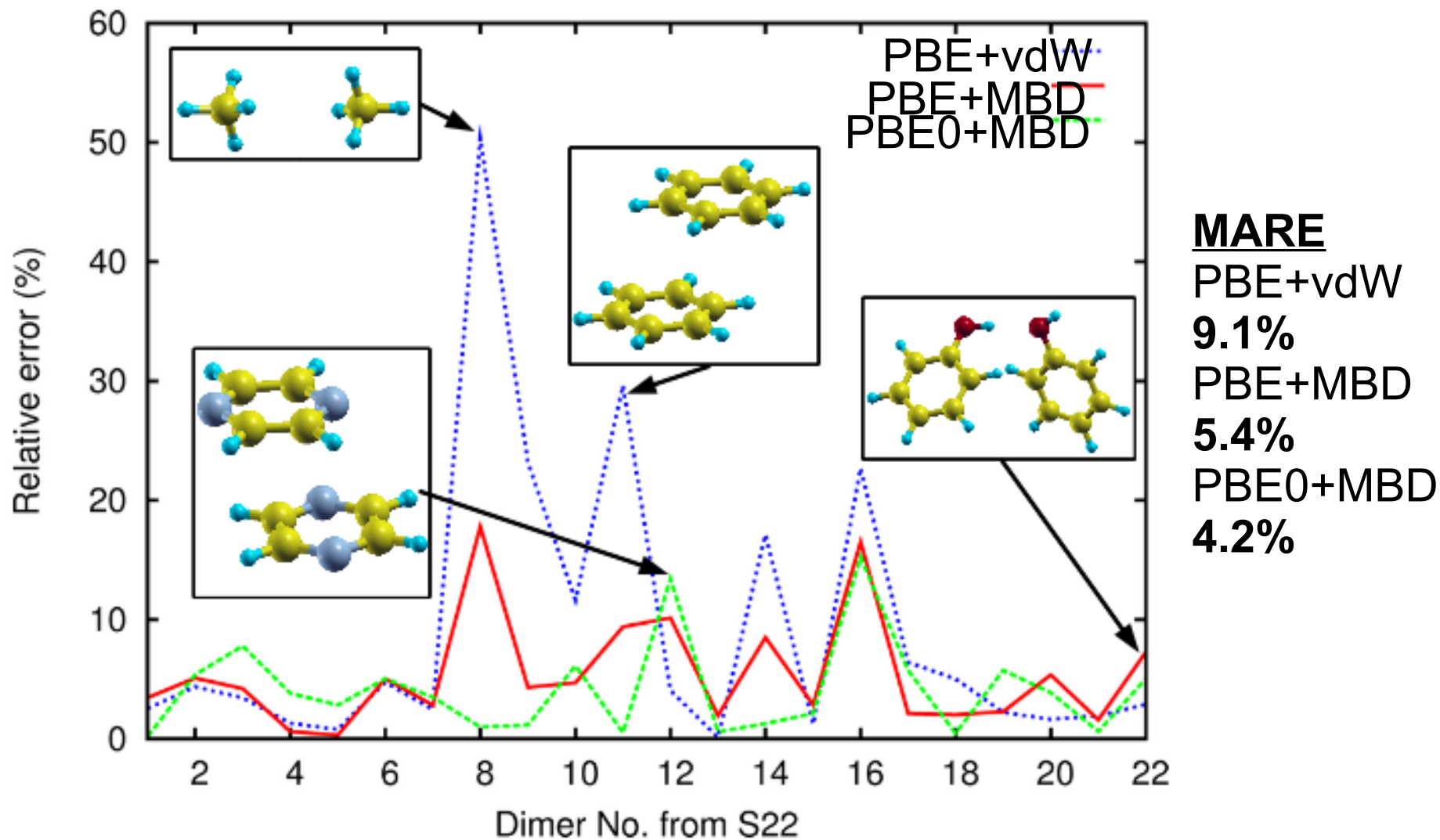
# Many-body dispersion effects

$C_6$  coefficients: from small to large silicon clusters

