Topological Field Theory of Time-Reversal Invariant Insulators and Beyond

Xiaoliang Qi (Stanford University)

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Abstract: We show that the fundamental time reversal invariant (TRI) insulator exists in 4+1 dimensions, where the effective field theory is described by the 4+1 dimensional Chern-Simons theory and the topological properties of the electronic structure is classified by the second Chern number. These topological properties are the natural generalizations of the time reversal breaking (TRB) quantum Hall insulator in 2+1 dimensions. The TRI quantum spin Hall insulator in 2+1 dimensions and the topological insulator in 3+1 dimension can be obtained as descendants from the fundamental TRI insulator in 4+1 dimensions through a dimensional reduction procedure. The effective topological field theory, and the Z_2 topological classification for the TRI insulators in 2+1 and 3+1 dimensions are naturally obtained from this procedure. All physically measurable topological response functions of the TRI insulators are completely described by the effective topological field theory. Our effective topological field theory predicts a number of novel and measurable phenomena, especially the topological magneto-electric effect, where an electric field generates a magnetic field in the same direction, with an universal constant of proportionality quantized in odd multiples of the fine structure constant $\alpha = e^2/\hbar c$. Our theory shows explicitly that a three-d Z₂ topological insulator is a condensed matter realization of the $g\Theta$ vacuum studied in the high energy physics, with $\Theta = \pi$. Finally, we present a general classification of all topological insulators in various dimensions, and describe them in terms of a unified topological Chern-Simons field theory in phase space.