

# Finite temperature calculations of the electronic and optical properties of solids and nanostructures: the role of electron-phonon coupling from an Ab-Initio perspective



**Andrea Marini**

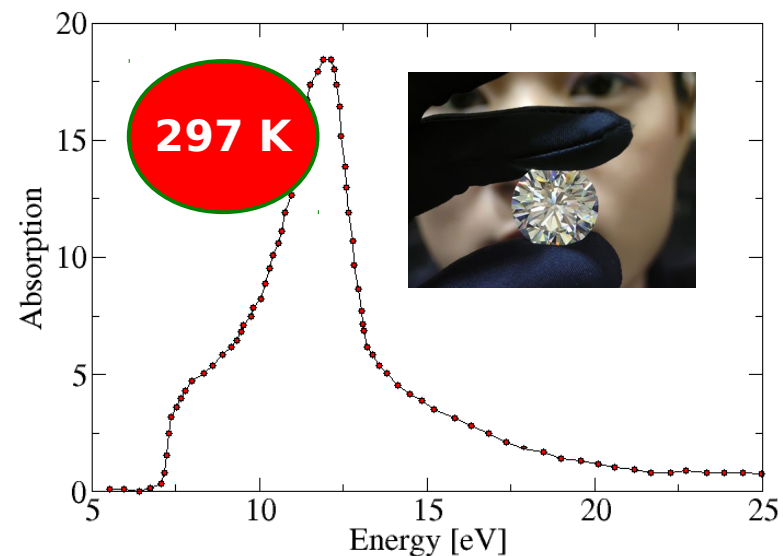
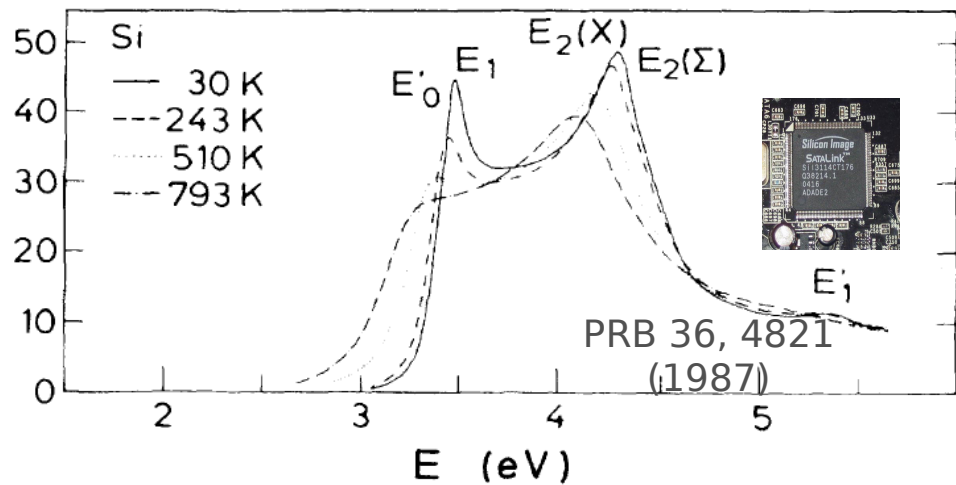
National Research Council (CNR), Italy

June 29<sup>th</sup> 2012, Tokyo

Yambo 



# Real life is at finite temperature (I)



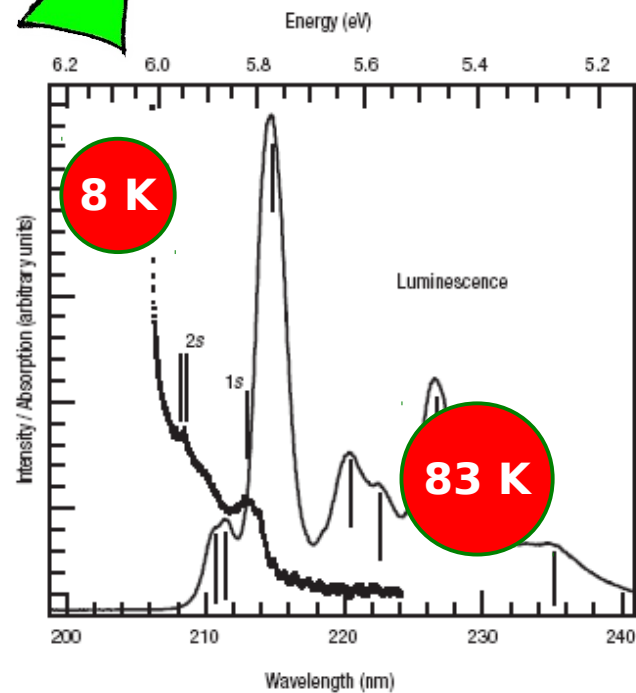
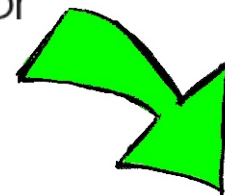
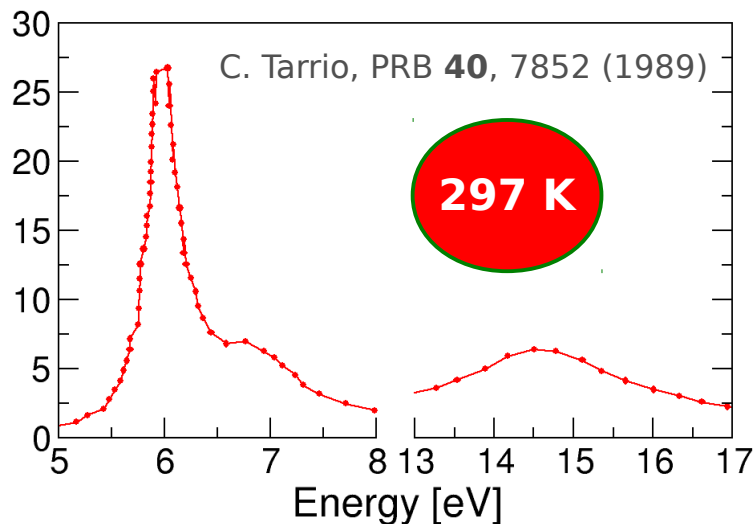
## ARTICLES

nature materials | VOL 3 | JUNE 2004 | www.nature.com/naturematerials

Direct-bandgap properties and evidence for ultraviolet lasing of hexagonal boron nitride single crystal

KENJI WATANABE\*, TAKASHI TANIGUCHI AND HISAO KANDA

Advanced Materials Laboratory, National Institute for Materials Science, 1-1 Namiki, Tsukuba, 305-0044, Japan  
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# Real life is at finite temperature (II)

PHYSICAL REVIEW B

VOLUME 36, NUMBER 9

15 SEPTEMBER 1987-II

## Temperature dependence of the dielectric function and interband critical points in silicon

P. Lautenschlager, M. Garriga, L. Viña,\* and M. Cardona

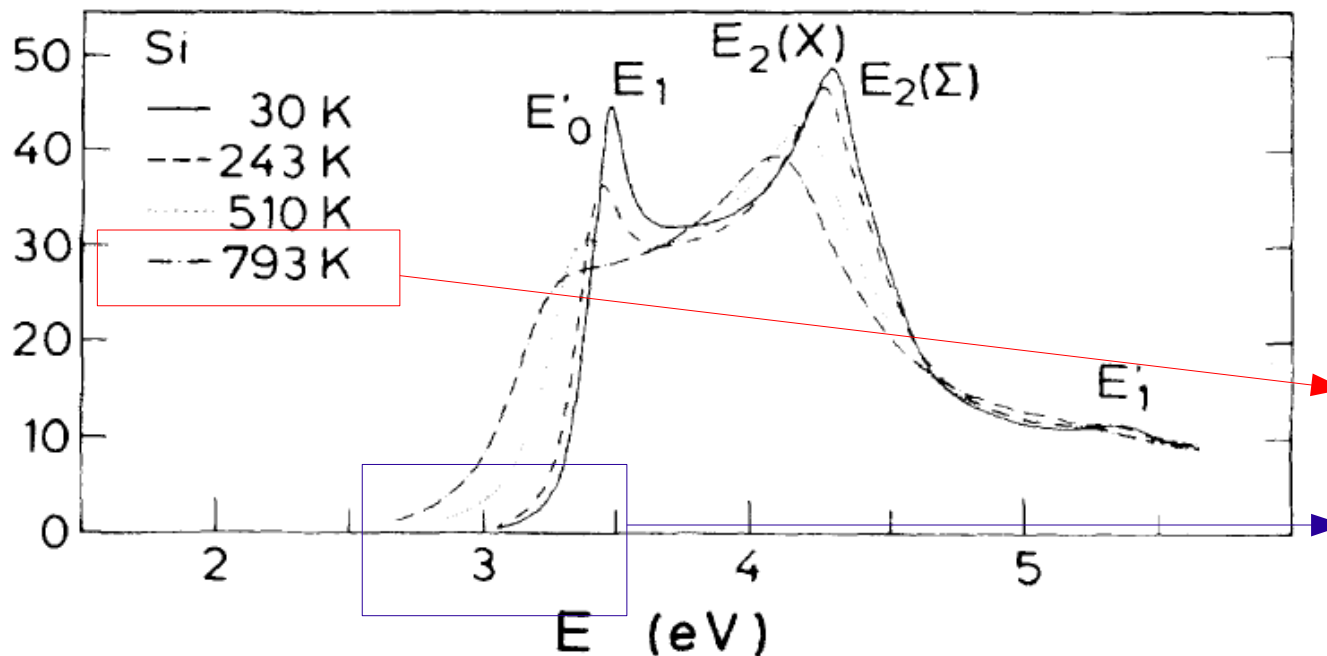
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany

(Received 30 April 1987)

The complex dielectric function  $\epsilon(\omega)$  of Si was measured ellipsometrically in the 1.7–5.7-eV photon-energy range at temperatures between 30 and 820 K. The observed structures are analyzed by fitting the second-derivative spectrum  $d^2\epsilon/d\omega^2$  with analytic critical-point line shapes. Results for the temperature dependence of the parameters of these critical points, labeled  $E'_0$ ,  $E_1$ ,  $E_2$ , and  $E'_1$ , are presented. The data show good agreement with microscopic calculations for the energy shift and the broadening of interband transitions with temperature based on the electron-phonon interaction. The character of the  $E_1$  transitions in semiconductors is analyzed. We find that for Si and light III-V or II-VI compounds an excitonic line shape represents best the experimental data, whereas for Ge,  $\alpha$ -Sn, and heavy III-V or II-VI compounds a two-dimensional critical point yields the best representation.



**“... unfortunately theorists do not even bother to compare their calculations with low-temperature measurements, using more easily accessible room temperature spectra.”**



**793 K = 68 meV**  
QP gap is 1200 meV

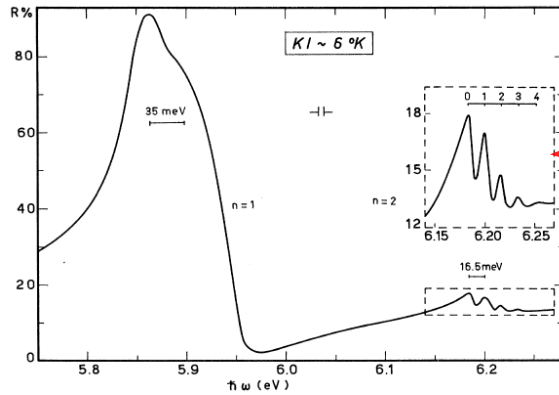
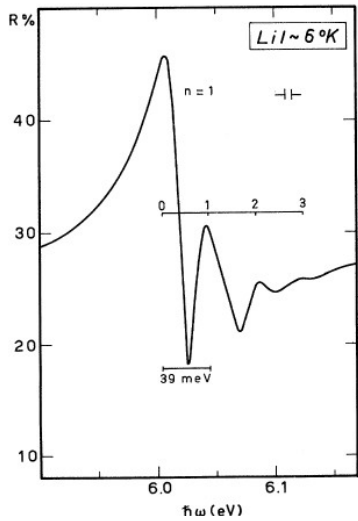


At the absorption threshold the QP lifetime is EXACTLY infinite



**YES ! We do need phonons.**

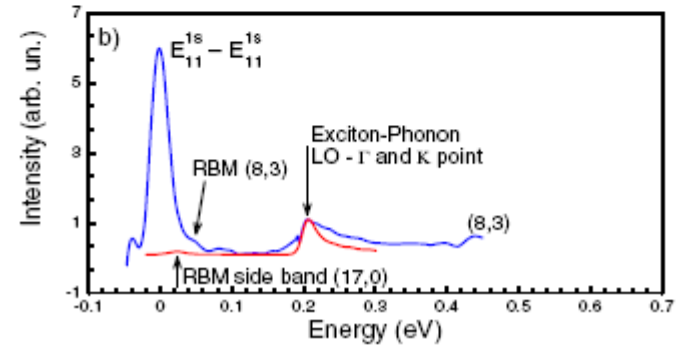
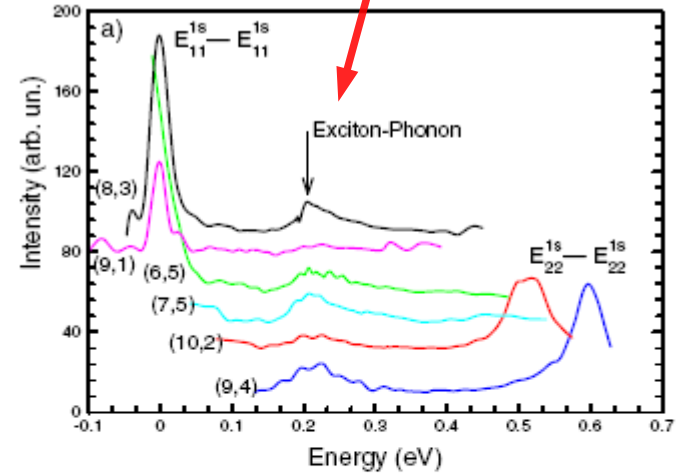
# Real life is at finite temperature (III)



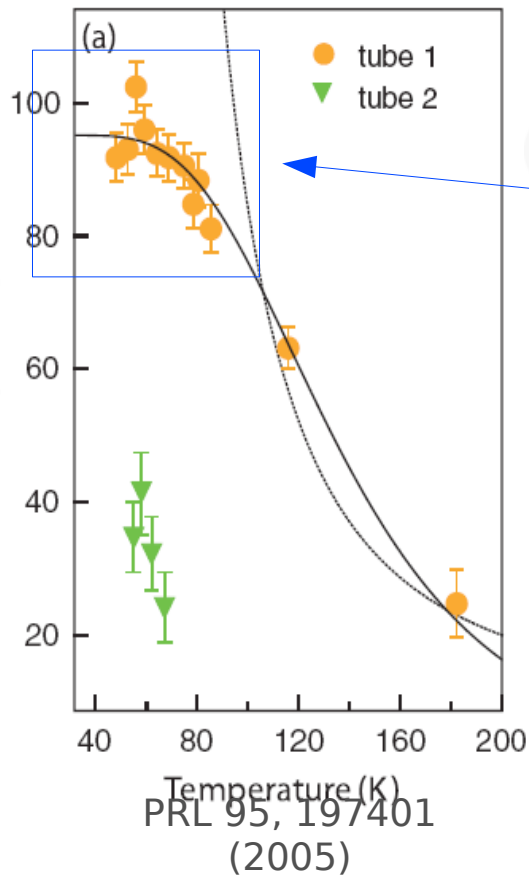
G. Baldini, A. Bosacchi, and B. Bosacchi, PRL, **23**, 846 (1969)



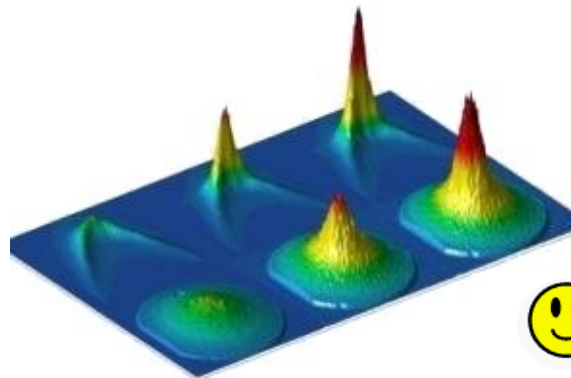
phonon sidebands



PRL 95, 247401 (2005)

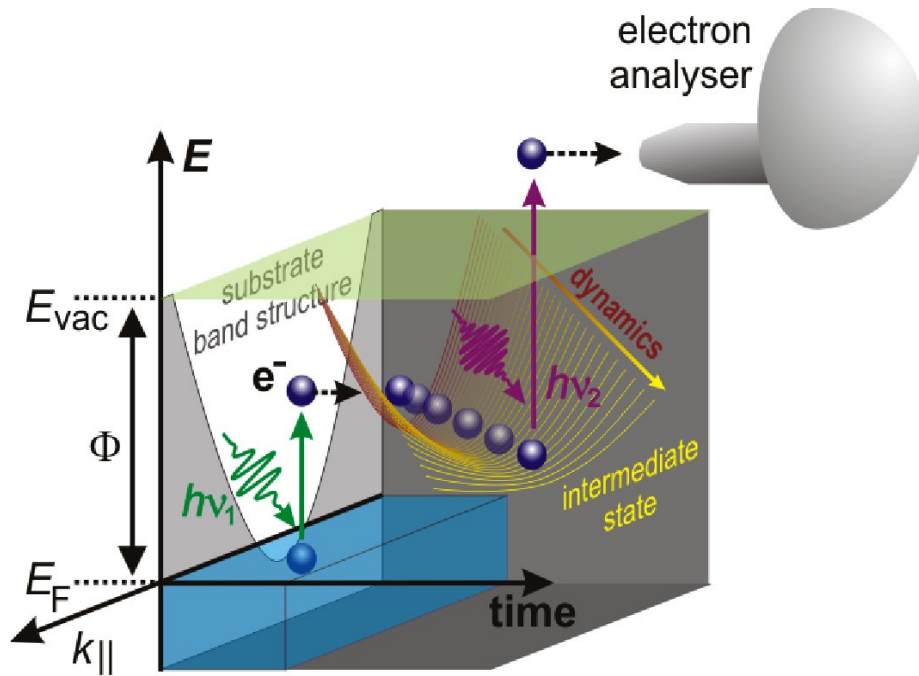


Phonons are responsible for the low-temperature saturation of the excitonic lifetime



There are many different "flavours" of excitons: polaritons, polaronic excitons... whose physical properties are influenced by the exciton-phonon interaction

# Electron-phonon today



1

An ultra-short laser pulse pumps electrons in the conduction

2

The non-thermal electronic distribution relaxes via e-e and e-ph scatterings

3

The electronic population is probed after varying delays with photoemission

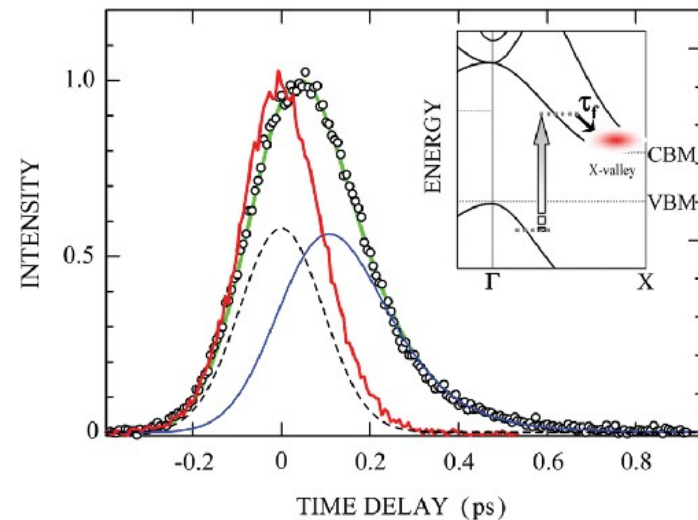
PRL 102, 087403 (2009)

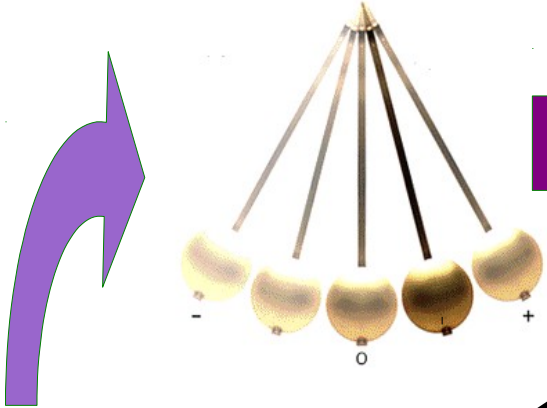
PHYSICAL REVIEW LETTERS

week ending  
27 FEBRUARY 2009

Ultrafast Carrier Relaxation in Si Studied by Time-Resolved Two-Photon Photoemission Spectroscopy: Intravalley Scattering and Energy Relaxation of Hot Electrons

