# (Topological Aspects of) Quantum Spin Nanotubes -S=1/2 Three-Leg Spin Tube-

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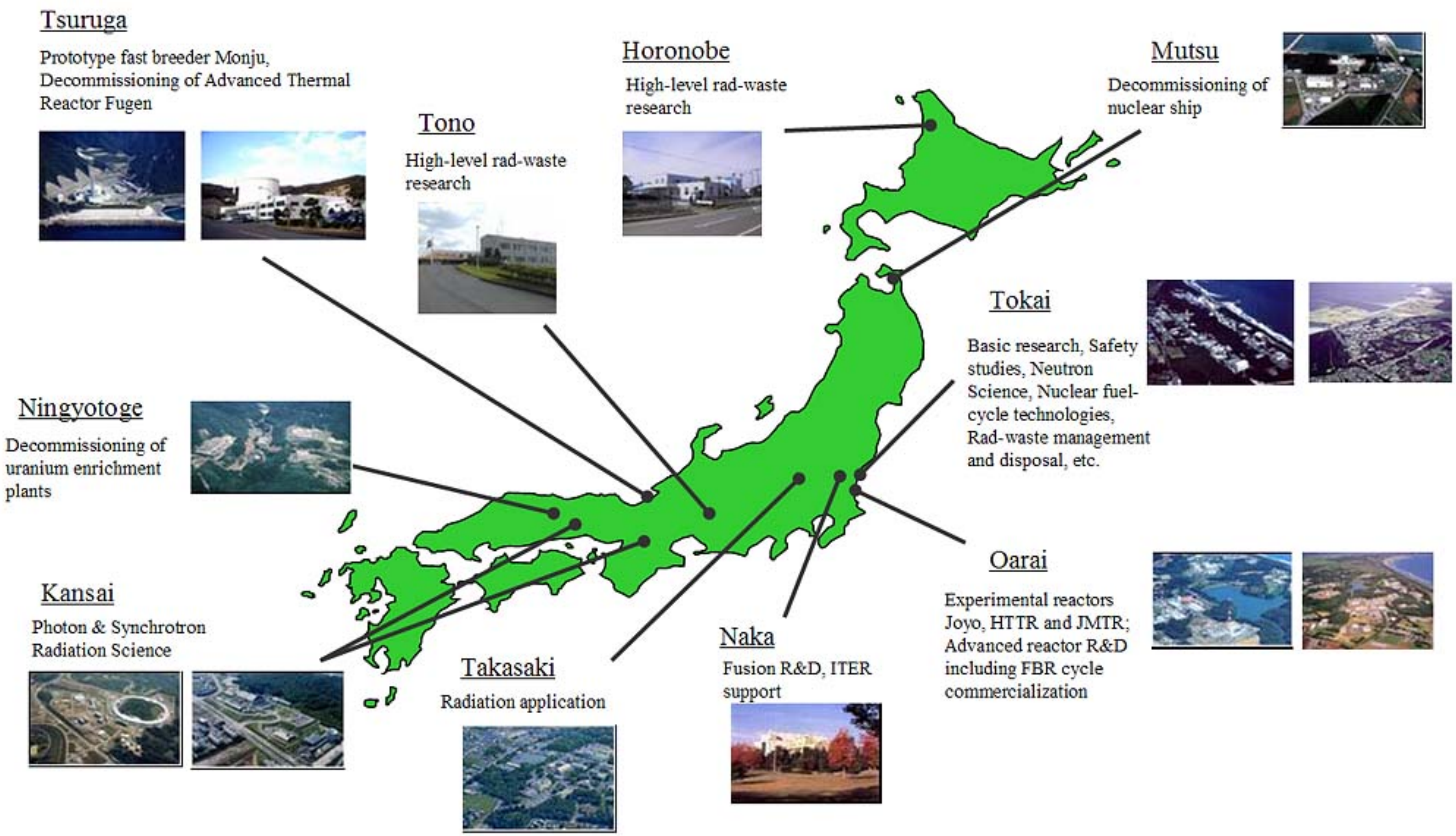
Review： T.Sakai et al. , J. Phys. Condensed Matter 22 (2010) 403201

# R&D Centers of JAEA



# SPring-8 (Super Photon Ring 8Gev)

SACLA(**S**Pring-8 **A**ngstrom **C**ompact Free Electron **La**ser)

# Contents

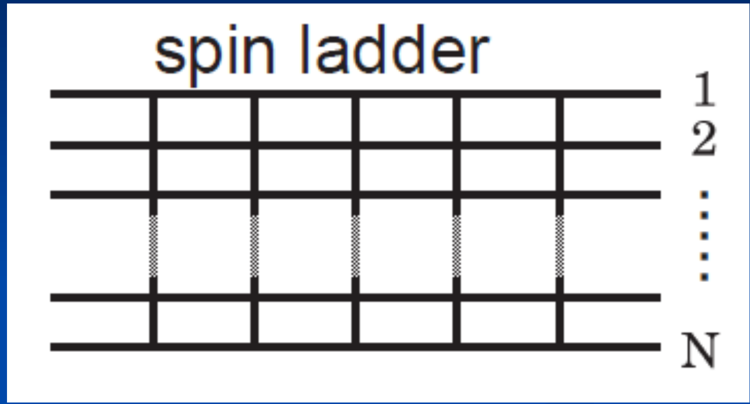
- Introduction
- Quantum Phase Transition between 3-leg tube and ladder
- Real Spin Tubes
- 1/3 Magnetization Plateau
- Carrier doped spin tubes

# Introduction

Spin Ladder

Two (even) legs : Spin gap

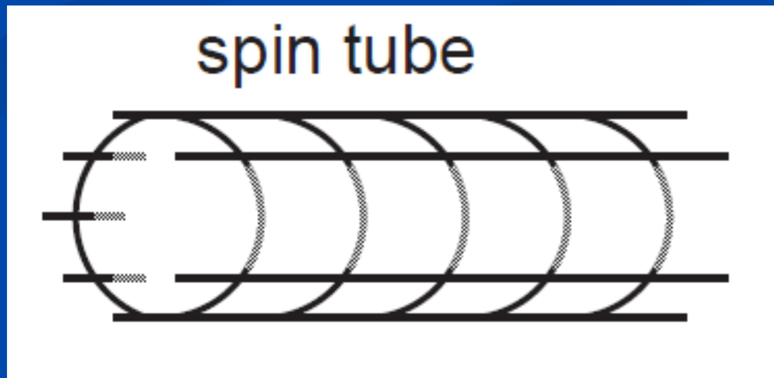
Three (odd) legs : Gapless




Spin Tube

Two (even) legs : Spin gap

Three (odd) legs : Spin gap



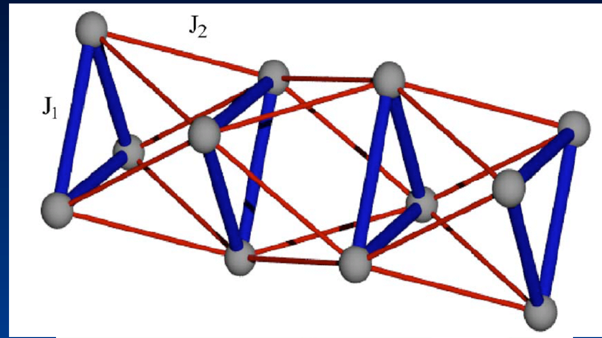

$$H = J_r \sum_{i=1}^{N} \sum_{j=1}^{L} S_{i,j} \cdot S_{i+1,j} + J_l \sum_{i=1}^{N} \sum_{j=1}^{L} S_{i,j} \cdot S_{i,j+1}$$

$S=1/2$

# Real spin tubes

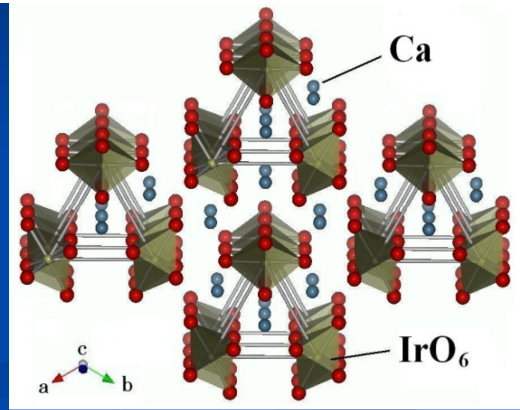
3-leg

$[(CuCl_2tachH)_3Cl]Cl_2$




*J. Schnack, et al, PRB70, 174420 (2004).*

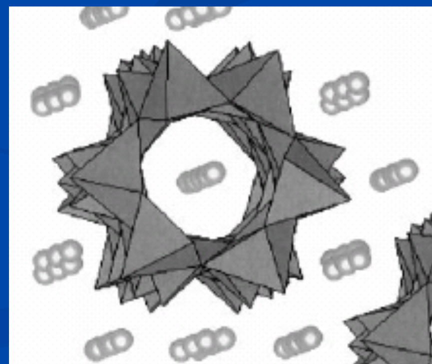
**$Ca_5Ir_3O_{12}$** **1/6**-filled tube




