Model for Breakdown of Laminar Flow of a Quantum Hall Fluid Around a Charged Impurity: Comparison with Experiment

A.M. Martin, K.A Benedict, F.W. Sheard and L. Eaves

School of Physics and Astronomy, University of Nottingham, Nottingham, NG7 2RD, UK

In the integer quantum Hall effect (QHE) regime, a two-dimensional electron fluid carries an almost dissipationless current and the ratio of the current, to the Hall voltage is quantized in units of e^2/h. However, above a critical current, the dissipative voltage, V_x, measured along the direction of current flow, increases rapidly, leading to breakdown of the QHE. For certain samples, including those used to maintain the US resistance standard at the National Institute of Standards and Technology (NIST) [1,2], breakdown occurs as a series of steps, in V_x, of height $\Delta V_x \approx 5mV$.

In this work we develop a theoretical model to account for the dissipative steps observed by the NIST group and others. We show that, in the presence of charged impurity-induced disorder, the quantum Hall fluid (QHF) is unstable when the local fluid velocity exceeds a critical value. Under these conditions, magneto-exciton or electron-hole (e-h) pair excitations are generated spontaneously near an impurity. The voltage step height, ΔV_x, is directly related to the rate of formation of the pairs. In addition we show how this type of excitation of the QHF is analogous to vortex-antivortex pair formation in classical and other quantum fluids [3].

Our starting point is the Fermi golden rule, which we use to calculate the rate of generation of e-h pairs due to a single impurity in a uniform electric field. From this we obtain following condition

$$\Delta\varepsilon = \hbar\omega_c + \frac{e^2}{4\pi\kappa l_b}\Delta_{n,(n+1)}[Q] - eE_y l_b^2(Q) = 0, \qquad (1)$$

where ω_c is the cyclotron frequency, l_b is the magnetic length, κ is the dielectric constant, E_y is the y component of the Hall electric field, $\hbar Q$ is the momentum change and $\Delta_{n,(n+1)}[Q]$ includes the local and exchange field corrections for an excitation from (n) to ($n+1$). This condition, see Fig 1, is met when the dashed line crosses the solid curve. This stipulates that energy is conserved in moving an electron from the filled lower Landau level (LL) to the unoccupied upper LL. For a given electric field, E_y, we then calculate the rate of generation of e-h pairs due to the presence of an impurity.

Consider a local region of the sample where E_y is large enough to create e-h pairs at a given rate, due to scattering from a charged impurity. A pair created close to an impurity drifts along the Hall bar at a velocity E_y/B, so for a fixed generation rate, a stream of e-h pairs moves along the Hall bar. Then, for a filling factor of 2 we have a situation in which a small fraction of electrons in the lower LL ($n=0$) have been replaced by holes and the previously empty upper LL ($n=1$) contains some electrons. As the e-h pairs move away from the high field region, the spacing between the electron and hole in a pair increases and most pairs eventually ionise by acoustic phonon emission. Due to the absence of empty states into which the excited electron can relax and neglecting weak, second order Auger processes, we can assume that all the generated e-h pairs eventually ionise, leading to a dissipative current $i=eW$, where W is the pair generation rate due to scattering from the charged impurity. This current flows across the Hall voltage equipotentials. At filling factor $\nu=2$, it gives rise to a dissipative voltage

$$V_x = eW\left(\frac{h}{2e^2}\right). \qquad (2)$$

The NIST experiments carried out at $\nu=2$ and $B=12.3T$ show a series of up to 20 dissipative steps in V_x of regular height $\Delta V_x \approx 5mV$. In Fig. 2 we plot our calculated dissipative voltage versus the background electric field E_y. Since W is strongly influenced by the overlap between the wave-functions in the occupied ($n=0$) and unoccupied ($n=1$) LLs, it increases rapidly at a *critical* electric field. This occurs when V_x is comparable to the small background dissipative voltage governed by σ_{xx}. The rate, and hence V_x, then reaches a maximum when the electric field is such that the e-h pairs are formed close to the roton minimum of the magneto-exciton dispersion curve. For a given E_y, for which W is finite, the pairs formed at the breakdown relax, so that electrons in the upper LL moving towards one side of the Hall bar, whilst holes move in the opposite direction. The presence of these carriers screens the Hall field over much of the Hall bar. Since the Hall voltage in the NIST experiments remains constant at its quantized value over the magnetic field range where the dissipative steps occur, this screening effect tends to enhance the electric field at the breakdown point. Thus, as the *critical* electric field is reached, the generation rate at the breakdown point increases rapidly, inducing a further increase in E_y due to the screening of the Hall field in other regions of the Hall bar. For breakdown at a single charged impurity the system should switch between two stable states, corresponding to $V_x=0$ and $V_x=5.6mV$, which corresponds to the maximum value of V_x in Fig. 2. This is in good agreement with NIST experiments where a series of steps is observed and we attribute each step to the formation of separate streams of e-h pairs generated by other impurities.