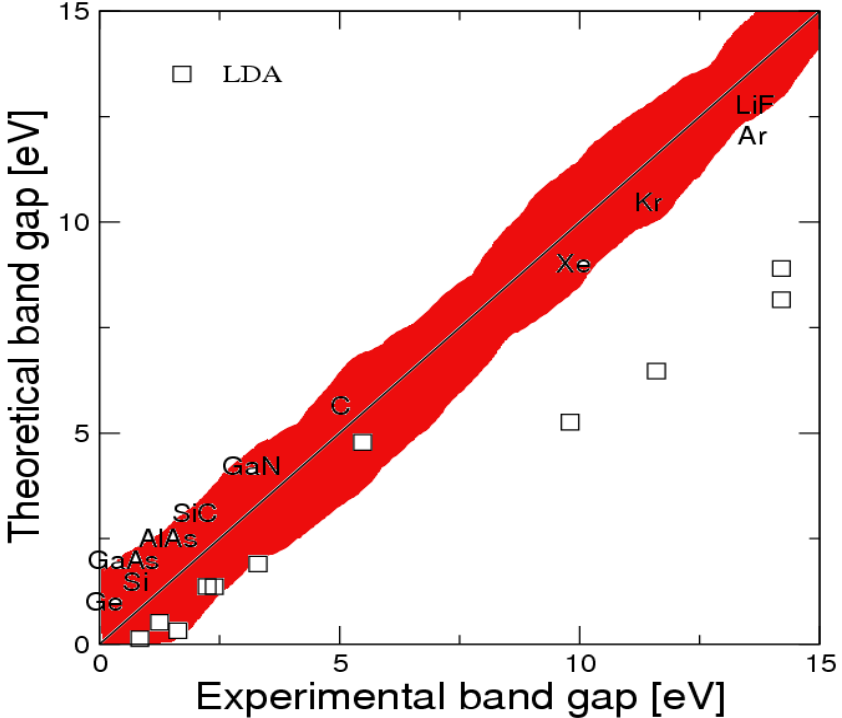
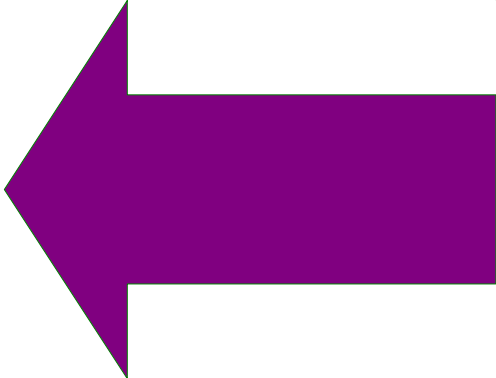
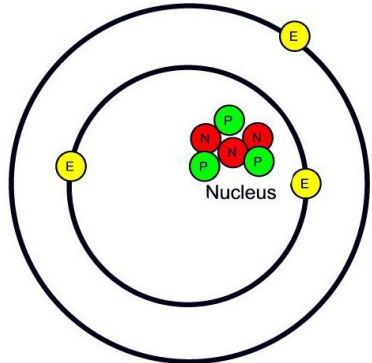
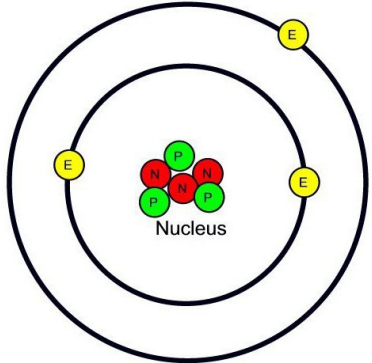
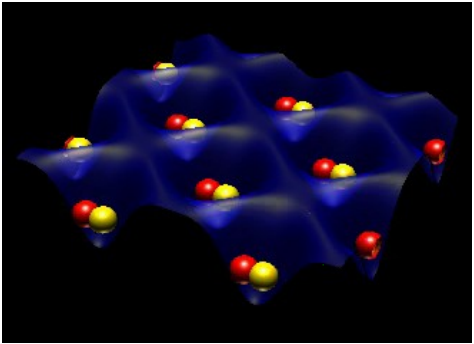
*See Monday's talk!*



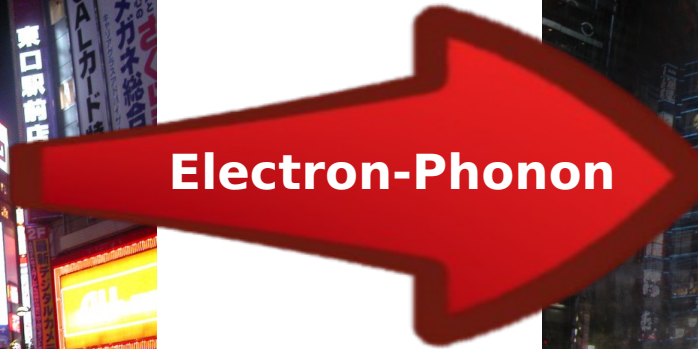


$$\langle u^2(T) \rangle \approx \frac{\hbar}{4M\Omega} \langle 1 + 2N_{Bose}(T) \rangle$$

The quantistic zero-point motion effect



# THE Motivation

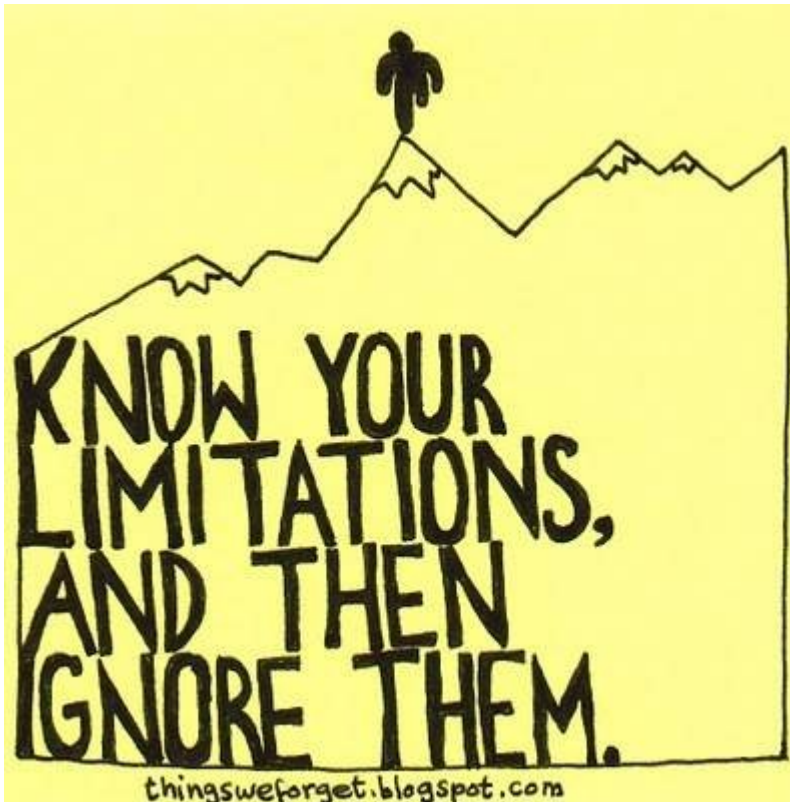


Are purely  
electronic theories  
reliable ???

# THE limitations...

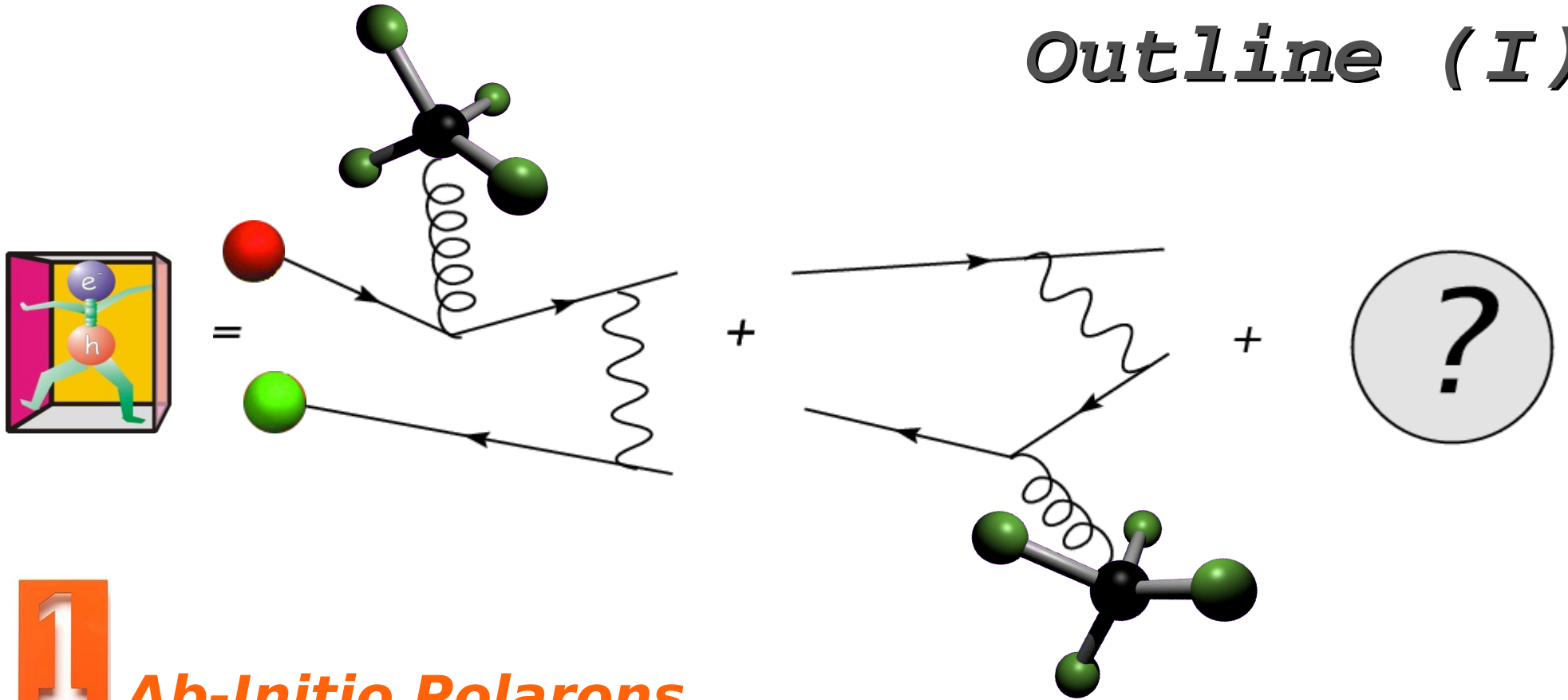
## In this talk ...

- I will not extensively discuss formal aspects (vertex, conserving props., ...)
- I will (try) not do extensive and heavy math (see references for that)
  - I will not talk about superconductivity, Fröhlich or Holstein Hamiltonians
  - I will use LDA or GGA, nothing more complicated (including SC-GW)





# Outline (I)



**1**

***Ab-Initio Polarons***

**2**

***Finite temperature excitons***

**3**

***Spectral functions and the QP-approximation***

## 1

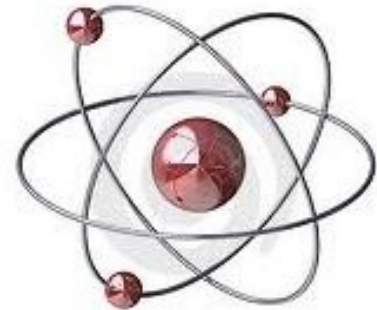
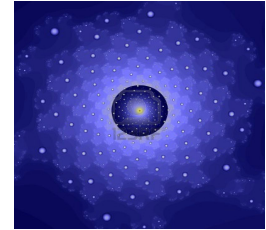
### **Ab-Initio Polarons**

*The Heine-Allen-Cardona Approach (static)*

*The Hedin-Lundqvist approach (dynamical)*

*The Diagrammatic approach (dynamical)*

*Density Functional Perturbation Theory*



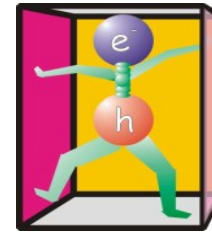
## 2

### **Finite temperature excitons**

*The Bethe-Salpeter equation in the polaronic basis*

*A phonon induced kernel for the Bethe-Salpeter equation*

*Finite temperature optical spectra of solids*

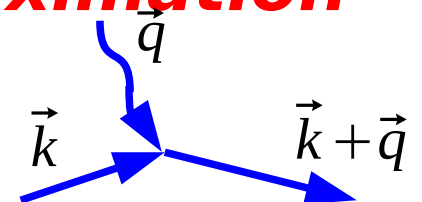


## 3

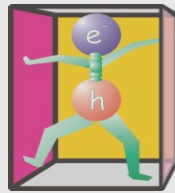
### **Spectral functions and the QP-approximation**

*Quasiparticles and spectral functions*

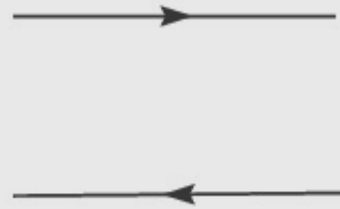
*Polarons as entangled electron-phonon states*



*Independent QPs*

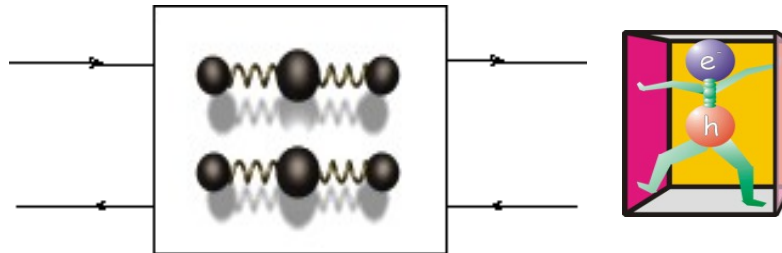
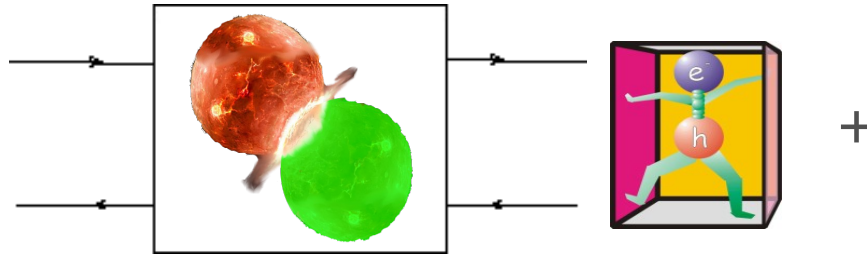
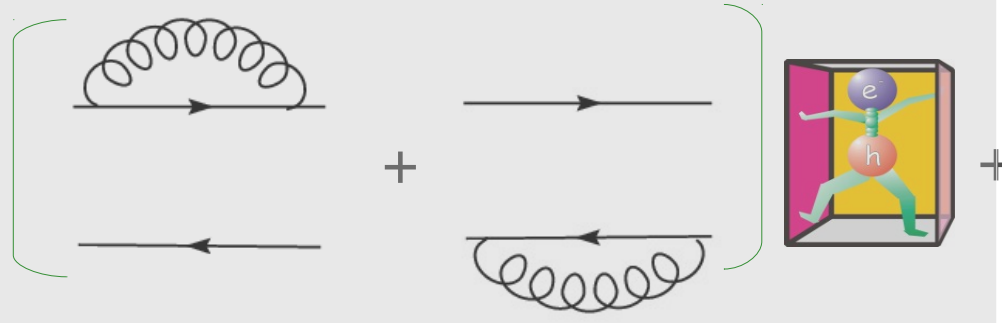


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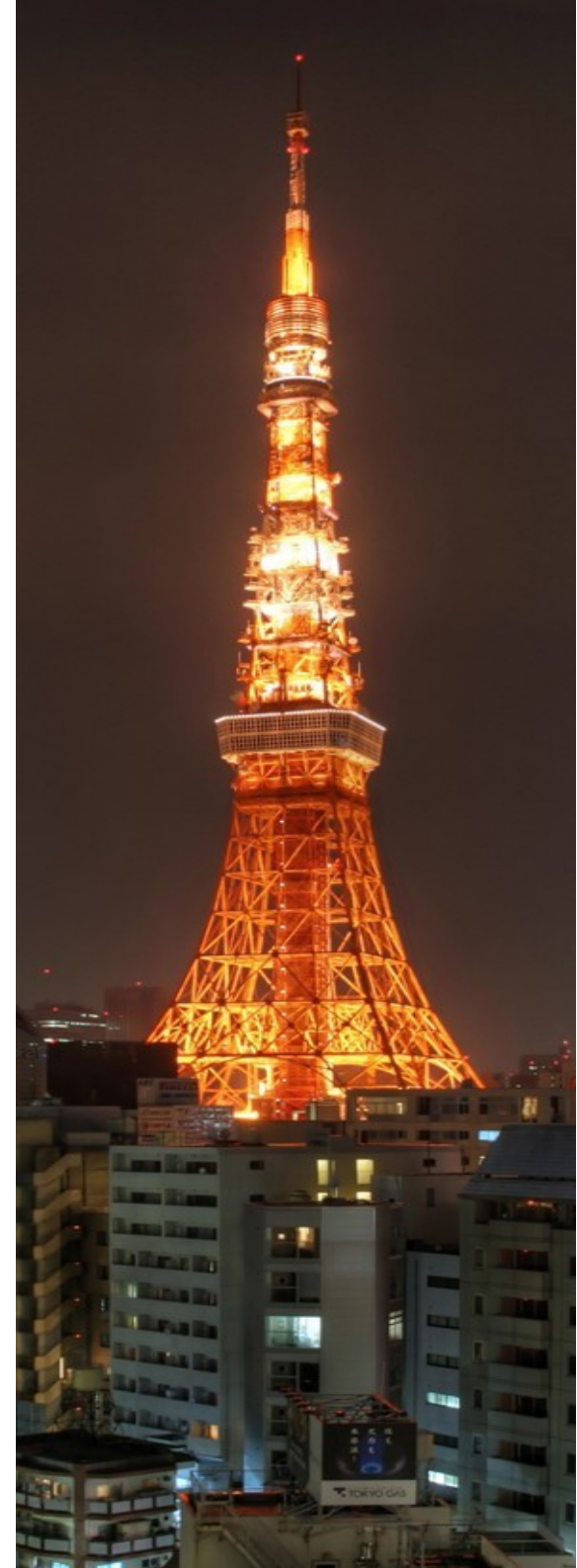


+

**“Indirect”  
exciton-phonon  
scattering**



**Ab-Initio Polarons**



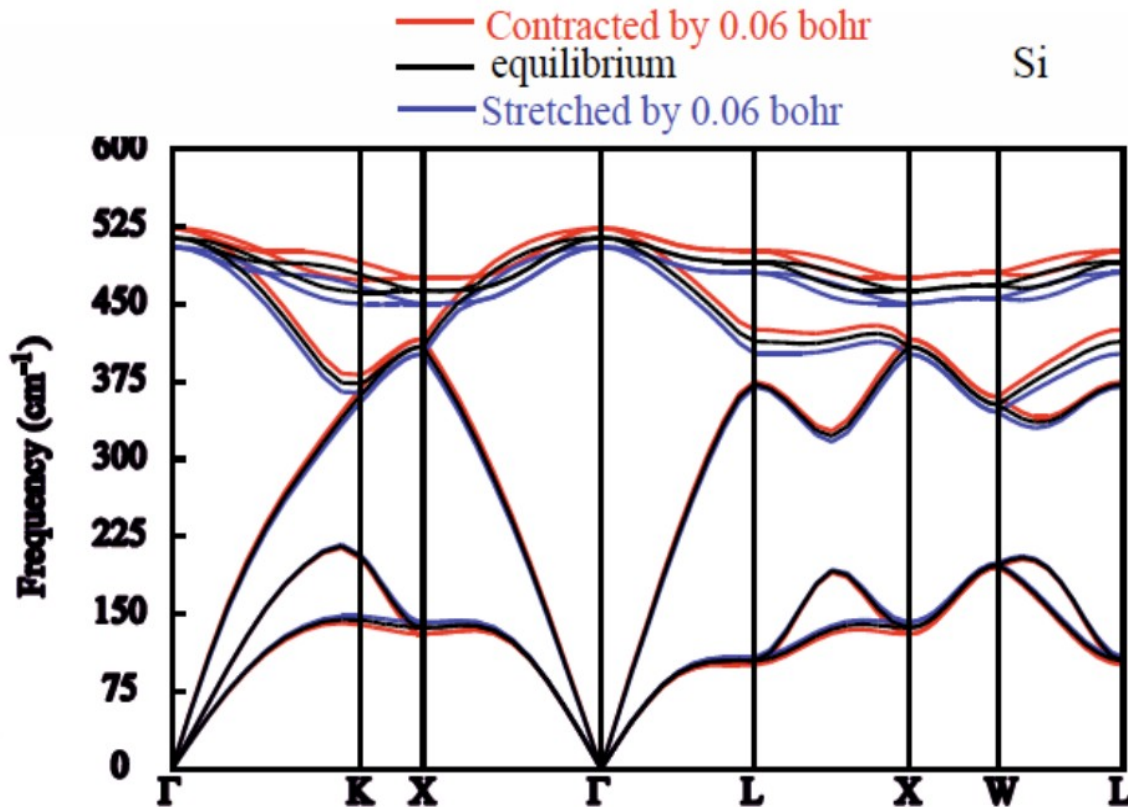
# The quasi-harmonic approximation

$$\left(\frac{\partial \varepsilon_{n\bar{k}}}{\partial T}\right)_P = \left(\frac{\partial \varepsilon_{n\bar{k}}}{\partial T}\right)_V + \left(\frac{\partial \varepsilon_{n\bar{k}}}{\partial \ln V}\right)_T \alpha_P(T)$$