*H. Sato, et al, (Chuo University)*

9-leg

**$Na_2V_3O_7$**



*P. Millet, et al, J. Solid State Chem. 147, 676 (1999)*

7-leg

**Organic compounds**

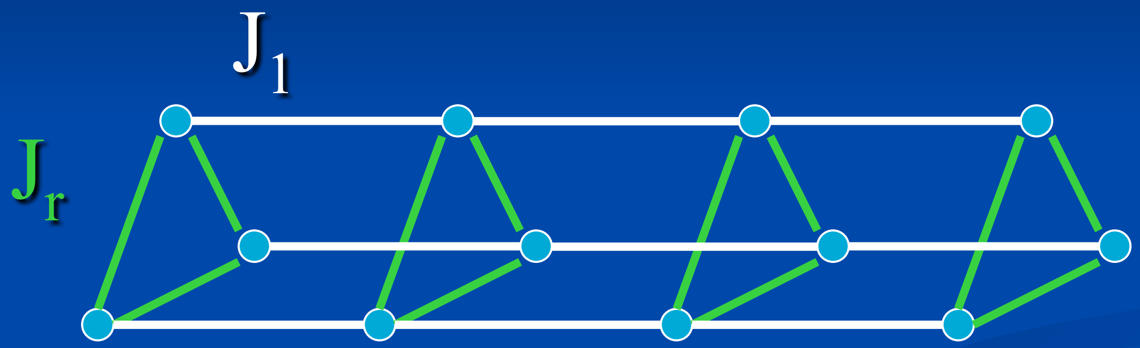
*Unpublished (Tsukuba University)*

# Theories on 3-leg spin tubes

- Schulz 1996: S=1/2 Bosonization
- Kawano-Takahashi 1997:S=1/2, Spin gap~0.28$J_\perp$
- Cabra-Honecker-Pujol 1998: S=1/2, 1/3 magnetization plateau
- Sato-TS 2007 : field-induced new phases
- TS-Sato-Okunishi-Okamoto-Itoi 2008, 2010: Isosceles triangle
- Nishimoto-Arikawa 2008: Isosceles triangle
- Charrier-Capponi-Oshikawa-Pujol 2010: Integer spin
- Nishimoto-Fuji-Ohta 2011: S=3/2, Spin gap
- Lajko-Sindzingre-Penc 2012: S=1/2, Multispin exchange
- Okunishi-Sato-TS-Okamoto-Itoi 2012: new phase at m=1/3

# S=1/2 Three-leg spin tube

Model



# S=1/2 Three-leg spin tube

Kawano-Takahashi: JPSJ 66 (1997) 4001

Effective model based on Chirality operators

Density Matrix Renormalization Group(DMRG)

$J_r > 0$ : Spin gap

$J_r/J_1 \to \infty$: $\Delta = 0.28 J_1$

I. Affleck, Phys. Rev. B 37, 5186 (1988)

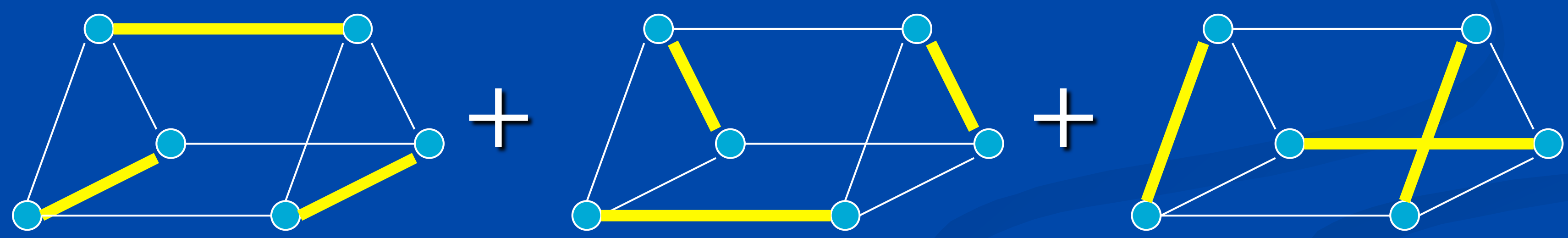
Lieb-Shultz-Mattis theorem

odd-leg ladders : gapless

or Gournd state is degenerated.

# Gap due to double periodicity

Singlet with double periodicity（Kawano-Takahashi)




⇒ Quantum phase transition

with respect to $J_r/J_1$

# Analysis of Quantum Phase Transition

Numerical diagonalization：Low-lying excitation

3×L spin systems： $L \leq 8$

Phenomenological renormalization

Scaled gap: $L\Delta$

$L\Delta$ is independent of size ⇒ Gapless

$L\Delta$ increasing with size ：Gapped

$L\Delta$ decreasing with size ：Degenerated with GS

（DMRG： Direct calculation of gap）

# Spin Gap

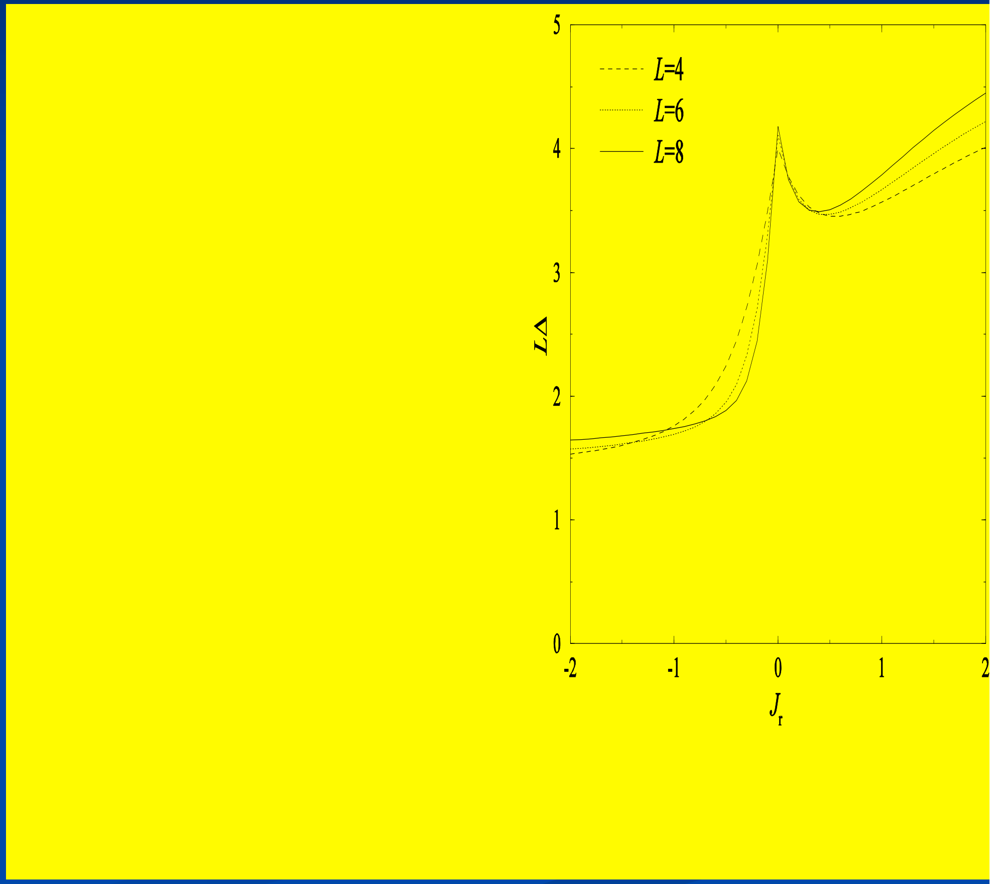
$J_l$=1 fixed, varying $J_r$

Ground state : singlet

1st excited state : triplet k=π

$J_r$=0 three chains : gapless

$J_r \to -\infty$(S=3/2 chain) : gapless




Result :

