

# Shuttle Instabilities: Semiclassical Phase Analysis

A. Donarini<sup>1</sup>, T. Novotný<sup>1,2</sup>, A. P. Jauho<sup>1</sup>

<sup>1</sup> *Mikroelektronik Centret,  
Building 345 east, Technical University of Denmark,  
DK-2800 Lyngby, Denmark*

<sup>2</sup> *Department of Electronic Structures,  
Faculty of Mathematics and Physics, Charles University,  
Ke Karlovu 5, 121 16 Prague, Czech Republic*

Nanoelectromechanics (NEMS) combines electrical and mechanical degrees of freedom to obtain novel functionalities. The specific application of NEMS we consider in this paper consists of a movable resonant tunnelling device in the Coulomb blockade regime. The connection of the device to the leads can be direct, as in the model introduced by Gorelik et al. [1], or mediated by two fixed quantum dots, as proposed by Armour and MacKinnon [2]. The charge distribution in the biased system gives rise to a force on the central dot, influencing its mechanical dynamics, while the central dot position determines the tunnelling rates and thereby influences the electrical transport through the system. A current through the device can sustain oscillations of the central dot even in presence of a mechanical damping.

With appropriate physical parameters, the system may support a shuttle regime, where a single electron per cycle is transported through it: due to the position dependent tunnelling amplitude the central dot gets charged when near to the left lead, then the electrostatic force pushes the dot towards the right lead where the now enhanced tunnelling rate allows the electron to escape to the right lead. Following [2], we describe the electronic part with the density matrix formalism and couple the resulting master equation to a classical equation of motion for the central dot position.

A linear instability analysis of the system shows where the equilibrium solution becomes unstable leading to a Hopf bifurcation. Thus in the instability regions two complex conjugate eigenvalues of the system get a positive real part and the dynamics can be reduced to the two-dimensional subspace defined by the two unstable eigenmodes. The relative phase between the charge, position and velocity components of these eigenmodes defines the relative phases in the unstable rotating solution and shows where this instability can be identified as shuttling (fig.1, right column).

The semiclassical approach is justified by the quantum-classical correspondence since the dot oscillations are bigger than the minimum quantum amplitude. The semiclassical analysis and the phase-space description for the pure quantum approach [3] yield consistent results. The present mean field approximation for the electric part neglects the effect of shot noise, and consequently the damping factor threshold for onset of shuttling is usually very much reduced compared to the quantum treatment.

- [1] L. Y. Gorelik, A. Isacsson, M. V. Voinova, B. Kasemo, R. I. Shekhter, and M. Jonson, Phys. Rev. Lett. **80**, 4526 (1998), cond-mat/9711196.
- [2] A. D. Armour and A. MacKinnon, Phys.Rev. B **66**, 035333 (2002), cond-mat/0204521.
- [3] T. Novotný, A. Donarini, and A. P. Jauho, cond-mat/0301441.

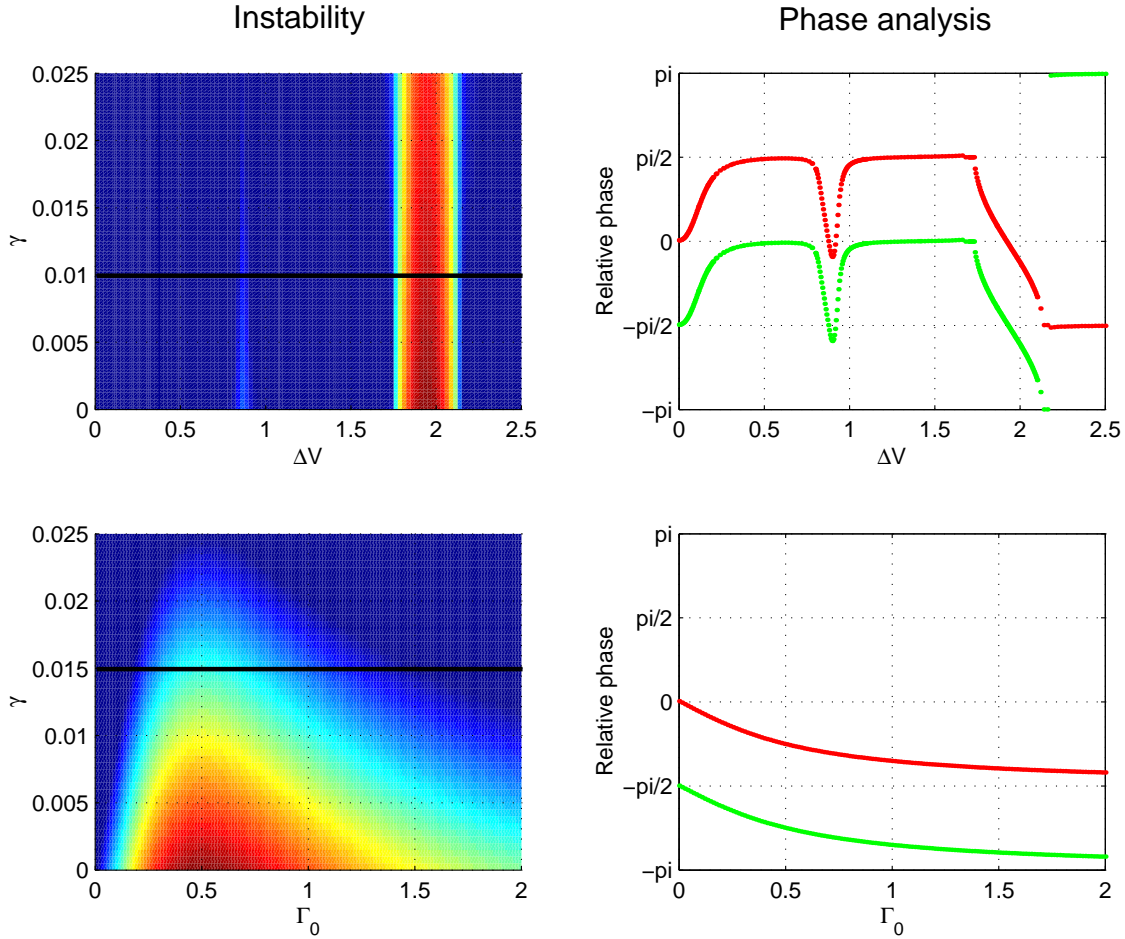


Figure 1: The left column displays the instability analysis in the triple- (top) and single-dot (bottom) model respectively. The real part of the highest eigenmode for the linearized system is plotted as a function of the device-bias  $\Delta V$  (difference between the first and third dot energies) and the mechanical damping  $\gamma$ , or, in the case of single dot shuttle, as a function of bare tunneling rate  $\Gamma_0$  and mechanical damping. The right column shows the phase of velocity (red) and position (green) with respect to the phase of the charge in the (central) dot. In both cases the phase has been calculated for one of the unstable eigenmodes at the damping rate indicated by the black lines (left column) and only the instability region is relevant. We identify a negative velocity-charge phase difference (position-charge  $< -\pi/2$ ) as an indication of shuttling regime. The single-dot model is shuttling whenever unstable, while the triple-dot model shows well-developed shuttling only in the vicinity of the  $\Delta V \approx 2\omega$  resonance.