# Stability of semi-metals : (partial) classification of semi-metals

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# *Collaborators*

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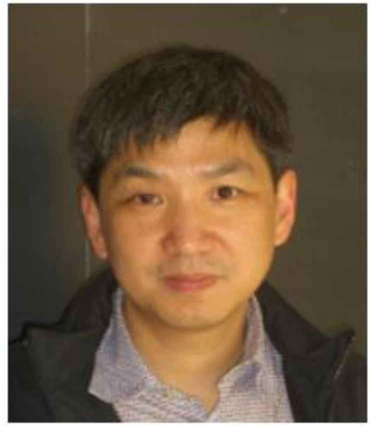

Yong Baek, Kim
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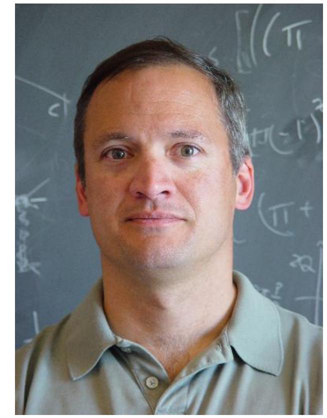

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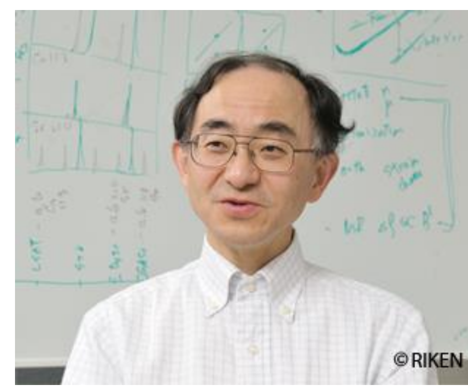



N. Nagaosa,
Riken

# Stability of semi-metals

Refs :

1. Non-Fermi liquids and topological phases :
   *EGM, Xu, Kim, and Balents (arXiv : 2012.1168)*

2. All-in All-out magnetic phase transition in pyrochlore structures :
   *Savary, EGM, and Balents (to appear)*

3. Topological quantum phase transitions in noncentrosymmetic systems :
   *Yang, et. al. (EGM) (PRL 110.086402)*

4. Stability of semi-metals :
   *EGM et. al. (to appear)*

# Stability of semi-metals

- Motivation

Amounts of
charge excitations

**Insulators : charge gapped systems**

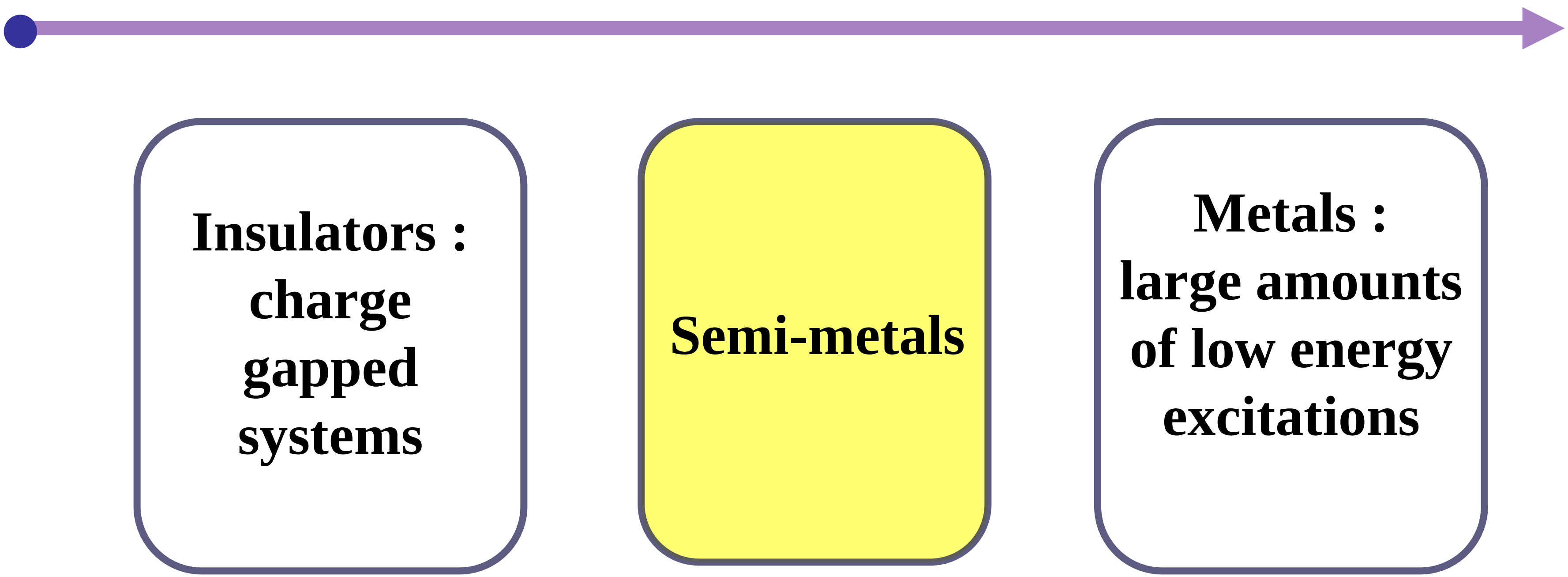
**Metals : large amounts of low energy excitations**