$J_r > 0$: Spin gap is opening with increasing $J_r$

$J_r < 0$ : Always gapless

# Doubly degenerated Ground State

Singlet excitations  k=π

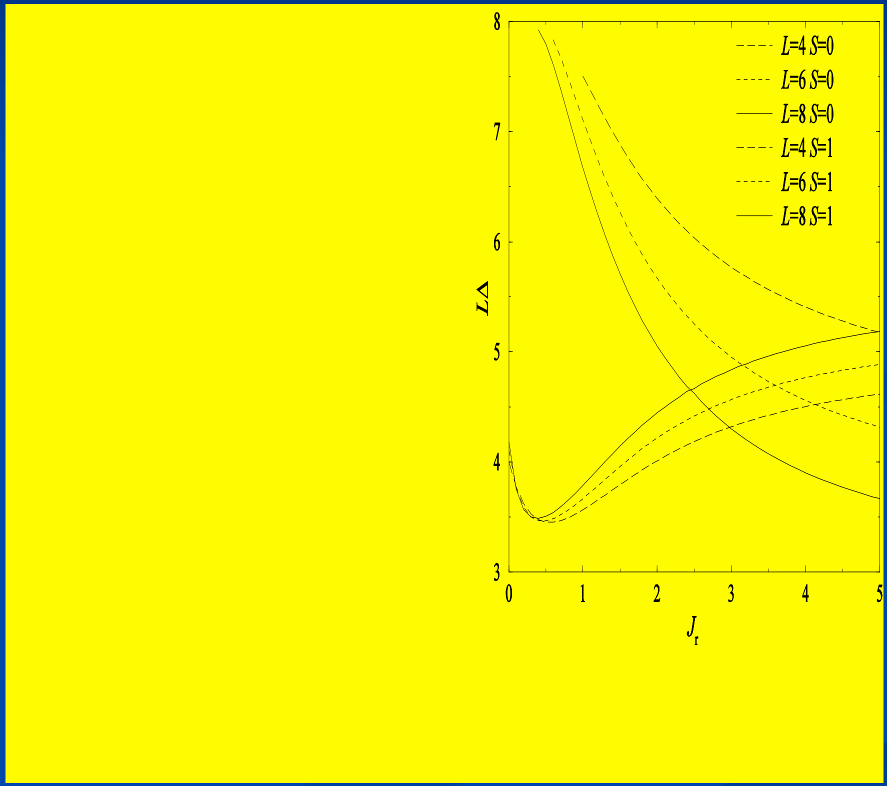
degenrated with GS

⇓

Double periodicity

⇓

Spin gap



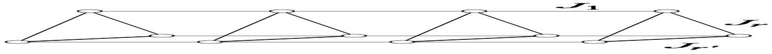

Critical point $J_{rc}$

$J_{rc}$ =0 or finite ?

# Quantum Phase Transition between 3-leg spin tube and ladder -Isosceles Triangle Tube-

Distortion from a regular triangle ($J_r > J_{rc}$)

$\alpha = J_r' / J_r$




$\alpha=0$ : Three-leg ladder
gapless

$\alpha=1$ : Spin gap

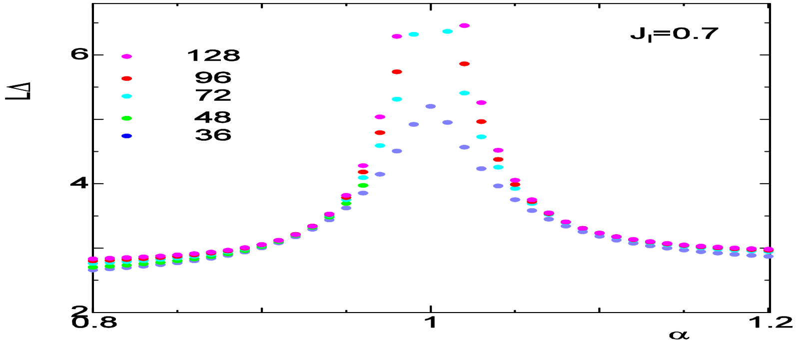
$\alpha \to \infty$ : Dimer and monomer
gapless

Quantum phase transition with respect to $\alpha$

# Spin Gap by DMRG

$J_1$=0.7 fixed

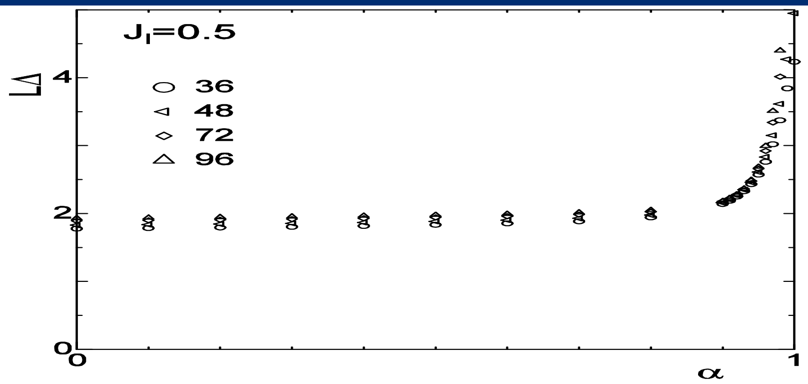
Spin gap is open
only in a very small region
$\alpha \sim 1$



# Large Size Corrections

- Logarithmic system size corrections

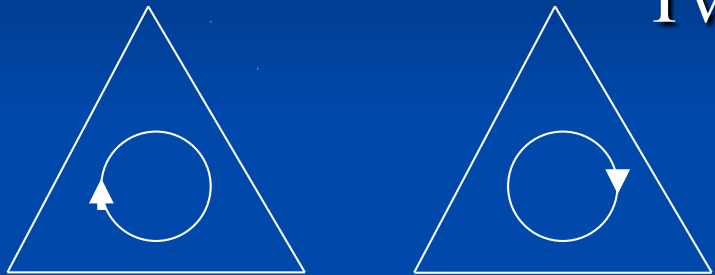
  ~ 1/log L
- 3-leg ladder looks gapped !



# Previous Effective Hamiltonian

Chiral symmetry: S=1/2 Regular triangle cluster

Two doublets are degenerated.



$$|\uparrow L\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^{-1}|\downarrow\uparrow\uparrow\rangle\right),$$

$$|\downarrow L\rangle = \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\downarrow\rangle + \omega^{-1}|\uparrow\downarrow\downarrow\rangle\right),$$

$$|\uparrow R\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega^{-1}|\uparrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\uparrow\rangle\right),$$

$$|\downarrow R\rangle = \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega^{-1}|\downarrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\downarrow\rangle\right), \quad \omega = \exp\left(\frac{2\pi i}{3}\right).$$

Chirality operators

$$\tau^+|\cdot L\rangle = 0, \qquad \tau^-|\cdot L\rangle = |\cdot R\rangle,$$

$$\tau^+|\cdot R\rangle = |\cdot L\rangle, \quad \tau^-|\cdot R\rangle = 0.$$

$J_r/J_1 \rightarrow \infty$

$$H = J_1/3\sum_j \vec{S}_j \cdot \vec{S}_{j+1}[1 + \gamma(\tau_j^+\tau_{j+1}^- + \tau_j^-\tau_{j+1}^+)]$$

SU(2) symmetry

# New effective theory

- Hubbard model on 3-leg tube

$$H = H_{\text{hop}} + H_{\text{int}},$$

$$H_{\text{hop}} = \sum_{n=1}^{L}\sum_{i=1}^{3}\sum_{\sigma=\uparrow,\downarrow}(tc^{\dagger}_{n+1,i,\sigma}c_{n,i,\sigma} + s_{i+1,i}c^{\dagger}_{n,i+1,\sigma}c_{n,i,\sigma} + h.c.),$$

$$H_{\text{int}} = U\sum_{n=1}^{L}\sum_{i=1}^{3} n_{n,i,\uparrow}n_{n,i,\downarrow},$$


Large U ⇒ Heisenberg model

$J_1=t^2/U$, $J_r=s^2/U$,

$\alpha=J_r'/J_r=\beta^2$

# Criterion of Spin Gap

- Hopping term: 3 bands

$$E_1(k) = -\beta s + 2t\cos k,$$
$$E_2(k) = \frac{1}{2}(\beta s - \sqrt{\beta^2+8} + 4t\cos k),$$
$$E_3(k) = \frac{1}{2}(\beta s + \sqrt{\beta^2+8} + 4t\cos k),$$