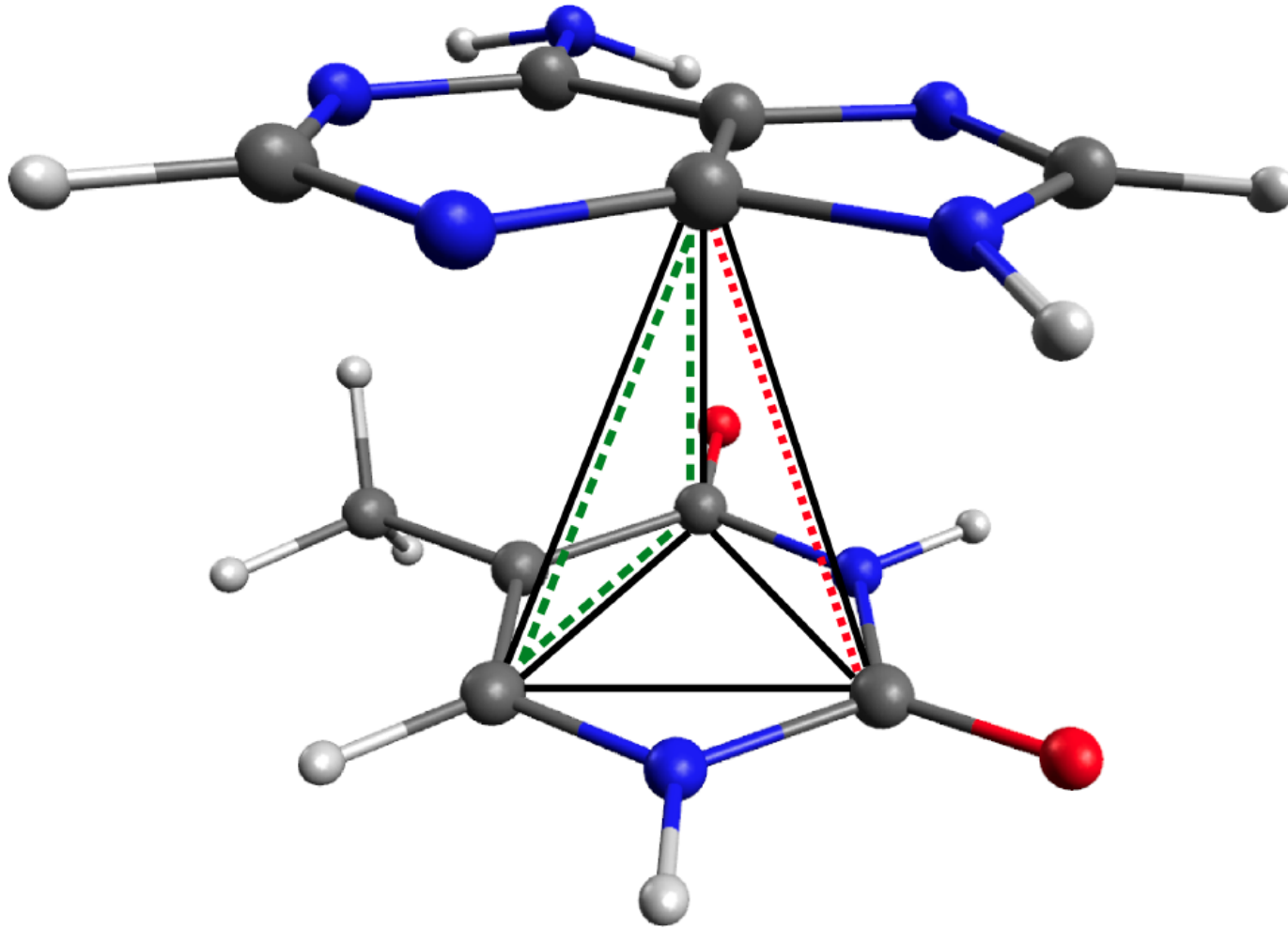


$$C_6^{AA} = \frac{3}{\pi} \int [\alpha_A(i\omega)]^2 d\omega$$

# PBE(0)+MBD: S22 database