Imura, Yoshida (06/07),
Furusaki (06/10), Maciejko (06/11),
Oshikawa (06/18),
and many other talks in the symposium

# Stability of semi-metals

- Motivation

Amounts of
charge excitations

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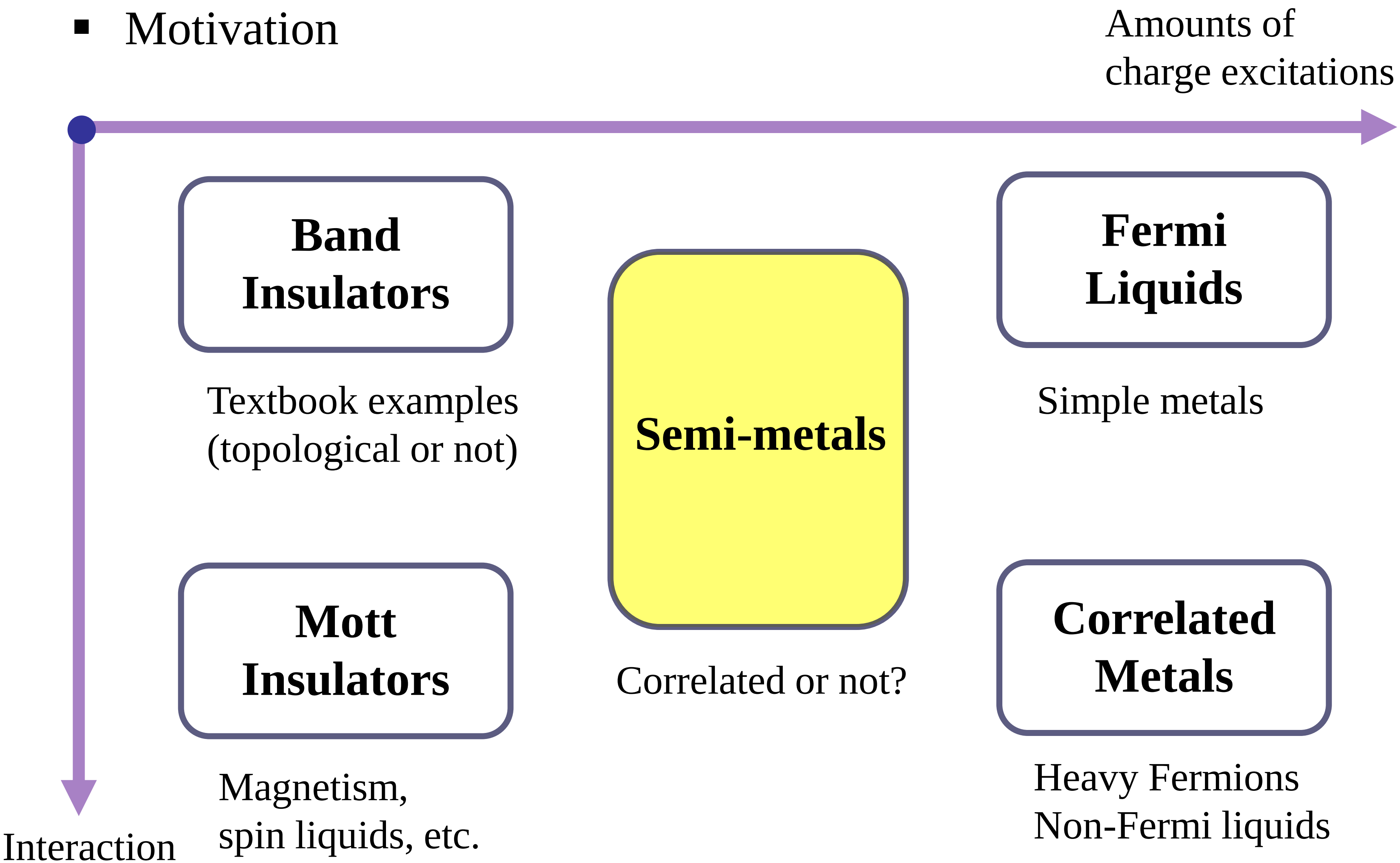
# Stability of semi-metals