Phonon population



Thermal expansion



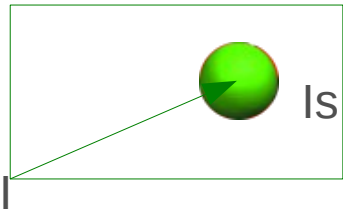
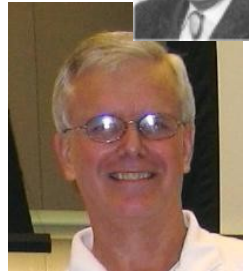
$$\gamma_{m\bar{q}} = -\frac{\partial(\ln \omega_{m\bar{q}})}{\partial(\ln V)} \quad \text{Grüneisen parameters}$$

$$\alpha(T) = \frac{V}{3B} \sum_{\bar{q}, m} \frac{1}{\hbar \omega_{m\bar{q}}} \gamma_{m\bar{q}} \frac{\partial n(\omega_{m\bar{q}})}{\partial T}$$

It can be easily calculated using DFT and finite-differences

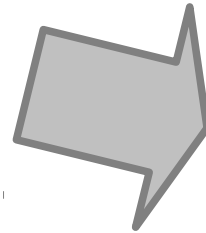
# The Heine-Allen-Cardona Approach (I)

For a review see M. Cardona, Solid State Commun. **133**, 3 (2005).



$$H = T + V_{SCF}(\{\mathbf{R}_{Is}\})$$

$$\mathbf{R}_{Is} = \mathbf{R}_{Is} + \mathbf{u}_{Is}$$



$$\delta H = \delta H^{(1)} + \delta H^{(2)}$$

$$\delta H^{(1)} = \sum_{Is} \frac{\partial V_{SCF}}{\partial \mathbf{R}_{Is}} \mathbf{u}_{Is}$$

$$\delta H^{(2)} = \frac{1}{2} \sum_{IsJt} \frac{\partial^2 V_{SCF}}{\partial \mathbf{R}_{Is} \partial \mathbf{R}_{Jt}} \mathbf{u}_{Is} \mathbf{u}_{Jt}$$

Using standard **1<sup>st</sup> and 2<sup>nd</sup> order** perturbation theory and the fact that  $E_{n\mathbf{k}} \equiv \langle n\mathbf{k} | H | n\mathbf{k} \rangle$

$$\delta E_{n\mathbf{k}} = \sum_{IsJt} \left[ \frac{1}{2} \left\langle \frac{\partial^2 V_{SCF}}{\partial \mathbf{R}_{Is} \partial \mathbf{R}_{Jt}} \right\rangle + \sum_{m\mathbf{p}} (E_{n\mathbf{k}} - E_{m\mathbf{p}})^{-1} \left\langle \frac{\partial V_{SCF}}{\partial \mathbf{R}_{Is}} \middle| m\mathbf{p} \right\rangle \left\langle m\mathbf{p} \middle| \frac{\partial V_{SCF}}{\partial \mathbf{R}_{Jt}} \right\rangle \right] \mathbf{u}_{Is} \mathbf{u}_{Jt}$$

Debye-Waller
Fan

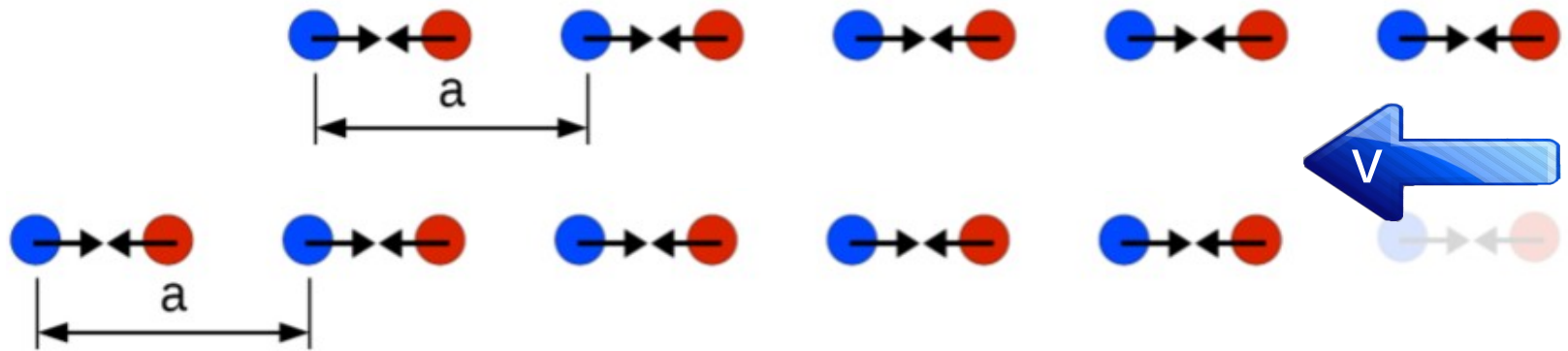
Now we can rewrite the displacement operator using the canonical operators

$$\sum_{Is} \mathbf{u}_{Is} \langle n'\mathbf{k} + \mathbf{q} | \frac{\partial V_{SCF}}{\partial \mathbf{R}_{Is}} | n\mathbf{k} \rangle = \sum_{\mathbf{q}\lambda} g_{n'n\mathbf{k}}^{\mathbf{q}\lambda} (b_{\mathbf{q}\lambda}^\dagger + b_{\mathbf{q}\lambda})$$

$$\delta E_{n\mathbf{k}} = \sum_{\mathbf{q}\lambda m} \frac{|g_{n'n\mathbf{k}}^{\mathbf{q}\lambda}|^2}{E_{n\mathbf{k}} - E_{m\mathbf{k}+\mathbf{q}}} (2N_{\mathbf{q}\lambda} + 1)$$

$$\langle (b_{\mathbf{q}\lambda}^\dagger + b_{\mathbf{q}\lambda})(b_{\mathbf{q}\lambda}^\dagger + b_{\mathbf{q}\lambda}) \rangle = 2N_{\mathbf{q}\lambda} + 1$$

# The Heine-Allen-Cardona Approach (II)



By imposing an *“acoustic sum rule”*  $\delta E_{n\mathbf{k}} [\{\mathbf{u}_{I_s} + \mathbf{v}\}] = \delta E_{n\mathbf{k}} [\{\mathbf{u}_{I_s}\}]$

$$\left\langle \frac{\partial^2 V_{SCF}}{\partial \mathbf{R}_{I_s} \partial \mathbf{R}_{J_t}} \right\rangle = \sum_m (E_{n\mathbf{k}} - E_{m\mathbf{k}})^{-1} F \left[ \langle n\mathbf{k} | \frac{\partial V_{SCF}}{\partial \mathbf{R}_{I_s}} | m\mathbf{p} \rangle \right]$$

**!** Non-Diagonal DW corrections can be important in isolated systems like molecules

$$\delta H^{(2)} = \frac{1}{2} \sum_{I_s J_t} \frac{\partial^2 V_{SCF}}{\partial \mathbf{R}_{I_s} \partial \mathbf{R}_{J_t}} \mathbf{u}_{I_s} \mathbf{u}_{J_t}$$

X. Gonze et al, Annalen der Physik **523**, 168 (2011)

$$\delta E_{n\mathbf{k}} = \sum_{\mathbf{q}\lambda m} \left[ \frac{|g_{n'n\mathbf{k}}^{\mathbf{q}\lambda}|^2}{E_{n\mathbf{k}} - E_{m\mathbf{k}+\mathbf{q}}} - \frac{\Lambda_{n'n\mathbf{k}}^{\mathbf{q}\lambda}}{E_{n\mathbf{k}} - E_{m\mathbf{k}}} \right] (2\langle N_{\mathbf{q}\lambda} \rangle + 1)$$



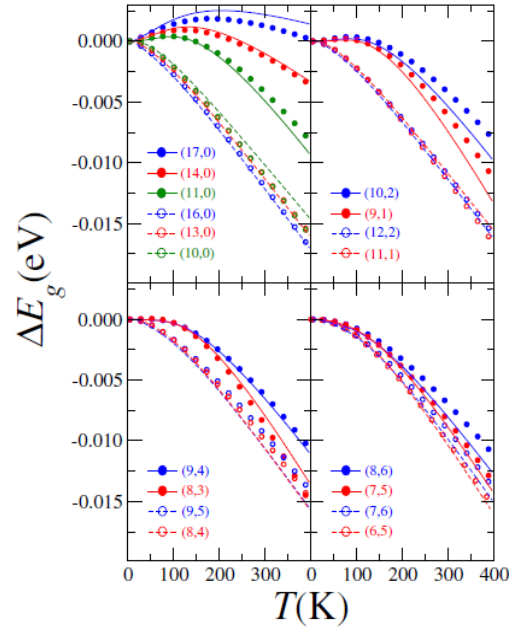
Clear dependence on the temperature



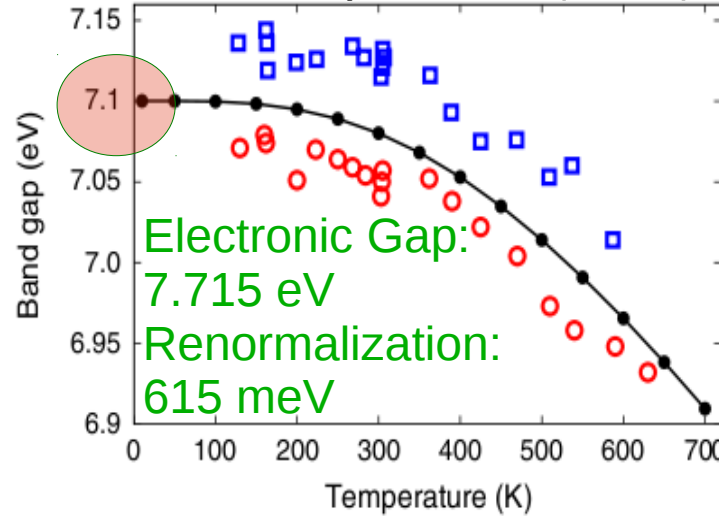
Polaron damping neglected

# The Heine-Allen-Cardona Approach (III)

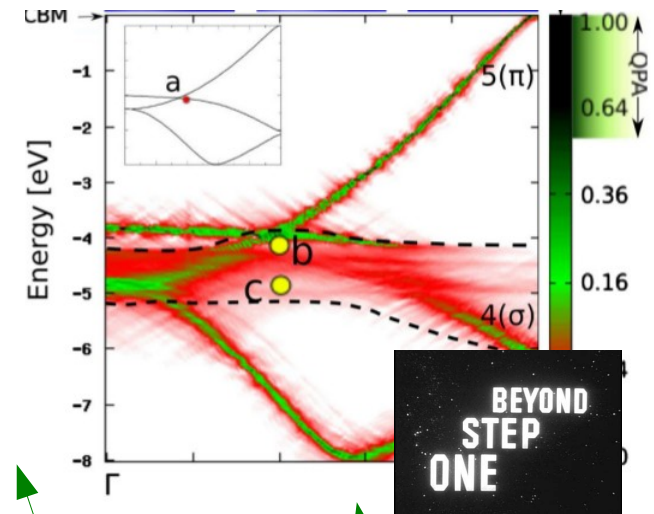
R. B. Capaz *et al.* PRL **94**, 36801 (2005)



F. Giustino, *et al.*  
PRL, **105**, 265501 (2010)



E. Cannuccia  
PRL **107**, 255501 (2011)



1992

2005

2008

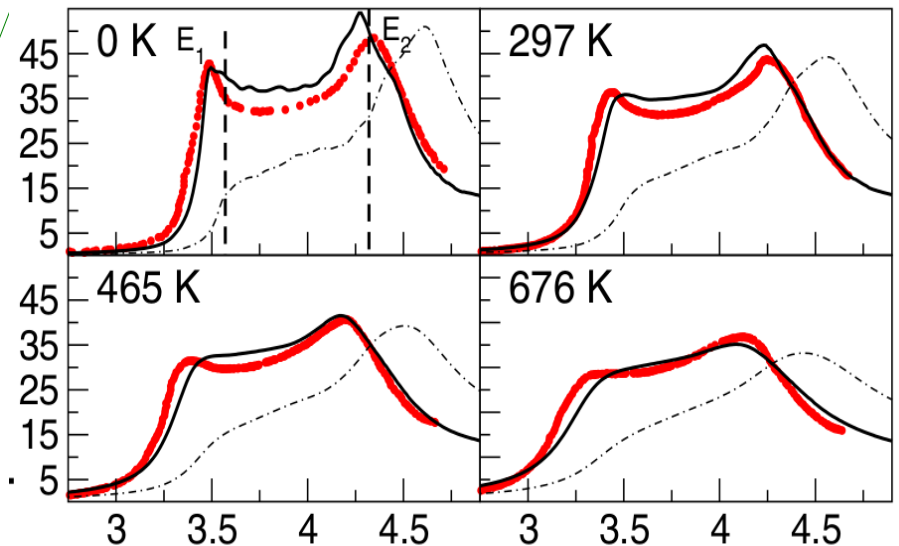
2010

2011

State	Mass	SE	DW	Total
$\Gamma_{25'}(v)$	12.00	0.0	0.0	0.0
$\Gamma_{15}(c)$	12.00	-547.8	-130.1	-677.9

S. Zollner *et al.*  
PRB **45**, 3376 (1992)

A. Marini  
PRL **101**, 106405 (2008)



# Green's functions: an (over)simplified picture (I)

G. Strinati, Nuovo Cimento **11**, 1 (1988)

$$G_{nk} = \langle nk | (\omega - H)^{-1} | nk \rangle$$

$$G_{nk}(\omega) = \sum_I | \langle \Psi | c_{nk}^\dagger | I \mathbf{k} \rangle |^2 (\omega - E_{I\mathbf{k}})^{-1}$$

$$E_{I\mathbf{k}} \rightarrow A_{nk}(\omega) = \sum_I | \langle \Psi | c_{nk}^\dagger | I \mathbf{k} \rangle |^2 \delta(\omega - E_{I\mathbf{k}})$$

**Spectral Functions**



$$G(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) \equiv (-i) \langle T \{ \hat{\psi}_H(\mathbf{x}_1, t_1) \hat{\psi}_H^\dagger(\mathbf{x}_2, t_2) \} \rangle$$



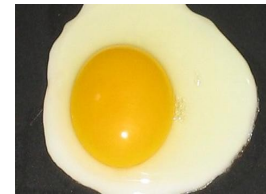
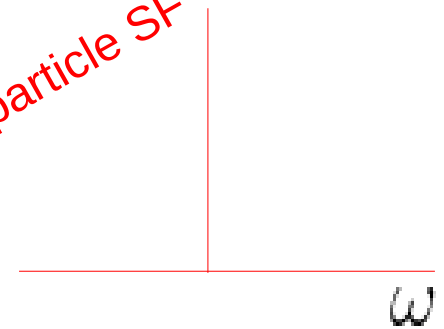
$$i\partial_t \hat{\psi}_H(\mathbf{x}, t) = (\dots) + \int d\mathbf{x}' \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} \hat{\psi}_H(\mathbf{x}, t)$$

$\rho(\mathbf{x}', t)$  Electronic & **Nuclear** density



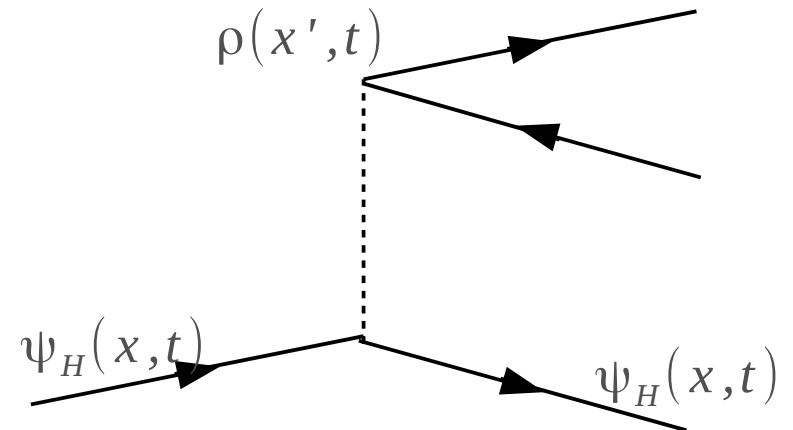
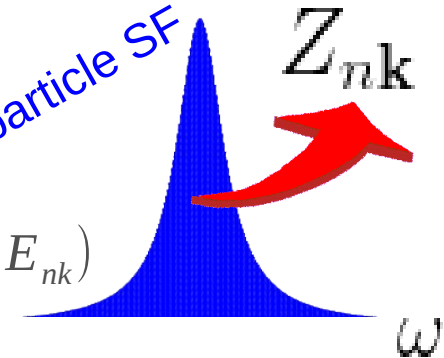
$$E_{I\mathbf{k}} = \epsilon_{nk}$$

Real particle SF



$$E_{I\mathbf{k}} = \Re(E_{nk}) + i\Im(E_{nk})$$

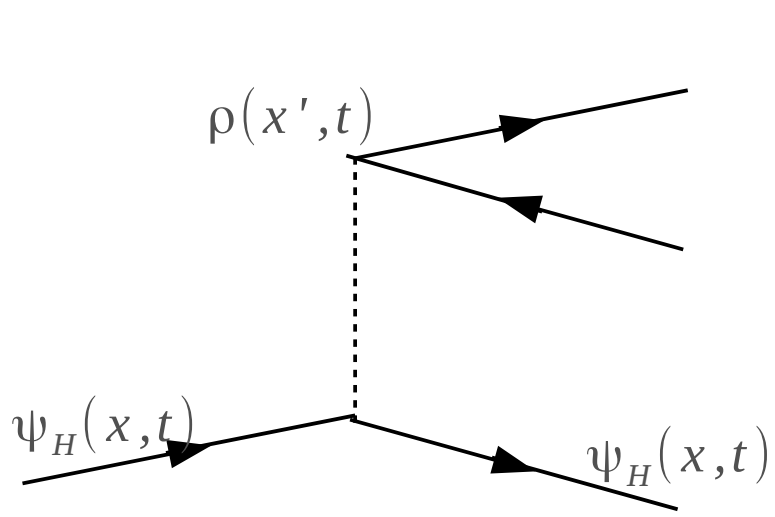
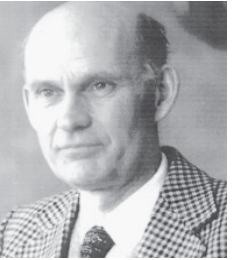
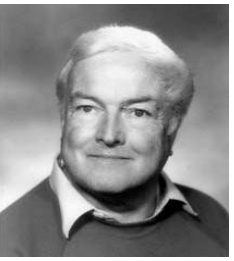
Quasi particle SF





# Diagrammatic vs Hedin-Lundqvist approach (I)

LH and SL, Solid. State Phys. **23**, 1 (1969); RvL, PRB **69**, 115110 (2004)

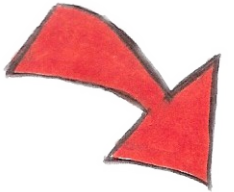


Feynmann diagrams



Equation of motion

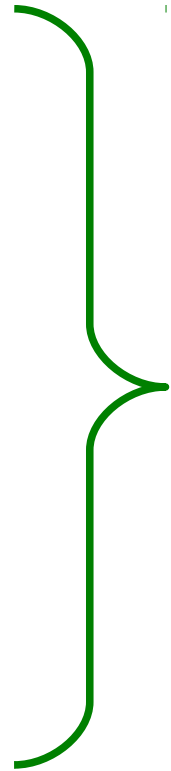
$$H \rightarrow H + \int \phi(x) \rho(x, t)$$



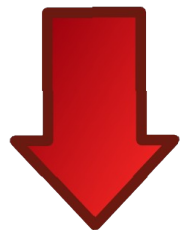
$$\psi_H(x, t) \rightarrow \psi_{H, \phi}(x, t)$$



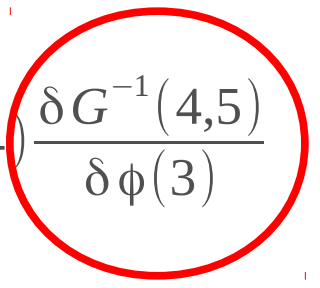
$$\frac{\delta \psi_H(x, t)}{\delta \phi} \sim \rho(x', t) \psi_H(x, t)$$



$$[i\partial_t - h(1)]G(1,2) = \delta(1,2) + \int dr \Sigma(1,3)G(3,2)$$

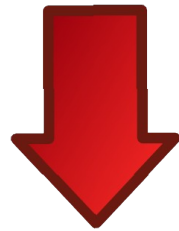


$$\Sigma(1,2) \sim \iint d3 d4 v(1,3) G(1,4) \frac{\delta G^{-1}(4,5)}{\delta \phi(3)}$$



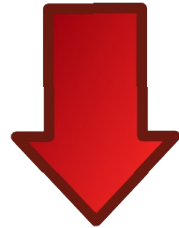
# Green's functions: an (over)simplified picture (II)

$$[i\partial_t - h(1)]G(1,2) = \delta(1,2) + \int dr \Sigma(1,3)G(3,2)$$



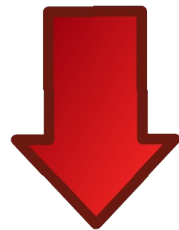
Time Fourier transformation

$$G_{nk}(\omega) = \frac{1}{\omega - \epsilon_{nk} - \Sigma_{nk}(\omega) + i\eta}$$



Linear expansion

$$E_{nk} = \epsilon_{nk} + \Sigma(\epsilon_{nk}) + \Sigma'(\epsilon_{nk})(E_{nk} - \epsilon_{nk})$$