- Number of Fermi points

  0 to 3 pairs of Fermi points

- Criterions

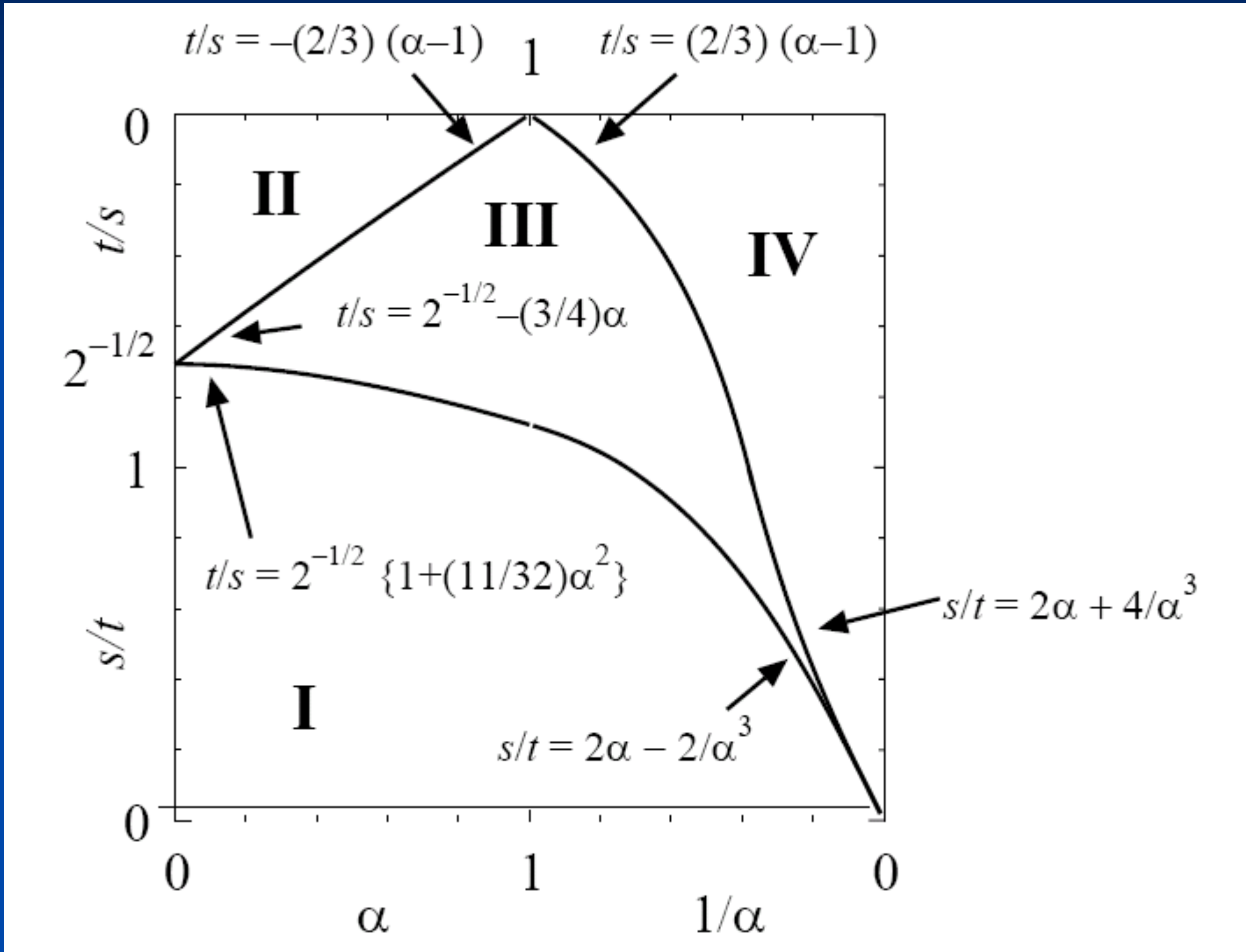
  Even pairs: Gapped

  Odd pairs: Gapless⇒ phase boundary

# Phase diagram by effective theory

Phase III: Gapped

Phase I ?



# Level spectroscopy

- KT phase boundary between gapped and gapless
  Logarithmic size corrections (Okamoto-Nomura)
  $J_1$-$J_2$ frustrated spin chain $(J_2/J_1)_c$=0.2411…

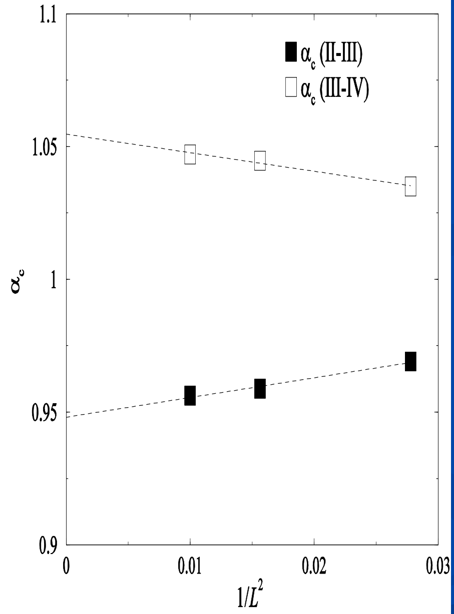
  Triplet excitation gap $\Delta_t \sim 1/L\,(1-C/\log L\ \ldots)$
  Singlet excitation gap $\Delta_s \sim 1/L\,(1+3C/\log L\ \ldots)$
  At the critical point C=0
  $\Delta_t = \Delta_s$ ⇒ precise phase boundary

# Phase boundary $\alpha_c$

- By level spectroscopy with numerical diagonalization up to L=10

  for $J_1$=0.2

  ($J_r$=1 fixed)

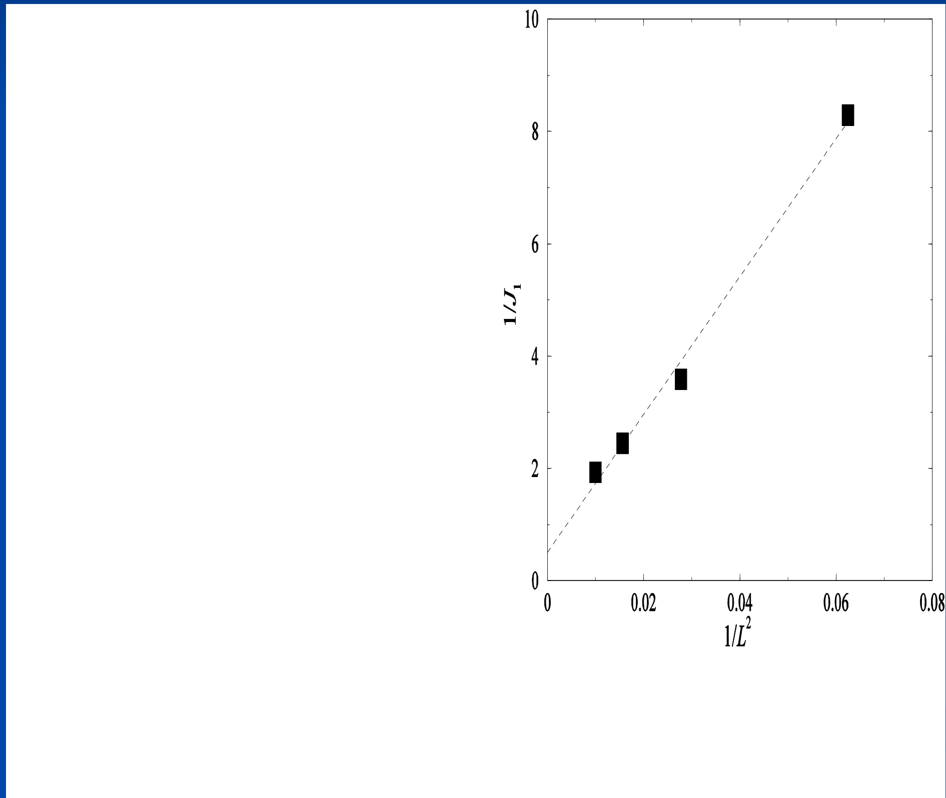


# $J_{1c}$ for symmetric case $\alpha$=1

- Large size dependence

  Critical point

  $1/J_1=0.51 \mp 0.4$




It is difficult to determine $J_{rc}$ =0 or not .

