Network models for 2D disordered superconductors

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The Bogoliubov-De-Gennes Hamiltonian which governs the dynamics of quasi-particles in disordered superconductors represents a new set of symmetry classes. The corresponding four random matrix ensembles, are called C, CI, D, and DIII and classified by Altland and Zirnbauer [1] according to their symmetries under spin-rotation and time-reversal inversion. They are distincnt from the three Gaussian ensembles which proved to be successful in elucidating the physics of normal disordered conductors. In order to expose the relation between these new symmetry classes and realistic physical systems we consider, for each symmetry class, the propagation of quasi-particles on a network of links and nodes such that the corresponding transfer matrix has the desired symmetry. The original network model [2] was proposed to describe transitions between plateaux in the quantum Hall effect. It described quasi-particles moving along unidirectional links forming closed loops in analogy with semi-classical motion electrons on contours of constant potential. Scattering between links is allowed at nodes in order to map tunneling through saddle point potentials, whence propagation along links yields a random phase. Hence it is termed as the U(1) network model.

Recently, we have studied two symmetry classes (out of the four mentioned above). 1) For class C the Hamiltonian has time reversal symmetry broken but spin rotational-symmetry remains intact. It can be realized in a material consisting of singlet superconductor grains surrounded by normal metal in a perpendicular magnetic field or by an order parameter that breaks time reversal, such as d+id[1]. The associated dynamics can be probed by spin transport, rather than charge transport, since the quasi-particle charge density is not conserved. This symmetry class is realized on the SU(2) network model with an SU(2) matrix on each link. The corresponding phase diagram and critical exponents were reported earlier [3]. 2) For class D, the Hamiltonian has neither time-reversal nor spin-rotation invariance. Two-dimensional systems in this category can display phases with three different types of quasi-particle dynamics: metallic, localized, and incompressible with quantized (thermal) Hall conductance. Correspondingly, they exhibit a variety of delocalization transitions. We have illustrated this behavior by investigating numerically phase diagrams of network models with the appropriate symmetry, and demonstrated the appearance of the metallic phase [4].

We present here preliminary results for the two other classes, CI and DIII. The Hamiltonian for both of them is invariant under time reversal symmetry operation, but spin-rotational symmetry is preserved only for class CI. Those symmetries can be realized in disordered s-wave (class CI) and p-wave (class DIII) superconductors in the absence of an external magnetic field. The method of calculations is based on the transfer matrix algorithm combined with finite size scaling applicable near the critical transition point.

Another quantity of great physical interest is nearest-neighbor spacing distribution (NNSD) of energy levels. Besides its usefulness in suggesting an alternative approach for treating quantum mechanical phase transitions, it represents correlations of any order between energy eigenvalues. The corresponding numerical procedure is distinct from that used in transfer matrix calculations. If the system it is wounded up as a torus then the S matrix expressing outgoing amplitudes on the links in terms of incoming one defines an eigenvalues problem. The unitarity of the S matrix implies that such a procedure can be regarded as an application of a discrete-time evolution operator. The action of that operator on a vector of flux amplitudes, emanating from each link maps the "wave function Ψ" (a collection of amplitudes) on itself. The (quasi) energy levels δ_n are used to compute the corresponding NNSD denoted as p(s) where $s=\delta_{n+1}-\delta_n$ [5]. It is expected to reflect the localization properties of the system, being close to the Wigner's surmise for extended states and to Poissonian statistics for deeply localized states. In the critical region, p(s) is anticipated to have a unique form depending on the nature of the physical system under study. We report calculations of p(s) for different energy domains and confront the resulting distribution with the expected ones as described above.

Summarizing our study of new symmetry classes in disordered superconductors, our main conclusions are: 1) Some of these classes reveal novel critical properties which are not encountered within the theory of disordered normal metals (with and without magnetic field). For example, class C (which can be realized when a disordered s-wave superconductor is subject to a magnetic field, or in superconductors with d+id[1] order parameter) exhibits a quantum Hall like transition for which the (spin) Hall conductance jumps by two units [3]. Or, class D (which can be realized when a p-wave disordered superconductor is subject to an external magnetic field) which at some points on the phase diagram exhibits a one dimensional metal [4]. 2) On the other hand, away from the critical points, the system is either diffusive (like in ordinary weakly disordered metal) or insulating like an Anderson insulator. From that point of view, we can say that although introduction of these new symmetries correspond to four different RMT ensembles (leading to a total number of 10 different ensembles), and manifest a variety of new phenomena, some basic characteristics remain intact, depending only on time reversibility and spin rotation invariance.

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