- Motivation

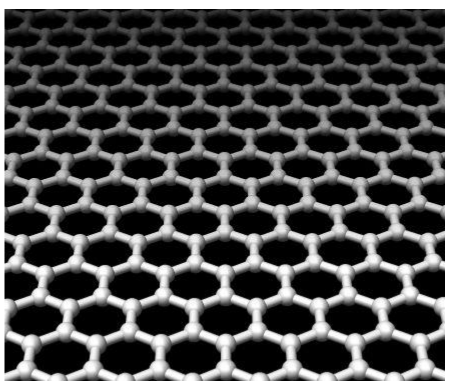


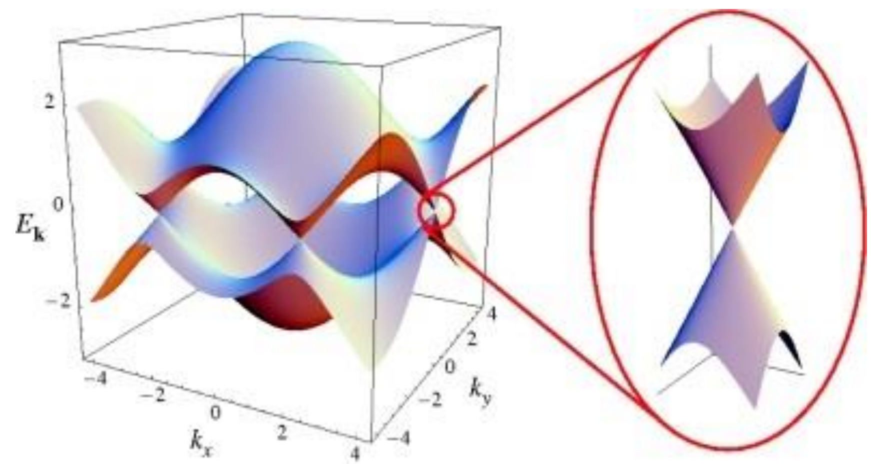
# Stability of semi-metals

- Characteristics of semi-metals

**Semi-metals**

Finite number of band touching points in Brillouin zone
(ex : graphene)






- Existence of gapless excitations — K and K' points
- Symmetry is more important. (protecting symmetries) — sub-lattice symmetry
- Interaction effect is more important than insulators. — quantum critical phase
- Physical properties show different behaviors. — zero Landau Level

# Stability of semi-metals

- Strategy

  - Consider "symmetric" semi-metals.
  - Investigate Coulomb interaction effects.
  - Break symmetries and obtain different class of semi-metals.

