Excitation Spectrum in Bilayer $\nu = 2$ Quantum Hall Systems

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In a bilayer $\nu = 2$ quantum Hall (QH) system, theoretical investigations have predicted the existence of canted antiferromagnetic phase (CAF) and experimental evidence have confirmed it [1]. The excitation properties in this system have been discussed in the Hartree-Fock approximation (HF) [2] and exact diagonalization (ED) [3] calculations. However, the HF calculation cannot fully treat the electron correlation effects and a clear picture of excitation spectra has yet to come. Moreover, the HF calculation is not applicable to a bilayer $\nu = 2/m$ fractional QH system (m: odd integer), which is a composite-fermion bilayer $\nu_{\mathrm{CF}} = 2$ QH system. We construct an effective Hartree-Fock-Bogoliubov (HFB) Hamiltonian. This Hamiltonian contains more correlation effects than the HF Hamiltonian and is easily applicable to the bilayer $\nu = 2/(4m+1)$ fractional QH system (m: integer). We succeeded in getting the SU(2) symmetrical excitation spectrum, *i.e.*, not only spin-triplet excitations but also spin-singlet ones came out. On the other hand, in the HF calculation, only the spin-triplet excitations have been discussed. We calculated the number of electrons occupying antisymmetric single-particle states, then compared with those by the ED and by the effective spin theory [4]. We find that the ground state properties by the HFB are better than those by the effective spin theory for large tunneling energy (Δ_{SAS}).

The HFB Hamiltonian is given by

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}} &= \sum_{K,N} [\, (\Delta_{\mathrm{SAS}} + e_K + \lambda_K)(C^\dagger_{KN} C_{KN} + H^\dagger_{KN} H_{KN}) \\
&\quad + (\lambda_K/2)(-1)^N (C^\dagger_{KN} C^\dagger_{K,-N} + H^\dagger_{KN} H^\dagger_{K,-N} + \mathrm{h.c.}) \\
&\quad + f_K (C^\dagger_{KN} H_{KN} + (-1)^N C^\dagger_{KN} H^\dagger_{K,-N} + \mathrm{h.c.}) \\
&\quad + (\Delta_{\mathrm{SAS}} + \Delta_{\mathrm{Z}} + e_K + \lambda_K - g_K) D^\dagger_{KN} D_{KN} \\
&\quad + (\Delta_{\mathrm{SAS}} - \Delta_{\mathrm{Z}} + e_K + \lambda_K - g_K) F^\dagger_{KN} F_{KN} \\
&\quad - g_K (-1)^N (D^\dagger_{KN} F^\dagger_{K,-N} + \mathrm{h.c.}) \,], \\
e_K &= \sum_{J=0}^{2S} (2J+1) V^J_{\mathrm{intra}} \left[\frac{(-1)^{2S-J}}{2S+1} - \begin{Bmatrix} SSJ \\ SSK \end{Bmatrix} \right], \\
\lambda_K &= \sum_{J=0}^{2S} (2J+1) \frac{V^J_{\mathrm{intra}} - V^J_{\mathrm{inter}}}{2} [(-1)^{2S-J} - 1] \begin{Bmatrix} SSJ \\ SSK \end{Bmatrix}, \\
f_K &= \sum_{J=0}^{2S} (2J+1) \frac{V^J_{\mathrm{intra}} - V^J_{\mathrm{inter}}}{2} (-1)^{2S-J} \begin{Bmatrix} SSJ \\ SSK \end{Bmatrix}, \\
g_K &= f_K - \lambda_K .
\end{aligned}
$$

The interaction matrix elements can be expressed in terms of Wigner's $6j$- symbol $\{ {}^{SSJ}_{SSK} \}$, while $C^\dagger_{KN}$, $D^\dagger_{KN}$, $F^\dagger_{KN}$, $H^\dagger_{KN}$ create an exciton (a hole in the $s\!\uparrow$, $s\!\uparrow$, $s\!\downarrow$, $s\!\downarrow$ band and a particle in the $a\!\uparrow$, $a\!\downarrow$, $a\!\uparrow$, $a\!\downarrow$ band with total angular momentum K ($0 \le K \le 2S$, $2S$: the number of flux quanta passing through the Haldane sphere) and its z component N, respectively, where s/a and $\uparrow/\downarrow$ denote symmetric/antisymmetric and spin-up/-down states, respectively), and $V^J_{\mathrm{intra/inter}}$ are the intra- and inter-layer pseudopotentials for relative angular momentum $2S - J$, respectively. We diagonalized this Hamiltonian and calculated various excitation spectra by changing layer separation d/l_{B} and tunneling energy $\Delta_{\mathrm{SAS}}/E_{\mathrm{C}}$ (where $l_{\mathrm{B}} = \sqrt{c\hbar/eB}$ and $E_{\mathrm{C}} = e^2/\epsilon l_{\mathrm{B}}$). We compared the HFB spectra with the ED ones in case of zero Zeeman energy ($\Delta_{\mathrm{Z}} = 0$). The HFB approximation reproduces the ED spectrum with high accuracy for large Δ_{SAS} (Fig.1).

