Dynamics of one-dimensional Bose liquids in Y-junction and its related system:

Andreev-like reflection and absence of the Aharonov-Bohm effect

Akiyuki Tokuno (Hokkaido Univ.)

Collaborator
Masaki Oshikawa (ISSP)
Eugene Demler (Harvard Univ.)

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OUTLINE

1. Introduction:
One-dimensional Bose liquid
and Tomonaga-Luttinger liquid

2. Y-junction systems for Bose liquid

3. Ring-type interferometer for Bose liquid

4. Summary

INTRODUCTION

1D bose system

Repulsively interacting bosons

Tight confinement by cylindrical trap potential

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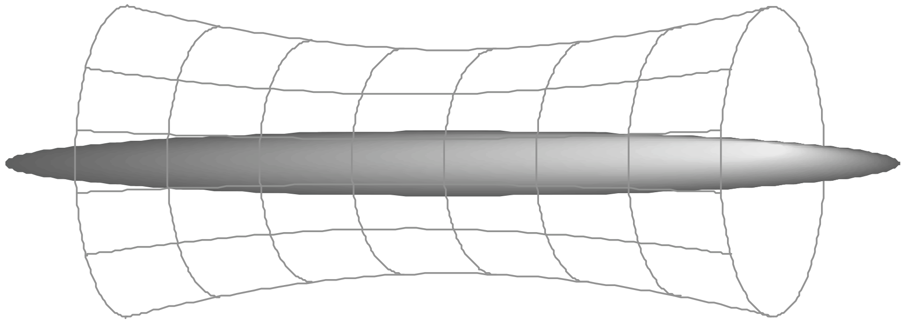
Effective 1D shaped systems



Effective 1D Hamiltonian

$$\mathcal{H} = \int dx \left[\frac{\hbar^2}{2m} \partial_x \psi^\dagger(x) \partial_x \psi(x) + \frac{U}{2} \rho(x) \rho(x) \right]$$

Effective 1D interaction for confined systems

$$U = \frac{u}{\pi a_\perp} \left(1 - C \frac{a}{a_\perp} \right)^{-1}$$

M. Olshanii (1998)

↑

Lieb-Liniger model (integrable)

E. H. Lieb, W. Liniger (1963), E. H. Leib (1963)

INTRODUCTION

Low-energy effective theory and Tomonaga-Luttinger liquid for Bose systems

TL liquid is universal description in low-energy physics of 1D quantum systems.

F. D. M. Haldane (1981)

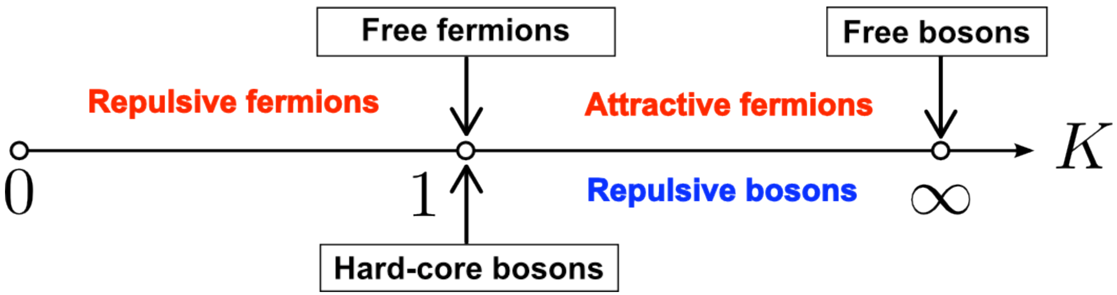
Extract collective excitation

$$\psi(x) \sim \sqrt{\bar{\rho}} e^{i\theta(x)} \qquad \rho(x) \sim \bar{\rho} + \frac{1}{\pi}\partial_x \varphi$$

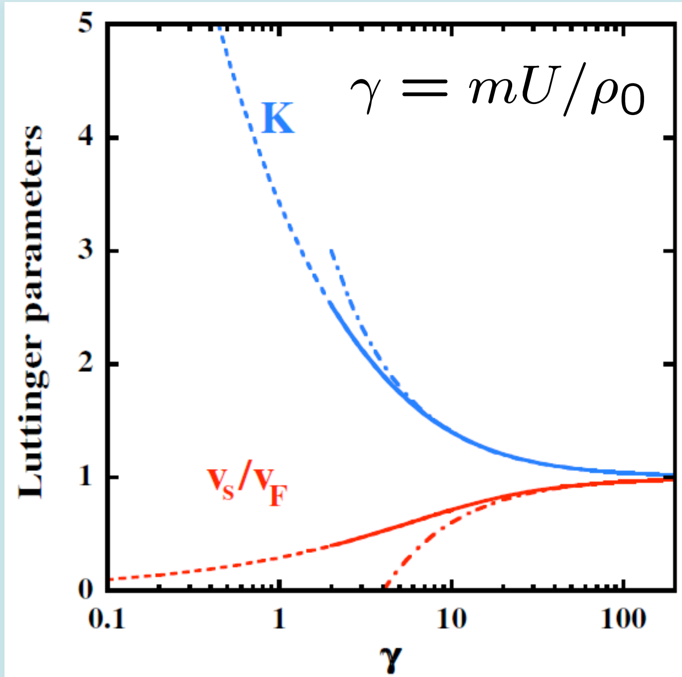
$$\mathcal{H} = \frac{v}{2\pi}\int dx \left[K\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{K}\left(\frac{\partial\theta}{\partial x}\right)^2 \right]$$

K: Luttinger parameter

v: velocity of collective mode



1D quantum systems corresponding to TL parameter.



