Coherent versus Sequential Electron Tunneling in Quantum Dots

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Sequential tunneling is the key hypothesis for the standard rate equations [1] used to explain the transmission spectrum of quantum dots in the Coulomb blockade regime [2, 3]. This probabilistic picture neglects non-resonant quantum virtual processes, under the assumption that the resonant decay widths Γ are much smaller than both k_BT and the energy separation between the quantum dot resonances $\delta\varepsilon$, namely, $\Gamma \ll k_BT$ and $\Gamma \ll \delta\varepsilon$, a condition often met by experiments in nearly isolated quantum dots. The early experimental data taken from ballistic chaotic quantum dots were successfully confronted with the sequential theory by using the random matrix theory (RMT) to model the dot statistical single-particle properties. More recently, the analysis of the measured conductance peak-heights in the Coulomb blockade regime [2, 3] show significant deviations from this theory, indicating that some physics is missing. The inclusion of inelastic scattering processes [4], spin-orbit coupling [5], and exchange interaction [6, 7] into the sequential approach expand in interesting ways the considered physical processes. These studies achieved only a limited success in reconciling theory with experiment.

We show that quantum coherence, so far overlooked, leads to important corrections to the sequential tunneling picture and explains some of the puzzles pointed out by the conductance experiments [2, 3]. The importance of coherent processes is justified by noticing that while the sequential theory requires $\Gamma \ll k_BT$, $\delta\varepsilon$, the experiments satisfy those conditions only in average, namely, $\langle\Gamma\rangle < \Delta \equiv \langle\delta\varepsilon\rangle$ and $\langle\Gamma\rangle \lesssim k_BT$. Since both the decay width Γ and the resonance spacings $\delta\varepsilon$ fluctuate, conductance peaks where Γ is larger than k_BT and $\delta\varepsilon$ are not exceptional. More importantly, the study of fully coherent transport, as opposed to the sequential tunneling limit, provides a better framework to understand the interplay between coherence and interactions.

The conductance through the quantum dot is expressed in terms of the interacting system retarded Green's function, which is written as a sum over terms containing different (and fixed) number of electrons in the dot

$$G^R = \sum_{N=0}^{\infty} P_N \Big\{ \Big[\varepsilon I - H_{\rm dot}^{(N)} - \Sigma^R(\varepsilon)\Big]^{-1} (I - n_N) + \Big[\varepsilon I - H_{\rm dot}^{(N-1)} - \Sigma^R(\varepsilon)\Big]^{-1} n_N \Big\}, \tag{1}$$

where the quantum dot Hamiltonian matrix elements are

$$\Big[H_{\rm dot}^{(N)}\Big]_{i,j} = (E_j - e\alpha V_g + UN)\delta_{i,j}, \tag{2}$$

U is the quantum dot charging energy, E_j stands for (closed) dot eigenenergies, and P_N is the thermal probability to find N electrons in the dot. In Eq. (1) we set $[n_N]_{i,j} = \langle n_i \rangle_N \, \delta_{i,j}$ given by the canonical occupation numbers of the (closed) dot eigenstates. The retarded self-energy matrix elements, due to the coupling to the leads, become

$$\left[\Sigma^R(\varepsilon)\right]_{i,j} = \sum_{k,{\rm a}\in{\rm L,R}} \frac{V_{i,(k,{\rm a})} V_{(k,{\rm a}),j}}{\varepsilon + {\rm i}0^+ - \varepsilon_{k,{\rm a}}} \approx -\frac{\rm i}{2}\left(\Gamma_{\rm L} + \Gamma_{\rm R}\right) . \tag{3}$$

since the lead-dot coupling matrix elements $V_{(k,{\rm a}),j}$ vary in the energy scale of ε_k and hence are practically constant in energy windows comprising several single-particle states. The linear-response conductance is

$$G = \frac{2e^2}{h} g \quad \text{with} \quad g = \int d\varepsilon \left(-\frac{\partial f_\mu}{\partial \varepsilon}\right) T_{\rm R,L}(\varepsilon) \quad \text{and} \quad T_{R,L}(\varepsilon) = \Big| \sum_{i,j} V_{(k,{\rm L}),i} \left[G^R\right]_{i,j} V_{j,(k,{\rm R})} \Big|^2 , \tag{4}$$

where f_μ is the Fermi distribution function in the leads with chemical potential μ.

The statistical study of the dimensionless conductance peak heights $g^{\max}$ allows for a comparison between the results of our approach, experiments and the sequential tunneling theory. The statistical ansatz is to assume that the underlying electronic dynamics in the quantum dot is very complex and hence the fluctuation properties of its single-particle eigenenergies and eigenfunctions coincide with those of an ensemble of random matrices. Accordingly, the single-particle levels display universal fluctuations and their spacings $\delta\varepsilon$ follow the Wigner-Dyson distribution.

