Entanglement of Nuclear Spins in Spin-Blocked Quantum Dots

Mikio Eto, Takashi Ashiwa and Mikio Murata

Faculty of Science and Technology, Keio University, Yokohama 223-8522, Japan

Entanglement—nonlocal correlation among more than one particle— is one of the basic concepts in quantum mechanics. Entangled states need to be realized and controlled in quantum computation and quantum communication. In this paper, we propose an entanglement of nuclear spins in quantum dots driven by the electric current accompanied by the spin flip. This situation is relevant for a leakage current in spin-blockade regions where electrons cannot be transported unless the spins are flipped [1]. We show that the entanglement of nuclear spins drastically enhances the leakage current which should be observable experimentally.

We examine the electric transport through a quantum dot in the spin-blockade region. From an external lead, an electron tunnels into the dot with spin $|\uparrow\rangle$ or $|\downarrow\rangle$, and occupies a single level with envelope wavefunction $\psi(\mathbf{r})$. The electron is coupled to N nuclear spins, $\mathbf{I}_k$, by the hyperfine contact interaction, $H_{\rm hf} = 2\sum_{k=1}^{N} \alpha_k \mathbf{S}\cdot\mathbf{I}_k$, where $\alpha_k \propto |\psi(\mathbf{r}_k)|^2$ (nuclear spins of 1/2 are assumed). After the spin is flipped, the electron tunnels out to another lead. We study the case of uniform coupling constants, $\alpha_k = \alpha$, first. A generic case with non-uniform α_k is examined later.

To illustrate the entanglement of nuclear spins, let us begin with the case of two nuclear spins ($N = 2$) and an initial state of $|I_z = 1/2\rangle_1 |I_z = 1/2\rangle_2$. An electron with spin $|\downarrow\rangle$ tunnels into the dot and is spin-flipped by the hyperfine interaction. Then the state of nuclear spins becomes

$$(|-1/2\rangle_1|1/2\rangle_2 + |1/2\rangle_1|-1/2\rangle_2)/\sqrt{2} = |J = 1, M = 0\rangle,$$

where J and M are the total spin and its z component, respectively. This is an entangled state: we do not know which of the nuclear spins is flipped. After the electron tunnels out of the dot, the next electron is injected with $|\downarrow\rangle$ or $|\uparrow\rangle$. The spin-flip rate of the second electron is proportional to $|\langle 1, \mp 1; \uparrow (\downarrow)|H_{\rm hf}|1, 0; \downarrow (\uparrow)\rangle|^2 = 2\alpha^2$. This value is twice the rate in the case of non-entangled states, $|1/2\rangle_1|-1/2\rangle_2$ or $|-1/2\rangle_1|1/2\rangle_2$. In general, the capability of state $|J, M\rangle$ to flip an electron spin is $\propto (J \pm M)(J \mp M + 1)$.

For N nuclear spins ($N \sim 10^5$ in GaAs quantum dots), we assume that nuclear spins are randomly oriented in the initial state. The state involves all the components of $|J, M\rangle$ for the total nuclear spin with random coefficients $\{C^{(0)}_{J,M,\lambda}\}$,

$$\Psi^{(0)} = \sum_{J,M,\lambda} C^{(0)}_{J,M,\lambda} |J, M, \lambda\rangle$$

(Index λ distinguishes states with the same J and M [2]). An electron with $|\downarrow\rangle$ (or $|\uparrow\rangle$) is injected to the dot, spin-flipped, and ejected out of the dot. Then the state of nuclear spins

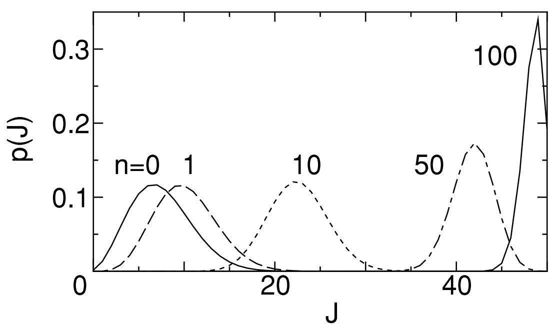

Fig. 1: The distribution of the total spin J in the state of nuclear spins, $p(J) = \sum_{M,\lambda} |C^{(n)}_{J,M,\lambda}|^2$, after n spin-flips of electrons. The number of nuclear spins is $N = 100$.