Luttinger parameter for LL model

M. A. Cazalilla (2004)

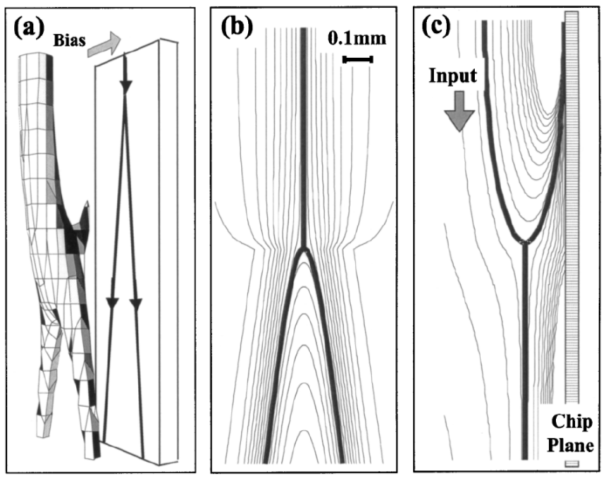
Y-JUNCTION PROBLEM FOR BOSE LIQUID

Y-junction system and experiment

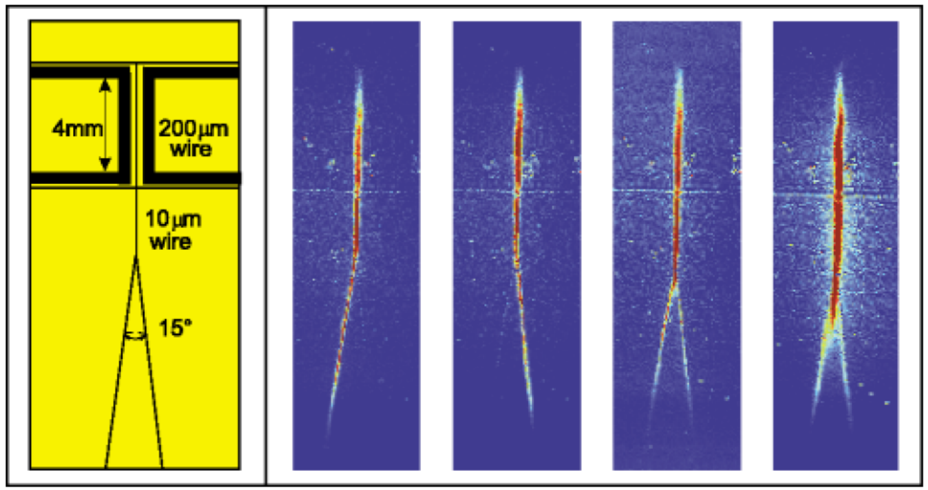
BEC beam splitter for guided trapping atoms.

D. Cassettari et al. (2000)

- Atom chip technology realizes Y-shape trap potential.
- Trapped BEC can be guided as well.
- Application as BEC splitter.



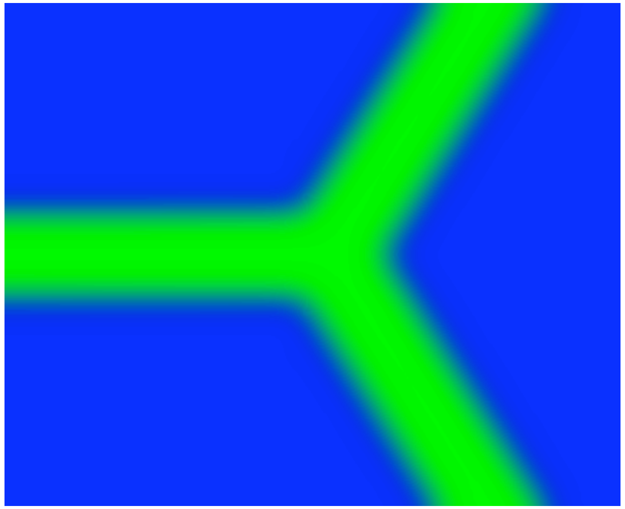
3D imaging and contour map for trap potential.

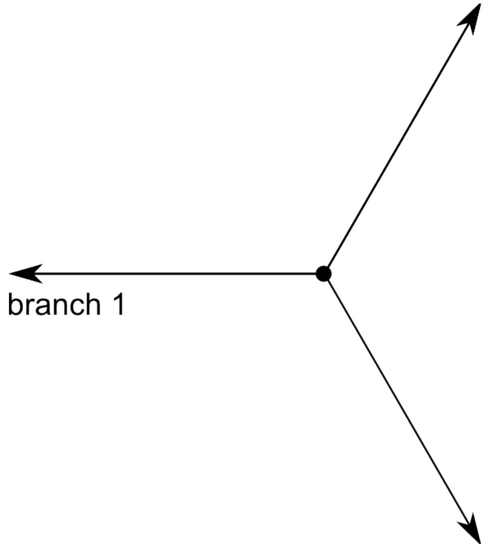


Circuit of Atom chip, and imaging of guiding Y-shaped trapped BEC.

Y-JUNCTION PROBLEM FOR BOSE LIQUID

Set up the system





Simplify the Y-shaped Bose liquid system.