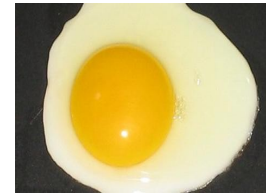
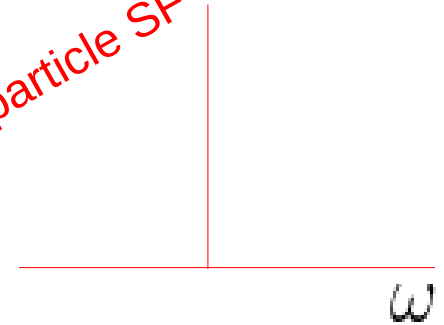
QP approximation

$$G_{nk}(\omega) = \frac{Z_{nk}}{(\omega - E_{nk})} \quad Z_{nk} = \frac{1}{(1 - \Sigma'(\epsilon_{nk}))}$$



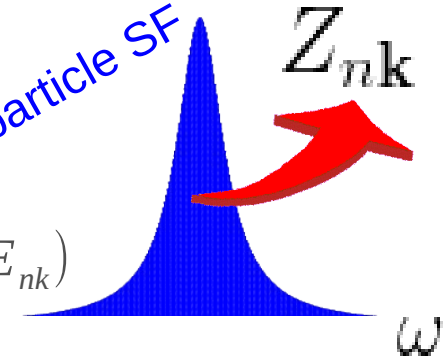
$$E_{Ik} = \epsilon_{nk}$$

Real particle SF



$$E_{Ik} = \Re(E_{nk}) + i\Im(E_{nk})$$

Quasi particle SF

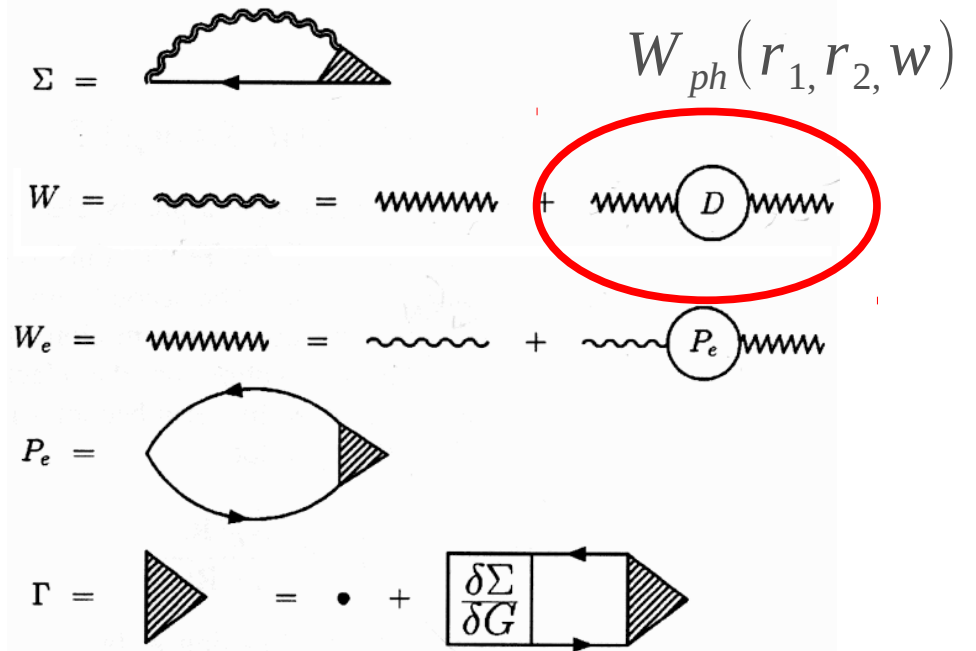


# Hedin-Lundqvist approach (II)

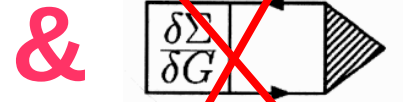
RvL, PRB **69**, 115110 (2004)

$$\frac{\delta n(x,t)}{\delta \phi(x',t')} \sim P_e(x,t;x',t)$$

$$\frac{\delta N(x,t)}{\delta \phi(x',t')} \sim D(x,t;x',t)$$



$$\text{wavy line with } D \text{ circle} = W_{ph}(\mathbf{r}_1, \mathbf{r}_2, i\omega) = \sum_{\mathbf{q}\lambda} \frac{2\omega_{\mathbf{q}\lambda}}{\omega^2 + \omega_{\mathbf{q}\lambda}^2} g_{\mathbf{q}\lambda}(\mathbf{r}_1, i\omega) g_{\mathbf{q}\lambda}^*(\mathbf{r}_2, i\omega)$$



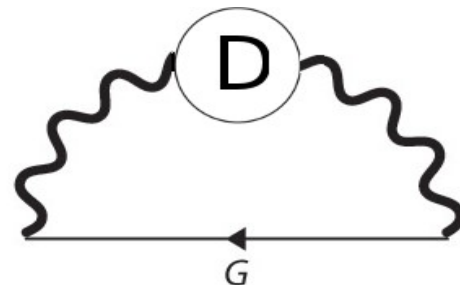
$$g_{\mathbf{q}\lambda}(\mathbf{r}, i\omega) \propto \omega_{\mathbf{q}\lambda}^{-1/2} \sum_{I_s} \int d\mathbf{r}_1 \epsilon_e^{-1}(\mathbf{r}, \mathbf{r}_1; i\omega) \epsilon(\mathbf{q}\lambda|s) \cdot \nabla V_{ion}(\mathbf{r}_1) e^{i\mathbf{q} \cdot (\mathbf{R}_I + \tau_s)}$$

+

=

Fan self-energy

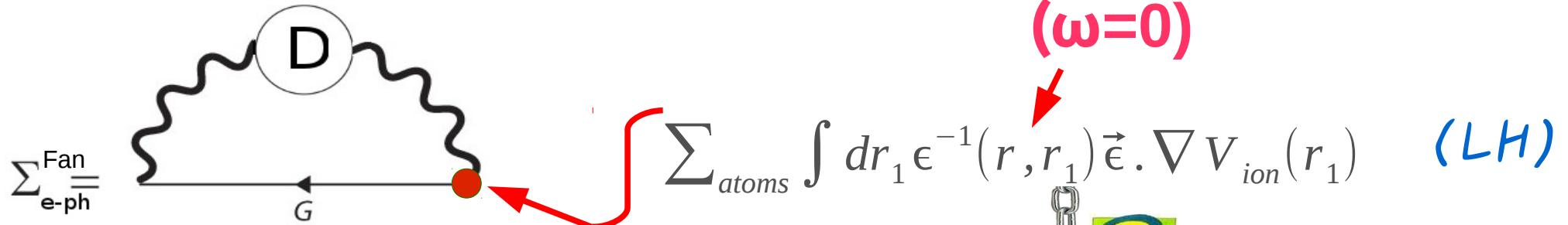
$\sum_{\text{e-ph}}^{\text{Fan}}$



$$\Gamma \sim \left( \frac{m_{\text{electron}}}{M_{\text{atom}}} \right)^{1/2}$$

Migdal's theorem

# Hedin-Lundqvist approach (III): the FAN self-energy



$$\sum_{atoms} \int dr_1 \epsilon^{-1}(r, r_1) \vec{\epsilon} \cdot \nabla V_{ion}(r_1) \quad (LH)$$

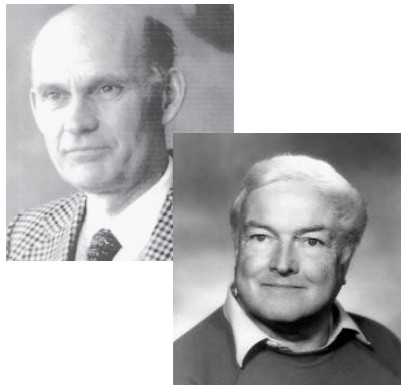


$$V_{SCF}(r) \quad (HAC)$$

$$G_{nk}(\omega) = \frac{1}{\omega - \epsilon_{nk} - \Sigma_{nk}(\omega) + i\eta}$$

$$\Sigma_{nk}^{Fan}(\omega) = \sum_{q\lambda} \frac{1}{N_q} \sum_{n'} |g_{n'nk}^{q\lambda}|^2 \left[ \frac{N_{q\lambda} + 1 - f_{n'k-q}}{\omega - \epsilon_{n'k-q} - \omega_{q\lambda} - i0^+} + \frac{N_{q\lambda} + f_{n'k-q}}{\omega - \epsilon_{n'k-q} + \omega_{q\lambda} - i0^+} \right] \quad (LH)$$

$$\delta E_{nk} = \sum_{q\lambda m} \left[ \frac{|g_{n'nk}^{q\lambda}|^2}{E_{nk} - E_{mk+q}} - \frac{\Lambda_{n'nk}^{q\lambda}}{E_{nk} - E_{mk}} \right] (2\langle N_{q\lambda} \rangle + 1) \quad (HAC)$$



**STATIC & ADIABATIC limit**

$$\omega \approx \epsilon_{nk}$$

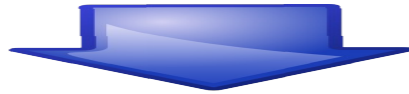
$$|\epsilon_{nk} - \epsilon_{n'k-q}| \gg \omega_{q\lambda}$$


**Fan term in Heine-Allen-Cardona Theory**

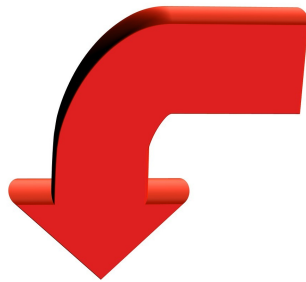
# Density Functional Perturbation Theory

[S. Baroni, REVIEWS OF MODERN PHYSICS, 2001 , 73, 515]

$$\frac{\partial V_{xc}}{\partial \mu} = \frac{dV_{xc}}{d\rho} \frac{\partial \rho(\mathbf{r})}{\partial \mu} \quad \frac{\partial V_H}{\partial \mu} = \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \frac{\partial \rho(\mathbf{r}')}{\partial \mu} d^3 r'$$



DFPT is composed by a self-consistent linear system



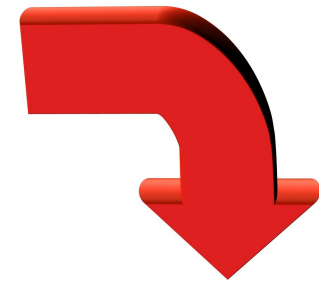
Dynamical  
Matrix

$$\omega_{q\lambda}$$

$$\left[ -\frac{1}{2} \nabla^2 + V_{KS}(\mathbf{r}) - \varepsilon_i \right] P_c \frac{\partial \psi_i(\mathbf{r})}{\partial \mu} = -P_c \frac{\partial V_{KS}}{\partial \mu} \psi_i(\mathbf{r})$$

$$\frac{\partial V_{KS}}{\partial \mu} = \frac{\partial V_{loc}}{\partial \mu} + \frac{\partial V_H}{\partial \mu} + \frac{\partial V_{xc}}{\partial \mu}$$

$$\frac{\partial \rho(\mathbf{r})}{\partial \mu} = \sum_i P_c \frac{\partial \psi_i^*(\mathbf{r})}{\partial \mu} \psi_i(\mathbf{r}) + \psi_i^*(\mathbf{r}) P_c \frac{\partial \psi_i(\mathbf{r})}{\partial \mu}$$

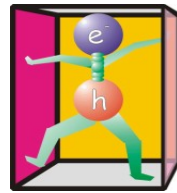


El-Ph  
matrix  
elements

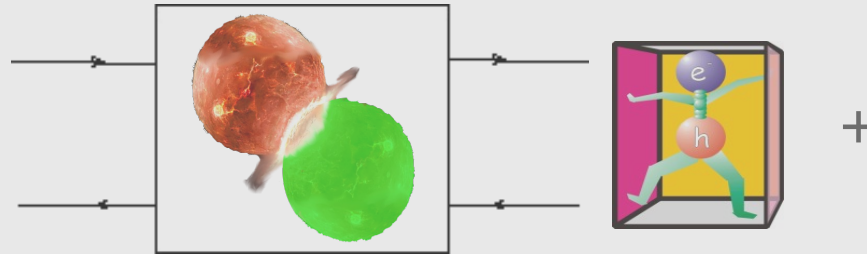
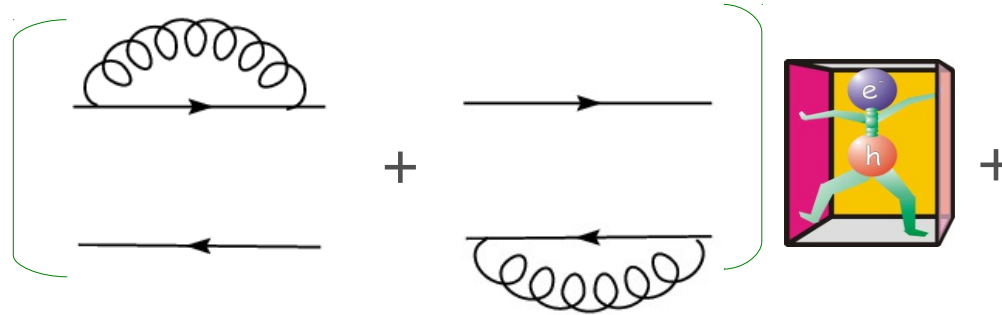
$$g_{nn'k}^{q\lambda} \sim (M \omega_{q\lambda})^{-1} \langle nk | \frac{dV_{KS}}{du_{q\lambda}} | n'k - q \rangle$$

By working out the DFPT definition of the KS potential it is possible to link DFPT to the static limit of the electron-phonon potential defined in MBPT

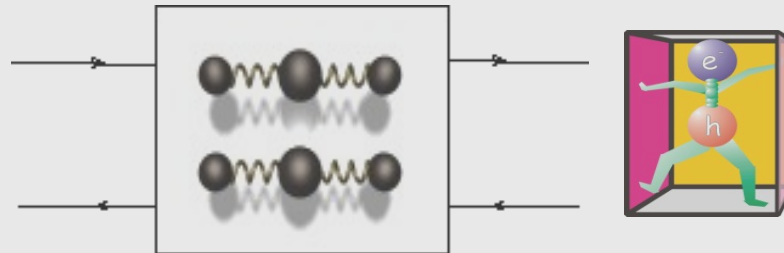
**Independent QPs**



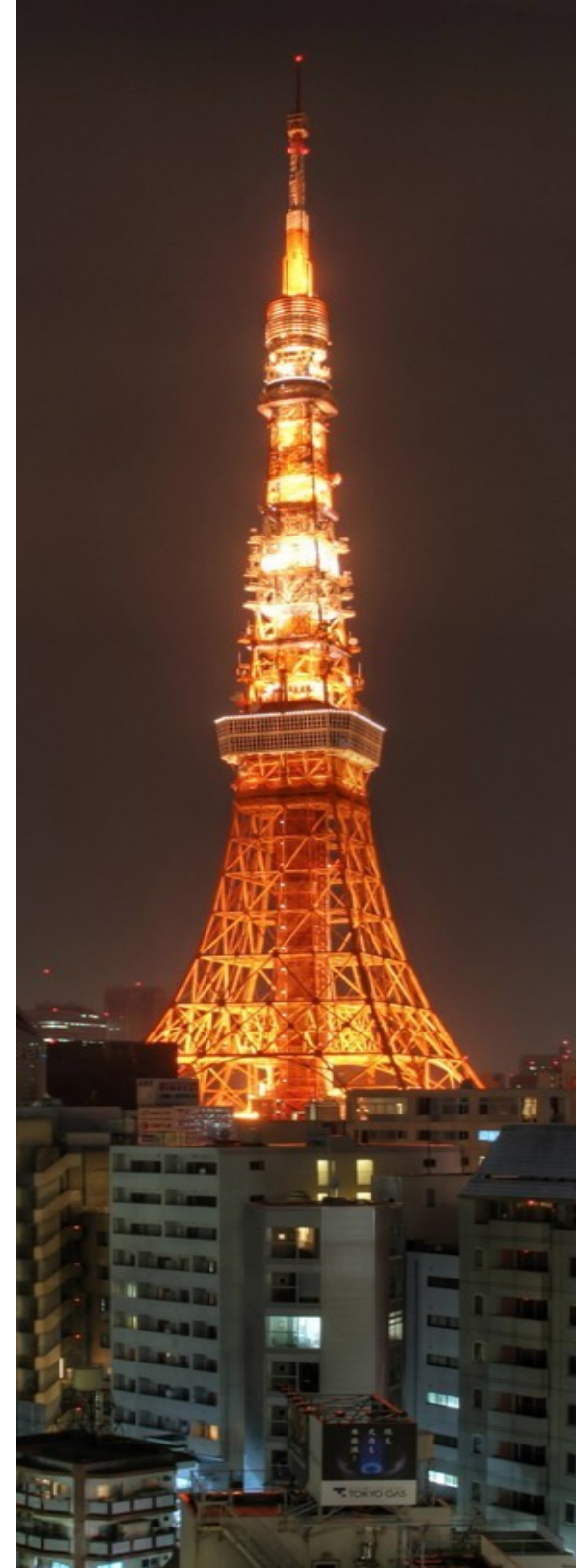
**“Indirect”  
exciton-phonon  
scattering**



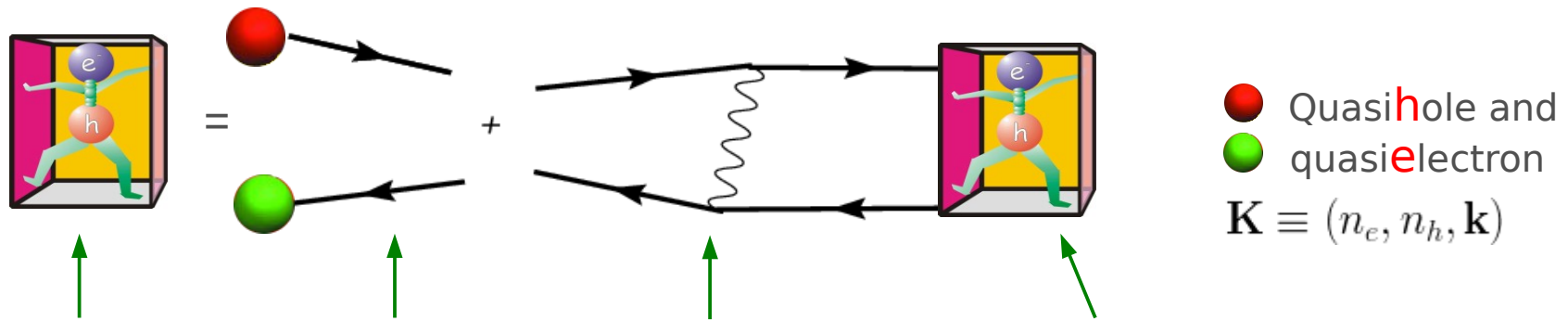
**“Direct”  
exciton-phonon  
scattering**



**Finite temperature excitons**



# Excitons: the polaronic picture



$$\tilde{L}_{K_1, K_2}(\tau) = \tilde{L}_{K_1, K_2}^{(0)}(\tau) [\delta_{2,3} + \Pi_{K_2, K_3}^{el}(\tau=0)] \tilde{L}_{K_3, K_2}(\tau)$$

The excitons are the poles of  $\tilde{L}$  and eigenstates of the Bethe-Salpeter Hamiltonian

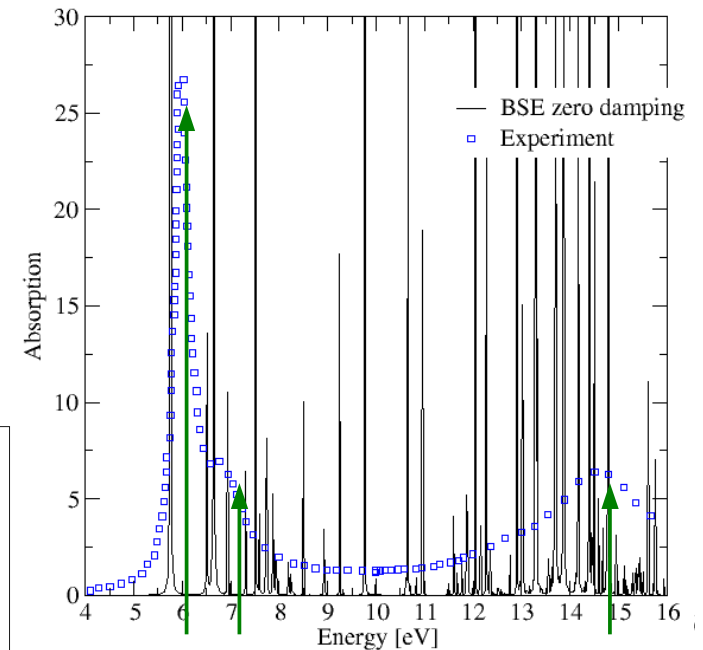
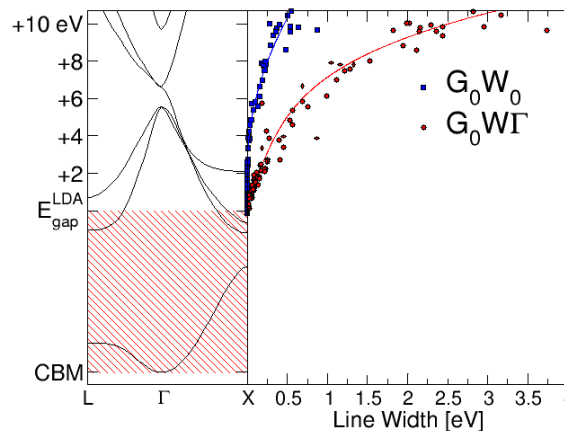
$$H_{K, K'}^{el} = (\epsilon_e - \epsilon_h) \delta_{K, K'} + \Pi_{K_1, K_2}^{el}$$



