Ground State of 2D Electrons in High Magnetic Field

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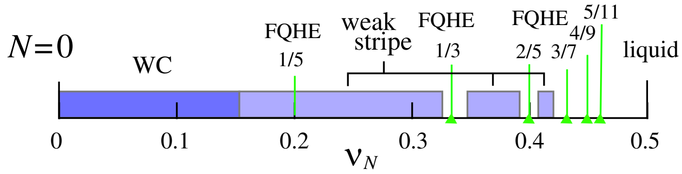
The ground state phase diagram of 2D electrons in high magnetic field is studied by the density matrix renormalization group (DMRG) method. The low energy excitations and pair correlation functions in Landau levels of N=0,1,2 are calculated for wide range of fillings. The obtained results for systems with up to 25 electrons confirm the existence of various electronic states. The ground-state phase diagram consisting of incompressible liquid, compressible liquid, charge density waves called stripe, bubble and Wigner crystal is determined.

Density matrix renormalization group method

In the present study we use the DMRG method to obtain the ground state wave function and low energy excitations. The DMRG method is a kind of variational method, which is combined with the real space renormalization group method. We apply the DMRG method to 2D quantum systems in magnetic field by using the eigenstates of free electrons in Landau gage as basis states. We iteratively expand the size of system by adding local orbitals at the center of the system with restricting the number of basis states using the density matrix calculated from the ground state wave function. The truncation error in the ground state wave function is estimated from the eigenvalues of the density matrix, and it is typically 10^{-4} for systems of 25 electrons with keeping 180 states in each block. In the present study we suppose completely spin polarized ground state and neglect Landau level mixing.

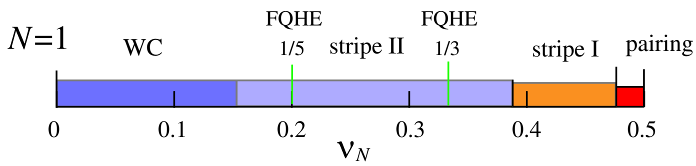
Phase diagram of N=0 Landau level

In the lowest Landau level, the existence of compressible liquid at ν=1/2, incompressible liquid at ν=5/11, 4/9, 3/7, 2/5, 1/3 and 1/5, and Wigner crystal below ν ~1/7 is confirmed. The vanishing of the excitation gap at ν=1/2 is shown in the size dependence of the gap for systems with up to 25 electrons in a square unit cell, and circularly symmetric pair correlation functions at ν=1/2 show liquid nature of the ground state. The estimated effective mass of composite fermions $1/m^*$ is around 0.2 in unit of $e^2/\varepsilon\ell$, which is consistent with existing results. Systematic study on the excitation gap at various fillings between ν=1/2 and 1/6 shows the existence of large excitation gap at $\nu = n/(2n+1)$. The size of the gap decreases with increasing n and $1/m^*$ of composite fermions estimated from the excitation gap is around 0.23, which is consistent with the value at ν=1/2. Although the pair correlation functions are circularly symmetric at $\nu = n/(2n+1)$, weak stripe correlation is observed below ν=0.42 with oscillations along the stripes. The weak stripe correlation is clearly different from the stripes observed in higher Landau levels, and the short range correlation is almost the same to that of Wigner crystal. We think the enhancement of the stripe correlation is due to the instability to Wigner crystal. At ν=1/5, the ground state is shown to be in the same phase characterized by Laughlin state. With further decreasing ν, energy of Wigner crystal comes down to the ground state energy and they seem to cross at around ν=1/7. Since these two energy levels belong to different momentum spaces, we expect first order transition to Wigner crystal at ν ~1/7.



Phase diagram of N=1 Landau level

In the second lowest Landau level, the existence of pairing state at $\nu_N = 1/2$, stripe state around $\nu_N = 0.4$, incompressible state at $\nu_N = 1/3$ and $1/5$, and Wigner crystal below $\nu_N \sim 1/7$ is confirmed with ν_N being the filling factor of the Landau level of index N. At $\nu_N = 1/2$, we calculate low energy spectrum and pair correlation functions for various Haldane's pseudo-potential V_1. The obtained results show that the ground state is different from neither stripe state in higher Landau levels nor compressible liquid state in the lowest Landau level. The pair correlation function is consistent with the pairing formation. Between $\nu_N \sim 0.47$ and 0.37 stripe ground state is obtained. The correlation function is similar to that observed in higher Landau levels, but the amplitude of the stripes is 50% smaller and the period is 30% shorter than those observed in $N = 2$ Landau level. Below $\nu_N = 0.37$, the ground state correlation functions become similar to that in the lowest Landau level and weak stripe correlation with oscillations along the stripes is observed. At $\nu_N = 1/3$ the ground state correlation function seems to be quite different from that of Laughlin state in the lowest Landau level. Although the ground state continuously connects to Laughlin state, the excitation gap is very small and the short range pair correlation function is enhanced. This is due to the large reduction of the short range repulsion V_1 in the second lowest Landau level. On the other hand, the ground state at $\nu_N = 1/5$ is almost identical to that in the lowest Landau level, because V_3 of the Coulomb interaction projected onto $N = 1$ Landau level is slightly increased from that of the lowest Landau level. The ground state at low fillings is almost the same to that in the lowest Landau level, and the first order transition to Wigner crystal is expected at $\nu_N \sim 1/7$.



Phase diagram of N=2 Landau level

In the third lowest Landau level, CDW states called stripe and bubble are obtained. From $\nu_N = 1/2$ to 0.42 the stripe state is the ground state. Compared with the stripes in the second lowest Landau level, the amplitude is much larger and the period is longer. Between $\nu_N = 0.42$ and 0.24 the two electron bubble state is realized in the ground state. Since the correlation function sharply changes at the transition, the transition between the stripe state and the bubble state is expected to be first order. With further decreasing ν_N, the bubble state becomes unstable and different ground state characterized by Wigner crystal is realized. The three electron bubbles are missing in the DMRG calculation, which is different from Hartree-Fock calculations. The reentrant integer quantum Hall effect state observed in experiments corresponds to the two electron bubble state.