- Branches are completely 1D.
- Bosons filled in entire system. → TL liquid description is applicable.
- Repulsive interacting bosons. → Luttinger parameter **K>1**.

$$\mathcal{H}_j = \int dx \left[-\frac{\hbar^2}{2m} \psi_j^\dagger \frac{\partial^2}{\partial x^2} \psi_j + \frac{U}{2} \rho_j^2 \right]$$

Y-JUNCTION PROBLEM FOR BOSE LIQUID

2008/6/10

Low-energy physics of Y-junction

$$\mathcal{H} = \mathcal{H}_{\text{bulk}} + \mathcal{H}_{\text{boundary}}$$

- **Bulk Hamiltonian: TL liquid**

$$\mathcal{H}_{\text{bulk}} = \sum_{j=1,2,3} \frac{\hbar v}{2\pi} \int dx \left[K \left(\frac{\partial \theta_j}{\partial x} \right)^2 + \frac{1}{K} \left(\frac{\partial \varphi_j}{\partial x} \right)^2 \right]$$

- **Boundary Hamiltonian**

  It is a non-trivial problem to express the concrete form of boundary Hamiltonian.

How to discuss the junction problem.

Transport through a potential barrier. Nayak et al. (1999)

Boundary physics：boundary condition + perturbation

Boundary Condition

Primary boundary condition: current conservation

$$J_1(0,t) + J_2(0,t) + J_3(0,t) = 0$$

What boundary conditions are suitable in low-energy limit?

Y-JUNCTION PROBLEM FOR BOSE LIQUID

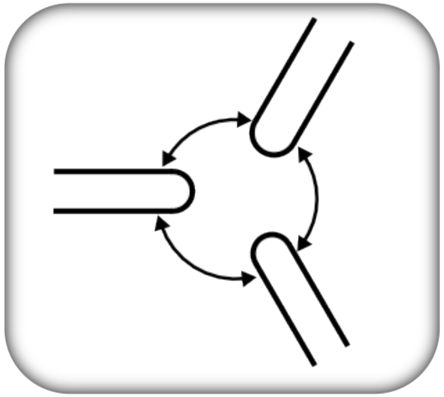
Boundary condition

Simplest boundary condition: decoupled branches

$$J_1(0,t) = J_2(0,t) = J_3(0,t) = 0$$

Perturbation: Tunneling between branches

Scaling dimension = 1/K < 1 **Relevant !!**



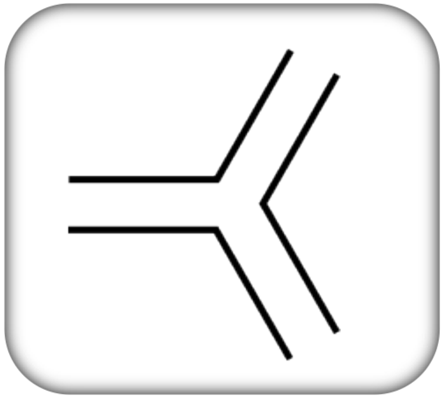
lower energy scale

Arrival point: strongly coupled branches

$$\psi_1(0,t) = \psi_2(0,t) = \psi_3(0,t)$$

Perturbation: Backscattering

Scaling dimension = 4K/3 > 1 **Irrelevant !!**



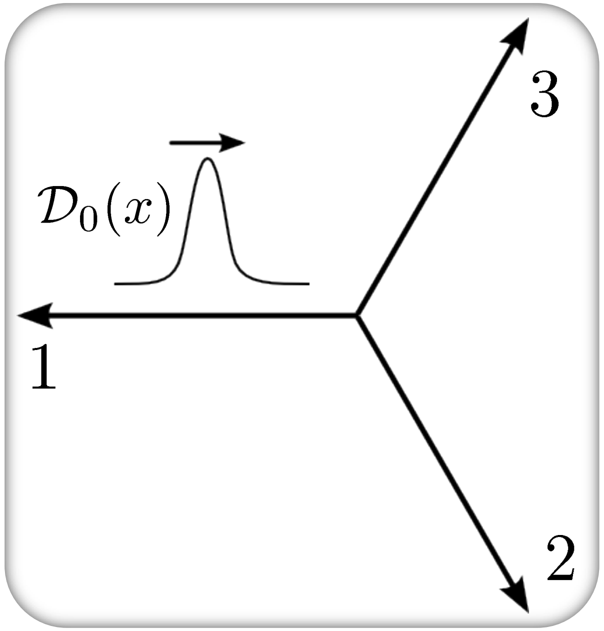
Renormalization group flow goes to the fixed point corresponding to "Strongly coupled limit".

Calculation of dynamics

Time evolution

For expectation values of fields → classical linear wave.

$$\langle \rho_j(x,t) \rangle = \bar{\rho} + \rho_j^L(x+vt) + \rho_j^R(x-vt).$$



Initial condition

Branch 1

$$\rho_1^L(x,0) = \mathcal{D}_0(x)$$

$$\rho_1^R(x,0) = 0$$

Branch 2 , 3

$$\rho_{2,3}^L(x,0) = 0$$

$$\rho_{2,3}^R(x,0) = 0$$

Boundary condition

Current conservation

$$J_1(0,t) + J_2(0,t) + J_3(0,t) = 0$$

Strongly coupled junction

$$\rho_1(0,t) = \rho_2(0,t) = \rho_3(0,t)$$

Reduced to classical linear wave problem with boundary conditions.