# Many-body dispersion effects

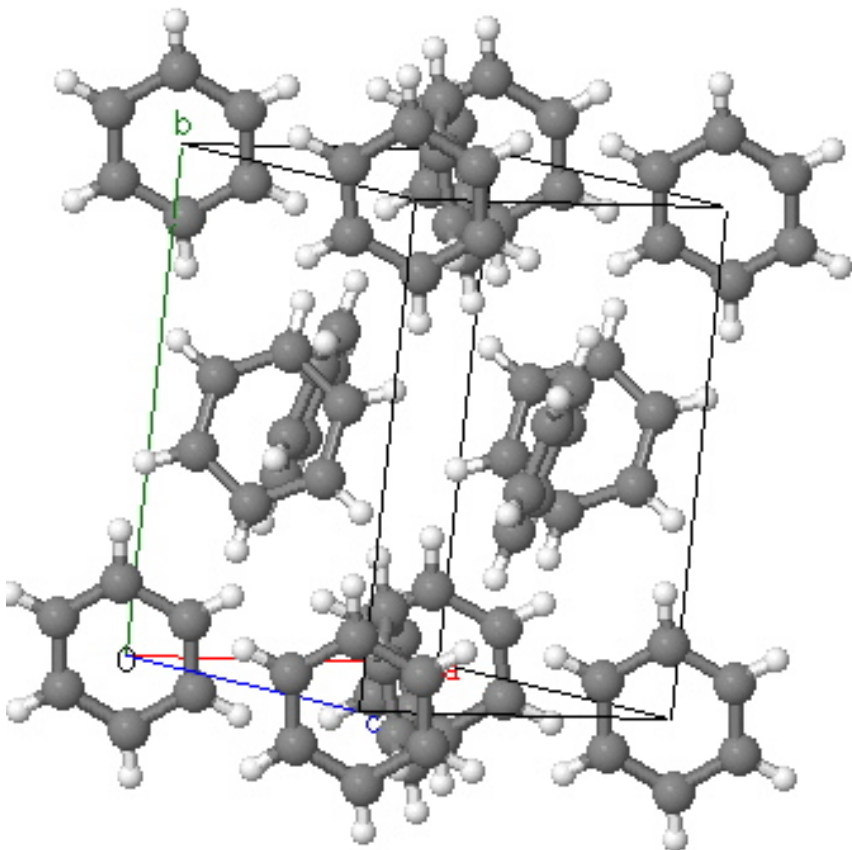


Tkatchenko, von Lilienfeld, DiStasio, submitted to PNAS (2012)



