Few-Particle Anyon Excitons in the Fractional Quantum Hall Regime

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We revisit the anyon exciton model (AEM) [1], which considers a neutral exciton made up of a valence hole and several fractionally-charged quasielectrons (anyons). The AEM is applicable at exact fractional filling factor ν, and for large separation between the photoexcited hole and a two-dimensional electron gas (2DEG), when the Coulomb field from the hole cannot destroy the incompressible quantum liquid (IQL). It has been applied to excitons against the background of $\nu = 1/3$ and 2/3 IQLs [2], providing a major insight into the role of electron-hole separation in determining the optical spectra, and giving a full classification of states for a four-particle anyon exciton. Recent developments in experimental techniques (see, e.g., [3]) have allowed the effective electron-hole separation (in units of magnetic length l_H) to be changed while keeping the filling factor constant, and thus direct verification of the AEM is now possible.

We generalise the model to an exciton consisting of a valence hole and N anyons with charge $-e/N$ and statistical factor α. The hole and anyons reside in different layers, separated by a distance of h magnetic lengths, and are subject to a magnetic field $\mathbf{H} = H\hat{\mathbf{z}}$ perpendicular to their planes of confinement. An exciton consisting of a hole and N anyons, all in the lowest Landau level, will have a total of $N+1$ degrees of freedom. As the exciton is neutral, we can assign it an in-plane momentum $\mathbf{k}$, which absorbs two of these degrees of freedom. For $N \geq 2$, the exciton will have $N-1$ *internal* degrees of freedom, which results in internal quantum numbers and a multiple-branch energy spectrum. For $\mathbf{k} = 0$ the problem has rotational symmetry and the angular momentum L_z of the exciton can be introduced. This momentum is related to the degree L of the polynomial, symmetric in anyon coordinates, which enters the exciton wavefunction $[L_z = -L - N(N-1)\alpha/2]$. We use a result from the theory of partitions to enumerate all possible symmetric polynomials, which provides a complete set of exciton basis functions.

We find some exact solutions of this $(N+1)$-particle problem in a boson approximation ($\alpha = 0$). For example, the binding energy for $\mathbf{k} = 0$ and $L = 0$ is given by

$$E_b = \sqrt{\frac{\pi}{2N}}\, e^{h^2/2N} \operatorname{erfc}\left(h/\sqrt{2N}\right) - \frac{(N-1)\sqrt{\pi}}{4N\sqrt{N}}, \qquad (1)$$

where $\operatorname{erfc}(x)$ is the complementary error function and the energy is measured in units of $e^2/(\epsilon\, l_H)$, where ϵ is the dielectric constant. The first term in Eq. 1 represents the anyon-hole attraction and the second term represents the anyon-anyon repulsion. Using the asymptotic expansion of $\operatorname{erfc}(x)$ it can be easily seen from Eq. 1 that the anyon-hole attraction potential tends to $1/h$ as $h \to \infty$, as expected. For $N = 1$, Eq. 1 reproduces the well-known result for the binding energy of a two-dimensional diamagnetic exciton [4]. The critical inter-plane separation h_c at which the $\mathbf{k} = 0$, $L = 0$ state becomes unbound can also be found from Eq. 1. For $N = 3$, the critical separation $h_c \approx 5.39\, l_H$, for $N = 5$ we find that $h_c \approx 5.59\, l_H$, and for $N \gg 1$ we have $h_c \approx 1.32\sqrt{2N}\, l_H$. Notably, these critical separations are well inside the region for which the AEM is applicable. It should be emphasised that the state with $L = 0$ is not the ground state for the anyon exciton at large separation h. For example, for a four-particle exciton [2], the ground states for large separation satisfy a superselection rule $L = 3m$, where m is an integer, and when $h \to \infty$ the ground state energy tends to its classical value, $E_c = -(2/3)^{3/2}/h$. Thus, at large 2DEG-hole separations the ground state at $\mathbf{k} = 0$ becomes optically inactive. However, at non-zero $\mathbf{k}$ the ground state is a mixture of states with different values of L_z, and hence magnetoroton-assisted transitions become possible. The evolution with increasing k of the ground state for a four-particle anyon exciton at $h = 3$ is shown in Fig. 1.