Y-JUNCTION PROBLEM FOR BOSE LIQUID

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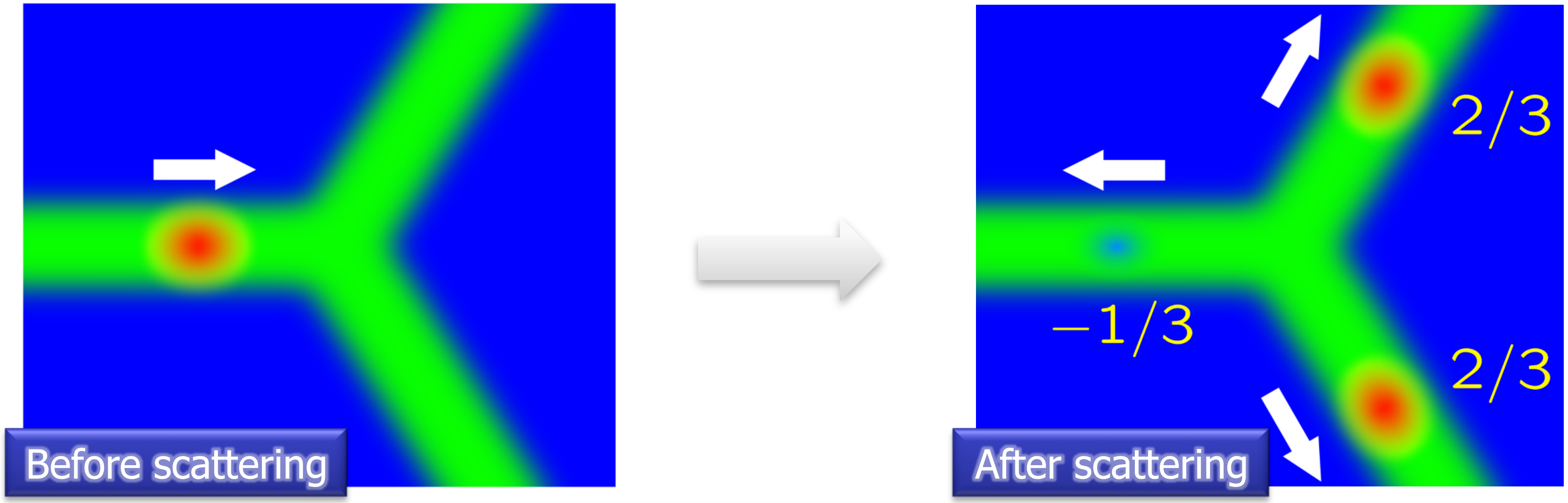
Results

$$\rho_1(x,t) = \rho^0 + \mathcal{D}_0(vt+x) - \underbrace{\frac{1}{3}\mathcal{D}_0(vt-x)\Theta_{\text{step}}(vt-x)}_{\text{Reflection}}$$

$$\rho_{2,3}(x,t) = \rho^0 + \underbrace{\frac{2}{3}\mathcal{D}_0(vt-x)\Theta_{\text{step}}(vt-x)}_{\text{Transmission}}$$

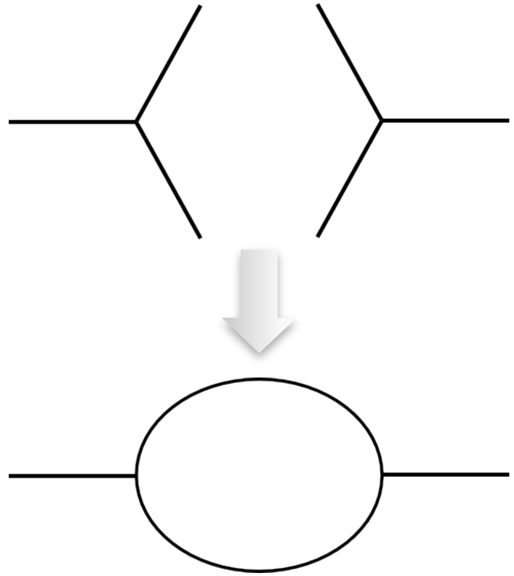


- Total transmission : **4/3** ***Enhanced!***
- Negative reflection : **−1/3** ***Hole-like reflection!***

Andreev-like reflection can be also observed in boson systems.

