# Many-Body Perturbation Theory: (3) More Examples and Further Issues

Michael Rohlfing

Fachbereich Physik Universität Osnabrück

MASP, June 29, 2012

- Examples
- Interrelation with geometry



• Expansion of the excitations:

$$|S
angle = {egin{array}{c} \mathsf{hole elec} \ \sum v & \sum c \ \mathbf{k} \end{array} } A^S_{vc\mathbf{k}} \left| vc\mathbf{k} 
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angle$$



 $|vc\mathbf{k}\rangle := \hat{a}_{v\mathbf{k}}^{\dagger} \hat{b}_{c\mathbf{k}+\mathbf{Q}}^{\dagger} |0\rangle$  free electron-hole interband transition

• Bethe-Salpeter equation for  $G_2$  or, resp.,  $|S\rangle$ :

$$(E_{c\mathbf{k}+\mathbf{Q}}^{\mathbf{QP}} - E_{v\mathbf{k}}^{\mathbf{QP}}) A_{vc\mathbf{k}}^{S} + \sum_{v'c'\mathbf{k}'} \langle vc\mathbf{k} | K^{eh} | v'c'\mathbf{k}' \rangle A_{v'c'\mathbf{k}'}^{S} = \Omega_{S} A_{vc\mathbf{k}}^{S}$$

$$E_{v\mathbf{k}}^{\mathbf{QP}}, E_{c\mathbf{k}+\mathbf{Q}}^{\mathbf{QP}} \quad \text{QP energies of the single-particle states} \\ E_{c\mathbf{k}-\mathbf{k}}^{eh} \quad \text{Electron-hole interaction} \\ \Omega_{S} \quad \text{Excitation energy} \quad \text{Excitation energy} \quad \text{Correlation}$$

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#### In here: Quasiparticle (QP) band-structure energies

$$G_1(E) = G_1^{(0)}(E) + G_1^{(0)}(E) \Sigma(E) G_1(E)$$
$$\Sigma(E) = \text{Self-energy operator}$$

**EOM** for a single **quasiparticle** (= excited electron / hole):

 $\left\{-\nabla^2 + V_{\mathsf{ext}} + V_{\mathsf{Coul}}^{[\rho]} + \Sigma(E_{\mathsf{m}}^{\mathsf{QP}})\right\}\psi_{\mathsf{m}}^{\mathsf{QP}} = E_{\mathsf{m}}^{\mathsf{QP}}\psi_{\mathsf{m}}^{\mathsf{QP}}$ 

 $\implies {\rm Quasiparticle\ wave\ functions} \quad \psi^{\rm QP}_{\rm m}({\bf r})$  ${\rm Quasiparticle\ energies} \quad E^{\rm QP}_{\rm m}$ 

 $Re(E_{m}^{QP}) = Energy \ level \ / \ Band-structure \ energy$  $Im(E_{m}^{QP}) = Spectral \ width \ \gamma_{m} \ / \ Inverse \ lifetime$ 





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$$\begin{array}{c} \langle vc\mathbf{k}|K^{eh}|v'c'\mathbf{k}'\rangle = & \iint d\mathbf{x}d\mathbf{x}'\,\psi^*_{c\mathbf{k}+\mathbf{Q}}(\mathbf{x})\psi_{v\mathbf{k}}(\mathbf{x})\,\mathbf{v}(\mathbf{r},\mathbf{r}')\,\psi_{c'\mathbf{k}'+\mathbf{Q}}(\mathbf{x}')\psi^*_{v'\mathbf{k}'}(\mathbf{x}') \\ & -\frac{i}{2\pi}\int d\omega e^{-i\omega 0^+}\iint d\mathbf{x}d\mathbf{x}'\,\psi^*_{c\mathbf{k}+\mathbf{Q}}(\mathbf{x})\psi_{c'\mathbf{k}'+\mathbf{Q}}(\mathbf{x})\,W(\mathbf{r},\mathbf{r}',\omega)\,\psi_{v\mathbf{k}}(\mathbf{x}')\psi^*_{v'\mathbf{k}'}(\mathbf{x}') \\ & \times \left[\frac{1}{\Omega_S-\omega-(E^{\mathsf{QP}}_{c'\mathbf{k}'+\mathbf{Q}}-E^{\mathsf{QP}}_{v\mathbf{k}})+i0^+}+\frac{1}{\Omega_S+\omega-(E^{\mathsf{QP}}_{c\mathbf{k}+\mathbf{Q}}-E^{\mathsf{QP}}_{v'\mathbf{k}})+i0^+}\right] \end{array}$$

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#### Relevance of molecular level alignment for electron conductance





- QP corrections move molecular states away from Fermi level
  - $\implies$  reduced transmission at  $E_F$
- $\bullet$  Include image-state effect in  $\Sigma$  !

[S.Y. Quek, H.J. Choi, S.G. Louie, and J.B. Neaton, Nano Lett. 9, 3949 (2009).]

# **Image-Potential Effects**

- Electron outside metal surface (at *z*) polarizes the metal substrate
  - $\implies$  Same field distribution as from an image charge
- $\implies$  attractive potential  $V(z) = -\frac{e^2}{4z}$
- Modification of empty-state energies by  $-\frac{e}{4z}$
- Opposite effect on occupied states: +
- Also valid in front of non-metals:  $\pm \frac{\epsilon_0 1}{\epsilon_0 + 1} \frac{e^2}{4z}$
- Automatically included in GW self energy !
- M. Rohlfing et al., PRL 91, 256802 (2003).
  J.B. Neaton et al., PRL 97, 216405 (2006).
  K.S. Thygesen and A. Rubio, PRL 102, 046802 (2009).
- J.M. Garcia-Lastra and K.S. Thygesen, PRL 106, 187402 (2011).



