When a helium surface is charged with electrons and a critical surface density of electrons \((\approx 3 \times 10^9 \text{ cm}^{-2})\) is reached, the helium surface becomes unstable and multielectron bubbles (MEBs) form\[^1\]. MEBs are cavities in liquid helium, with a radius determined by balancing the Coulomb repulsion of the electrons with the surface tension of the helium and the pressure applied on the helium. For \(N=10^4\) electrons in the bubble at zero externally applied pressure, the radius is 1.064 microns. The MEB radius scales as \(N^{2/3}\). The electrons inside the bubble are not distributed homogeneously throughout the volume of the bubble, but collect in a nanometer thin film at the surface of the bubble and form a spherical two-dimensional electron system. The (quasi-)2D spherical geometry is also present in buckyballs and nanoshells, but the realization in a multielectron bubble is particularly interesting since the surface density of electrons can be changed in a continuous way over four orders of magnitude (in the range \(10^9 - 10^{13} \text{ cm}^{-2}\)) by changing the pressure on the helium. This will allow recently proposed experiments\[^2\] to investigate hitherto inaccessible regimes of surface density of electrons on liquid helium.

In this contribution, we theoretically investigate the properties of this curved two-dimensional electron system. First, the modes of oscillation of the multielectron bubbles are derived\[^3\], and the dispersion of the ripplons and longitudinal plasmon modes on the curved surface are presented as a function of the angular momentum of the mode and the radius of curvature of the surface. The study of these modes sheds new light on the stability of multielectron bubbles and we briefly discuss the origin of the energy barrier preventing the fissioning of MEBs\[^4\].

Next, we derive the interaction energy between these modes of oscillation and the electrons in the bubble, and calculate the polaronic effects arising from the electron-ripplon coupling in a curved geometry. Using the path-integral technique, we derive expressions for the free energy and the dimple shape of ripplon-polarons in a Wigner lattice in a multielectron bubble, as a function of temperature, externally applied pressure and number of electrons in the bubble. We find that, owing to the difference in the ripplon and longitudinal plasmon frequencies, the ripplon-polarons exist only in a Wigner crystallized state. This state differs from the Wigner lattice of electrons, in that the electrons in the ripplon-polaron Wigner lattice are localized by the electron-ripplon interaction rather than the Coulomb repulsion, and in that the melting occurs through the dissociation of the ripplon-polarons. As electron-ripplon interaction is weakened (for example by reducing the externally applied pressure on the multielectron bubble) the electrons can shed their localized dimple and the ripplon-polaron Wigner state is destroyed. The melting transition (from a ripplon-polaron solid to an electron liquid) is shown to occur in a region of phase space that is accessible to recently proposed experiments for stabilizing multielectron bubbles. The phase diagram for ripplon-polarons in multielectron bubbles is presented\[^5\].