RING-TYPE INTERFEROMETER

Extension to double Y-junction problem

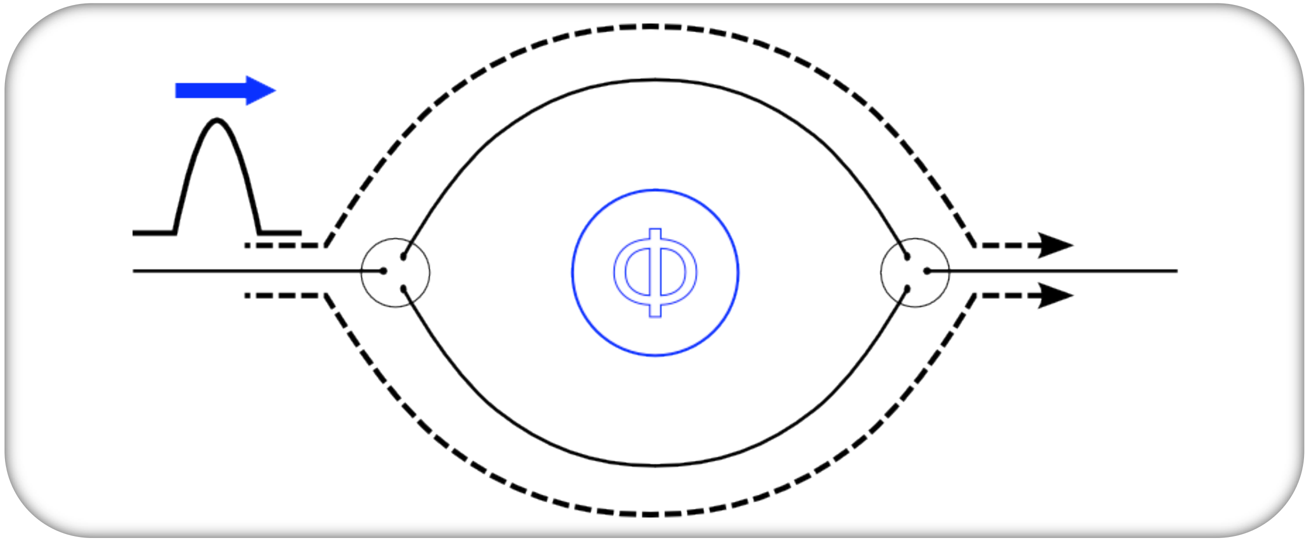


Symmetrically connect two of branches of each Y-junction.

↓

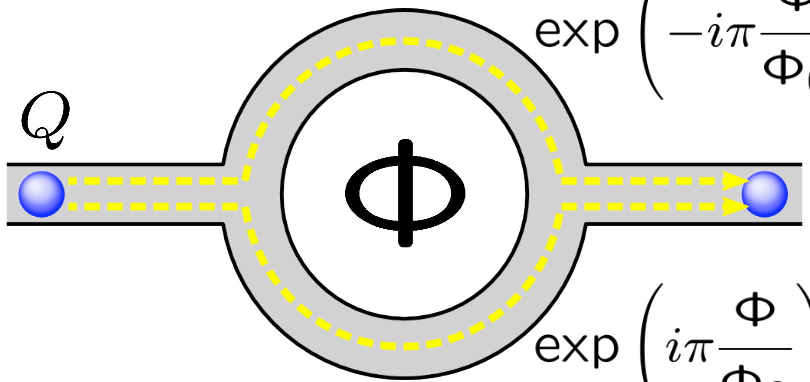
Quantum ring with leads (Ring-type interferometer) system

- "Effective" magnetic flux inside ring.
  → Bosons couple with gauge field.
- Sufficiently large size 2L of the ring.
- Bosons are filled in entire system.



RING-TYPE INTERFEROMETER

Example: single particle problem



$\exp\left(-i\pi\frac{\Phi}{\Phi_0}\right)$

$\exp\left(i\pi\frac{\Phi}{\Phi_0}\right)$

Probability

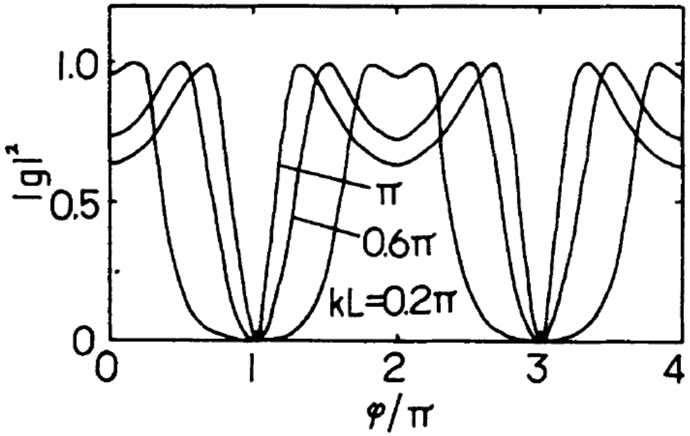
$$|\psi_0 e^{i\pi\frac{\Phi}{\Phi_0}} + \psi_0 e^{-i\pi\frac{\Phi}{\Phi_0}}|^2$$

$$= 2|\psi|^2\left[1+\cos 2\pi\frac{\Phi}{\Phi_0}\right]$$

Interference

$$\Phi = \Phi_0 \times (\mathbb{Z} + 1/2)$$

Then, transmission is zero.



Transmission for free fermion

J.-B. Xia (1992)

