Quantum magnetotransport of a 2DEG in a weak, two-dimensional periodic modulation

X. F. Wang[a], P. Vasilopoulos[a], and F. M. Peeters[b]

[a]Concordia University, Department of Physics,
1455 de Maisonneuve Ouest, Montréal, Québec, H3G 1M8, Canada
[b]Departement Natuurkunde, Universiteit Antwerpen (UIA),
Universiteitsplein 1, B-2610, Belgium

Transport properties of the two-dimensional electron gas (2DEG) are investigated in the presence of a perpendicular magnetic field B and of a *weak*, two-dimensional (2D) *periodic* potential modulation in the 2DEG plane $U(x, y) = V_x \cos(2\pi x/a) + V_y \cos(2\pi y/b)$, with a, b the periodicities along the x and y directions, respectively. The solution of the corresponding tight-binding equation has shown[1] that each Landau level splits into several subbands, whose number depends on the magnetic field, and that the gaps between them are exponentially small. Assuming the latter are closed due to disorder and following Ref. 2 gives analytical wave functions and simplifies considerably the evaluation of the magnetoresistivity tensor $\rho_{\mu\nu}$. The relative phase of the oscillations in ρ_{xx} and ρ_{yy} depends on the modulation periods involved. For a 2D modulation with **short** period $\leq$ 100 nm, the tensor $\rho_{\mu\nu}$ shows *prominent peaks when one flux quantum* $\hbar/e$ *passes through an integral number of unit cells* in very good agreement with recent experiments[3]. For 1D modulations these peaks are absent while for 2D modulations, with period $300 - 400$ nm and the usual densities, they occur at magnetic fields $10 - 25$ times smaller than those of the Weiss oscillations ($B \leq 0.4$ T) and appear unresolved in early experiments[4].

1. D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976); F. H. Claro *et al.*, **19**, 6068 (1979).
2. J. Labbe, Phys. Rev. B **35**, 1373 (1987).
3. S. Chowdhury *et al.*, EP2DS-14, Prague (2001); A. R. Long, private communication
4. R. R. Gerhardts, D. Weiss, and U. Wulf, Phys. Rev. B **43**, 5192 (1991).

SUPPORTING MATERIAL

For a 2DEG in a normal magnetic field B we use the one-electron Hamiltonian

$$H^0 = (\mathbf{p} + e\,\mathbf{A})/2m^* + V_x \cos(K_x x) + V_y \cos(K_y y), \quad (1)$$

with $K_x=2\pi/a$, $K_y= 2\pi/b$, a, b the periodicities along the x and y directions and V_x ,V_y the modulation strengths. For $\mathbf{A}$ = (0, Bx, 0) and $|n, k_y>$ the eigenstate for $V_x = V_y=0$, the solutions of Eq. (1) are sought in the form[1] $|\phi_{n,ky}> = \Sigma_p A_p \mid n,k_y+pK_y y>$, p *integer*. Then the tight-binding equation $<n,k_y+ pK_y y\,|H^0 - E|\,\phi_{n,ky}> = 0$, which neglects mixing of Landau levels, takes the form (Φ_0= h/e, Φ=Bab, x_0= (h/eB) k_y)

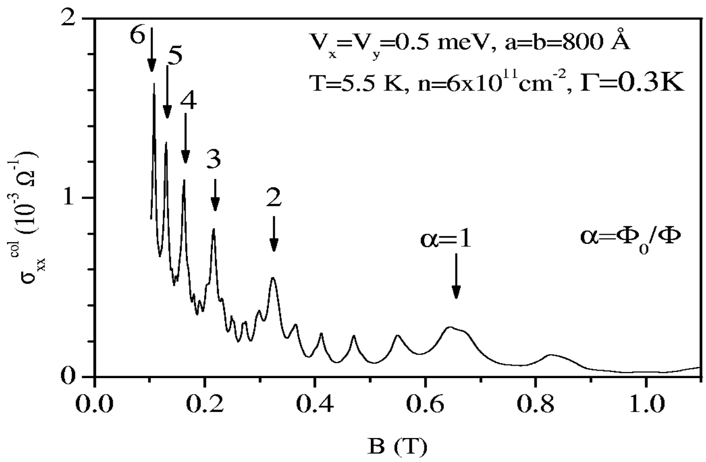
$$V_x F_n(u_x+u_y) \cos(2\pi p\alpha+K_x x_0)A_p+V_y F_n (u_y)[A_{p+1}+A_{p-1}]/2 = (E - E_n)A_p , \quad (2)$$

here $\alpha = \Phi_0 /\Phi$, Φ is the flux through the unit cell, $E_n=(n +1/2)\hbar\omega_c$ the eigenvalue for $V_x = V_y=0$, and ω_c the cyclotron frequency. Further, $F_n(u_\mu) = \exp(-u_\mu/2) L_n(u_\mu)$, $L_n(u_\mu)$ is the Laguerre polynomial, and $u_\mu= (h/eB)K^2_\mu/2$. For α = *integer* or *half-integer,* Eq. (2) admits the solutions $A_p = A_0 e^{i\xi p}$ with ξ=(h/eB)$K_y k_x$. The resulting eigenvalues are

$$E_{nk_y\xi} = E_n \pm V_x F_n (u_x)\cos(K_x x_0) +V_y F_n(u_y)\cos\xi \quad (3)$$

Thus, the unperturbed Landau levels broaden into bands, with width equal to $2(V_x|F_n(u_x)| + V_y|F_n(u_y)|)$, that oscillates with magnetic field B.

Eq. (3) misses the fine structure of the exact solution of Eq. (2) for $\alpha \neq$ *integer.* Assuming the latter is absent due to disorder, we can use Eq. (3) for all fields B and evaluate the resistivity tensor along the lines of Ref. 2. The result for the collisional contribution to the conductivity, $\sigma_{xx}\sim\rho_{yy}/B^2$, is shown in the figure for the parameters of Ref. 3. The prominent peaks marked by the arrows show the *integral* number of unit cells through which *one* flux quantum Φ_0 passes. The experimental field values at which these peaks occur, respectively, for α=1-6, are B (T) =0.64, 0.32, 0.21, 0.16, 0.13, 0.11. The smaller-amplitude oscillations between, e.g., α=1 and α=2, are the Weiss oscillations. For 2D modulations with period 300-400 nm these peaks occur at B fields 10-25 times smaller than those of the Weiss oscillations (B≤0.4T) and appear unresolved in early experiments[4].



1. J. Labbe, Phys. Rev. B **35**, 1373 (1987).
2. P. Vasilopoulos and F. M. Peeters, Phys. Rev. Lett. **63**, 2120 (1989)
3. S. Chowdhury *et al.,* EP2DS-14, Prague (2001); A. R. Long, private communication
4. R. R. Gerhardts *et al.,* Phys. Rev. B **43**, 5192 (1991).