# Many-body dispersion effects

Many-body vdW effects at play: Benzene molecular crystal



**PBE+vdW**

690 meV/molecule

**PBE+MBD**

565 meV/molecule

**Experiment**

518-560 meV/molecule

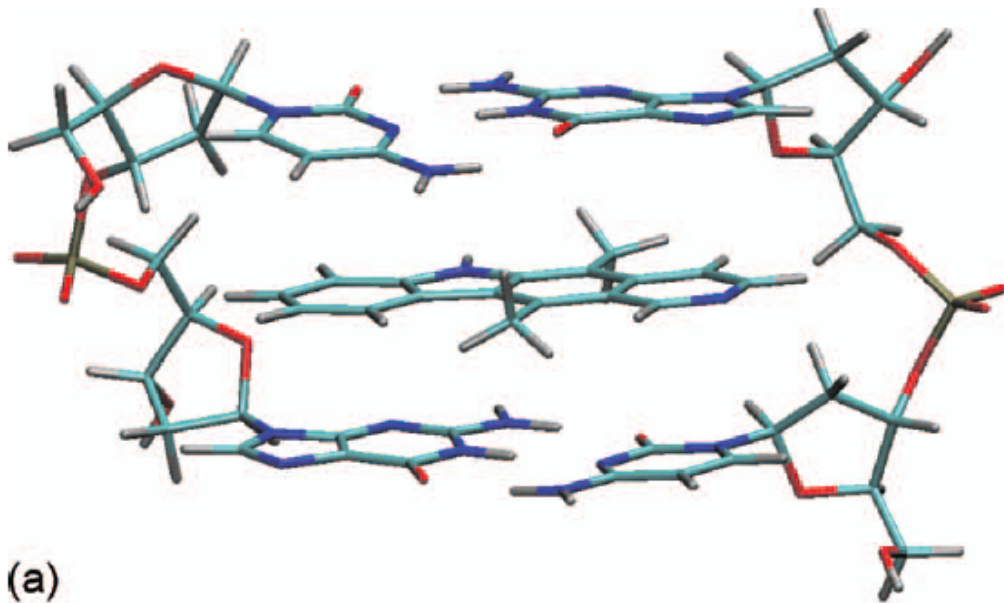
2.9 kcal/mol  
due to screening  
and many-body  
VdW effects





# Many-body dispersion effects

Many-body vdW effects at play: Ellipticine



BLYP+DCACP  
**-37 kcal/mol**

PBE+vdW  
**-46.6 kcal/mol**

PBE+MBD  
**-55.0 kcal/mol**

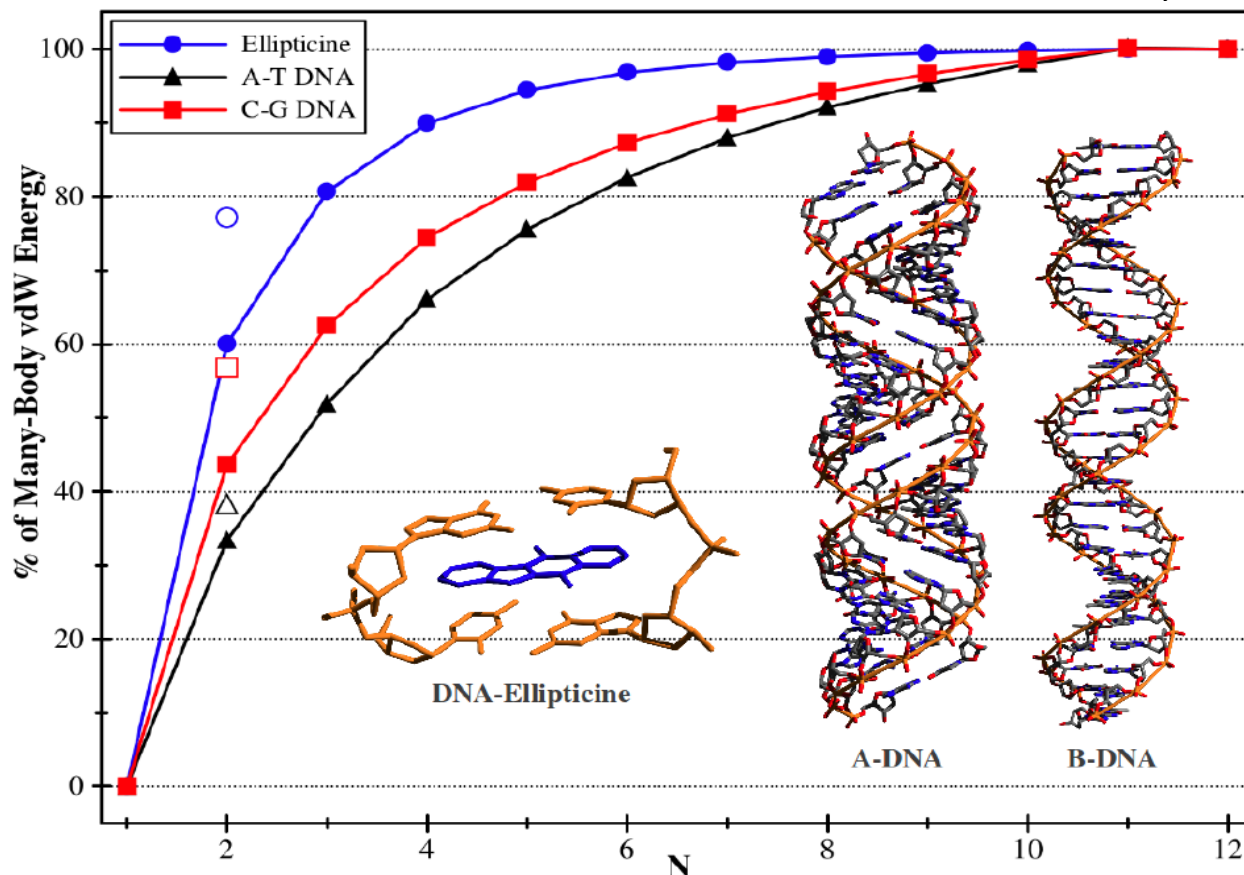
*Lin, von Lilienfeld, Coutinho-Neto, Tavernelli, Rothlisberger, J. Phys. Chem. B* (2007); *von Lilienfeld and Tkatchenko, J. Chem. Phys.* (2010).