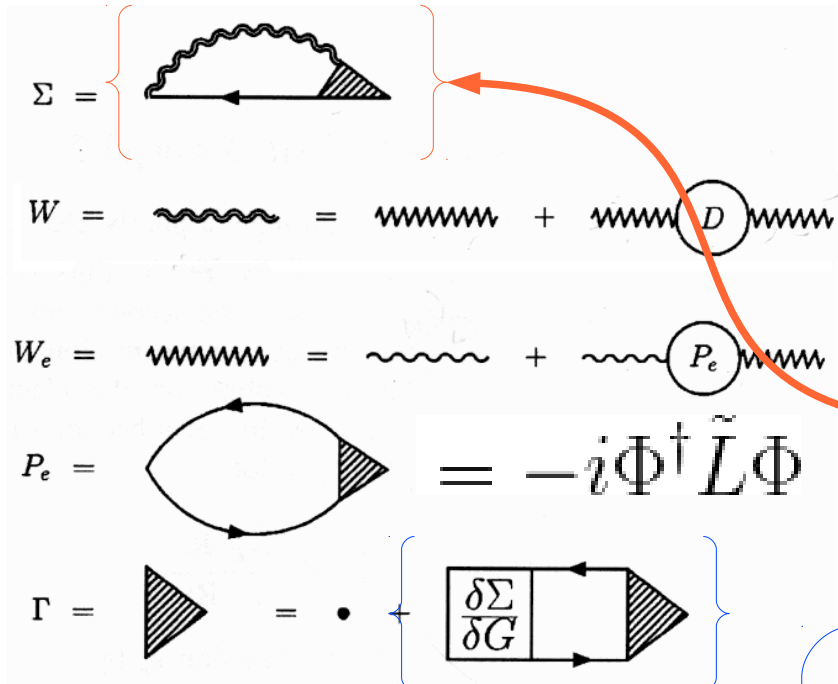
Quasiparticle energies are real in the optical range



The BS Hamiltonian is Hermitian

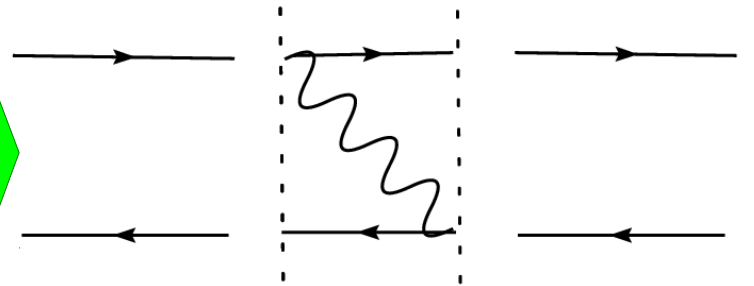
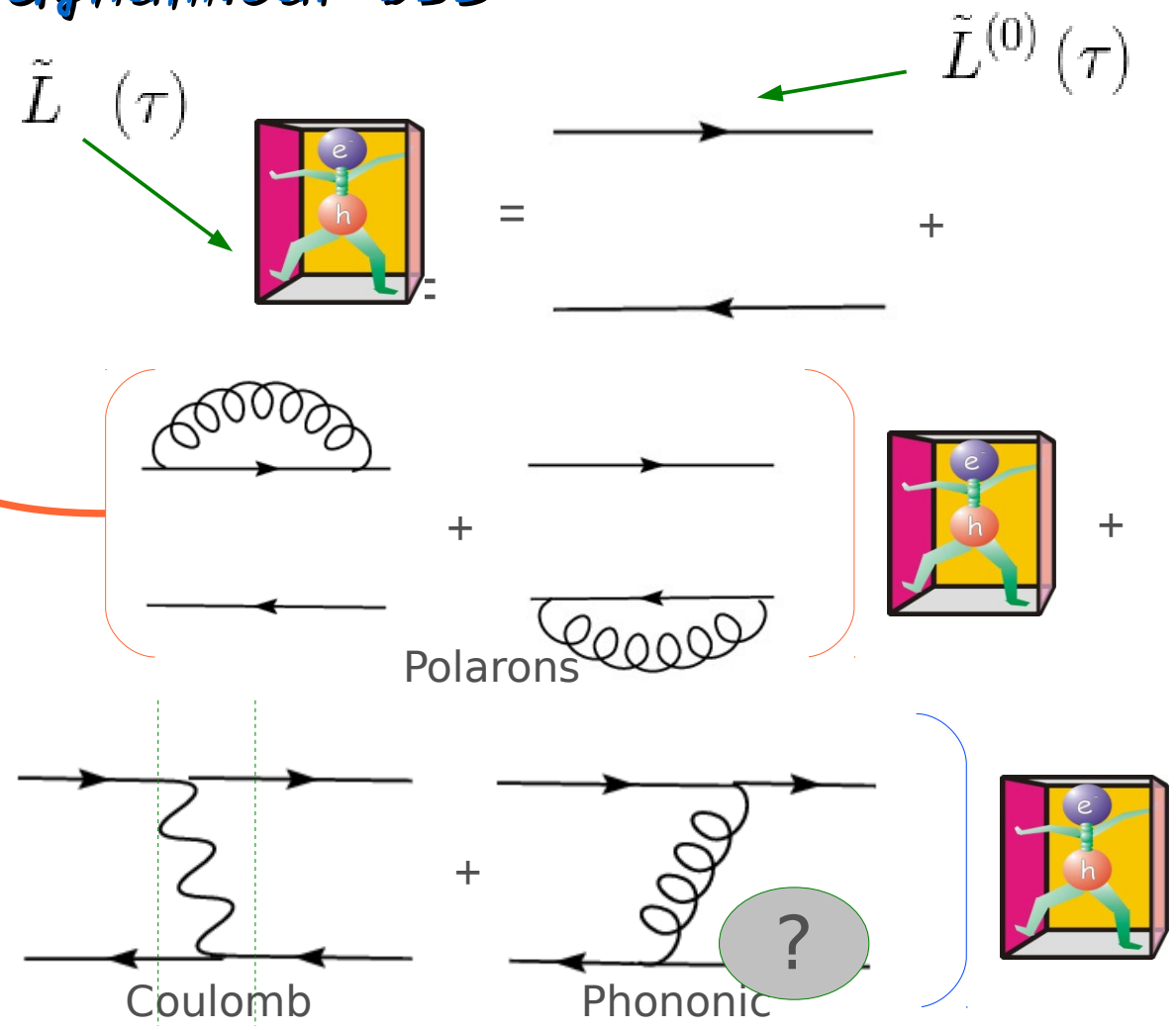


# The dynamical BSE



$$P_e = -i\Phi^\dagger \tilde{L} \Phi$$

$$\epsilon(i\omega) \equiv 1 - 4\pi P_e(i\omega)$$



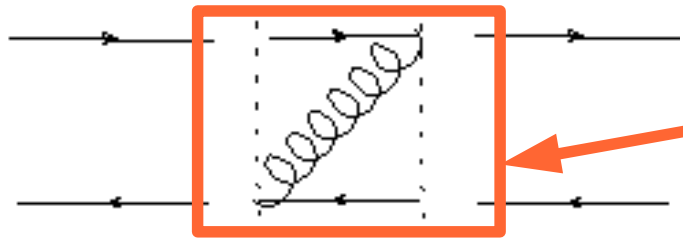
$$\mathbf{K} \equiv (n_e, n_h, \mathbf{k}) \quad L_{\mathbf{K}_1}^{(0)}(\tau - \tau_1) \quad \Pi_{\mathbf{K}_1, \mathbf{K}_2}^{el}(\tau_1 - \tau_2) \quad L_{\mathbf{K}_2}^{(0)}(\tau_2)$$



The **dynamical BSE** [AM, R. Del Sole, PRL **91**, 176402 (2003)] was introduced to sum the frequency dependent Coulomb interaction



# The dynamical BSE II: the phonon term



$$\Pi_{\mathbf{K}_1, \mathbf{K}_2}^{ph}(i\omega) = - \sum_{\lambda} g_{c_2 c_1 \mathbf{k}_1}^{q\lambda} \left( g_{v_2 v_1 \mathbf{k}_1}^{q\lambda} \right)^* \sum_I \left[ \frac{1 + \langle N_{q\lambda} \rangle}{i\omega + \Delta_I - \omega_{q\lambda}} + \frac{\langle N_{q\lambda} \rangle}{i\omega + \Delta_I + \omega_{q\lambda}} \right]$$

$$\Delta_1 = \epsilon_{v_2 \mathbf{k}_1 - \mathbf{q}} - \epsilon_{c_1 \mathbf{k}_1}$$

$$\Delta_2 = \epsilon_{v_1 \mathbf{k}_1} - \epsilon_{c_2 \mathbf{k}_1 - \mathbf{q}}$$

Partial summation of the electronic part

$$\tilde{L}(i\omega) = \tilde{L}^{(0)}(i\omega) [1 + (\Pi^{el}(i\omega) + \Pi^{ph}(i\omega)) \tilde{L}(i\omega)] = \tilde{L}^{el}(i\omega) [1 + \Pi^{ph}(i\omega) \tilde{L}(i\omega)]$$

wit<sub>h</sub>  $\tilde{L}^{el}(i\omega) = \tilde{L}^{(0)}(i\omega) [1 + \Pi^{el}(i\omega) \tilde{L}^{el}(i\omega)]$

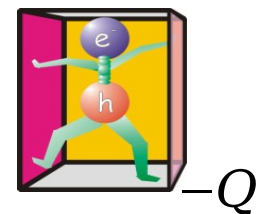
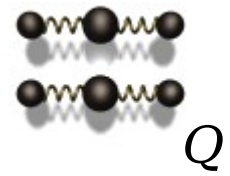
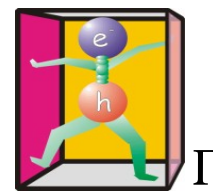
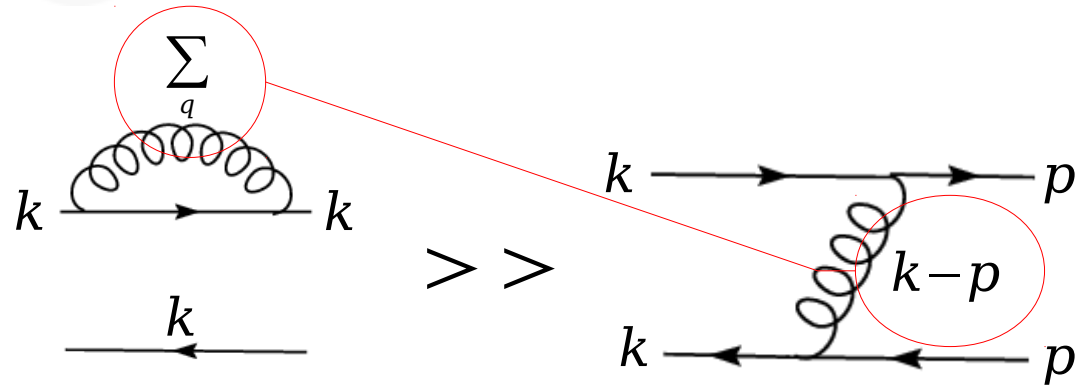


The phononic part of the Bethe-Salpeter kernel reduces to a **“self-energy-like”** operator (without introducing bosonic coordinates)

**Excitonic “Self-Energy”**

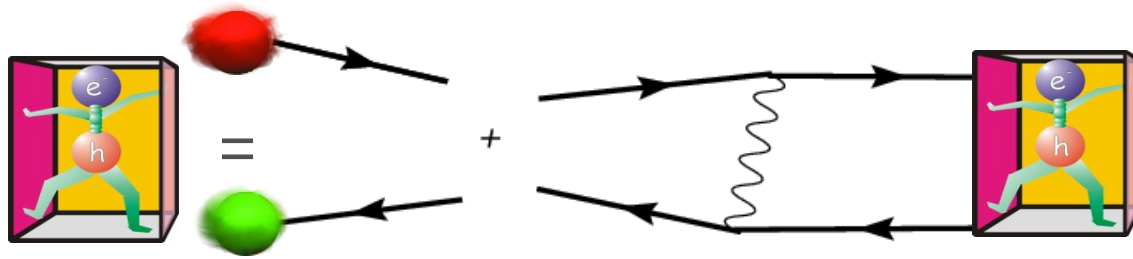




The momenta conservation makes the polaronic (indirect) term dominant



“There is no Ab-Initio justification for simple bosonic scattering pictures”

# Excitons: the polaronic picture

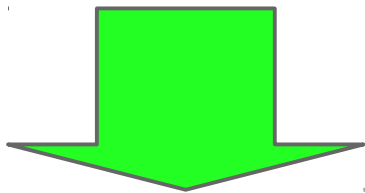
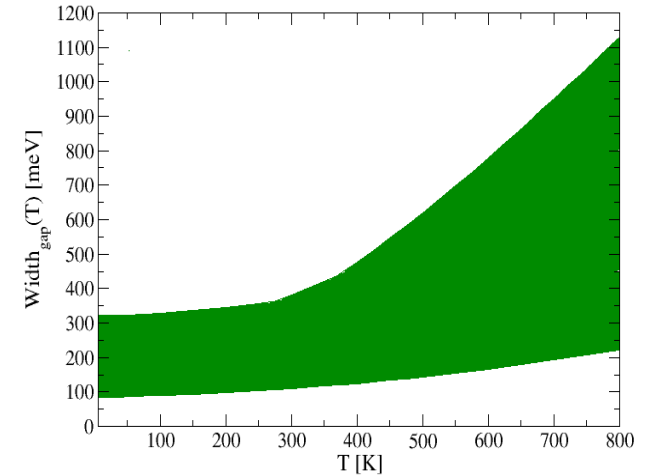


 Quasihole and  
 quasidelectron  
**polarons**  
 $\mathbf{K} \equiv (n_e, n_h, \mathbf{k})$

$$H_{\mathbf{K}, \mathbf{K}'}(T) = [E_e(T) - E_h(T)] + i[\Gamma_e(T) - \Gamma_h(T)] + \Pi_{\mathbf{K}_1, \mathbf{K}_2}^{el}$$



The BS Hamiltonian is  
**NOT Hermitian**



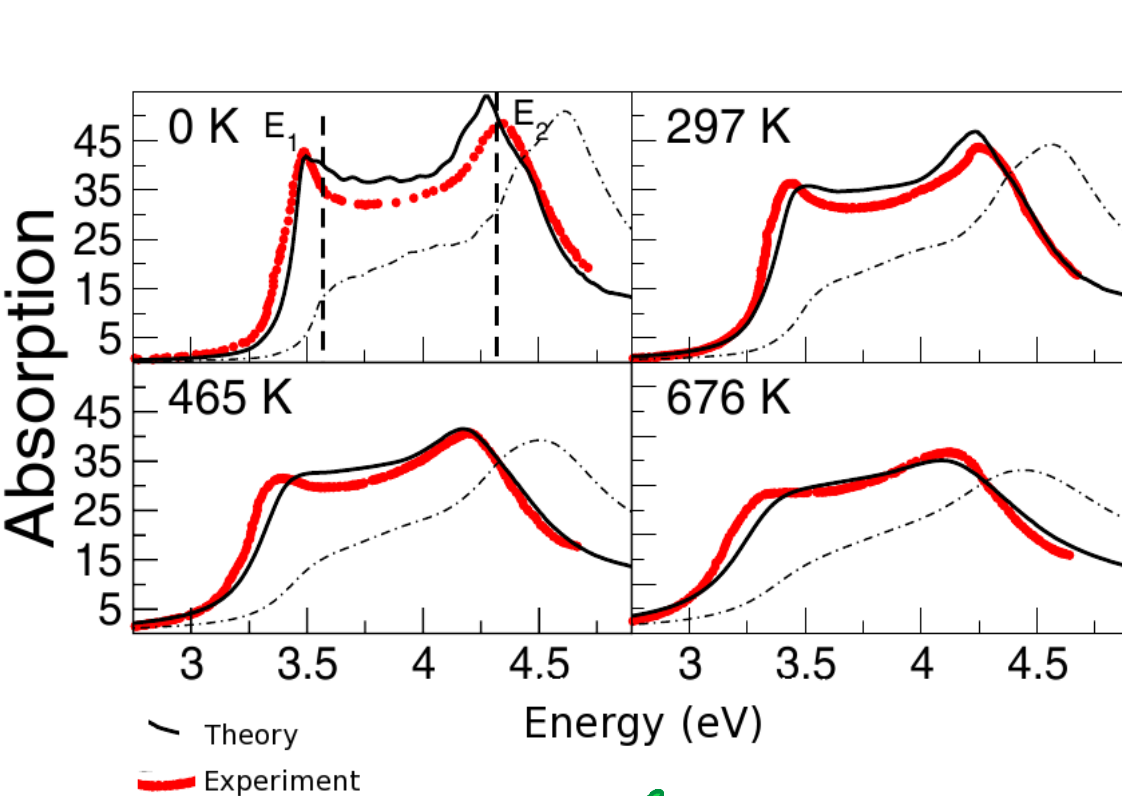
$$\epsilon_2(\omega, T) \propto \sum_{\lambda} S_{\lambda}(T) (\omega - E_{\lambda}(T))^{-1}$$

$$H_{\mathbf{K}, \mathbf{K}'}(T) \lambda_{\mathbf{K}'}(T) = E_{\lambda}(T) \lambda_{\mathbf{K}}(T)$$

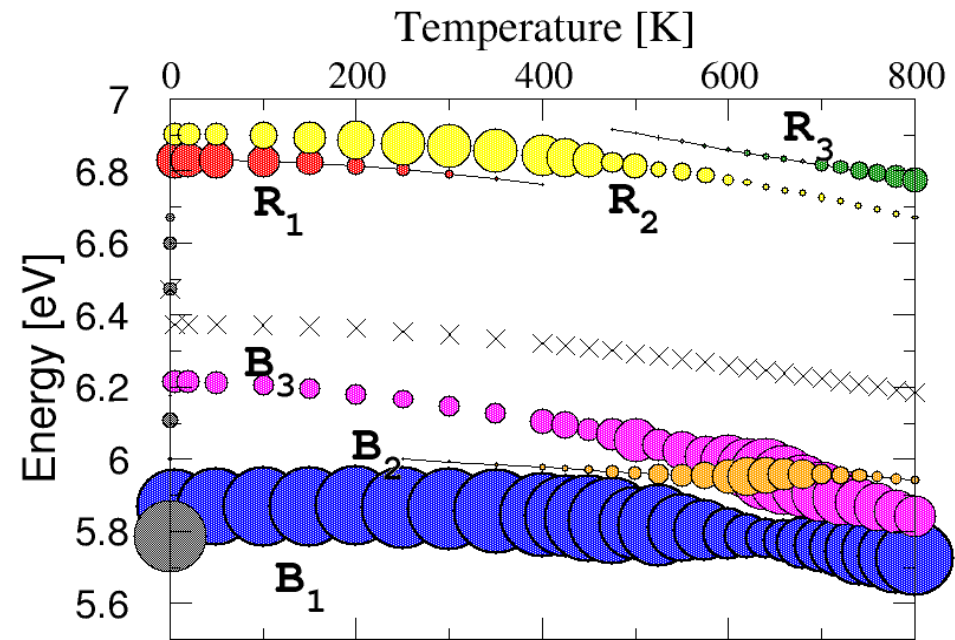
$$\tau^{\lambda}(T) = [2 \Im(E_{\lambda}(T))]^{-1}$$

# Finite T excitons

AM, *Phys. Rev. Lett.* **101**, 106405 (2008)



✓ Bright to dark (and vice versa) transitions



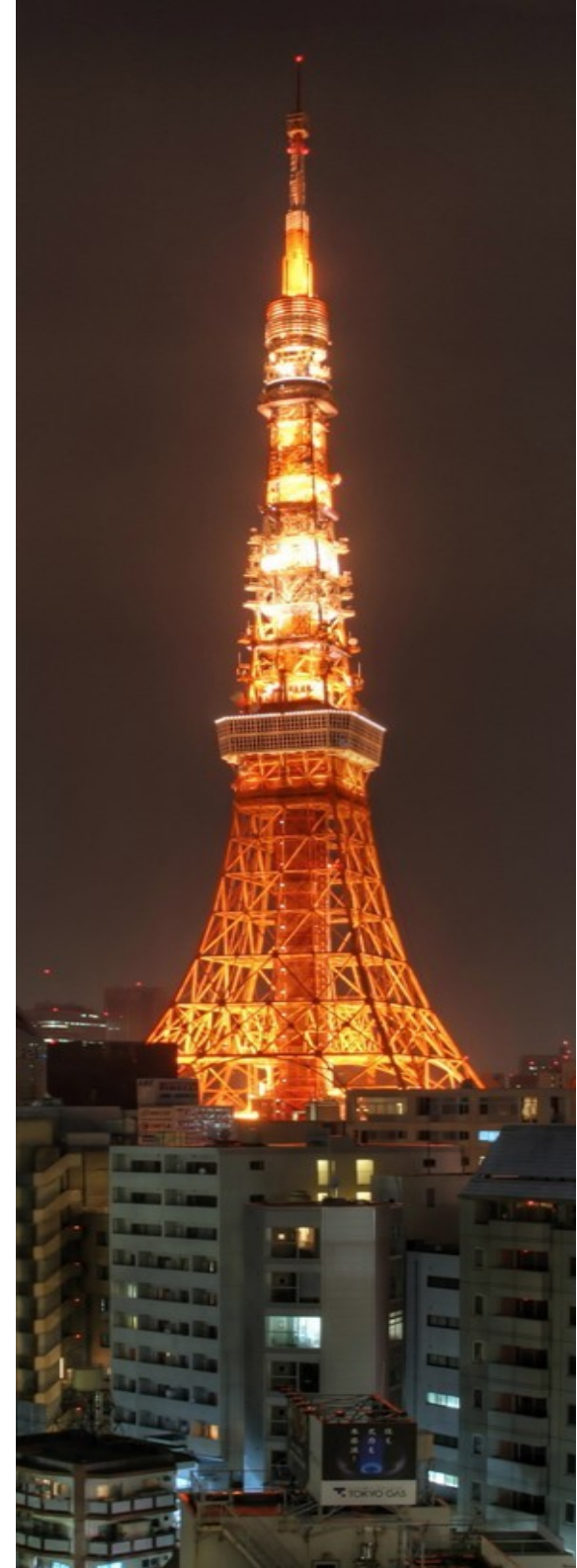
✓ ...gradual worsening of optical efficiency

