# Spin-transistor action in waveguides with periodically modulated strength of the spin-orbit interaction 

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Spin-polarized electron transport through waveguides, in which the strength of the spin-orbit interaction (SOI) $\alpha$ is varied periodically, is studied using the transfer-matrix technique. It is shown that the transmission $T$ exhibits a spin-transistor action, as a function of the strength or of the length of one of the two subunits of the unit cell, provided only one mode is allowed to propagate in the waveguide. A similar but not periodic behavior is shown by $T$ as a function of the incident electron energy $E$. In a waveguide with only in one segment, of strength $\alpha_{2}$ and length $l_{2}$, comprised between two segments of strength $\alpha_{1}$, the total transmission, obtained as $T=1 /\left[\cos ^{2}\left(\Delta_{2} l_{2}\right)+r \sin ^{2}\left(\Delta_{2} l_{2}\right)\right]$, with $r$ a function of $\Delta_{1}, \Delta_{2}$ and $\Delta_{j}=\left[m^{* 2} \alpha_{j}^{2}+2 m^{*}\left(E-E_{1}\right)\right]^{1 / 2}$, shows an explicit sinusoidal dependence. The corresponding spin-up $\left(T^{+}\right)$and spin-down $\left(T^{-}\right)$transmissions are given by $T^{+}=T \cos ^{2} \phi$ and $T^{-}=T \sin ^{2} \phi$, where $\phi$ is a measure of the spin precession. The total phase acquired by electrons in different branches during propagation is $\phi=2\left[\delta_{1}\left(L-l_{2}\right)+\delta_{2} l_{2}\right]$ with $^{1}$ $\delta_{i}=2 m^{*} \alpha_{i} / \hbar^{2}$ and $L$ the waveguide length. The transmission through a superlattice, with alternating segments of lengths $l_{1}, l_{2}$, and corresponding SOI strengths $\alpha_{1}, \alpha_{2}$, is also a periodic function of $\alpha_{j}$ and $l_{j}, j=1,2$. As the strength $\alpha$ can be well controlled by applying gates or adjusted with the help of band engineering ${ }^{2}$, the structure considered is a good candidate for the establishment of a realistic spin transistor. The recently developed spin-detection technique ${ }^{3}$ could be used to observe this transistor action also reported for periodically stubbed waveguides of constant ${ }^{4}$ strength $\alpha$.
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## DESCRIPTION

(a) Schematics of a waveguide, of width c , with periodically modulated strength of the SOI. Within one unit, $l_{1}, l_{2}$ and $\alpha_{1}, \alpha_{2}$ are the lengths and SOI strengths of the subunits AB and BC , respectively. (b) Dispersion relation for a waveguide. Neglecting subband mixing the energy levels are given by

$$
\begin{equation*}
E^{ \pm}\left(k_{y}\right)=E_{n}+\hbar^{2} k_{y}^{2} / 2 m^{*} \pm \alpha k_{y} \tag{1}
\end{equation*}
$$

and $\mathrm{E}_{\mathrm{n}}$ is the energy of the nth subband due to the confinement along the x axis. The dashed and dotted curves show the + and - branches for finite strength $\alpha$, the solid curve is for $\alpha=0$.
(c) Transmission versus length $1_{2}$. N is the number of units, $1_{1}=1050 \AA, \alpha_{2}=5 \times 10^{-11} \mathrm{eV} \mathrm{m}$, and $\mathrm{E}=3.2$ meV . The dash-dotted curve shows the spin-down transmission $\mathrm{T}^{-}$for $\mathrm{N}=1$, the other curves show the total transmission. The incident carriers are assumed to be spin-up polarized. For only one waveguide segment, of strength $\alpha_{2}$ and length $l_{2}$, comprised between two segments of strength $\alpha_{1}=0$, the total transmission at zero temperature is given by ( $\left.\Delta_{j}=\left[m^{* 2} \alpha_{j}^{2}+2 m^{*}\left(E-E_{1}\right)\right]^{1 / 2}, \mathrm{j}=1,2\right)$

$$
\begin{equation*}
T=\frac{1}{\cos ^{2}\left(\Delta_{2} l_{2}\right)+r \sin ^{2}\left(\Delta_{2} l_{2}\right)} \tag{2}
\end{equation*}
$$

where $r=\left(\Delta_{1}^{2}+\Delta_{2}^{2}\right)^{2} / 4 \Delta_{1}^{2} \Delta_{2}^{2}$. The periodicity of T with $\mathrm{l}_{2}$ or $\Delta_{2}$ is evident. As shown, T is also periodic for $\mathrm{N}>1$. Its approximate square-wave form, pertinent to a spin transistor, is rounded off with increasing temperature. The spin-up (+) (spin-down) (-) transmission is $\mathrm{T}^{+}=\mathrm{T} \cos ^{2} \phi, \mathrm{~T}^{-}=\mathrm{T} \sin ^{2} \phi$. The phase difference is $\phi=2\left[\delta_{1}\left(\mathrm{~L}-\mathrm{l}_{2}\right)+\delta_{2} \mathrm{l}_{2}\right]$ with L the waveguide length and $\delta_{i}=2 m^{*} \alpha / \hbar^{2}=k_{y}^{-}-k_{y}^{+}$.
(d) As in (c) for $1_{1}=100 \AA, \mathrm{~N}=8, \alpha_{2}=6 \times 10^{-11} \mathrm{eV} \mathrm{m}$, and $\mathrm{E}_{\mathrm{F}}=3.2 \mathrm{meV}$. The solid curve is for temperature $\mathrm{T}=0.2 \mathrm{~K}$, the dotted one for $\mathrm{T}=0.5 \mathrm{~K}$.
(e) Transmission as a function of the strength $\alpha_{2}$ for $1_{1}=1_{2}=900 \AA, \alpha_{1}=0, N=8$, and $E_{F}=3.3 \mathrm{meV}$, at temperature $\mathrm{T}=0.2 \mathrm{~K}$. The solid (dotted) curve is the total (spin-up) transmission.