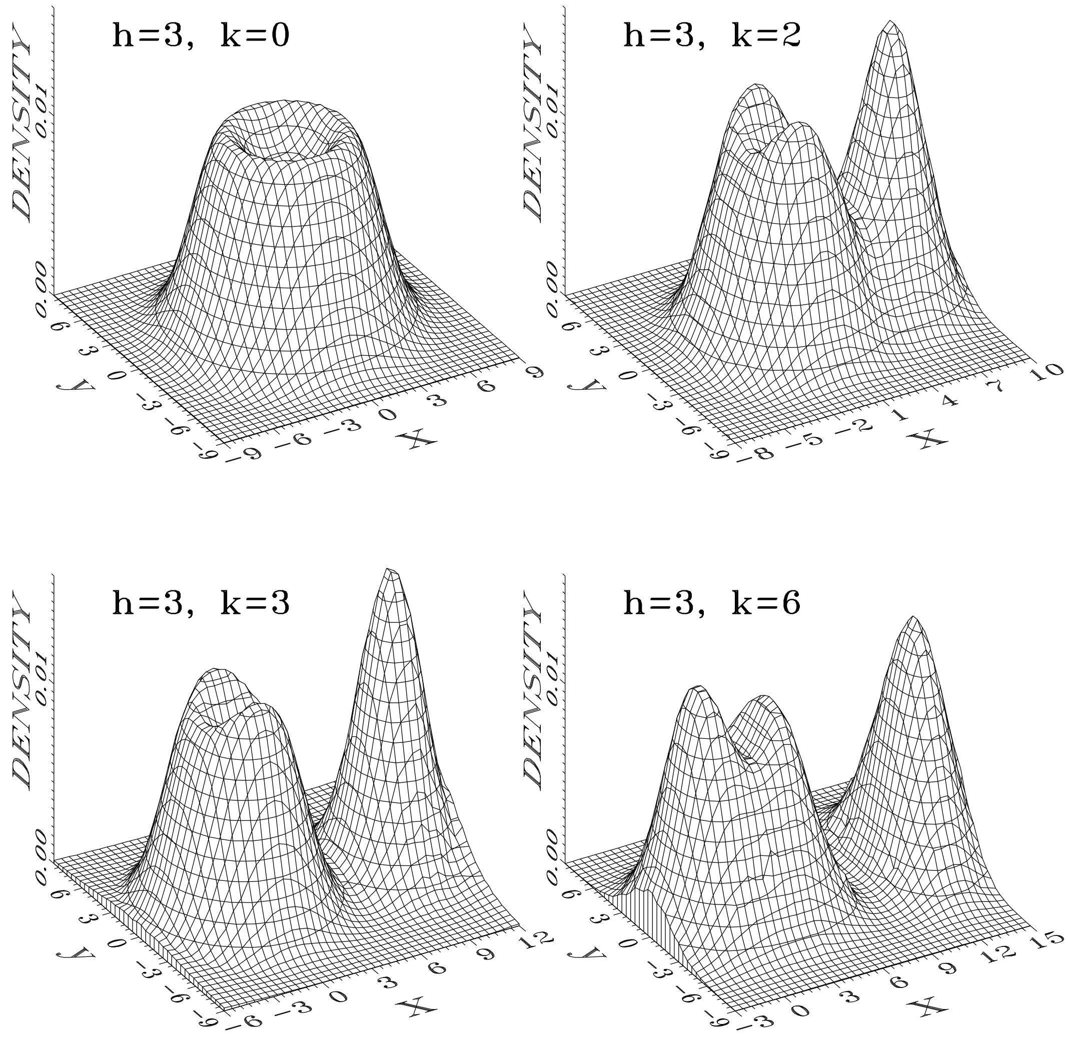

Figure 1: Electron density distribution in an anyon exciton for different values of the exciton in-plane momentum k. The distance h between the hole and the incompressible electron liquid is equal to three magnetic lengths. The hole is at the origin; the x-axis is chosen along the in-plane component of the exciton dipole moment.

We show that a neutral $(N + 1)$-particle exciton remains bound for 2DEG-hole separations exceeding several magnetic lengths, which contradicts the recent statement of Wójs and Quinn [5]. It is evident that the introduction of realistic form factors, which reduce the anyon-anyon repulsion at small distances, would not change this fundamental result. We believe that the appearance of fractionally-charged anyon ions at the bottom of numerically calculated excitation spectra [5] is an artefact caused by finite-size effects in the spherical geometry.

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