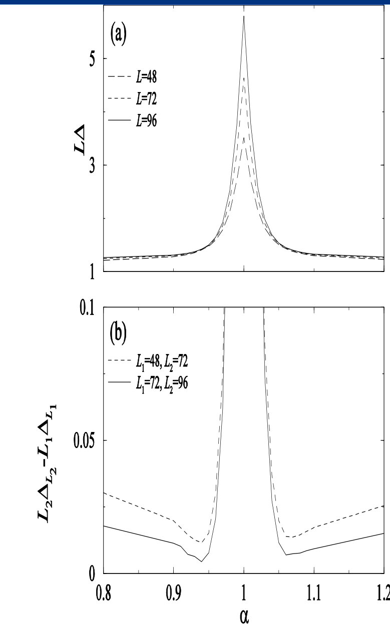
# Phenomenological renomalization

- DMRG up to L=128
- Diagonalization up to L=10
- No fixed points by

  $L_1\Delta_{L1} = L_2\Delta_{L2}$
- Gapless points determined

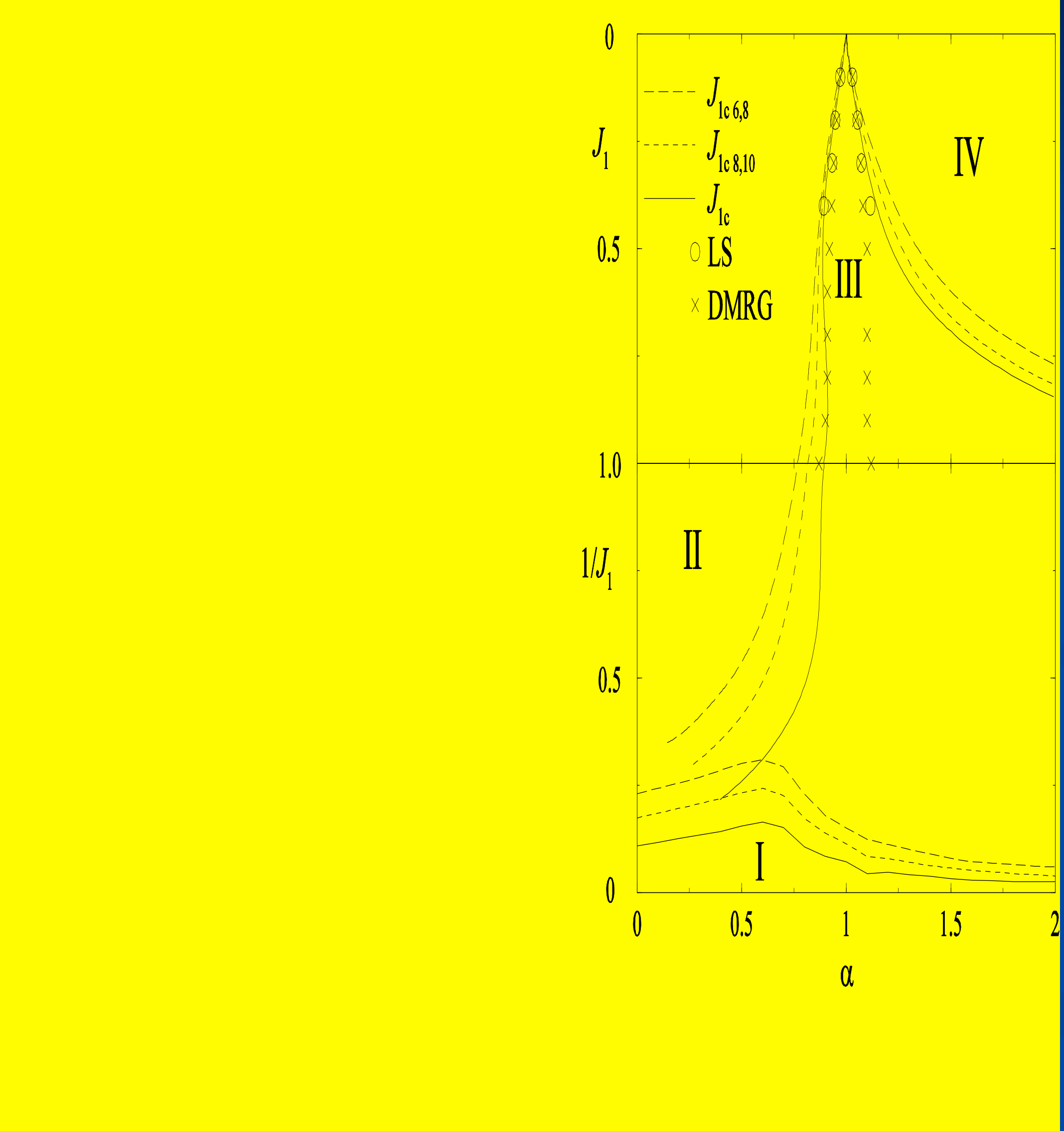
  by the minimum of

  $L_1\Delta_{L1} - L_2\Delta_{L2}$

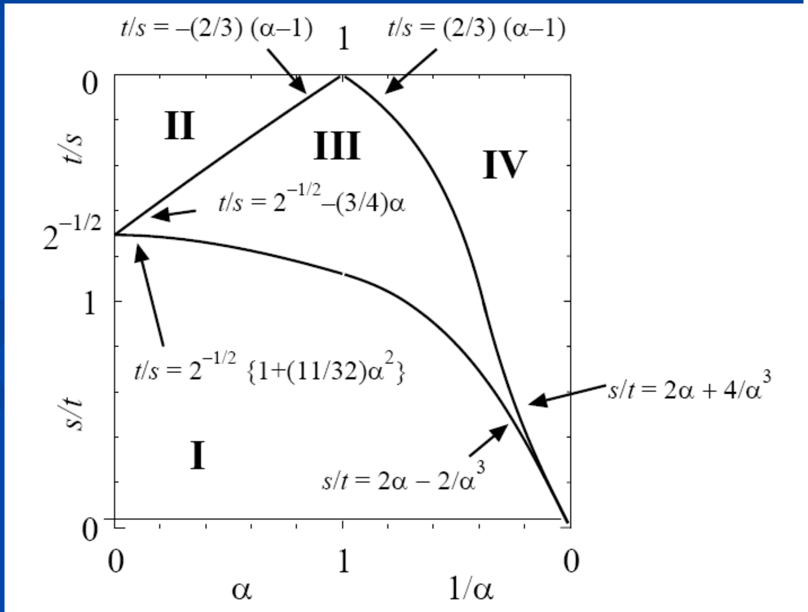



DMRG for $J_1$=0.3

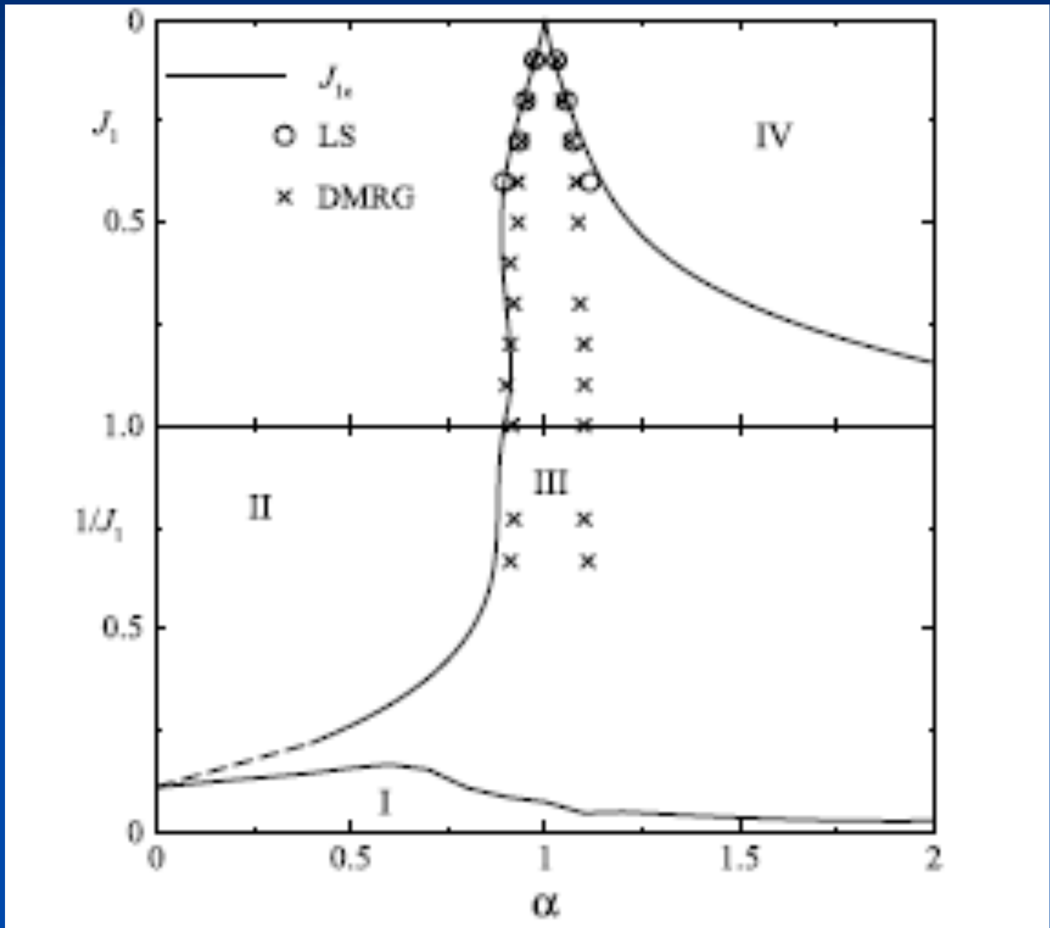
# Numerical Phase Diagram



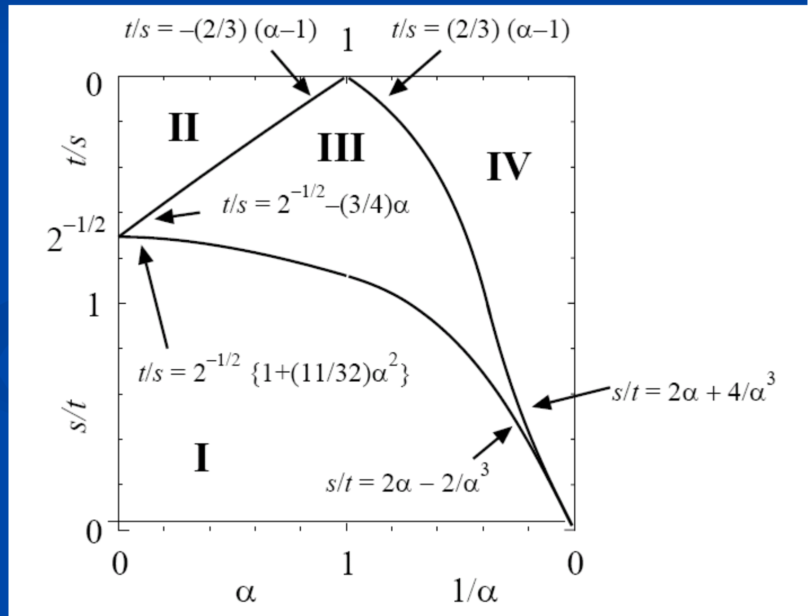

Effective theory



# Numerical Phase Diagram




Effective theory



# BKT transition

Conformal field theory analysis

$\langle S_0^z S_r^z \rangle \sim (-1)^r r^{-\eta}$

$\Delta \sim \pi v_s \eta / L \rightarrow \eta$

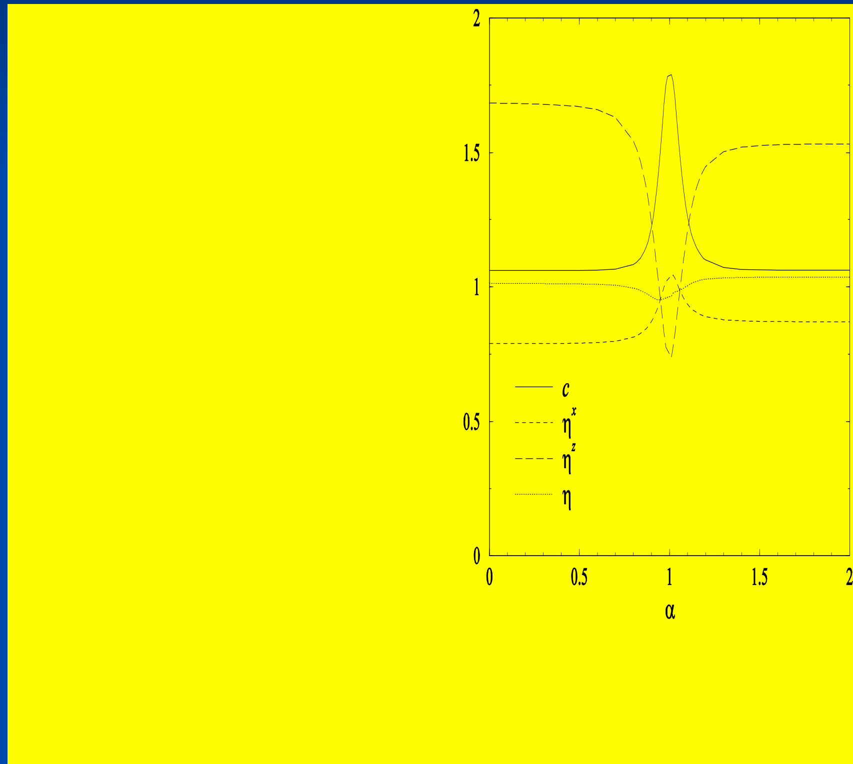
Central charge : c

Numerical diagonalization:

$\eta$=1  c=1 for gapless phases

$\Rightarrow$ BKT transition

Berezinskii-Kosterlitz-Thouless



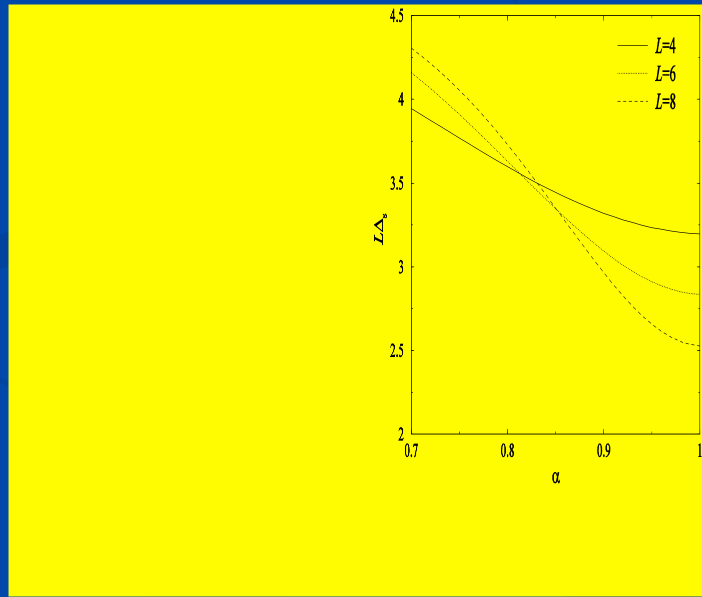