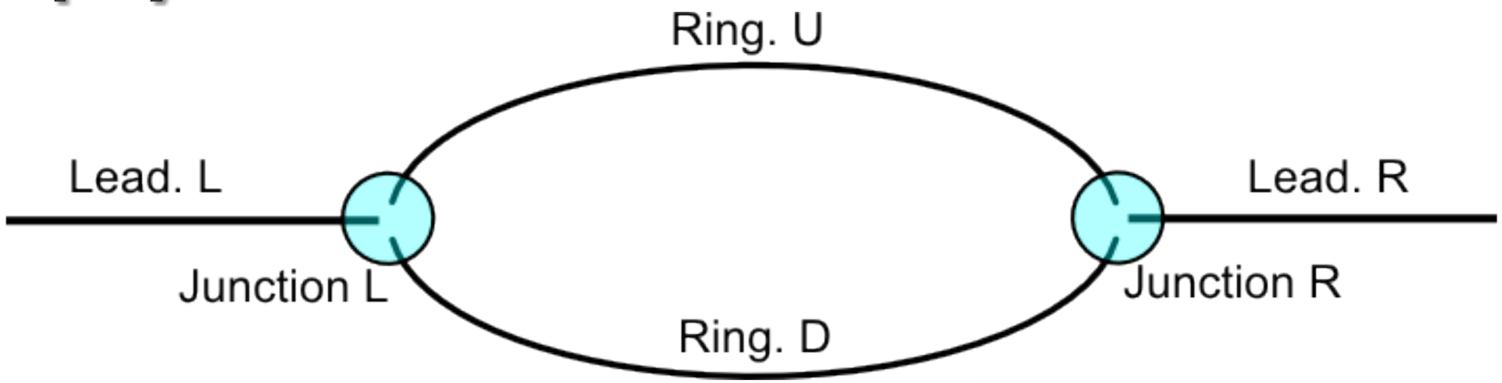
Flux vs Transmission

No transmission at $\Phi = \Phi_0 \times (Z+1/2)$.
Independent of momentum.

Aharonov-Bohm effect

RING-TYPE INTERFEROMETER

Set up system



Hamiltonian

$$\mathcal{H} = \mathcal{H}^U_{ring} + \mathcal{H}^D_{ring} + \mathcal{H}^L_{lead} + \mathcal{H}^R_{lead}$$

$$\tilde{\psi}_{U,D}(x) = \psi_{U,D}(x) \exp\left(\pm i \frac{\pi\Phi}{L\Phi_0} x\right)$$

$$\mathcal{H}^{L,R}_{lead} = \int dx \left[-\frac{1}{2m} \psi^\dagger_{L,R}(x) \frac{\partial^2}{\partial x^2} \psi_{L,R}(x) + \frac{U}{2} \rho_{L,R}(x) \rho_{L,R}(x) \right]$$

$$\mathcal{H}^{U,D}_{ring} = \int_0^L dx \left[-\frac{1}{2m} \tilde{\psi}^\dagger_{U,D}(x) \frac{\partial^2}{\partial x^2} \tilde{\psi}_{U,D}(x) + \frac{U}{2} \rho_{ring}(x) \rho_{ring}(x) \right]$$

Boundary condition: Current conservation + strongly coupled limit

Left side boundary condition

$$J_L(0,t) = J_U(0,t) + J_D(0,t)$$

$$\psi_L = \tilde{\psi}_U = \tilde{\psi}_D$$

Right side boundary condition

$$J_R(0,t) = J_U(L,t) + J_D(L,t)$$

$$\psi_R = \tilde{\psi}_U e^{i\pi\Phi/\Phi_0} = \tilde{\psi}_D e^{-i\pi\Phi/\Phi_0}$$

RING-TYPE INTERFEROMETER

Low-energy effective theory

Bosonization formula

$$\psi \sim e^{i\theta}, \qquad \rho = \bar{\rho} + \pi^{-1}\partial_x \varphi$$

Branch Hamiltonian

$$\mathcal{H}^{U,D}_{ring} = \frac{v}{2\pi}\int_0^{L/2} dx \left[K\left(\frac{\partial \theta_{U,D}}{\partial x}\right)^2 + \frac{1}{K}\left(\frac{\partial \varphi_{U,D}}{\partial x}\right)^2 \right]$$

$$\mathcal{H}^{L,R}_{lead} = \frac{v}{2\pi}\int dx \left[K\left(\frac{\partial \theta_{L,R}}{\partial x}\right)^2 + \frac{1}{K}\left(\frac{\partial \varphi_{L,R}}{\partial x}\right)^2 \right]$$

Bosonized boundary conditions

Left side boundary condition

$$\partial_x \theta_L = \partial_x \theta_U + \partial_x \theta_D$$

$$\varphi_L = \varphi_U + \varphi_D$$

Right side boundary condition

$$\partial_x \theta_R = \partial_x \theta_U + \partial_x \theta_D$$

$$\varphi_R = \varphi_U + \pi\Phi/\Phi_0 = \varphi_D - \pi\Phi/\Phi_0$$

<u>**Flux appears only in boundary condition.**</u>

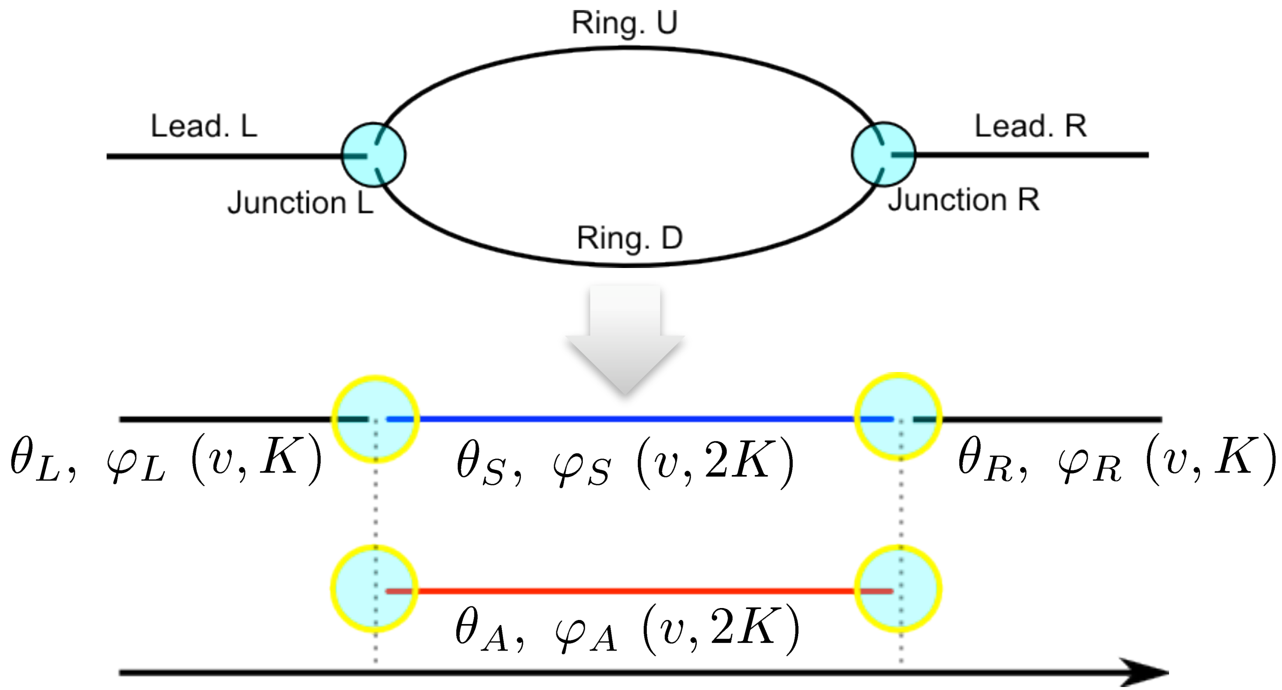
→ **absence in boundary conditions for density.**