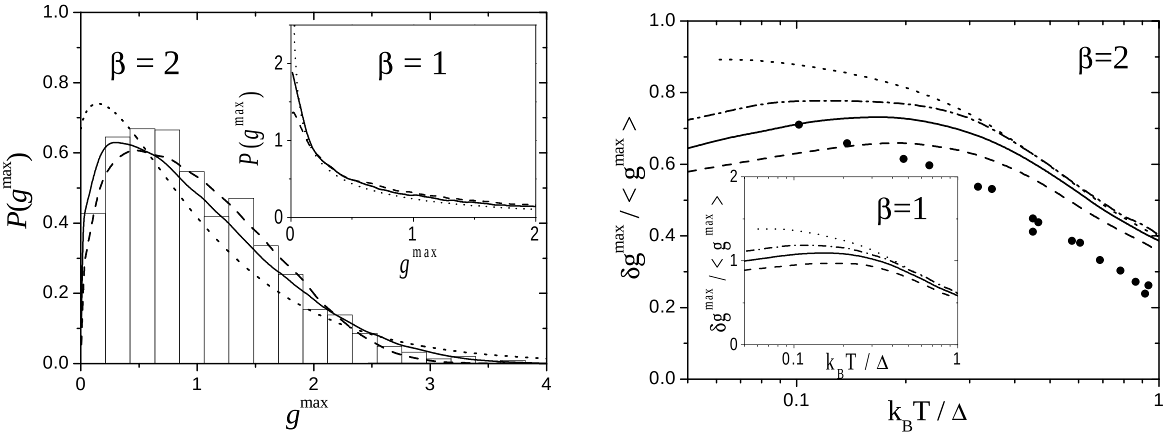

FIG. 1: Left: Peak height probability distribution $P(g^{\max})$ for $k_BT = 0.1\Delta$ and $\beta = 2$. The same for $\beta = 1$ in the inset. Our theory for $\langle\Gamma\rangle/\Delta = 0.1$ (solid line) and 0.2 (dashed line) is compared with the standard sequential tunneling result (dotted line), and the experimental distribution (histogram) [2]. Right: Normalized peak heights distribution width σ_g for the unitary case (the orthogonal case is shown in the inset) as a function of k_BT/Δ, for $\langle\Gamma\rangle/\Delta = 0.05, 0.1, 0.2$ (dashed-dot, solid and dashed lines respectively). Symbols correspond to the experimental results of Ref. [2] *for different dots* and the dotted lines to the standard sequential theory.

Likewise, the decay widths Γ are Porter-Thomas distributed. The physical inputs of the statistical theory are only the mean level spacing Δ and the average decay width $\langle\Gamma\rangle$. We consider both the orthogonal time-reversal invariant case ($\beta = 1$) and the unitary case ($\beta = 2$) of broken time-reversal symmetry. The later is the relevant one for comparison with avaliable experimental data.

The numerical implementation is straightforward, but costly. The evaluation G^R requires matrix inversions for each realization. The canonical thermal quantities P_N and $\langle n_i\rangle_N$ are computed using a quadrature formula. For $k_BT \lesssim \Delta$ good accuracy requires taking into account at least 30 levels around the resonant one. The charging energy U is taken to be 50Δ (the results are quite insensitive to U, provided $U \gg \Delta$).

The data of Ref. [2] show that at very low temperatures, $k_BT \ll \Delta$, the conductance peak-height distribution does not follow the standard random matrix theory. By accounting for quantum coherent tunneling we obtain a very nice agreement with the experimental distributions. This is illustrated in Fig. 1 for $\beta = 2$. In the inset we present our results for the distribution of $g^{\max}$ for $\beta = 1$. In Fig. reffig-PG the dimensionless conductance peak heights $g^{\max}$ are scaled to unit mean. We show the peak heights distribution for $k_BT = 0.1\Delta$, $\langle\Gamma\rangle = 0.1\Delta$ (solid line) and $\langle\Gamma\rangle = 0.2\Delta$ (dashed line). The histogram corresponds to the experimental result of Ref. [2] avaliable only for $\beta = 2$. Different dots have different $\langle\Gamma\rangle/\Delta$, a ratio that can be determined from the experimental $g^{\max}$. $\langle\Gamma\rangle/\Delta \sim 0.1$ is representative of the analyzed experiments. We find that as the ratio $\langle\Gamma\rangle/\Delta$ is increased, the probability to obtain small conductances is suppressed in comparison with the standard sequential theory (dotted line). This can be understood as follows: If a given resonance has small tunneling rates, the contributions due to virtual processes through its neighbors will reduce the chance to obtain a very small peak. Thus, we expect $P(g^{\max} = 0) = 0$.

Our results show the importance of coherent contributions in the Coulomb blockade electronic transport. Other approaches show similar level of agreement with the most recent experiments [4, 6, 7].

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