Electron Dynamics in a Quantum Dot on Helium Surface

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The system of electrons on the surface of superfluid ^{4}He is attractive for making a scalable quantum computer (QC) [1]. The electrons have an extremely long relaxation time and display the highest mobility known in a condensed-matter system [2]. In addition, the typical inter-electron distance is comparatively large, $\sim 1\,\mu$m. A QC can be made by fabricating a system of micro-electrodes that should be submerged beneath the helium surface. The electrodes create potential wells for electrons on the surface, which become single-electron quantum dots, see Fig. 1. Localized electrons can be excited by resonant microwave radiation, and their energy spectrum can be electrostatically controlled.

Here we study dissipation processes for electrons in quantum dots on helium surface. We investigate mechanisms of coupling to excitations in helium, phonons and ripplons, and analyze how the electron relaxation rates depend on the parameters of the confining potential. Decay and decoherence of electron states result also from classical and quantum electrode noise. We relate the corresponding relaxation rates to the power spectrum of the fluctuating electric field on the electron. The dependence of the rates on the electrode parameters is obtained.

Electron states in a quantum dot $|n,\nu,m_\nu\rangle$ are characterized by the quantum number n of motion normal to the helium surface, the principal quantum number of in-plane vibrations ν, and the number m_ν that enumerates degenerate vibrational states. The working states of a qubit based on a confined electron are $|1,0,0\rangle$ and $|2,0,0\rangle$. The energy difference between these states E_2-E_1 can be Stark-shifted by the electric field from the electrode $\mathcal{E}_\perp$. This field also determines the in-plane vibrational frequency $\omega_\parallel$, see Fig. 1. A simple estimate of the field can be done assuming that the electrodes are spheres of radius $r_{\rm el}$. In this case $\omega_\parallel = (e\mathcal{E}_\perp/mh)^{1/2}$. Typically $\omega_\parallel/2\pi \sim 20$ GHz, whereas E_2-E_1 may be 6-10 times larger.

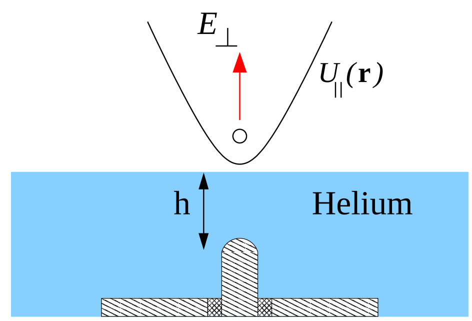

Figure 1. A single-electron quantum dot on helium surface. A micro-electrode is submerged by the depth $h\sim 0.5\,\mu$m beneath the helium surface. The electron is driven by a field $E_\perp$ normal to the surface. This field comes from the electrode and the parallel-plate capacitor (only the lower plate of the capacitor is shown). The confining in-plane potential $U_\parallel(\mathbf{r}) \approx m\omega_\parallel^2 r^2/2$ [$\mathbf{r}=(x,y)$] is determined by the electrode potential and geometry.

Because of the discreteness of the electron energy spectrum in a dot, the mechanisms of electron decay and dephasing are qualitatively different from those studied for a 2D electron system on helium in the absence of in-plane confinement. Decay of the excited state $|2,0,0\rangle$ is most likely to occur via a ripplon- or phonon-induced transition to the closest in energy excited vibrational state $|1,\nu_c,m_\nu\rangle$, with $\nu_c = {\rm int}[(E_2-E_1)/\hbar\omega_\parallel]$. The involved energy transfer is $\sim \hbar\omega_\parallel$. It largely exceeds the energy of ripplons with typical wave numbers $q \lesssim (\hbar/m\omega_\parallel)^{1/2}$. This makes one-ripplon decay exponentially improbable and strongly reduces the decay rate compared to the case of unconfined electrons.

It is more probable for an electron to decay by emitting two ripplons which propagate in opposite directions with nearly same wave numbers $q_{1,2}$ [$|\mathbf{q}_1+\mathbf{q}_2| \lesssim (\hbar/m\omega_\parallel)^{1/2} \ll q_{1,2}$]. Alternatively, decay may be due to emission of a phonon. The appropriate phonons propagate nearly normal to the helium surface. We show that the mechanisms of coupling to phonons are phonon-induced deformation of the helium surface and modulation of the dielectric constant of helium and thus of the electrostatic energy of the electron above helium. The contributions of these two mechanisms are comparable for typical parameter values.