**Symmetric semi-metals** :
energy dispersion is isotropic near gapless points.
(only one dynamic critical exponent is necessary.)

$$E_{\pm}(\vec{k}) = \pm k^{z}$$

**Coulomb interaction effects** : two types of interactions
- Short range Coulomb interaction (ex : Hubbard U)
- Long range Coulomb interaction (V(r) ~ 1/r )

*High momentum (energy) degrees of freedom induces the two types of interactions*

# Stability of semi-metals

- Effective action of the symmetric semi-metals $E_\pm(\vec{k}) = \pm k^z$

$$\mathcal{S}_{tot} = \int d\tau d^d x\, \psi^\dagger \left[\partial_\tau + \mathcal{H}_0(-i\nabla)\right] \psi + g_i(\psi^\dagger M_i \psi)^2$$

*spatial dimension* *Short-range Coulomb*

$$+\ e^2 \int d\tau d^d x d^d y\, \frac{\psi^\dagger(x)\psi(x)\, \psi^\dagger(y)\psi(y)}{|x-y|} \qquad (1)$$

*Long-range Coulomb*

**Scaling dimensions**

Non-interacting fixed point :

$$x \to x\, e^{-l}$$
$$\tau \to \tau\, e^{-zl}$$

$$[g_i] = z - d \quad , \quad [e^2] = z - 1$$

**Stability critical dimension** :

$$d_c = z$$

$d > z$ : *irrelevant contact interactions*

$d < z$ : *relevant contact interactions*

# Stability of semi-metals

- Classification of the symmetric semi-metals

**Notation** : ($d$nzm) phase (n spatial dimension and m dynamical critical exponent)

Non-interacting fixed points under contact interactions
- stable for (d2z1, d3z1, d3z2)
- marginal for (d1z1, d2z2, d3z3)
- unstable for (d1z2, d1z3, d2z3)

| | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|
| $z=1$ | $(0,0,E^0)$ | $(-1,0,E)$ | $(-2,0,E^2)$ |
| $z=2$ | $(1,1,E^{-\frac{1}{2}})$ unstable | $(0,1,E^0)$ | $(-1,1,E^{\frac{1}{2}})$ |
| $z=3$ | $(2,2,E^{-\frac{2}{3}})$ unstable | $(1,2,E^{-\frac{1}{3}})$ unstable | $(0,2,E^0)$ |

$$[g_i] = z - d \quad , \quad [e^2] = z - 1$$

Density of states :

$$\mathcal{D}(E) \sim E^{\frac{d}{z}-1}$$

Long-range Coulomb interaction effects ??

# Stability of semi-metals

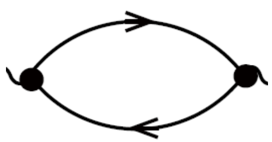
- Classification of the symmetric semi-metals

Density of states : **constant** at the stability critical dimensions.

$$\mathcal{D}(E) \sim E^{\frac{d}{z}-1}$$

**screening** of the long range Coulomb interaction.
*cf ) Thomas-Fermi screening in metallic systems.*

Polarization function



$$= \quad \Pi(q, i\Omega) = e^2 \int_k \frac{n_F(\alpha E_{k+q}) - n_F(\beta E_k)}{-i\Omega + \alpha E_{k+q} - \beta E_k} \mathrm{tr}(P_\alpha(k+q) P_\beta(k))$$

$$V_{coulomb}(q) \sim \frac{1}{q^2 - \Pi(0,0)} \sim \frac{1}{q^2 + e^2 \mathcal{D}(0)}$$

Long-range Coulomb interaction :
irrelevant at the stability critical dimensions.

