Integer quantum Hall effect in isotropic 3D systems

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It is intriguing to ask whether a kind of the integer quantum Hall effect (IQHE) can exist in three-dimensional (3D) systems. Usually the IQHE is considered to be peculiar to two dimensions (2D), since the existence of the Landau gaps is essential. However, it has been pointed out that 3D Bloch systems (i.e., systems having some periodicity) can have in IQHE *provided that* there is a gap in the energy spectrum, where the three components of the Hall conductivity tensor are individually quantized when the Fermi energy is in that gap[1].

In 3D Bloch systems it is highly nontrivial whether and how the interplay of Bragg's reflection and Landau's quantization results in energy gaps as in Hofstadter's butterfly in 2D, and if so, how the QHE integers behave systematically. We have previously shown[2] that butterfly-like spectra in fact appear in 3D when the system is anisotropic (quasi-1D), and have calculated the quantized 3D Hall tensor (topological invariants).

A more basic question, of both theoretical and experimental interests, is: can *isotropic* 3D systems in magnetic fields have butterfly-like spectra that are accompanied by the 3D QHE? Here we have investigated this possibility from two well-defined limits: the weak periodic potential and the strong periodic potential (tight-binding model).

In the former case, where a continuous (hence isotropic) 3D electron system is subject to a modulation (e.g. produced by a standing acoustic wave), we show that a weak periodic potential with a wavevector $\mathbf{G}$ quite generally gives rise to energy gaps in the quantum region ($E_F \sim \hbar\omega_c$), which seems to have been overlooked so far. There the Hall tensor $\boldsymbol{\sigma} \equiv (\sigma_{xy}, \sigma_{yz}, \sigma_{zx})$, with the Hall current given as $\mathbf{j} = -\boldsymbol{\sigma} \times \mathbf{E}$, becomes $\boldsymbol{\sigma} = (e^2/2\pi h)N\mathbf{G}$ with an integer N when the Fermi energy is in each of the energy gaps. If we superpose two modulations having different wave vectors, the energy spectrum becomes a Hofstadter's butterfly, where the Hall tensor wildly depends on the gap in which E_F lies.

In the opposite limit of the (isotropic) 3D tight-binding lattice, energy gaps appear universally as well in that all we have to have is a magnetic field $\mathbf{B}$ pointing to *general directions* (i.e., off the high-symmetry axes). Interestingly, the Hall tensor $\boldsymbol{\sigma}$ change directions in 3D space even for a fixed direction of $\mathbf{B}$ (accompanying Fig.). The tight-binding limit crosses over to the weak-potential limit in the region where the effective-mass approximation holds. Intuitively, the gapful spectra and the quantized Hall conductivity come from the quantum mechanical hopping between semiclassical orbits in the 3D k-space, so the effect is beyond the semiclassical picture.

[1] M. Kohmoto, B. I. Halperin, and Y. Wu, *Phys. Rev. B* **45**, 13488 (1992).
[2] M. Koshino, H. Aoki, K. Kuroki, S. Kagoshima, and T. Osada, *Phys. Rev. Lett.* **86**, 1062 (2001).

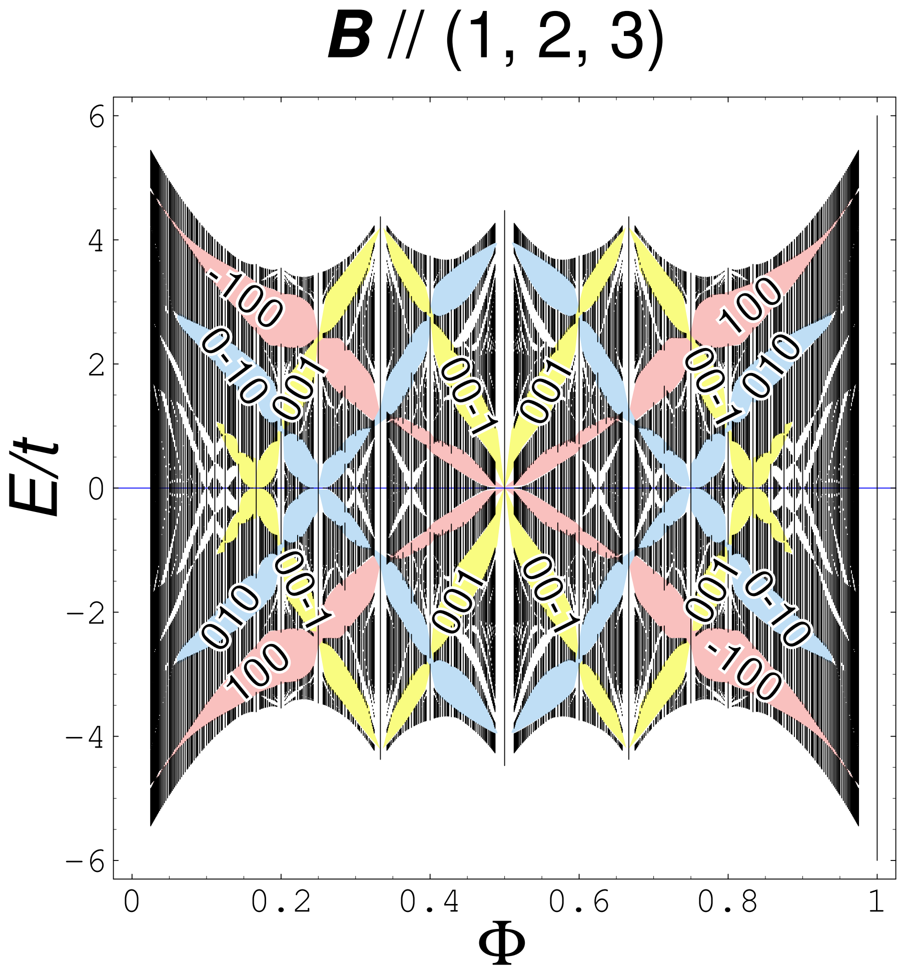

Figure 1. Energy spectrum for the isotropic simple-cubic tight-binding lattice in magnetic field $(B_x, B_y, B_z) = (\frac{h}{ea^2})\Phi \times (1, 2, 3)$, where a is the lattice constant and the energy is in units of the transfer energy t. Triple integers represent the Hall integers, $\sigma \equiv (\sigma_{xy}, \sigma_{yz}, \sigma_{zx})$ in units of $(-e^2/ha)$.