We also explore the relationship between our quantum model with an hydrodynamic model for breakdown. Our starting point is an effective Euler equation, derived by Stone [4],

$$m^*\left[\dot{\mathbf{v}} - \mathbf{v}\times\Omega\right] = e(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla\left(\frac{m^*}{2}|\mathbf{v}|^2 + \mu\right), \qquad (3)$$

where $\mathbf{v}$ is the velocity field, μ is the local chemical potential containing all the interaction terms and Ω is the fluid vorticity. Combining the above equation with the continuity equation for the density of the QHF we find the frequency ω of small perturbations in the velocity field,

$$\hbar\omega = \Delta\varepsilon = \hbar\omega_c - eE_y l_b^2 Q. \qquad (4)$$

For $\omega=0$ this result is equivalent to Eq. (1) in the absence of interactions and corresponds exactly to the elastic inter-Landau level tunneling condition. This hydrodynamic description corresponds to the condition required to generate a vortex-antivortex pairs at zero energy. Pursuing the analogy with the classical von-Karman vortex street, we can estimate the vortex pair generation rate one would expect to observe in an experiment on the QHE. Remarkably, the classical hydrodynamical model gives a value of W (and hence $\Delta V_x=4.7mV$) which agrees closely with both the NIST experiment and our model.

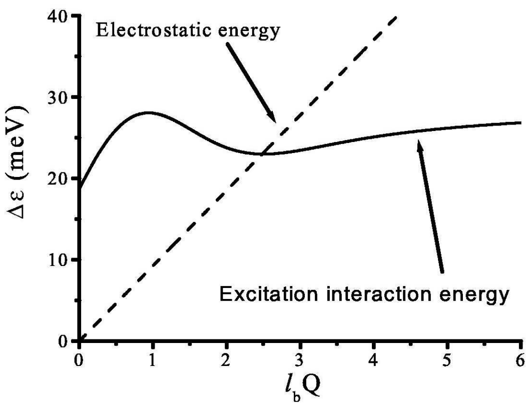

Figure 1: The magneto-exciton mode energy (solid line) the electrostatic energy (dashed line), under the conditions of the NIST experiment.

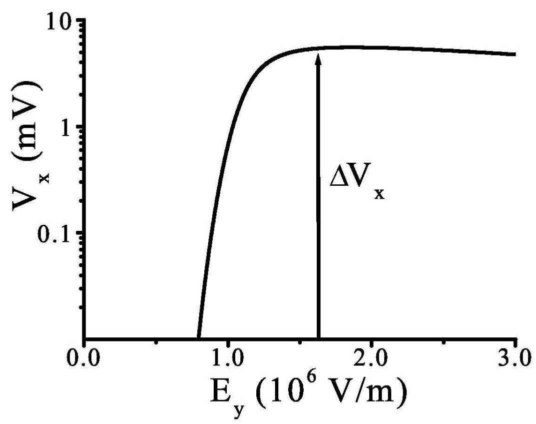

Figure 2 : Calculated dissipative voltage vs E_y due to breakdown at a single impurity for the NIST experiment.

[1] M.E. Cage *et al.*, Phys. Rev. Lett. **51**, 1374 (1983).
[2] C.F. Lavine *et al.*, J. Res. Nat. Inst. Stand. Technol., **99**, 757 (1994).
[3] L. Eaves, Physica B **298**, 1 (2001).
[4] M. Stone, Phys. Rev B **42**, 212 (1990).