## **PTCDA:** electronic spectrum





Metal surface reduces the fundamental gap

## **The C**<sub>2</sub>**H**<sub>4</sub>:**Si(001)-(2x1) surface**



## **The C**<sub>2</sub>**H**<sub>4</sub>:**Si(001)-(2x1) surface**



# LDA+*GdW* Electronic spectrum of the C2H4:Si(001)-(2x1) surface



Exp.: W. Widdra et al., PRL 80, 4269 (1998).

#### **Excitons in Semiconducting Carbon Nanotubes**





• Anisotropic e-h wave function

[C.D. Spataru, S. Ismail-Beigi, L.X. Benedict, and S.G. Louie, PRL 92, 077402 (2004).]

## **Red-shift of CNT Excitons from Physical Environment**



P. Finnie et al., PRL 94, 247401 (2005): CNT in He / Air / N<sub>2</sub> (at room temp.)



- Red-shifts of 20-50 meV
- Band-gap renormalization?
- Electronic coupling between excitons?
- Environmental polarizability?

## Polarizability effect from a neighboring tube



# Polarizability effect from a neighboring tube (2)



#### Exciton on tube 1:

Effective charge of the exciton

 $\Delta \rho^{[Exc]}(\mathbf{r}) := \rho_v(\mathbf{r}) - \rho_c(\mathbf{r})$ 

(Slightly positive inside, negative outside)

- $(c) \qquad \Delta \rho^{[ind]}(\mathbf{r}) \\ (1) \qquad \mathbf{2} \\ \mathbf{2$ 
  - Induced charge on tube 2:Positive in contact area,Negative elsewhere $\implies$  Interaction  $\implies$  Redshift
- Significant effect for touching systems, even without dipoles
- Should always lead to redshift















[A. Alexandrov et al., PRL 86, 536 (2001)]











# Si(111)-(2×1): Differential Reflectivity Spectrum (DRS)



# Si(111)-(2×1): Geometry and electronic structure

- Full treatment: Coupling to all phonon modes  $\longrightarrow$  very demanding.
- Instead: Coupling to that deformation mode which is created by the exciton itself
  - $\longleftrightarrow$  Relaxation in the excited state





# Si(111)-(2×1) Surface Exciton: DRS Line Width

• Vibrational sublevels ( $\sim 25~meV$ )

Franck-Condon factors / temperature dependent

- $\implies$  Line broadening
- plus: inhomogeneous broadening / further modes
  - $\implies$  Vibronic lines cannot be resolved.





• • Exp.: Ciccacci et al., PRL 56, 2411 (1986).

# A conjugated polymer: PPP / Poly-(Para-Phenylene) (C<sub>6</sub>H<sub>4</sub>)<sub>n</sub>

[E. Artacho, M. Rohlfing, M. Côté, P.D. Haynes, R.J. Needs, C. Molteni, PRL 93, 116401 (2004). ]



- Small H-H distance (1.9 Å)  $\implies$  Repulsion / Twisting
- Delocalized  $\pi$ -electron system
- Semiconducting band structure
- Exciton at 3.6 eV



Ground-state geometry:



**HOMO** / **VBM** (at Γ):



LUMO / CBM (at  $\Gamma$ ):



HOMO $\rightarrow$ LUMO transition: favours planar,  $\pi$ -like "A" bond  $\implies$  Exciton causes "planarization" of the polymer and shortening of "A" bond

Excited-state geometry: **O Exciton wave function:**

# **PPP: Exciton self trapping / relaxation in the excited state**

#### Locally (in one unit cell): DFT + MBPT

Excited state has a different band occupation / charge distribution

- $\implies$  Structural relaxation
- $\implies$  gap is lowered /

excitation energy becomes smaller:



#### This drives the relaxation !

#### Globally (entire polymer): constrained DFT

Self-trapped exciton [balence between kin. energy of the exciton and relaxation energy]



## **PPP: Relaxation / Stokes Shift**



- **1.** Optical absorption:  $\Omega = 3.5 \text{ eV}$  (MBPT)
- 2. Relaxation in the excited state:  $\Delta E = 0.2 \text{ eV}$  (constrained DFT)
- 3. "Lattice energy" in the ground state:  $\Delta E' = 0.2 \text{ eV}$  (DFT)

 $\implies$  Emission (4.):  $\Omega' = 3.1 \text{ eV}$ ; Stokes shift = 0.4 eV

## **Point defects** — **Example: F Center in CaF**<sub>2</sub>

[Y. Ma and M.R., Phys. Rev. B 77, 115118 (2008).]



- F center in CaF<sub>2</sub>: Missing F atom
- Charge neutral  $\iff$

Vacancy hosts one unpaired electron

- Spin-polarized GW/BSE approach
- $\bullet$  Deep defect level  $1s\uparrow$
- "2s", "2p": Strong hybridisation with bulk bands

Cf. Surh, Chacham, Louie, PRB 51, 7464 (1995); T Rohlfing, Louie, PRB 51, 2312 (1998): bulk LiF; Ma



Tiago, Chelikowsky, PRB 73, 2053 (2006); Ma, Rohlfing, PRB 75, 205114 (2007): bulk  $CaF_2$ 

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[Y. Ma and M.R., Phys. Rev. B 77, 115118 (2008).]



Exp.: Arends, PSS 7, 805 (1964); Patterson, Fuller, PRL 18, 1123 (1967)

### **Desorption of Iodine Atoms from Optically Excited KI**

Iodine emission from KI after laser excitation: [A. Alexandrov et al., PRL 86, 536 (2001)]



#### Possible mechanism:





#### **Desorption of lodine atoms from optically excited KI**



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[ C. Carbogno, A. Groß, and M. Rohlfing, Applied Physics A 88, 579 (2007).]