**CHANGE** in the excitonic state

**INCOHERENT** contribution

$$\Re[\Delta E_\lambda(T)] = [\langle \lambda(T) | \mathbf{H}^{FA} | \lambda(T) \rangle - \langle \lambda_{FA} | \mathbf{H}^{FA} | \lambda_{FA} \rangle] + \int d\omega \Re[g^2 F_\lambda(\omega, T)] [N(\omega, T) + 1/2]$$

***Giant polaronic effects in nanostructures***



# Spectral Functions and QP picture

$$G_{nk}(\omega) = \frac{1}{\omega - \epsilon_{nk} - \Sigma_{nk}(\omega) + i\eta}$$

$$G_{nk} = \langle nk | (\omega - H)^{-1} | nk \rangle$$

$$G_{nk}(\omega) = \sum_I | \langle \Psi | c_{nk}^\dagger | Ik \rangle |^2 (\omega - E_{Ik})^{-1}$$

$$E_{Ik} \rightarrow A_{nk}(\omega) = \sum_I | \langle \Psi | c_{nk}^\dagger | Ik \rangle |^2 \delta(\omega - E_{Ik})$$

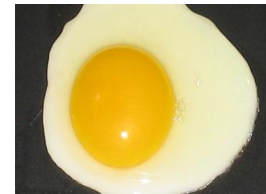
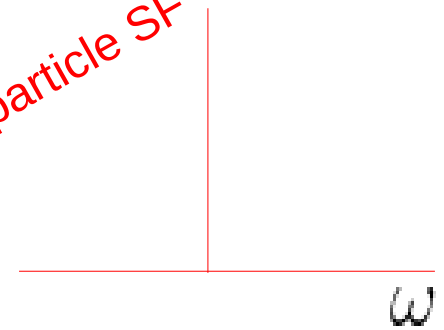
## Spectral Functions

Expanding in eigenstates of the total Hamiltonian the QP picture holds when there is a **dominant (and sharp!) pole that collects most of electronic charge**



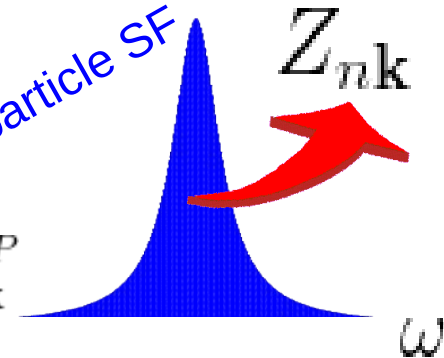
$$E_{Ik} = E_{nk}^{(0)}$$

Real particle SF

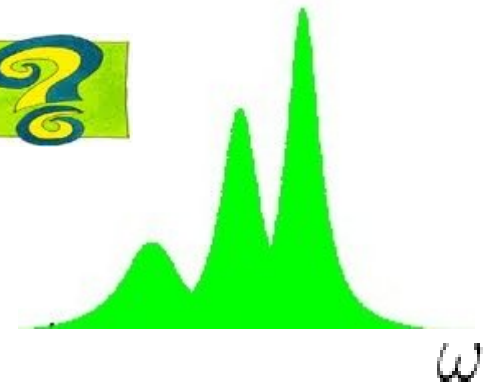


$$E_{Ik} = E_{nk}^{QP} + i\Gamma_{nk}^{QP}$$

Quasi particle SF



$$E_{Ik} = \dots$$



# Spectral Functions in the HEG (I)

PHYSICAL REVIEW

VOLUME 131, NUMBER 3

1 AUGUST 1963

## Coupled Electron-Phonon System\*

S. ENGELSBERG

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

AND

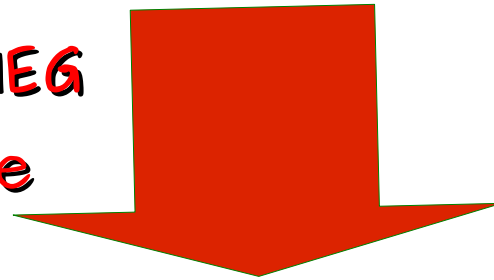
J. R. SCHRIEFFER

University of Pennsylvania, Philadelphia, Pennsylvania

(Received 26 March 1963)

$$\Sigma_{nk}^{Fan}(\omega) = \sum_{q\lambda} \frac{1}{N_q} \sum_{n'} |g_{n'nk}^{q,\lambda}|^2 \left[ \frac{B(\omega_{q\lambda}) + 1 - f_{n'k-q}}{\omega - \epsilon_{n'k-q} - \omega_{q\lambda} - i0^+} + \frac{B(\omega_{q\lambda}) + f_{n'k-q}}{\omega - \epsilon_{n'k-q} + \omega_{q\lambda} - i0^+} \right]$$

Debye Model in the HEG  
At zero temperature



$$\omega_{q\lambda} \sim \omega$$

$$g_{nn'k}^{q\lambda} \sim g$$

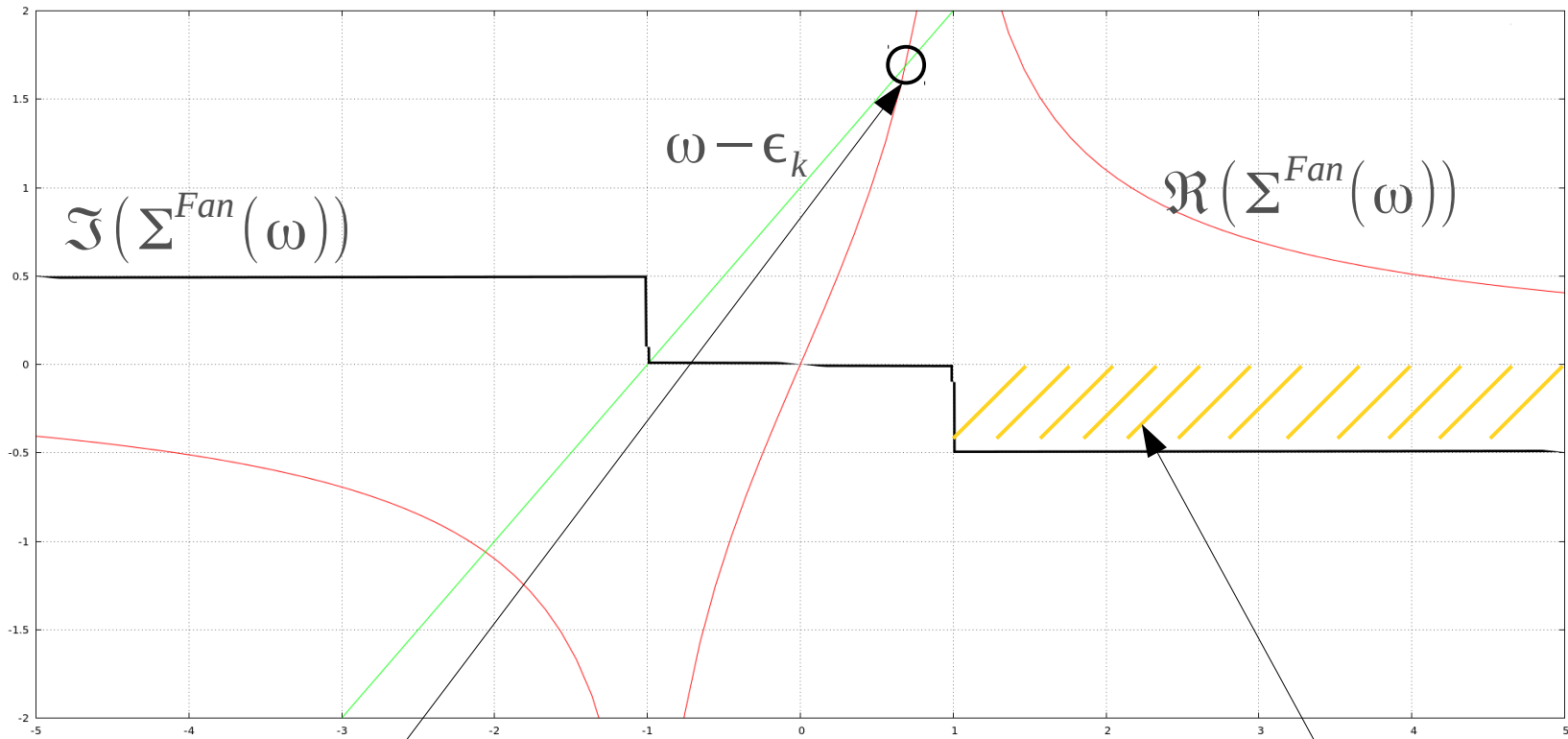
$$\epsilon_{nk} = \epsilon_k \sim \frac{k^2}{2}$$

$$\Sigma^{Fan}(\omega) = i g^2 \int d^4 k (2\pi)^{-4} D(p-k) G(k)$$

$$D(p-k) = ((p_0 - k_0)^2 - \omega^2 + i\eta)^{-1}$$

$$G(k) = (k_0 - \epsilon_k \pm i\eta)^{-1}$$

# Spectral Functions in the HEG (II)



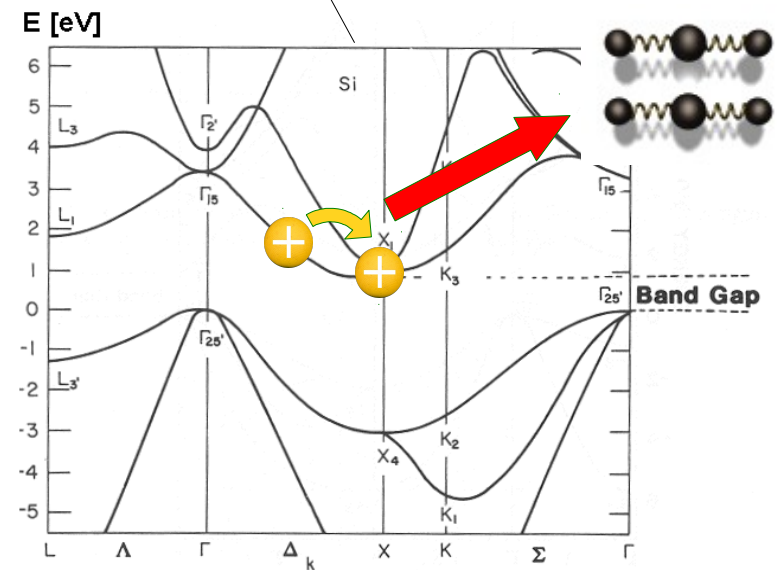
$$E_k = \epsilon_k + \Sigma_k(E_k)$$

QP

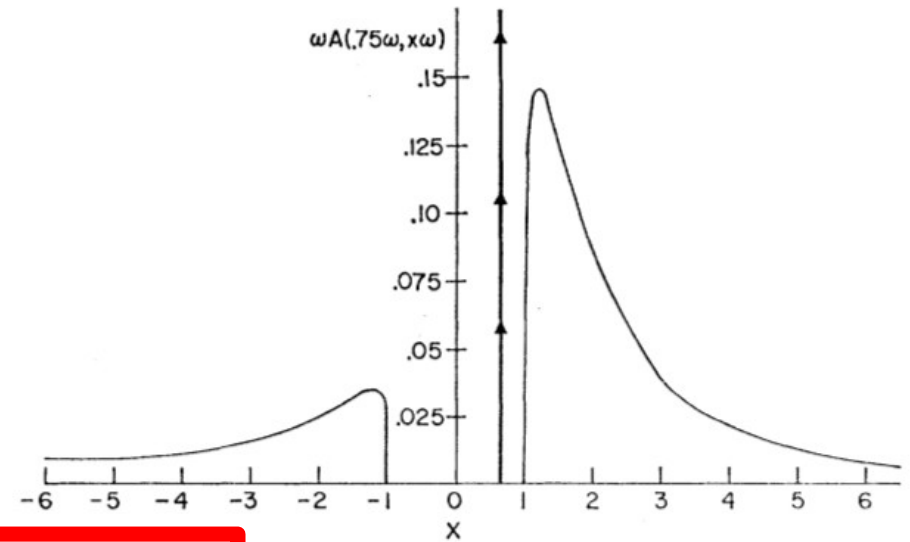
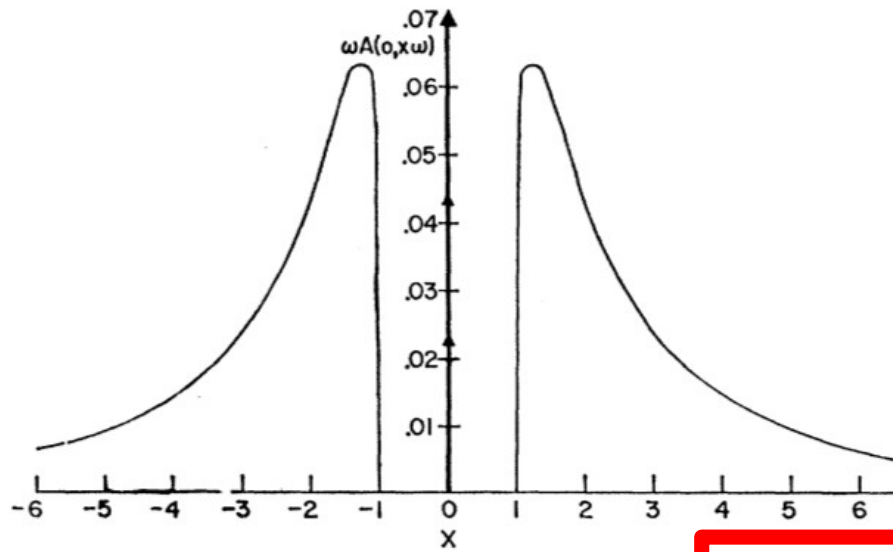
$$E_k = \frac{\epsilon_k}{(1 + g^2 N \omega^{-2})}$$

$$G_{nk}(\omega) = \frac{Z_{nk}}{(\omega - E_{nk})}$$

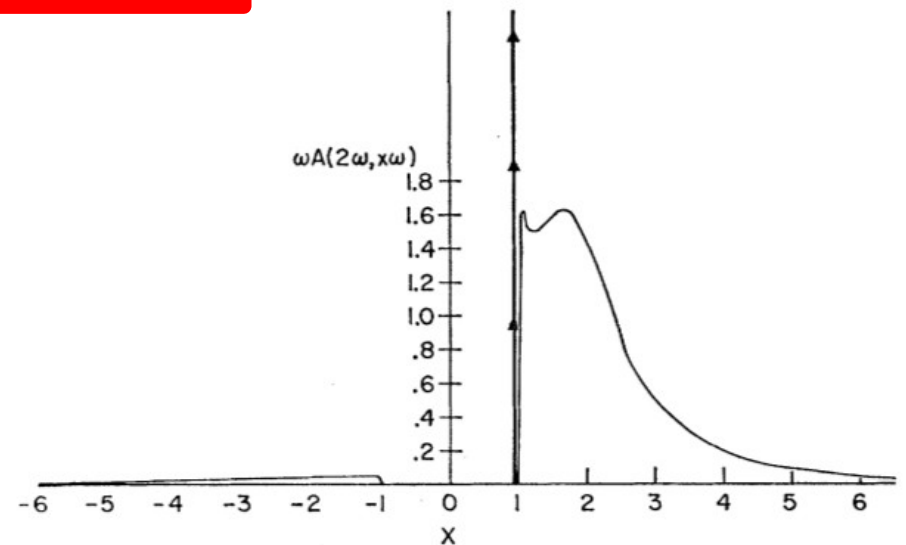
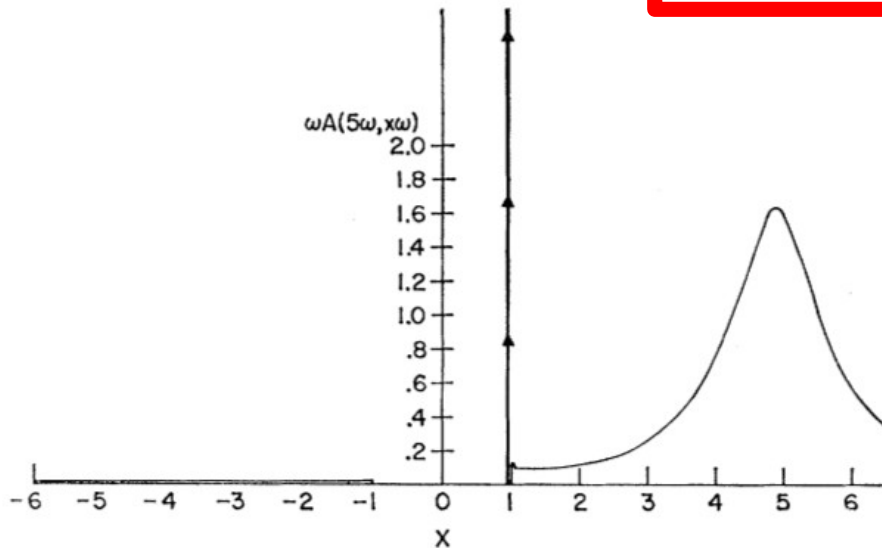
$$Z_{nk} = \frac{1}{(1 - \Sigma'(\epsilon_{nk}))}$$



