Rashba Spin Splitting and Ehrenfest's Theorem

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In quasi two-dimensional (2D) systems with a broken inversion symmetry spin-orbit interaction gives rise to a spin splitting of the electron states known as the Rashba effect [1]. The Rashba spin splitting has been observed in many experiments. However, the theory of the Rashba effect has been discussed controversially for many years. It appears reasonable that Rashba spin splitting should be proportional to the electric field $\boldsymbol{\mathcal{E}}$ that characterizes the inversion asymmetry of the system. Yet it was first pointed out by Ando *et al.* [2] that Rashba spin splitting should be very small because for a bound state the expectation value of the electric field vanishes, $\langle \boldsymbol{\mathcal{E}} \rangle = 0$. This reflects Ehrenfest's Theorem which says that on average there is no force acting on a bound state.

We resolve this controversy by establishing a strict one-to-one correspondence between the effective 2×2 Hamiltonian $\mathcal{H}$ for the electrons in the conduction band and the well-known Pauli equation. We show that the different terms in $\mathcal{H}$ emerge from the 8×8 Kane model in much the same way as the different terms in the Pauli equation emerge from a nonrelativistic approximation to the Dirac equation. This holds for the kinetic energy term $p^2/(2m)$, the Zeeman term, the Pauli-Rashba spin-orbit term, and the Darwin term. But it holds also for all higher-order terms that are usually neglected.

The present rigorous analysis reveals that the Rashba spin-orbit coupling of electrons in the conduction band is proportional to the electric field $\boldsymbol{\mathcal{E}}_V$ in the valence band. On the other hand, Ehrenfest's Theorem, when applied to electrons in the conduction band, refers to the electric field $\boldsymbol{\mathcal{E}}_C$ in the conduction band. As we usually have $\boldsymbol{\mathcal{E}}_C \neq \boldsymbol{\mathcal{E}}_V$ in quasi 2D systems this yields a natural answer to the controversy about Rashba spin splitting and Ehrenfest's Theorem.

[1] Y. A. Bychkov and E. I. Rashba, J. Phys. C: Solid State Phys. **17**, 6039 (1984).

[2] A. Därr, J. P. Kotthaus, and T. Ando, in *Proceedings of the 13th International Conference on the Physics of Semiconductors Semiconductors*, edited by F. G. Fumi (North-Holland, Amsterdam, 1976), p. 774.