RING-TYPE INTERFEROMETER

Low-energy effective theory

Symmetric & anti-symmetric fields

$$\theta_{S,A} = \theta_U \pm \theta_D, \ \varphi_{S,A} = \frac{\varphi_U \pm \varphi_D}{2}$$



Symmetric & anti-symmetric combination separate the system into two parts.

RING-TYPE INTERFEROMETER

Physics of symmetric part

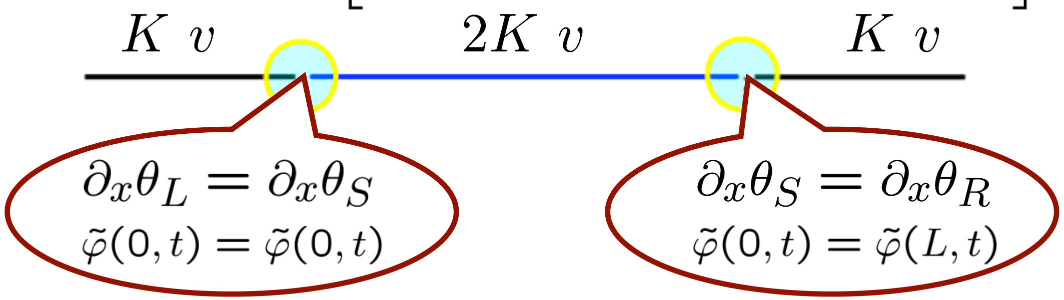
Analogous to inhomogeneous TL liquid

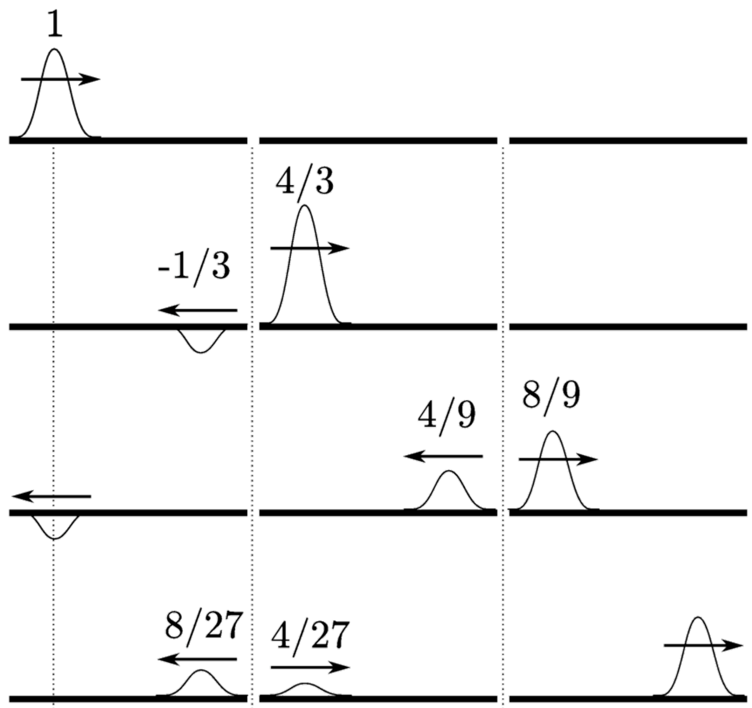
D. L. Maslov and M. Stone (1995), I. Safi and H. J. Schultz (1995)

Symmetric field: center of mass of packets in the ring.

→ Transport between the leads.

$$\mathcal{H} = \frac{v}{2\pi}\int dx \left[K(x)\left(\frac{\partial\tilde{\theta}}{\partial x}\right)^2 + \frac{1}{K(x)}\left(\frac{\partial\tilde{\varphi}}{\partial x}\right)^2 \right]$$





In symmetric field + lead sector,
effect of magnetic flux does not appear.
→ Transport between both side leads is independent of magnetic flux.

RING-TYPE INTERFEROMETER

Physics of anti-symmetric part

TL liquid with twisted boundary condition.

Quantum ring problem and persistent current.