$J_1$=0.3

# Phenomenological Renormalization -Singlet gap-

- Spin gap (singlet-triplet) : large size correction
- Singlet-singlet gap

Degenerated in gapped phase

$J_1$=0.5




Size-dependent fixed point

$(L+2)\,\Delta(L+2, \alpha_c) = L\,\Delta(L, \alpha_c)$

Extrapolate $\alpha_c(L) \Rightarrow \alpha_c(\infty)$

# Quantized Berry phase

Y.Hatsugai, JPSJ (2006)

"Local characterization of quantum liquids"

For spin systems (without magnetic field),

- make a local $SU(2)$ twist $\theta$ only at one link as,

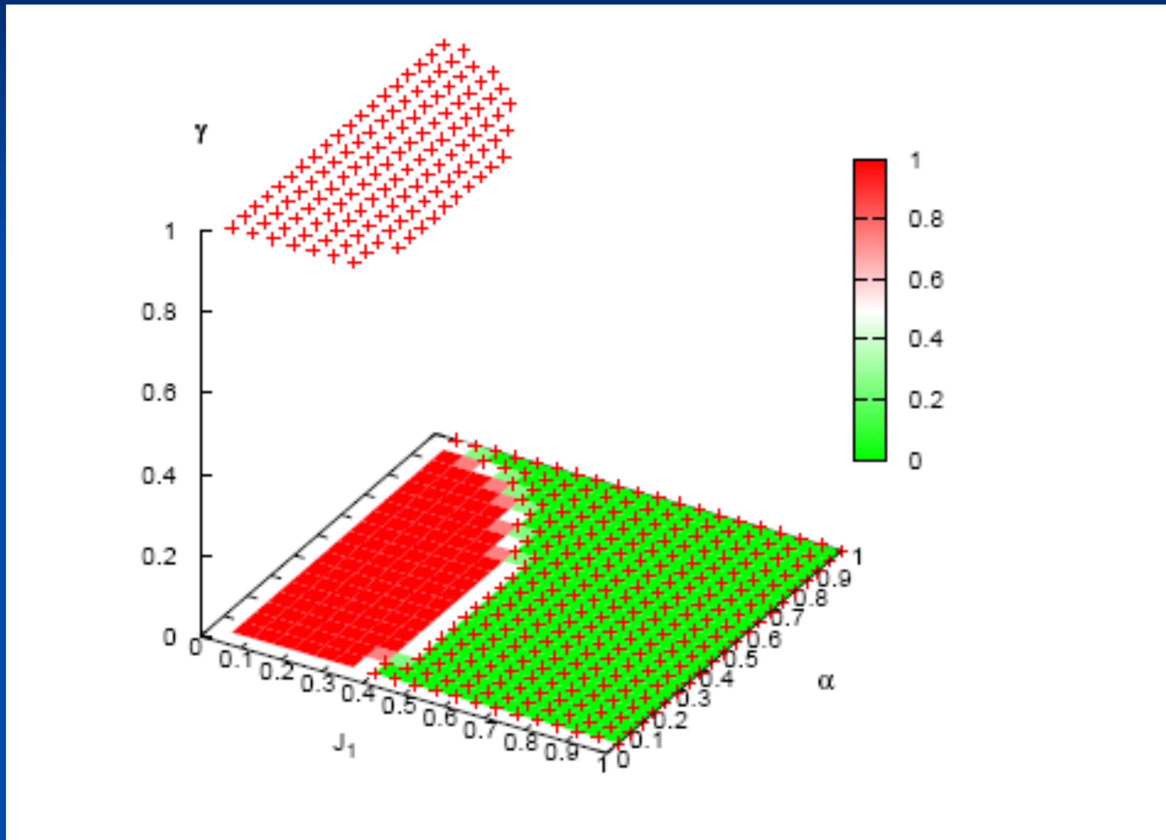
$$J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \to J_{ij}/2 \left( e^{-i\theta} S_i^+ S_j^- + e^{i\theta} S_i^- S_j^+ + 2 S_i^z S_j^z \right)$$

- sum up the (lattice) Berry connection,

$$\gamma = \mathrm{Arg} \prod_C \langle \psi_{\theta_j}^U | \psi_{\theta_{j+1}}^U \rangle, \qquad |\psi_{\theta_j}^U\rangle = |\psi_{\theta_j}\rangle\langle\psi_{\theta_j}|\phi\rangle$$

