

Fractal Behavior in Magnetoconductance in a Coupled Quantum Dot

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Fractal behavior in the magnetoconductance (MC) fluctuations has been observed and studied by two kinds of analysis methods; geometrically (exact self-similarity) and statistically (box counting), in order to clarify the fractal transport behavior in a coupled quantum dot. We have observed clear three or four fold self-similar structure in the MC of this system. Such a fractal behavior has previously been observed also in a low temperature MC of chaotic cavities [1,2]. We measured the MC in a coupled dot system so as to investigate the relationship between the inter-dot coupling and the fractal features in the MC. The geometrical design of the split gate is $1.0 \times 1.0 \mu\text{m}^2$ and $1.2 \times 1.2 \mu\text{m}^2$ coupled by a $0.3 \mu\text{m}$ -wide quantum point contact (QPC) as shown in the inset of Fig. 1. The low power measurement was performed using a lock-in amp. technique at 100 mK in a dilution refrigerator. Typical results of the MC at different gate voltages are shown in Fig. 1.

In the experiment considered here, the voltages for all the split gates were fixed at appropriate voltage, however, we varied that only for the central gate, modifying the coupling strength between the two dots [3]. In our analysis of the fractal behavior, two methods have been used; self-similar scaling for hierarchical structure and box counting for the volume dimension in the conductance fluctuations. The result of the factual analysis obtained using both methods are shown in Fig. 2. Comparing both set up data, the fractal dimension determined from these method show different dependence on the gate-voltage variations. One reason for this discrepancy can be considered to be that one of them seems to be able to engraft the non-fractal signals in the analysis. As the transport characteristic varied with both magnetic field, and the gate voltage at the coupling QPC, the fractal dimension including the non-fractal signal should also differ significantly for these two methods. Nevertheless, since the specific pattern in the hierarchical structure is important for the self-similar method, we believe that we can obtain a reasonable fractal dimension by using of the self-similar scaling. Since, this kind of hierarchical structure can be discussed in a scattering process from a saddle point potential as well as a kinetic process in the mixed phase space in chaotic cavities, the self-similar and unstable periodic orbits can be expected by isochroous pitchfork bifurcations due to harmonic saddles in curved walls of the dot.

[1] R. P. Taylor et al: Phys. Rev. Lett. **78** (1997) 1952.

[2] A. P. Micolich et al: Phys. Rev. Lett. **87** (2001) 036802.

[3] N. Aoki et al: Appl. Phys. Lett. **80** (2002) 2970.

[4] A. Budiyo et al: J. Phys. Soc. Jpn. **71** (2002) 2090.

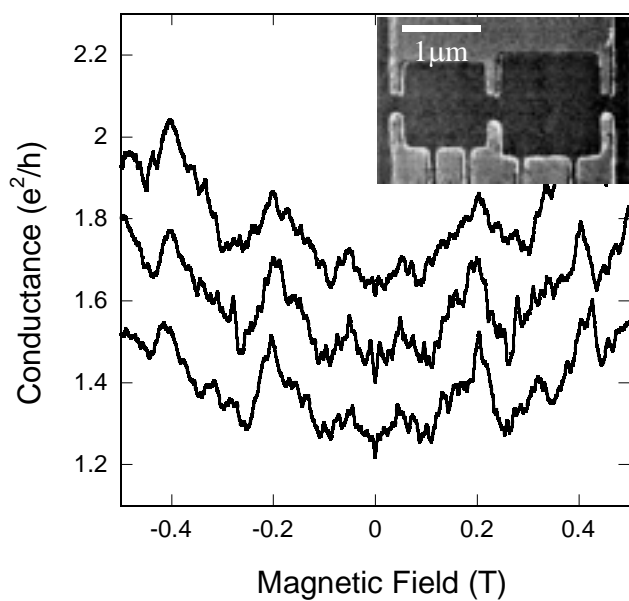


Fig. 1. The magnetoconductance traces at several gate voltage (from the bottom -2.8V, -2.4 V, -1.6 V). The inset is the SEM image of the sample.

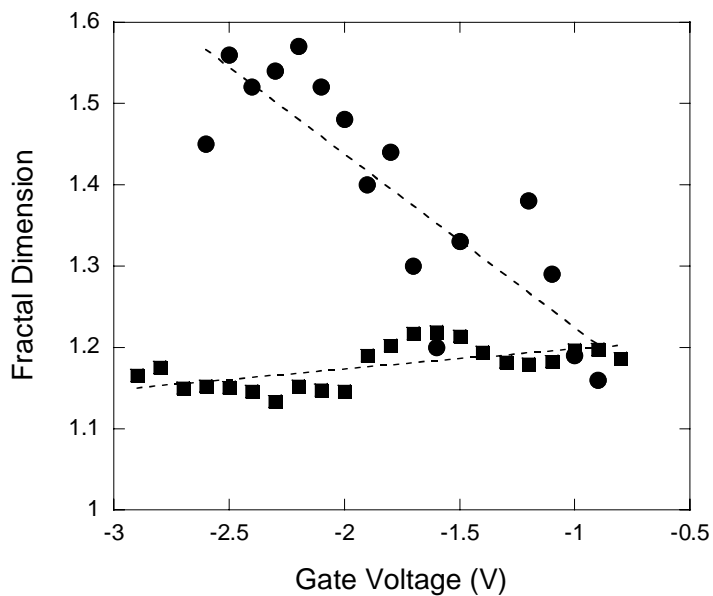


Fig. 2. The gate voltage dependences of fractal dimensions obtained from the self-similarity (filled circles) and box-counting method (filled squares).