# Stability of semi-metals

- Classification of the symmetric semi-metals

| | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|
| $z=1$ | $(0,0,E^0)$ | $(-1,0,E)$ | $(-2,0,E^2)$ |
| $z=2$ | $(1,1,E^{-\frac{1}{2}})$ unstable | $(0,1,E^0)$ | $(-1,1,E^{\frac{1}{2}})$ |
| $z=3$ | $(2,2,E^{-\frac{2}{3}})$ unstable | $(1,2,E^{-\frac{1}{3}})$ unstable | $(0,2,E^0)$ |

Coulomb interaction :
marginally irrelevant (QED types)

Coulomb interaction :
relevant (interaction is cruical.)

Simple estimation

$$E_{kin} \sim \frac{1}{mr^2} \quad , \quad E_C \sim \frac{e^2}{r}$$

$$E_{kin} \ll E_C \quad r \to \infty$$

At long distance (low energy),
Coulomb interaction is dominant.

# Stability of semi-metals

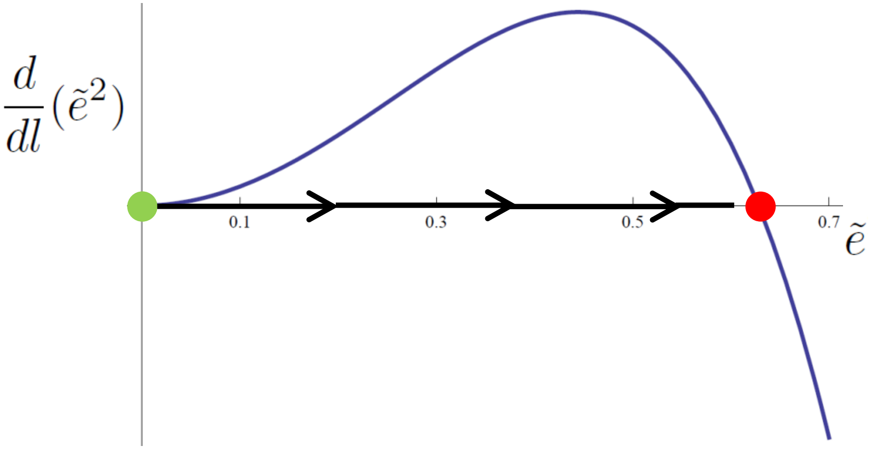
- Classification of the symmetric semi-metals

(d3z2) semi-metals
- short-range Coulomb interaction : irrelevant
- long-range Coulomb interaction : **relevant**

But,
virtual screening process eventually becomes dominant at long range physics.

Thus, the ground state becomes non-Fermi liquid.