Tkatchenko, von Lilienfeld, DiStasio, submitted to PNAS (2012)



# Many-body dispersion effects

Level of Theory	$\Delta E_{\text{bind}}$	$\Delta E_{\text{B-A}}^{\text{A:T}}$	$\Delta E_{\text{B-A}}^{\text{C:G}}$
<b>DFT</b>	+5.2	+4.2	+1.9
<b>vdW-MB(2)</b>	-39.1	+2.6	-3.5
<b>vdW-MB</b>	-50.7	-0.1	-8.2



Tkatchenko, von Lilienfeld, DiStasio, submitted to PNAS (2012)



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# Outlook

BG/Q  
(Mira)

## PETAFLUPS POWER

48 racks  
1,024 nodes per rack  
1.6 GHz 16-way core processor  
16 GB RAM per node  
384 I/O nodes  
240 GB/s, 35 PB storage

That's a total of **768K cores**,  
**768 terabytes** of RAM,  
and a peak performance of  
**10 petaflops**.

The system is capable of carrying out  
**10 quadrillion floating-point**  
**operations per second**.

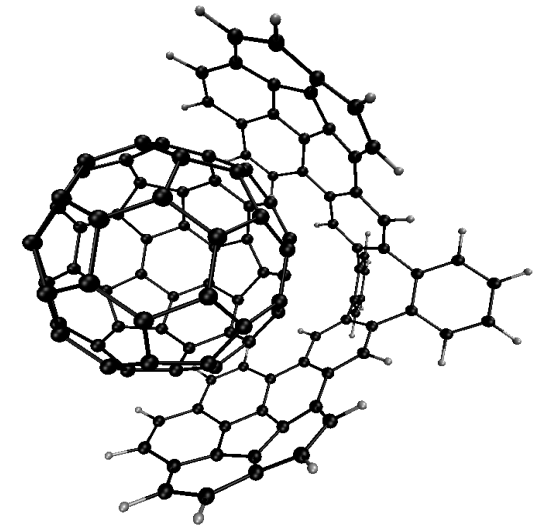


→ Argonne Leadership Computing Facility promotes open science internationally through INCITE program (<http://www.alcf.anl.gov/>)

→ Systems:

- Large and crowded
- Many

→ Methods: High quality (QMC, MBPT, CCSD(T) ...)



# Acknowledgments



Rothlisberger, EPFL



Tavernelli, EPFL



Sebastiani, FU Berlin



Tkatchenko, FHI Berlin



DiStasio, Princeton





# S22 database: Comparison with other approaches