Then the ground state is a spin-singlet (SS) state, while the mode softening occur in the HFB spectrum for $\Delta_{\rm SAS}/E_{\rm C} \leq 0.6$, *i.e.*, the CAF phase is favored.

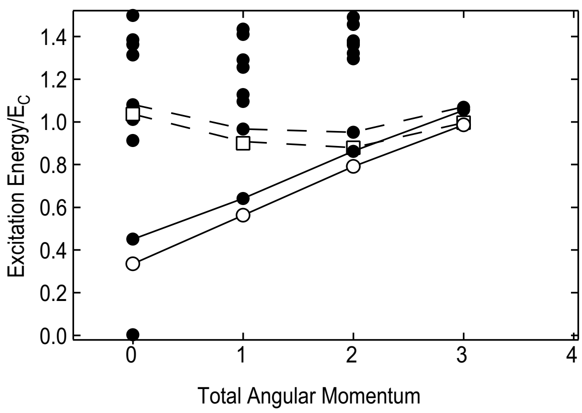

Fig.1: The low-lying excitation spectrum as a function of total angular momentum for eight electrons, $d/l_{\rm B} = 1$, and $\Delta_{\rm SAS}/E_{\rm C} = 0.7$. Open circles with solid line (open squares with dashed line) show spin-triplet (-singlet) excitation spectrum by the HFB approximation, and spin-triplet (-singlet) excitations obtained by the ED are linked by solid (dashed) line, too.

In order to investigate the ground state (GS) properties, we calculated the number of electrons ($N_{\rm A}$) occupying antisymmetric single-particle states in GS, then compared with those of the ED and the effective spin theory for large tunneling energy (Fig.2). It is found that the HFB approximation reproduces the ED result better than the effective spin theory does (especially for $\Delta_{\rm SAS}/E_{\rm C} \geq 0.9$ in Fig.2).

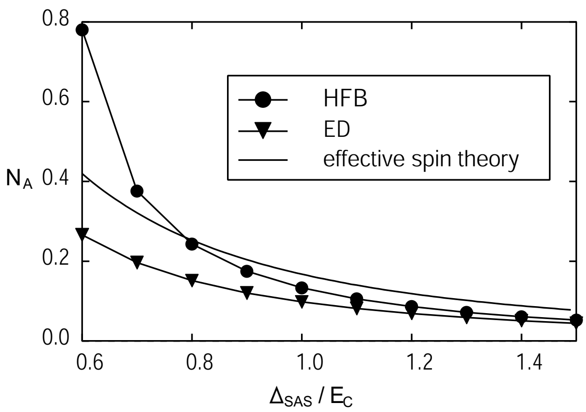

Fig.2: The number of electrons occupying antisymmetric states in the ground state as a function of $\Delta_{\rm SAS}/E_{\rm C}$ for eight electrons and $d/l_{\rm B} = 1$.

In conclusion, we succeeded in the construction of the effective Hamiltonian which is easily applicable to the fractional $\nu = 2/(4m+1)$ QH system (m: integer) and the low-lying excitation spectrum calculated from this Hamiltonian preserves the SU(2) symmetry. On the large tunneling energy region, this Hamiltonian describes the bilayer $\nu = 2$ QH system better than other approximate theories.

[1] A. Sawada *et al.*, Phys. Rev. Lett **80**, 4534(1998); V. Pellegrini *et al.*, *ibid.* **78**, 310(1997).
[2] S. Das Sarma *et al.*, Phys. Rev. B **58**, 4672(1998).
[3] J. Schliemann *et al.*, Phys. Rev. Lett **84**, 4437(2000).
[4] E. Demler *et al.*, Phys. Rev. Lett **82**, 3895(1999).