$$\beta(e^2) = \frac{d}{dl}e^2 = e^2 - c\,e^4$$



## Stability of semi-metals

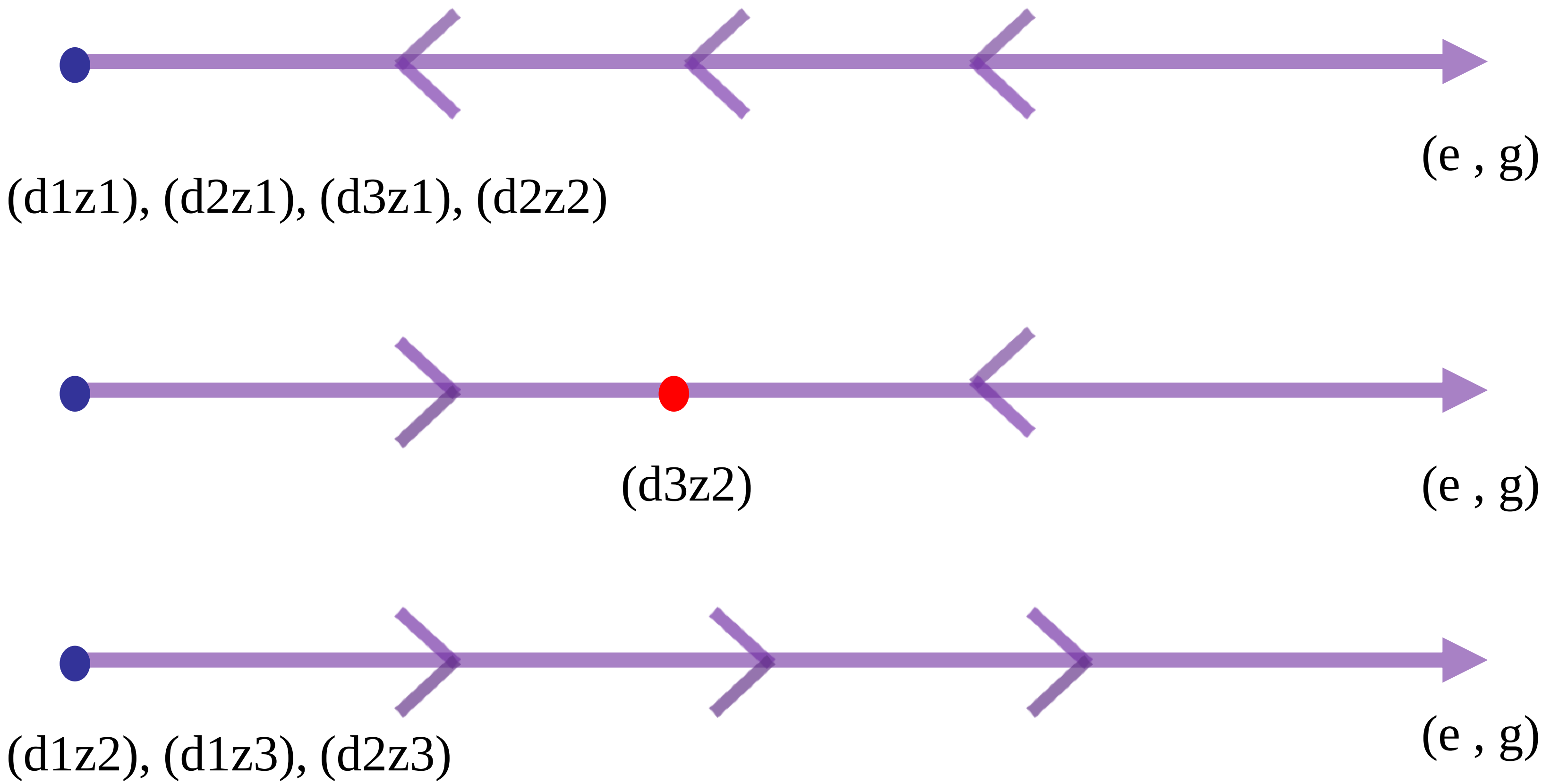
- Classification of the symmetric semi-metals

| | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|
| $z=1$ | $(0,0,E^0)$ Luttinger | $(-1,0,E)$ mono-graphene | $(-2,0,E^2)$ Weyl |
| $z=2$ | $(1,1,E^{-\frac{1}{2}})$ unstable | $(0,1,E^0)$ bi-graphene | $(-1,1,E^{\frac{1}{2}})$ LAB |
| $z=3$ | $(2,2,E^{-\frac{2}{3}})$ unstable | $(1,2,E^{-\frac{1}{3}})$ unstable | $(0,2,E^0)$ 3d-marginal |

# **Stability of semi-metals**

- Classification of the symmetric semi-metals

Renormalization group picture :



# Stability of semi-metals

- Derivative phases from the symmetric semi-metals

By breaking protecting symmetries, anisotropic semi-metals are achieved. Spatial scaling is anisotropic.

| | $\mathcal{H}_{(1,2)}$ | $\mathcal{H}_{(1,1,2)}$ | $\mathcal{H}_{(1,2,2)}$ |
|---|---|---|---|
| $z=2$ | $(-1,0,E^{\frac{1}{2}})$ $\text{TQPT}_2$ | $(-2,0,E)$ double Weyl | $(-3,0,E^{\frac{3}{2}})$ $\text{TQPT}_3$ |

