Symmetry, Berry connection and bulk-edge correspondence

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Symmetry has been highly respected by physicists since it governs universal features of matter and phenomena over their vast variety. As the best-known example, the concept of the symmetry breaking has been quite success for classification of phases except quantum/spin liquids which are characterized by the absence of fundamental local order parameters. Such typical and historical exceptions are quantum Hall (QH) states and Haldane spin chain where the ground states are gapped and any correlations of local objects are short-ranged in space. Strong quantum effects prevent from formation of classical order in any form.

Although the symmetry breaking does not play any fundamental role there, still the symmetry is crucial as the symmetry protection. Totally generic states are boring since they can be adiabatically connected with each other according to the Von Neumann-Wigner theorem. Symmetries such as gauge symmetry and time-reversal protect physically interesting classes from the others. Then adiabatic/topological invariants such as quantized quantities are useful labels of the adiabatically connected classes. This is a topological classification, which is a variation of the historical adiabatic heuristic argument for the quantum Hall states by Wilczek.

Such labels of the topological phases are explicitly constructed by the Berry connections, such as the Chern numbers and quantized Berry phases. They successfully characterize the above historical examples of the QH states and the gapped quantum spin chains. Let me make a self-contained (hopefully) introduction for these topological quantities putting a stress on the gauge structure of the Berry connection and its validity.

As for the gapped topological phases, one may think the absence of low energy modes as a bulk may cause difficulty in experimental identification of the phases. However it is not the case, there exist low energy modes as edge states for topological phases with boundaries or impurities. Geometrical perturbation induces low energy modes in the gap of the bulk when the system is topologically non trivial. This is the bulk-edge correspondence, which was discovered for the QH state and its validity/consistency is confirmed for various topological phases, such as topological insulators, graphene, superconductors, photonic systems and cold atoms. Generic surface states of semiconductors are in this category as well. Based on the bulk-edge correspondence, one may consider the behaviour of the edge states as a topological order parameter. It should be noted that they are negligible when one considers thermodynamic quantities. This principle is emergent in the sense that the edge states are well defined and have distinction from the bulk only by taking an infinite system limit still with boundaries. This bulk-edge correspondence is discussed in connection with the Chern numbers and quantized Berry phases.

The talk will be a pedagogical introduction of the basic concepts and some of recent applications will be mentioned as well.