$|\phi\rangle$ : (arbitrary) reference state to fix the gauge

# Calculation of quantized Berry phase for L=4 (Otsuka-TS 2012)

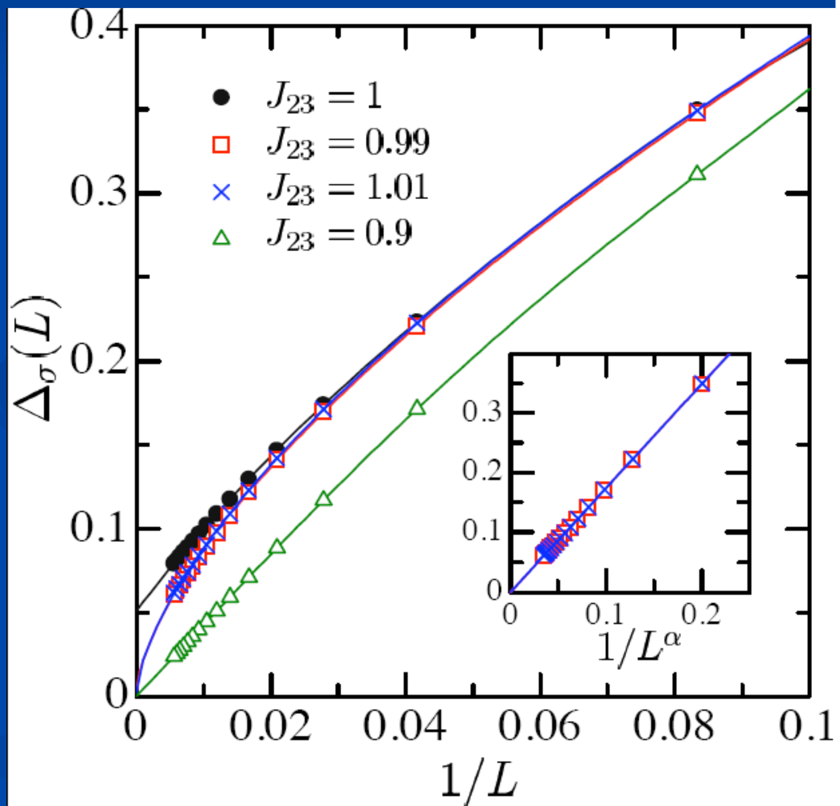



It supports the mechanism of spin gap.

# Another work

- DNRG by Nishimoto and Arikawa :
  PRB 2008

  Spin gap is open just for the symmetric tube(α=1), otherwise gapless, assuming the size correction has a power law.



# Summary1

S=1/2 Three-leg spin tube

Regular triangle tube　　Chiral symmetry

$J_r > 0 \Rightarrow$ Spin gap

Isosceles triangle tube　　No chiral symmetry

Spin gap in a small region $\alpha = J_r' / J_r \sim 1$

Quantum phase transition (BKT transition)

| | | |
|---|---|---|
| $\alpha=1$ | | $\alpha= 0$ |
| Spin tube | $\leftrightarrow$ | Spin ladder |
| Spin gap | | Gapless |

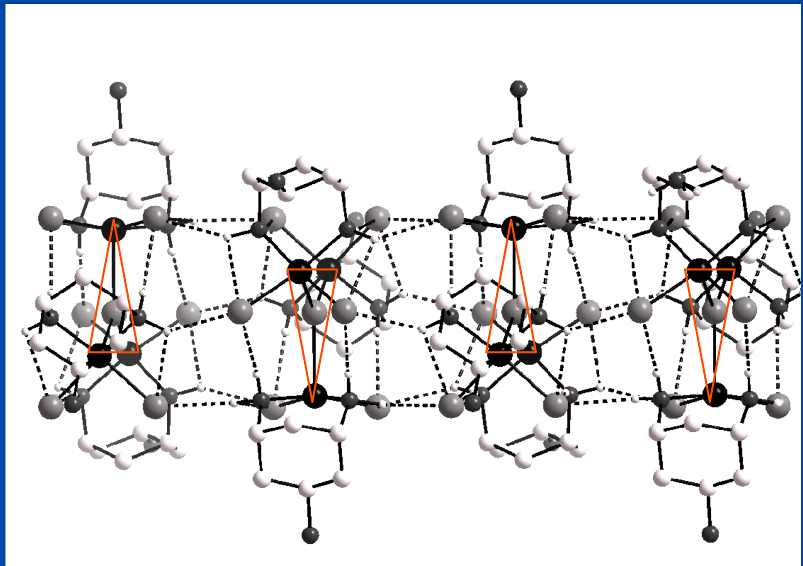
Distortion from regular triangle $\Rightarrow$ Gap vanishes rapidly

$J_{rc}$=finite or 0 ? still an open question

# 2. A real spin tube  $[(CuCl_2tachH)_3Cl]Cl_2$

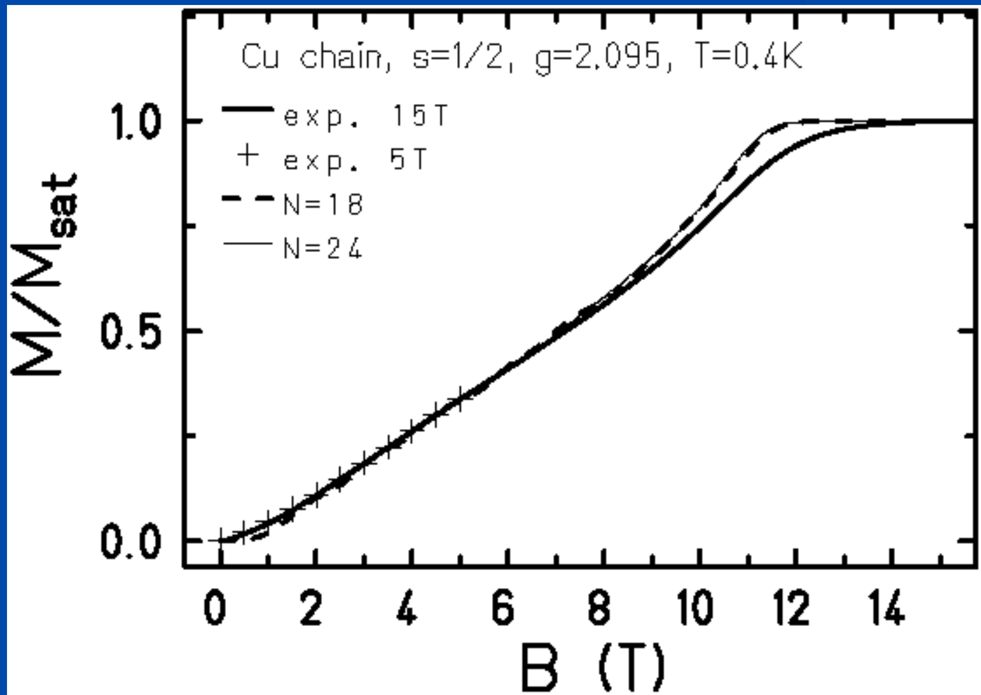
J. Schnack, H. Nojiri, P. Kögerler, G.J. T. Cooper and L.  Cronin
Phys. Rev. B **70**, 174420 (2004)

Assembly of  triangular  clusters



Phys. Rev. B **70**, 174420 (2004)

MH-curve




The model parameter is about J'/J=2

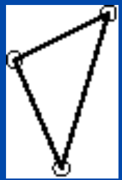
# What is the triangular lattice quantum spin tube?



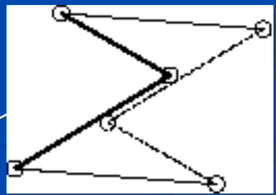
## Hamiltonian:

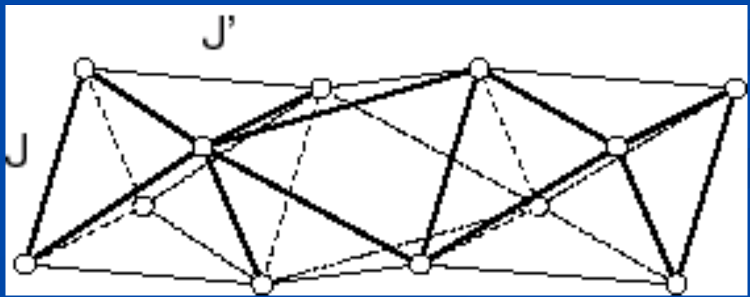
$$H = J\sum_{i}\sum_{j=1}^{3} S_{i,j}\cdot S_{i,j+1} + J'\sum_{i}\sum_{j=1}^{3}[S_{i,j}\cdot S_{i+1,j} + S_{i,j}\cdot S_{i+1,j+1}]$$