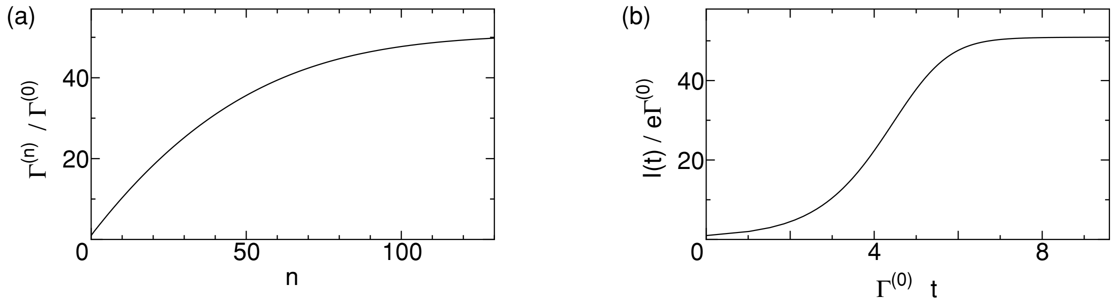

Fig. 2: (a) The spin-flip rate $\Gamma^{(n)}$ for the $(n+1)$st electron. (b) The current $I(t)$ as a function of time t. The number of nuclear spins is $N = 100$.

becomes

$$\Psi^{(1)} \propto \sum_{J,M,\lambda} C^{(0)}_{J,M,\lambda} \sqrt{(J \pm M)(J \mp M + 1)} |J, M \mp 1, \lambda\rangle.$$

This indicates that components of larger J increase by the factor of $(J \pm M)(J \mp M + 1)$. After n spin-flips of electrons, the distribution of J is shown Fig. 1. This correlated state of nuclear spins enhances the spin-flip rate of electrons. As shown in Fig. 2(a), the rate for the $(n+1)$st electron, $\Gamma^{(n)}$, increases with n linearly ($\Gamma^{(n)} \approx n\Gamma^{(0)}$) and finally saturates [$\Gamma^{(n)} \approx (N/2)\Gamma^{(0)}$]. This results in an enhancement of the leakage current in the spin-blockade region. The current as a function of time t is written as

$$\begin{cases} I(t) & \approx & e\Gamma^{(0)} e^{\Gamma^{(0)} t} & \quad (t \ll t_{\rm sat}) \\ I(t) & \approx & e(N/2)\Gamma^{(0)} & \quad (t_{\rm sat} \ll t). \end{cases}$$

The saturation time is given by $t_{\rm sat} = \ln(N/2)/\Gamma^{(0)}$, where $\Gamma^{(0)}$ is the spin-flip rate of the first electron by randomly oriented nuclear spins [2]. (In actual quantum dots, the increase in the current continues until the entangled state is broken by dephasing effects.) The time dependence of the leakage current in Fig. 2(b) may be observed experimentally, using NMR pulse to randomize the nuclear spins, when the interval time of the pulses is tuned.

Generally, the hyperfine coupling constants α_k depend on the position of nuclei due to the spatial variation of the envelope wavefunction $\psi(\mathbf{r})$ in quantum dots. We perform numerical calculations in the case of $\alpha_k \propto \exp(-k/x_0)$ ($k = 0, 1, 2, \cdots, N-1$) with the normalization of $\frac{1}{N}\sum_{k=0}^{N-1} \alpha_k = \alpha$. The initial state of nuclear spins is randomly averaged. Figure 3 presents the spin-flip rate $\Gamma^{(n)}$ with $x_0 = 1, 2, 5$, and ∞ ($N = 10$). Even with finite x_0, the spin-flip rate is significantly enhanced by the entanglement of nuclear spins. The behavior of $\Gamma^{(n)}$ is determined by the number of nuclear spins which are effectively coupled to the electron in the dot.

[1] K. Ono *et al.*, Science **297**, 1313 (2002); K. Ono *et al.*, in preparation.
[2] M. Eto, cond-mat/0210231.

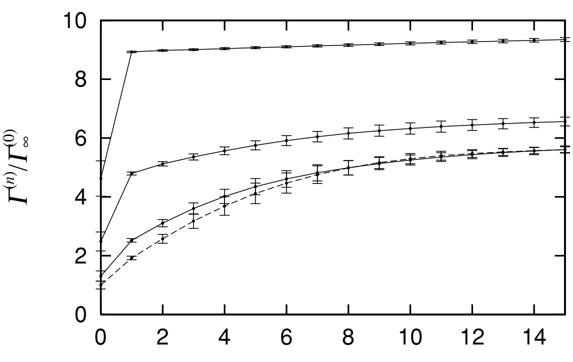

Fig. 3: The spin-flip rate $\Gamma^{(n)}$ for the $(n+1)$st electron, when the hyperfine coupling constants are $\alpha_k \propto \exp(-k/x_0)$ ($k = 0, 1, 2, \cdots, N-1$). $x_0 = 1, 2, 5$ (solid lines, from upper to lower) and ∞ (broken line) with $N = 10$. $\Gamma^{(n)}$ is normalized by $\Gamma^{(0)}$ with $x_0 = \infty$. The initial state of nuclear spins is randomly averaged (variation is also indicated).