# Spectral Functions in the HEG (III)



$$g^2 N \omega^{-2} = .5$$

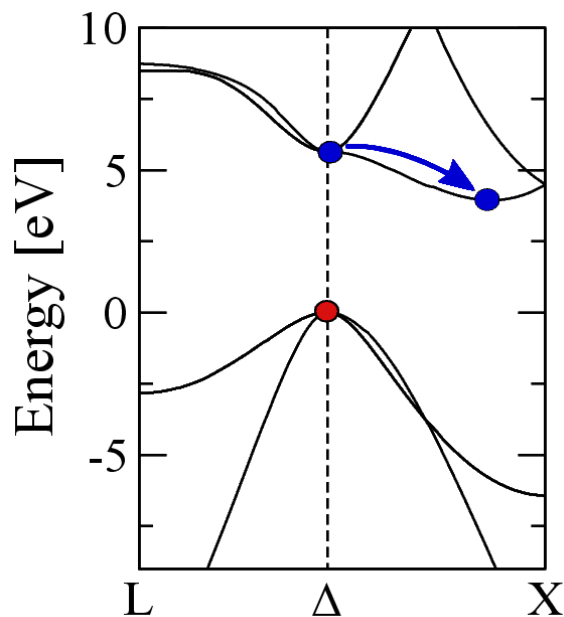
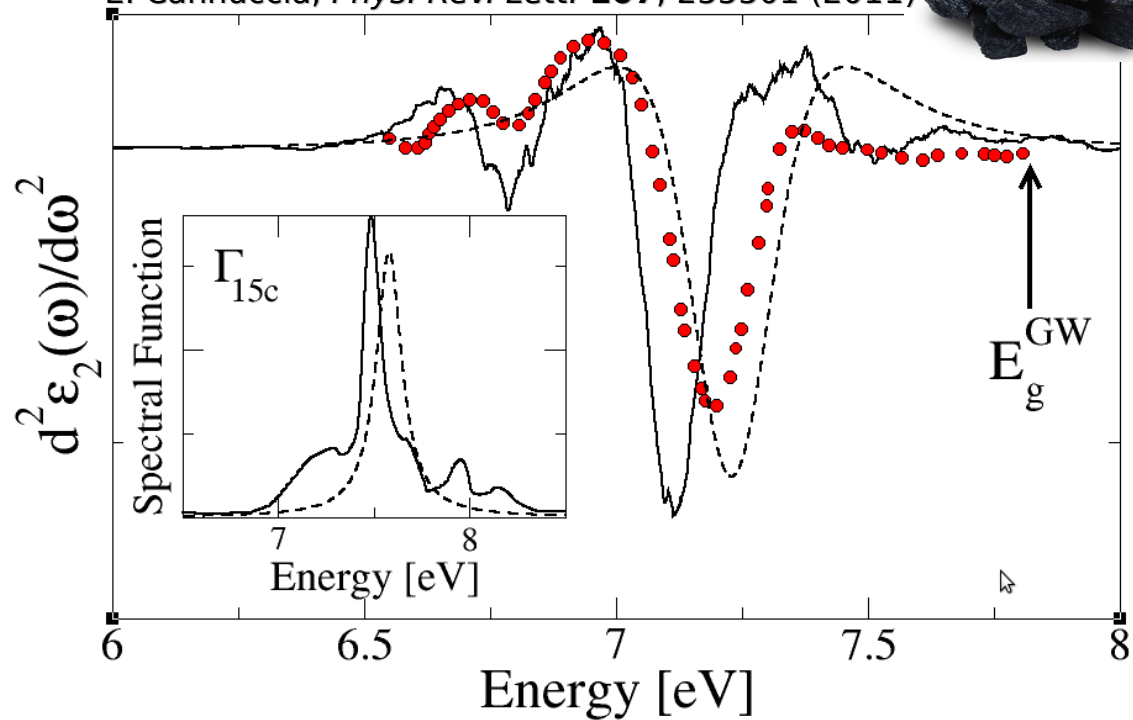
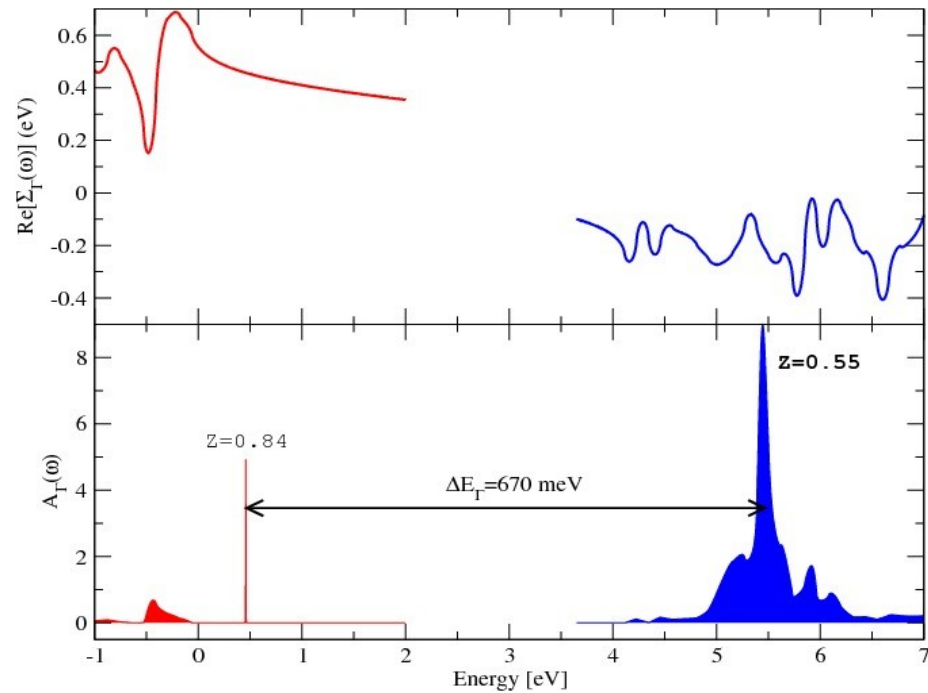




# Dynamical effects in Diamond !



E. Cannuccia, *Phys. Rev. Lett.* **107**, 255501 (2011)



$$\epsilon_2(\omega) \approx \int d\omega' \Im[G_{\Gamma_{15c}}(\omega - \omega')] \Im[G_{\Gamma_{25c}}(\omega')]$$

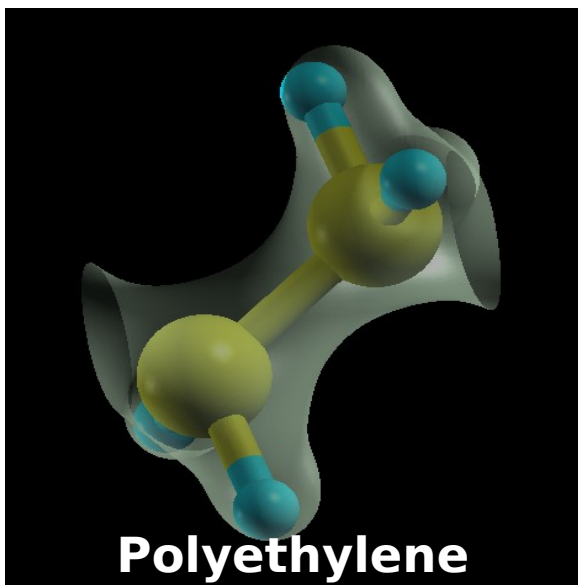
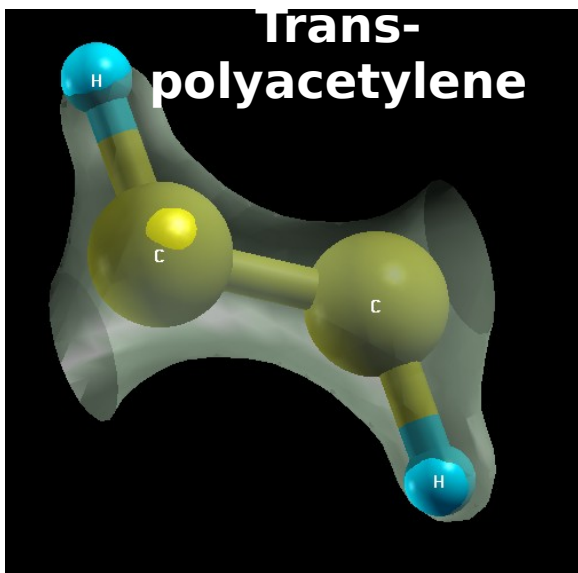
$$\Delta E_g(T \rightarrow 0)$$

**On-Mass shell (HAC)**  
**620 meV**

(**615 meV** in F. Giustino, S.G. Louie and M.L. Cohen, *PRL* **105**, 265501 (2010))

**Quasiparticle approximation**  
**670 meV**

# C-based nanostructures: polymers



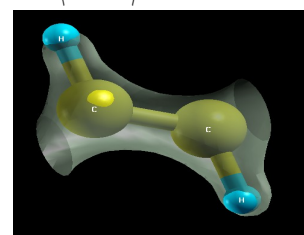
$$\langle u^2(T) \rangle \simeq \frac{\hbar}{4M_s\Omega} \langle 1 + 2\mathcal{N}_{bosc}(T) \rangle$$

$$\sqrt{\langle u^2 \rangle} \approx 0.1 \text{ a.u.}$$

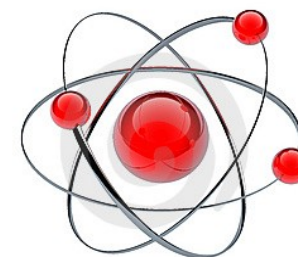


$$\sqrt{\langle u_C^2 \rangle} \approx 0.2 \text{ a.u.}$$

$$\sqrt{\langle u_H^2 \rangle} \approx 0.3 \text{ a.u.}$$



$$\sqrt{\langle u^2 \rangle} \approx 0.4 \text{ a.u.}$$



## Integrated Optoelectronic Devices Based on Conjugated Polymers

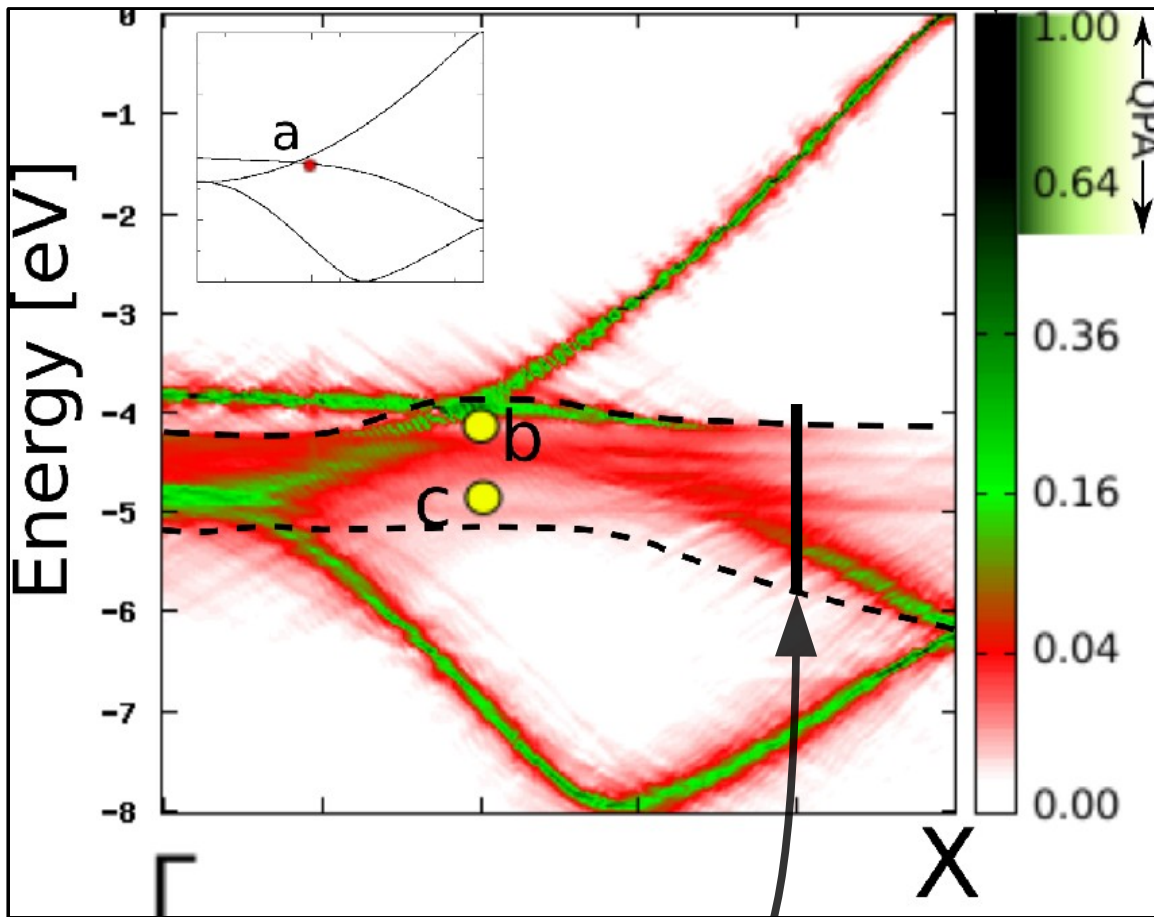
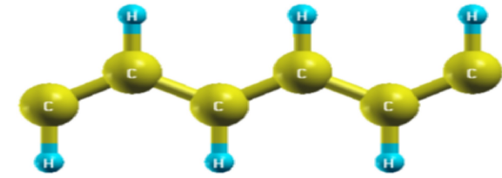
Henning Sirringhaus, \* Nir Tessler, Richard H. Friend \*

**Science 280**  
1741 (1998)

An all-polymer semiconductor integrated device is demonstrated with a high-mobility conjugated polymer field-effect transistor (FET) driving a polymer light-emitting diode (LED) of similar size. The FET uses regioregular poly(hexylthiophene). Its performance approaches that of inorganic amorphous silicon FETs, with field-effect mobilities of 0.05 to 0.1 square centimeters per volt second and ON-OFF current ratios of  $>10^6$ . The high mobility is attributed to the formation of extended polaron states as a result of local self-organization, in contrast to the variable-range hopping of self-localized polarons found in more disordered polymers. The FET-LED device represents a step toward all-polymer optoelectronic integrated circuits such as active-matrix polymer LED displays.

# Breakdown of the QP picture

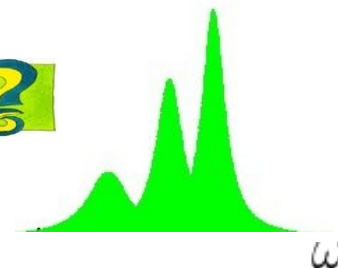
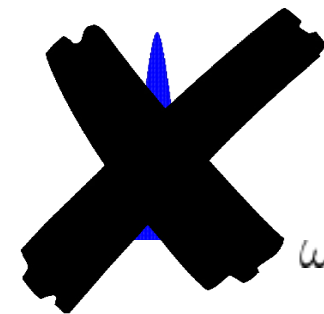
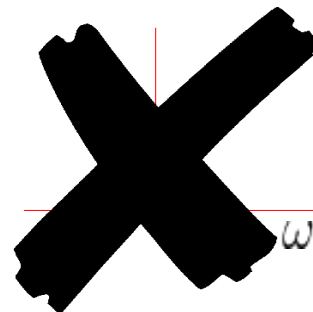
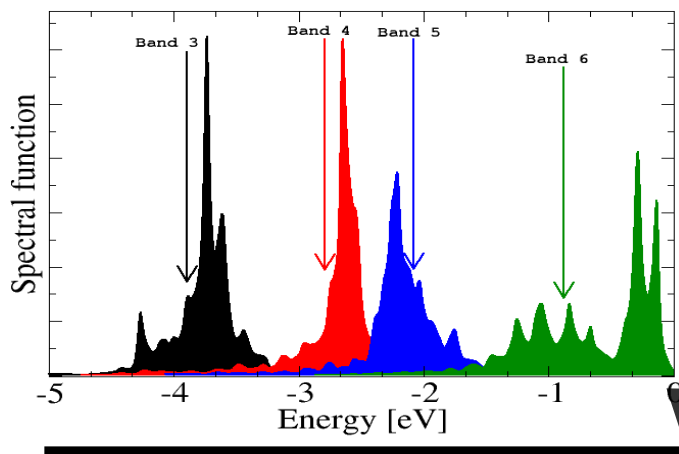
E. Cannuccia, *Phys. Rev. Lett.* **107**, 255501 (2011)



$$A(k, \omega) \equiv \sum_n \frac{1}{\pi} |\Im [G_{nk}(\omega)]|$$

$$\Delta Z \equiv A(k, \omega) \Delta\omega$$

←  $\Delta\omega = 50 \text{ meV}$ .



# Polarons in an Hamiltonian representation (I)

$$H = H_e + H_{ph} + \frac{1}{N_{\mathbf{q}}} \sum_{n', \mathbf{q}, \lambda} g_{n' n \mathbf{k}}^{\mathbf{q}, \lambda} c_{n' \mathbf{k} + \mathbf{q}}^\dagger c_{n \mathbf{k}} (b_{\mathbf{q}, \lambda}^\dagger + b_{-\mathbf{q}, \lambda})$$

**Phonon population:**  
 $N_{ph} = (e^{\beta \omega_{\mathbf{q}, \lambda}} - 1)^{-1}$

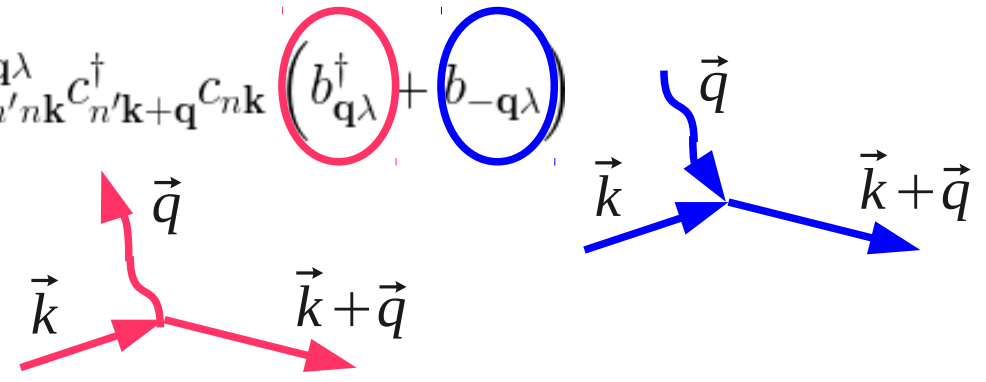
Basis set

$T = 0 \text{ }^\circ\text{K}$

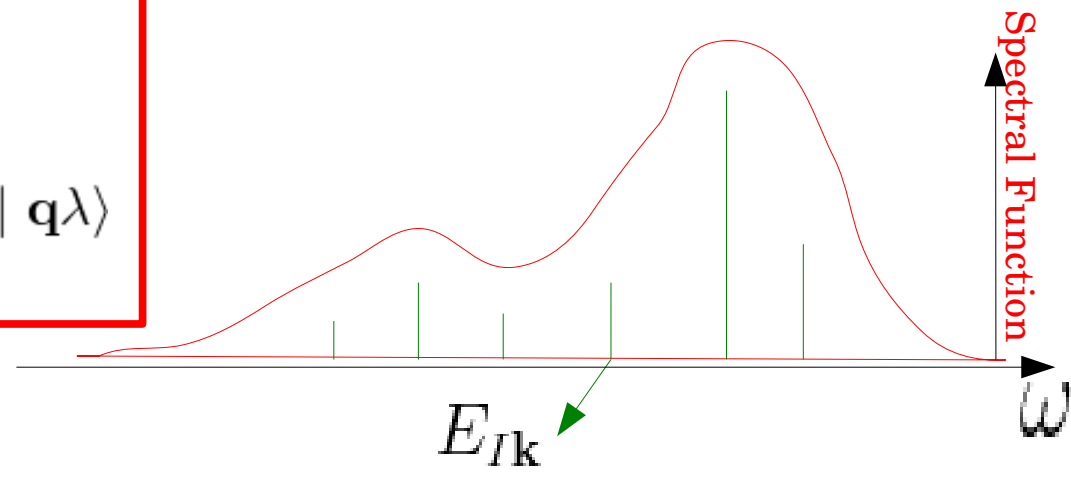
$$|n \mathbf{k}\rangle \otimes |vac\rangle \xrightarrow{H} |n \mathbf{k}\rangle \otimes |vac\rangle, |n \mathbf{k} - \mathbf{q}\rangle \otimes |\mathbf{q}, \lambda\rangle$$

$$H |I \mathbf{k}\rangle = E_{I \mathbf{k}} |I \mathbf{k}\rangle$$

$$|I \mathbf{k}\rangle = A_{n \mathbf{k}} |n \mathbf{k}\rangle + \sum_{n \mathbf{q}, \lambda} B_{n \mathbf{q}, \lambda} |n \mathbf{k} - \mathbf{q}\rangle \otimes |\mathbf{q}, \lambda\rangle$$



$$G_{n \mathbf{k}}(\omega) = \sum_I |\langle \Psi | c_{n \mathbf{k}}^\dagger | I \mathbf{k}\rangle|^2 \frac{1}{\omega - E_{I \mathbf{k}}}$$

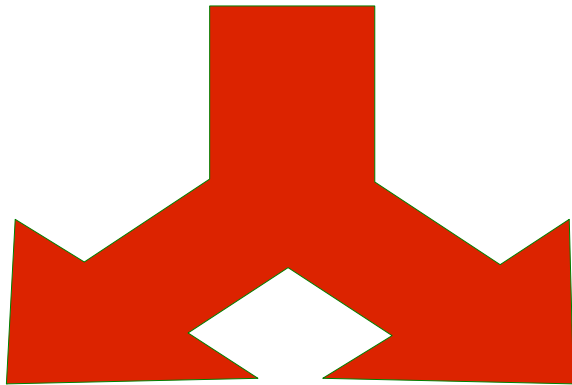


$$G_{n \mathbf{k}}(\omega) = \frac{G_{n \mathbf{k}}^0(\omega)}{G_{n \mathbf{k}}^0(\omega)^{-1} - \Sigma_{n \mathbf{k}}(\omega)}$$

# Polarons in an Hamiltonian representation (II)

$$G_{n\mathbf{k}}(\omega) = \sum_I |\langle \Psi | c_{n\mathbf{k}}^\dagger | I\mathbf{k} \rangle|^2 \frac{1}{\omega - E_{I\mathbf{k}}}$$

$$|I\mathbf{k}\rangle = A_{n\mathbf{k}} |n\mathbf{k}\rangle + \sum_{n\mathbf{q}\lambda} B_{n\mathbf{q}\lambda} |n\mathbf{k} - \mathbf{q}\rangle \otimes |\mathbf{q}\lambda\rangle$$