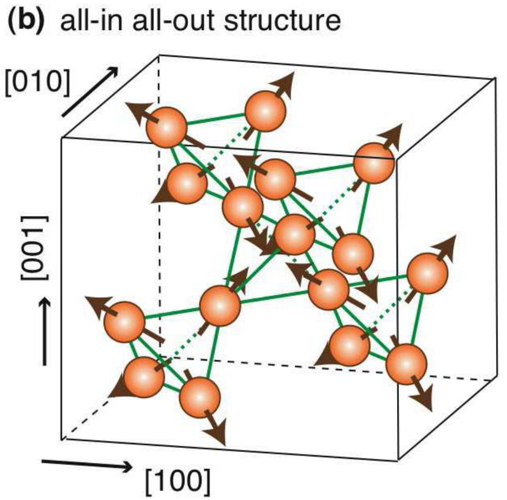
$$
\begin{aligned}
\mathcal{H}_{(1,1,2)} &= \psi^\dagger((k_x^2-k_y^2)\sigma^x + 2k_xky_y\sigma^y + k_z\sigma^z)\psi \\
\mathcal{H}_{(1,2,2)} &= \psi^\dagger(k_x\sigma^x + k_y\sigma^y + k_z^2\sigma^z)\psi \\
\mathcal{H}_{(1,2)} &= \psi^\dagger(k_x^2\sigma^x + k_z\sigma^z)\psi
\end{aligned}
$$

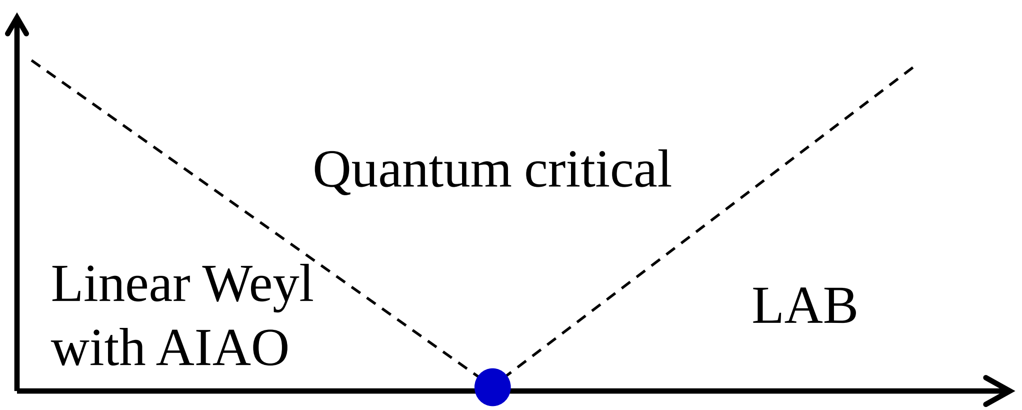
# Stability of semi-metals

- Quantum phase transitions around the semi-metals

Especially, the (d3z2) semi-metal has exotic phase transitions due to its special characteristics.

Example : AIAO phase transition around the (d3z2)




Emergent anisotropy and emergent symmetries.

*Savary et. al. (to appear)*

# Stability of semi-metals

- Summary

Stability of semi-metals under Coulomb interactions are studied.

In symmetric semi-metals,
four stable weakly(non) interacting semi-metals are possible.

In symmetric semi-metals, one interacting semi-metal phase is found.

Unstable phases might be gapped or might become interacting semi-metals.
(Fractionalization of electrons is possible.)

Anisotropic semi-metals are achieved by breaking protecting symmetries.

Exotic quantum phase transitions around the semi-metals are realized.

# Stability of semi-metals

- Future work

  Strongly interacting semi-metals

  Quantum critical behavior in semi-metals

  Impurity problems in semi-metals

  Physical quantities (optical conductivity, susceptibility, etc.)

  Crossover to small Fermi pocket systems.

***Thank you for your attention.***