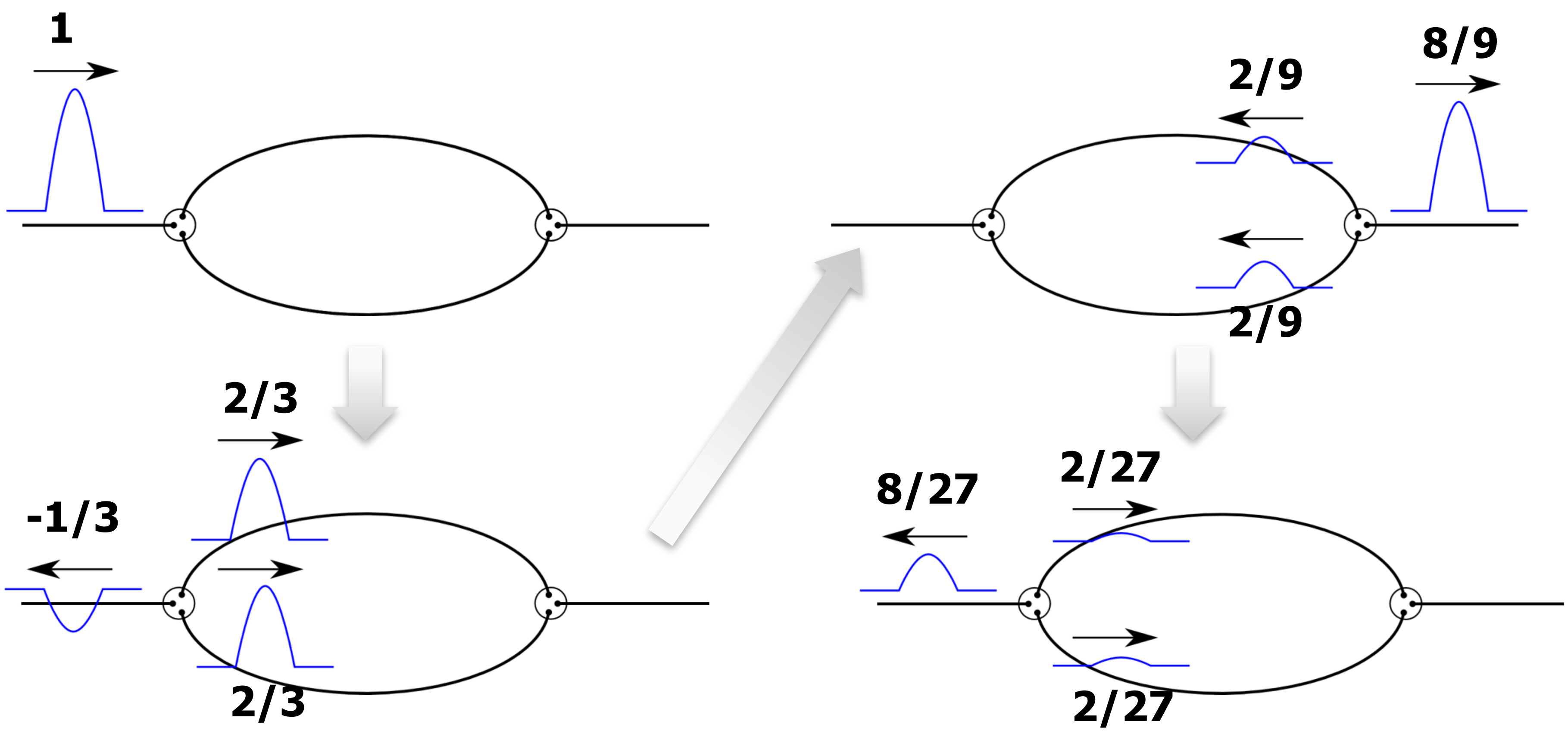
$$\mathcal{H} = \frac{v}{2\pi}\int_0^L dx \left[2K\left(\frac{\partial\theta_A}{\partial x}\right)^2 + \frac{1}{2K}\left(\frac{\partial\varphi_A}{\partial x}\right)^2\right]$$

Magnetic flux works as twisting anti-symmetric field.

→ **drive persistent current.**

RING-TYPE INTERFEROMETER

Summary of dynamics



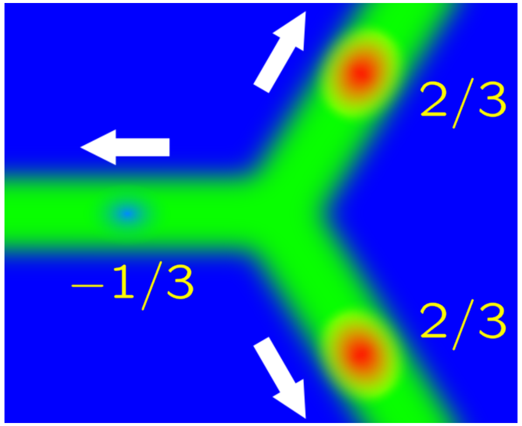
No interference → Absence of Aharonov-Bohm effect.
Magnetic flux inside ring → Just driving persistent current

SUMMARY

Y-junction for Bose liquid

- Bosons filled in Y-shaped potential.
- Dynamics of density packet in steady boson sea.

Negative reflection &
Enhancement of total transmission



Ring-type interferometer for Bose liquid

- Dynamics of density packet in ring-type interferometer.
- Effective magnetic flux inside ring.
- Strongly coupled boundary condition at junctions.

Absence of AB effect.