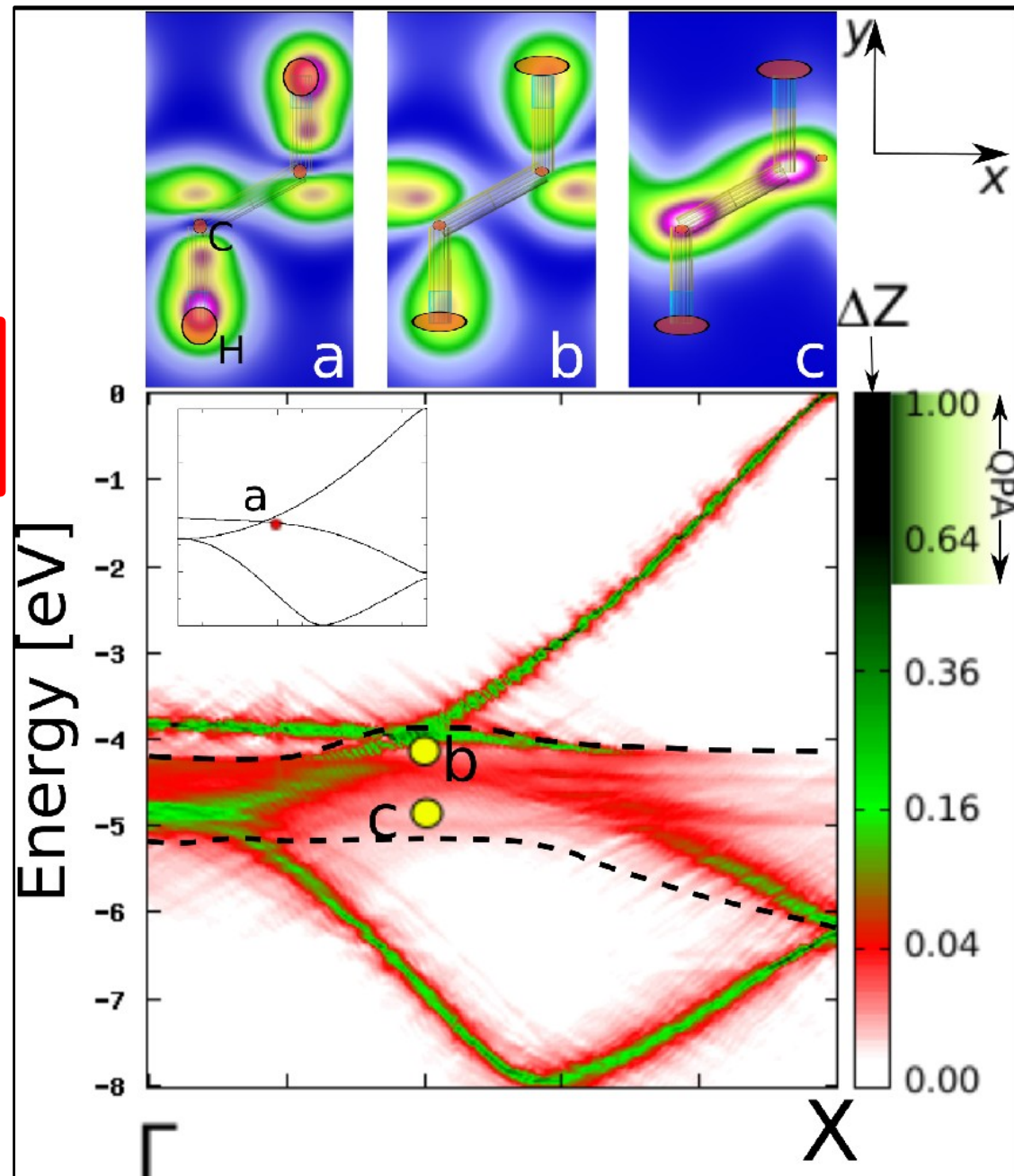


$$|\langle \vec{r} | I\mathbf{k} \rangle|^2$$

The polaronic wave-function

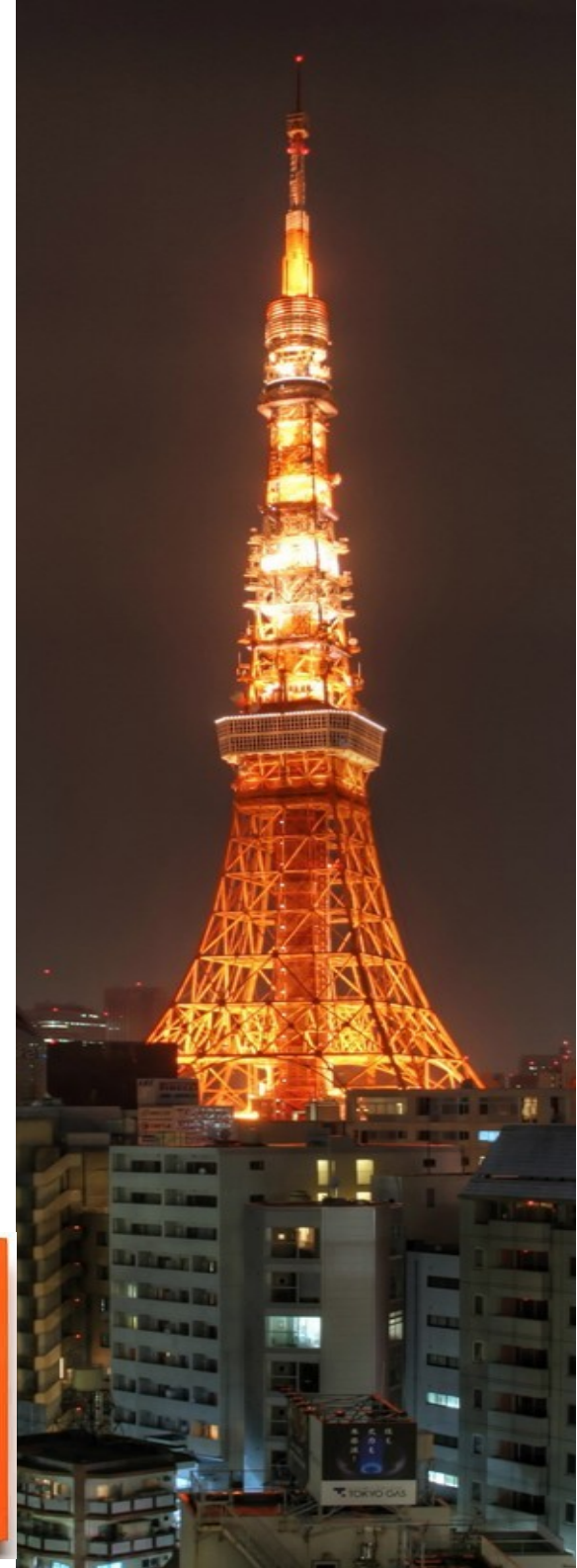
$$\langle I\mathbf{k} | u^2 | I\mathbf{k} \rangle$$

The atomic indetermination  
IN the polaronic state

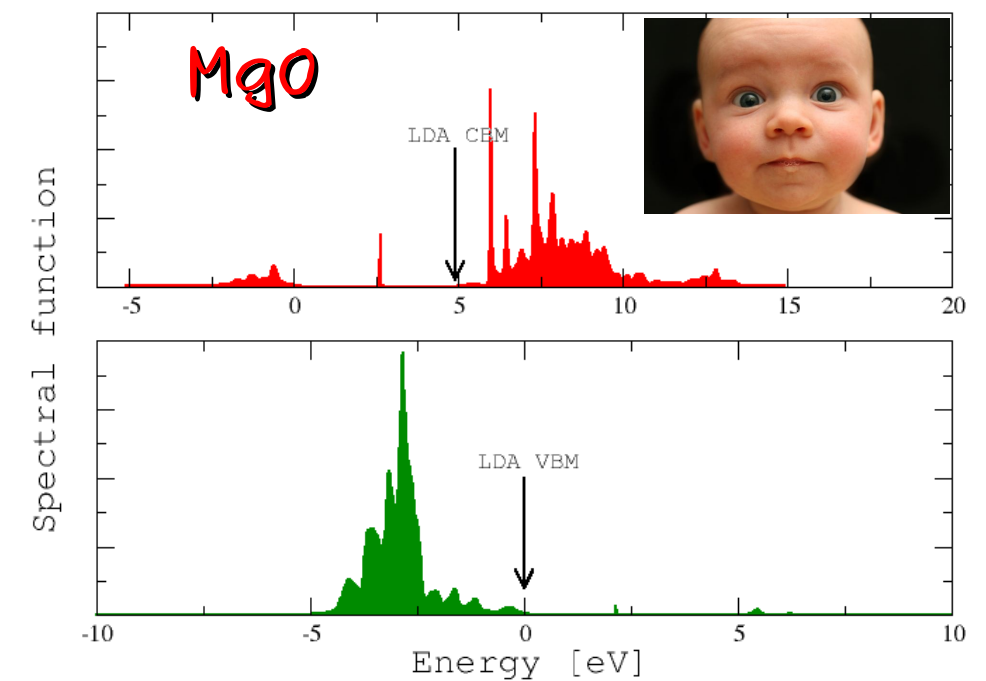
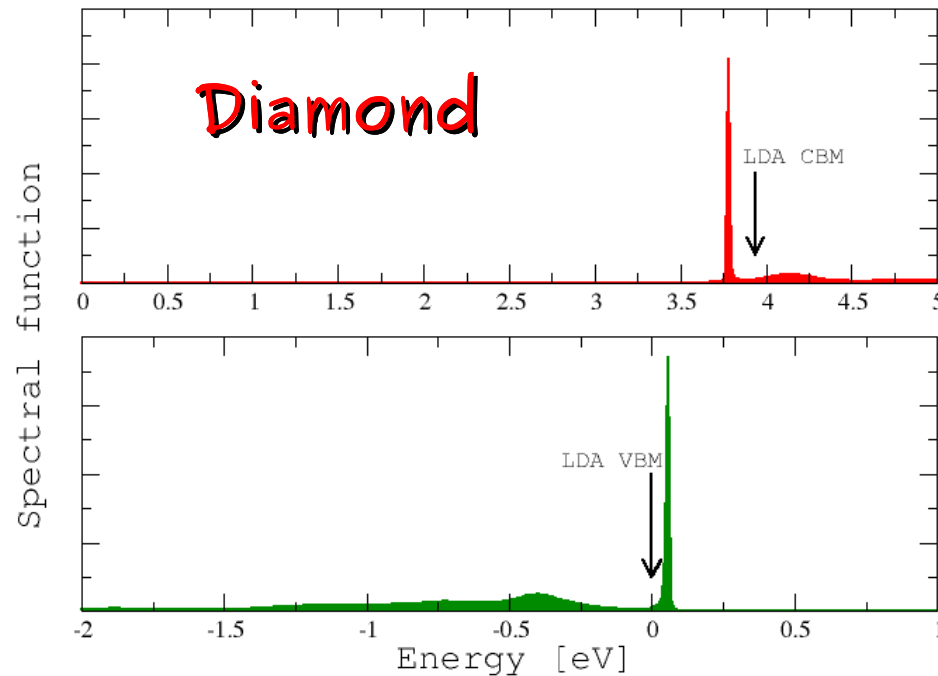
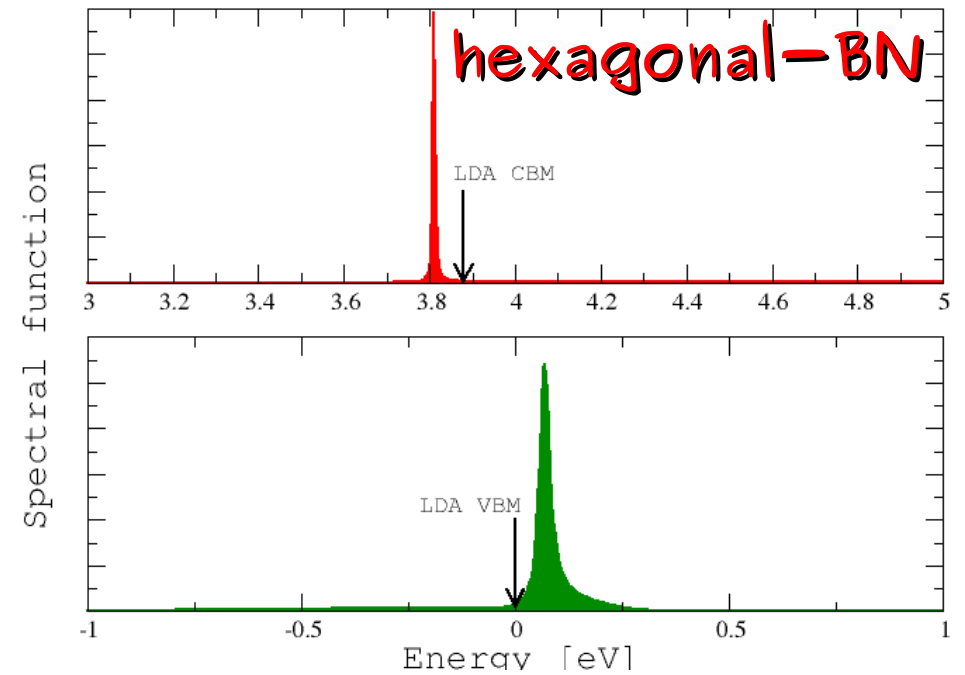
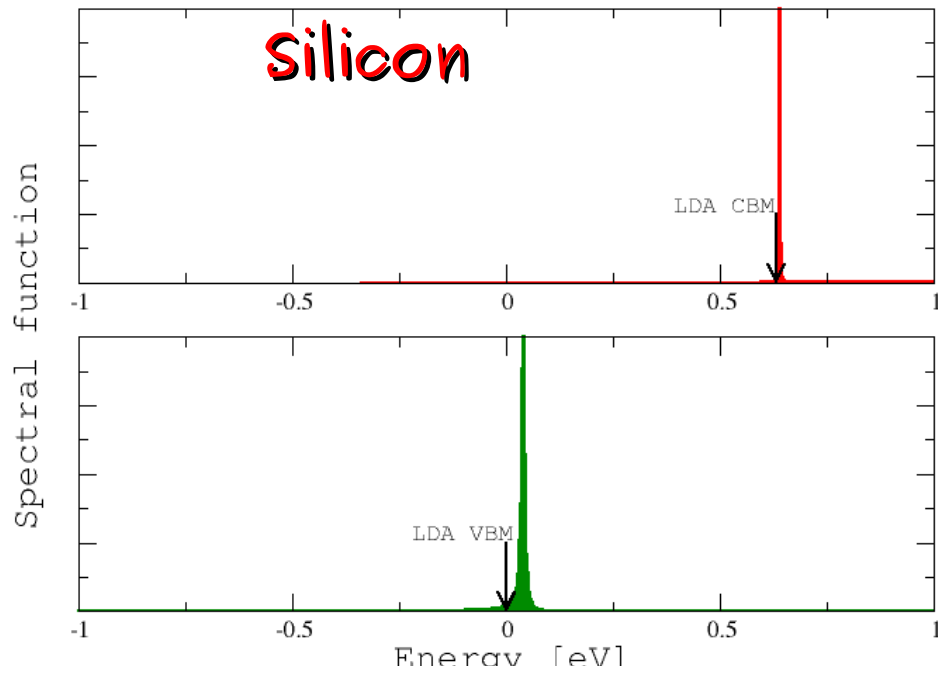


***Are polymers an isolated case ?***

4



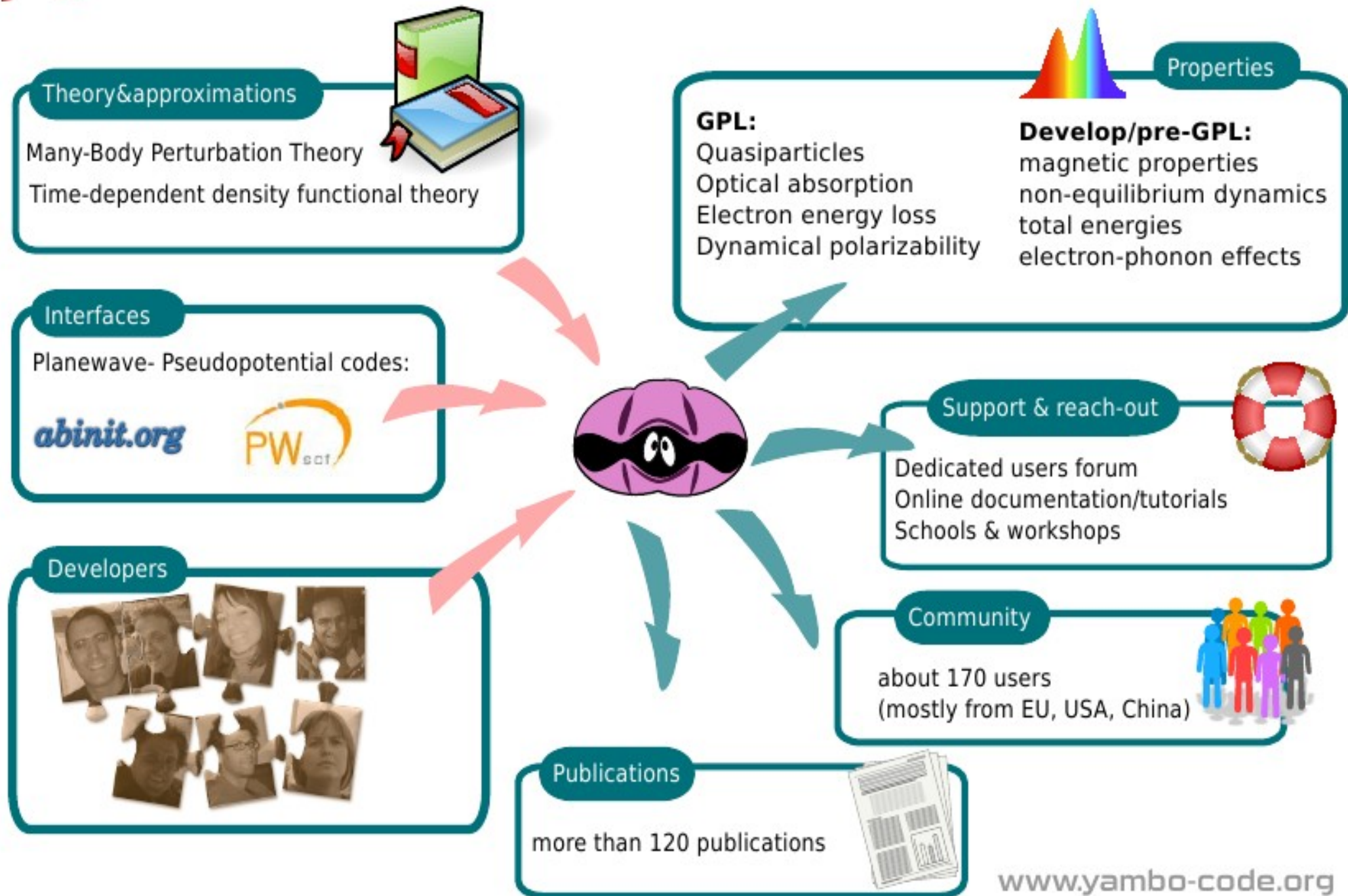
# Spectral Functions solids @ the CBM and VBM







# Yambo: an ab-initio tool for excited state calculations



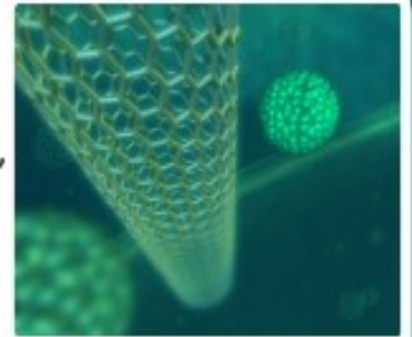
# Yambo: an ab-initio tool for R&D

## Material Science



applications to  
e.g. photovoltaics,  
lithium batteries,  
microelectronics

## Nano Science



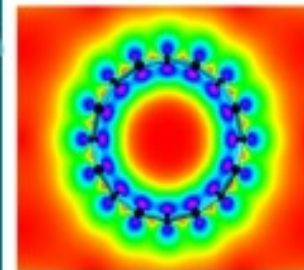
applications to  
e.g. nanophotonics,  
nanoelectronics

## Biology



studies of  
photoactive  
molecules  
and molecular  
complexes

## Physics



fundamental  
understanding  
of physical  
processes



# Yambo<sup>©</sup>



Optimal MPI (and, shortly, OpenMP) Paralellization

Improved and efficient memory distribution

Interfaced to common and well-known ground-state GPL codes: abinit, QEspresso, CPMD

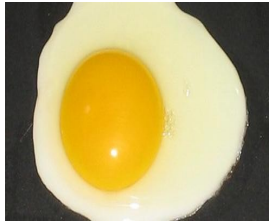
Development work-in-progress within the PRACE (Partnership for Advanced Computing in Europe) in collaboration with the Italian and Portuguese Super-Computing centers

**Yambo: an *ab-initio* tool for excited state calculations,**  
Andrea Marini, Conor Hogan, Myrta Grüning, Daniele Varsano,  
Comp. Phys. Comm. **180**, 1392 (2009)

Electronic Correlation



Real Particles



Quasi Particles



Multi-component particles

Electron-Phonon Coupling

# Yambo



<http://www.yambo-code.org/>

[:: project](#) [:: team](#) [:: links](#) [:: contact us](#)

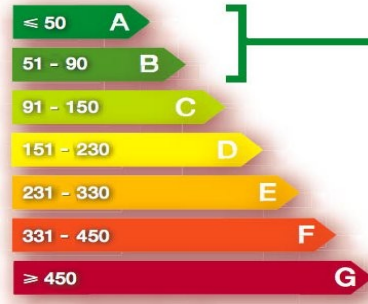
**PWSCF** Plane-Wave Self-Consistent Field

*"The computer is a tool for clear thinking"*  
Freeman J. Dyson

<http://www.pwscf.org>

# Conclusions

Polaronic-induced effect can be HUGE. They can even lead to the breakdown of the electronic picture



The weak correlation is counterbalanced by the low-energy activation process (~Debye energy)

Potential ground-breaking consequences on mobility, optical properties, transport...



Critical re-examination of ALL purely electronic band-structure calculations on C-based nanostructures ?

# People and references...



**Elena Cannuccia**

PhD Thesis (2011)

Giant polaronic effects in polymers: breakdown of the quasiparticle picture

[http://www.yambo-code.org/papers/Thesis\\_Elena\\_Cannuccia.pdf](http://www.yambo-code.org/papers/Thesis_Elena_Cannuccia.pdf)

E. Cannuccia and AM "Ab initio study of entangled electron-phonon states in polymers", submitted to Phys. Rev. B (2012)

E. Cannuccia and AM "Effect of the quantistic zero-point atomic motion on the opto- electronic properties of diamond and trans-polyacetylene", **107**, 255501 (2011)

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