Both the two-ripplon and phonon decay rates are very sensitive to the in-plane electron frequency $\omega_{\parallel}$. The increase of $\omega_{\parallel}$ leads to the decrease of the ripplon and phonon wavelengths. Helium vibrations with wavelengths smaller than the width of the diffuse layer on the helium surface should be essentially decoupled from electrons. For $\omega_{\parallel}/2\pi = 20$ GHz the overall decay rate is $\lesssim 10^4\ \mathrm{s}^{-1}$.

Electron dephasing due to coupling to excitations in helium comes primarily from quasi-elastic scattering of ripplons off the localized electron. Scattering amplitudes are different in different electron states, and therefore scattering leads to diffusion of the phase difference between the wave functions $|2,0,0\rangle$ and $|1,0,0\rangle$. For different mechanisms of coupling to ripplons the dephasing rate $\Gamma_\phi^{(\mathrm{r})}$ displays same temperature dependence $\Gamma_\phi^{(\mathrm{r})} \propto T^3$, which is much slower than the standard T^7 law for defects in solids. Numerically, $\Gamma_\phi^{(\mathrm{r})} \lesssim 10^2\ \mathrm{s}^{-1}$ for $T = 10$ mK.

An important source of dephasing of electron states is noise from the underlying electrode, see Fig. 1. Coupling to this noise is dipolar, with the Hamiltonian

$$H_{\mathrm{dip}} = -e\,\delta\hat{\mathcal{E}}_\perp z, \tag{1}$$

where $\delta\hat{\mathcal{E}}_\perp$ is the normal to the surface component of the field from quantum or classical charge density fluctuations in the electrode.

The rates $\Gamma_\phi^{(\mathrm{el})}$ and $\Gamma_{12}^{(\mathrm{el})}$ of the electron dephasing and decay can be expressed in terms of the field correlation function

$$Q(\omega) = \int_0^\infty dt\, e^{i\omega t} \langle \delta\hat{\mathcal{E}}_\perp(t)\delta\hat{\mathcal{E}}_\perp(0)\rangle \tag{2}$$

as

$$\Gamma_\phi^{(\mathrm{el})} = e^2(z_{22}-z_{11})^2 \mathrm{Re}\ Q(0)/\hbar^2, \quad \Gamma_{12}^{(\mathrm{el})} = e^2|z_{12}|^2 \mathrm{Re}\ Q(\Omega_{12})/\hbar^2. \tag{3}$$

Here, $z_{ij} = \langle i,0,0|z|j,0,0\rangle$, with $i,j = 1,2$, and $\Omega_{12} = (E_2 - E_1)/\hbar$.

The major contribution to the dephasing rate comes from Johnson's noise in the external leads. A simple estimate can be made assuming that the confining electrode is a sphere of radius r_{el}. It gives

$$\Gamma_\phi^{(\mathrm{el})} = 2k_B T_{\mathrm{ext}} \mathcal{R}_{\mathrm{ext}} e^2 (z_{22}-z_{11})^2 r_{\mathrm{el}}^2/\hbar^2 h^4, \tag{4}$$

where T_{ext} and $\mathcal{R}_{\mathrm{ext}}$ are the lead temperature and resistance. For $\mathcal{R}_{\mathrm{ext}} = 25\ \Omega$, $T_{\mathrm{ext}} = 1$ K, $r_{\mathrm{el}} = 0.1\,\mu$m, and $h = 0.5\,\mu$m we obtain $\Gamma_\phi^{(\mathrm{el})} \approx 1\times 10^4\ \mathrm{s}^{-1}$. This shows that thermal electrode noise may be the main source of dephasing for a qubit. Eq. (4) suggests how to reduce the rate $\Gamma_\phi^{(\mathrm{el})}$. The decay rate $\Gamma_{12}^{(\mathrm{el})}$ is much less than the phonon-induced decay rate. The estimates show that the overall relaxation rate can be made 4-5 orders of magnitude smaller than the clock frequency of the quantum computer based on electrons on helium.

[1] P.M. Platzman and M.I. Dykman, Science **284**, 1967 (1999); M.I. Dykman, P.M. Platzman, and P. Seddighrad, Phys. Rev. B **67** (2003).

[2] K. Shirahama, S. Ito, H. Suto, and K. Kono, J. Low Temp. Phys. **101**, 439 (1995).