Electronic-Nuclear Dynamics in Double Quantum Dots in the Spin-Blockade Regime

Takeshi Inoshita¹ and Seigo Tarucha¹,²

¹ERATO, JST, 3-1, Morinosato-Wakamiya, Atsugi, 243-0198, Japan.
²Dept. of Physics, Univ. of Tokyo, 7-3-1 Hongo, Tokyo 113-0033, Japan

Recent years have witnessed renewed interest in the dynamics of coupled electronic-nuclear systems owing to their potential usefulness for quantum information processing. Particularly attractive in this context are double quantum dots (DQDs) in the spin blockade regime, which have been shown to exhibit nuclear-induced current instability (hysteresis, temporal oscillation, etc.) [1]. Here we present a theory of electronic-nuclear dynamics and current instability in vertical double quantum dots in transverse magnetic fields \( B = (0, 0, B) \) (current direction) [2].

The starting point is the density matrix equation of motion for the total system

\[
\frac{d\rho}{dt} = -i[H, \rho],
\]

where \( H \) is the sum of the DQD Hamiltonian (described by a Hubbard-like many-electron model), nuclear Zeeman energy, hyperfine interaction between the dot electrons and the nuclei, the energy of the two leads (free electron gases with Fermi levels \( \mu_1 \) and \( \mu_2 \)), lead-dot tunnel coupling, and the heat bath that induces nuclear relaxation. The lead and heat bath degrees of freedom are traced out by the Born-Markov approximation.

It is to be noted that a dot contains a huge number of nuclei \( N \approx 10^7 \); This allows one to treat the collective nuclear spin \( J \) in the semiclassical approximation. Also, we are interested in the slow dynamics (\( t > \text{ms} \)) whereas the system contains rapidly changing parts (mainly electronic) with time scales \( \sim \text{ps} \). These fast degrees of freedom can be eliminated by an adiabatic approximation. Combining these two approximations (semiclassical + adiabatic), we could systematically derive a classical equation of motion for \( J \) that incorporates dissipation.

We examined numerically the fixed point solutions \( (dJ/dt = 0) \) and their stability using parameters for the InGaAs DQDs used in [1]. For simplicity, the hyperfine interaction was considered in only one of the two dots. Appropriate tuning of the bias voltage allows the system to be in the spin blockade regime and, furthermore, locates the two-electron singlet (S) and triplet (T) states in the transport window (between \( \mu_1 \) and \( \mu_2 \)). By sweeping \( B \), the two states can be made to cross: One expects the hyperfine coupling and leakage current \( I \) to be enhanced near this crossover. Thus we focused our study on this crossover region.

Our main findings are: (i) The fixed point in the absence of the hyperfine interaction \( (J_x = J_y = 0, J_z \neq 0, \text{i.e. nuclei polarized by } B \text{ alone}) \) remains to be a fixed point. However, near the crossover, this solution may become unstable. (ii) In this crossover region, additional fixed points with \( J_x, J_y \neq 0 \) appear, which may be stable or unstable (depending on temperature, etc.). When there are more than one stable fixed point, one would see hysteresis in the \( I(B) \) curve. And if all the fixed points are unstable, the only stationary solution is a limit cycle with \( I \) oscillating in time. These are in agreement with the experiment.

In summary, our theory explains the observed instability of leakage current (hysteresis and temporal oscillation) as due to the enhanced hyperfine coupling near the S-T crossover and the ensuing nonlinear dynamics of the coupled electronic-nuclear system. The implications of the result for quantum information processing will also be discussed.