intra triangle coupling



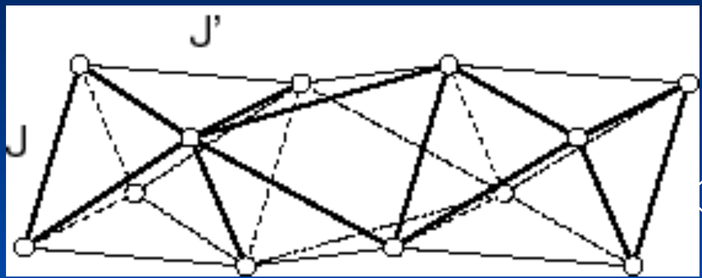
inter triangle coupling



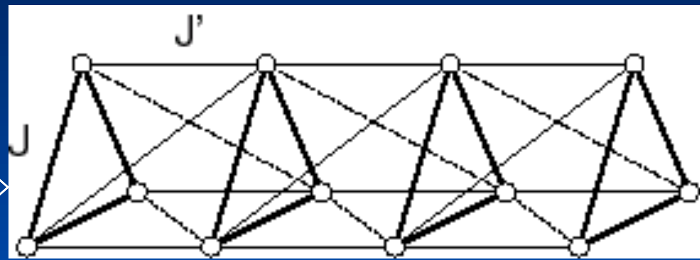


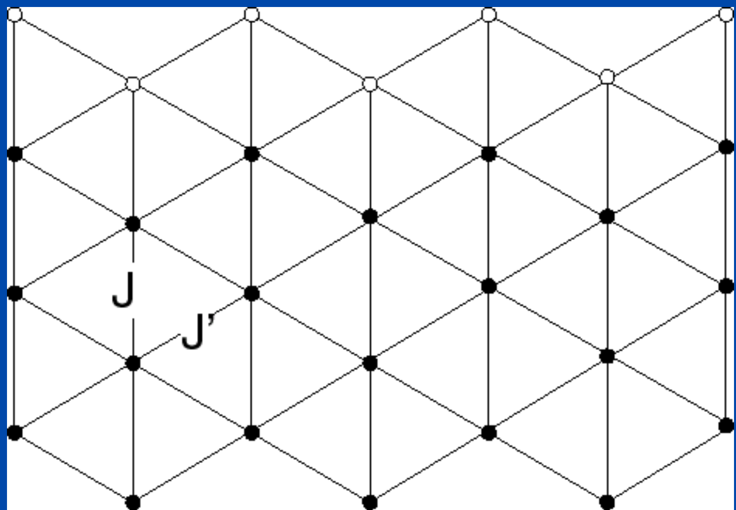
# Expand the tube






π/3 rotation

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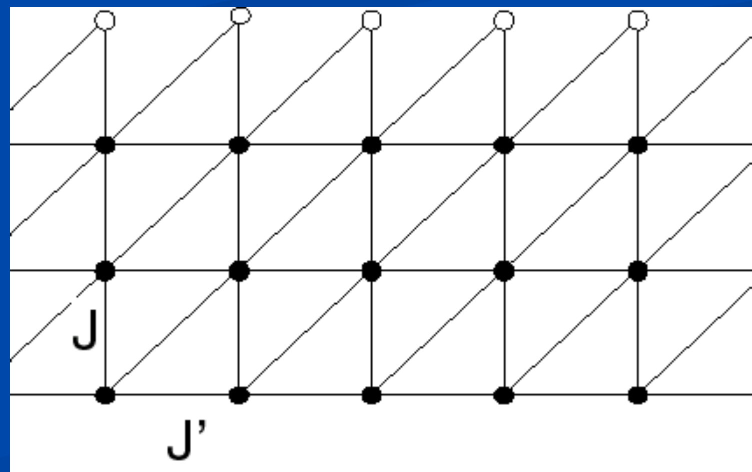
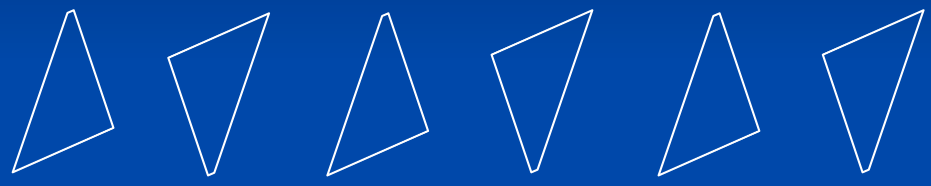



Triangular lattice structure

# basics

We can expect two phases

**(A)** J>>J' weakly coupled triangles



quantum phase transition

**(B)** J<<J' rhombic lattice(modulated square lattice)

If J=0, no frustration!

# Numerical calculation



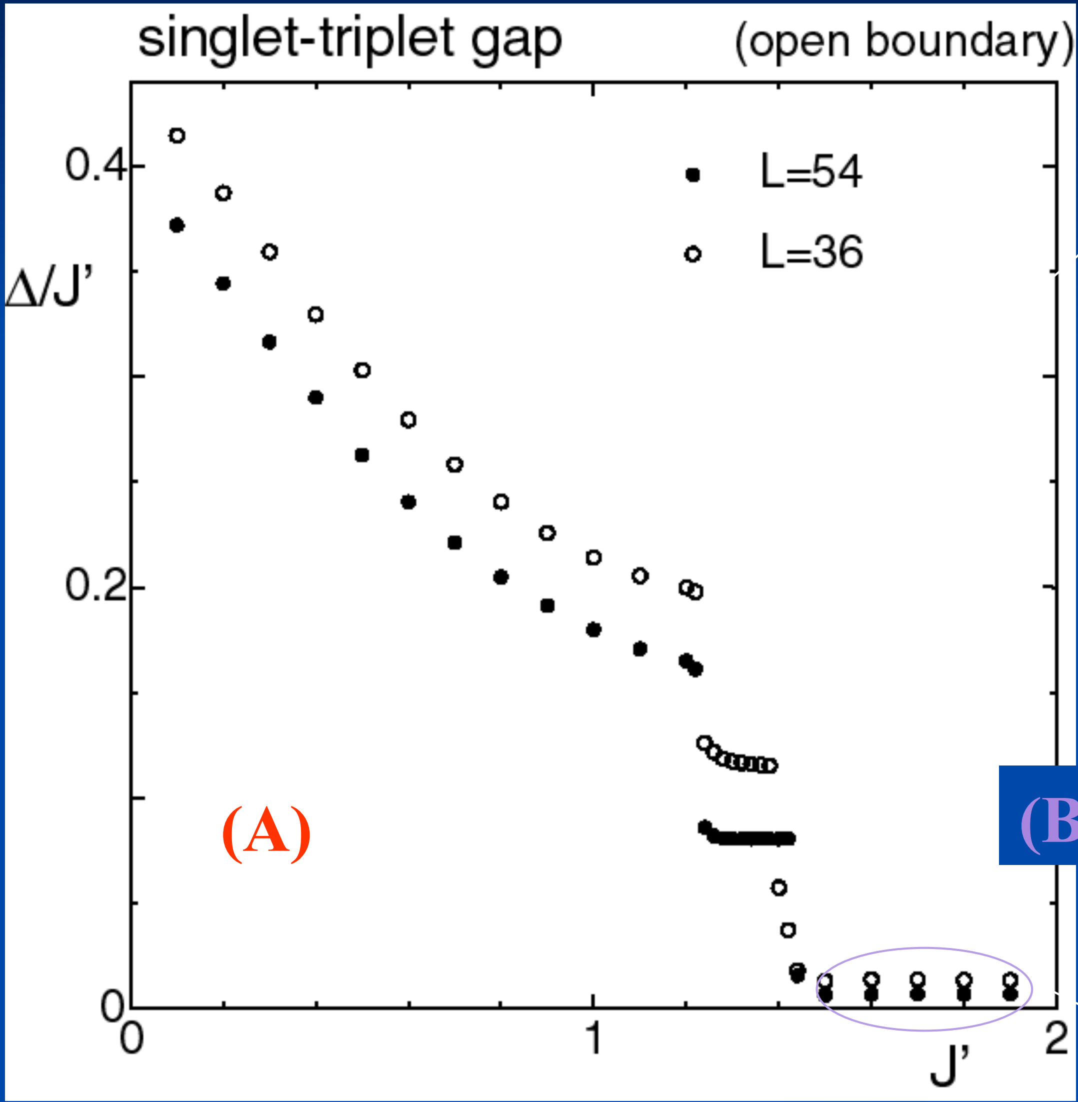
DMRG, finite size method

MH-curve : 36 x 3 spins, m=80

Spin gap : up to 144 x 3 spins,
up to m=220 with m extrapolation
(empirical)

# Spin gap (Okunishi et al. 2005, Fouet et al. 2006)






(C) intermediate phase??

First order transition is expected, since the gap exhibits disconti-nuities

(B) phase

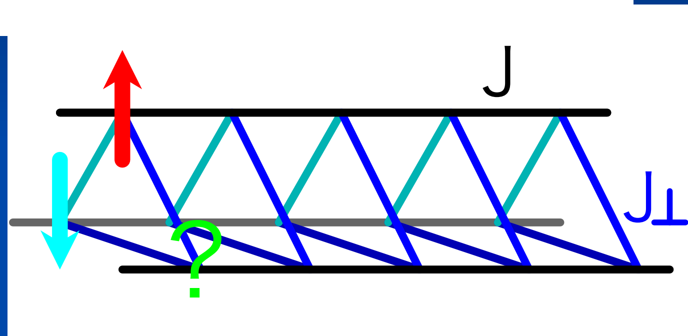
This is a boundary excitation

# 1/3 magnetization plateau

Three-leg spin tube

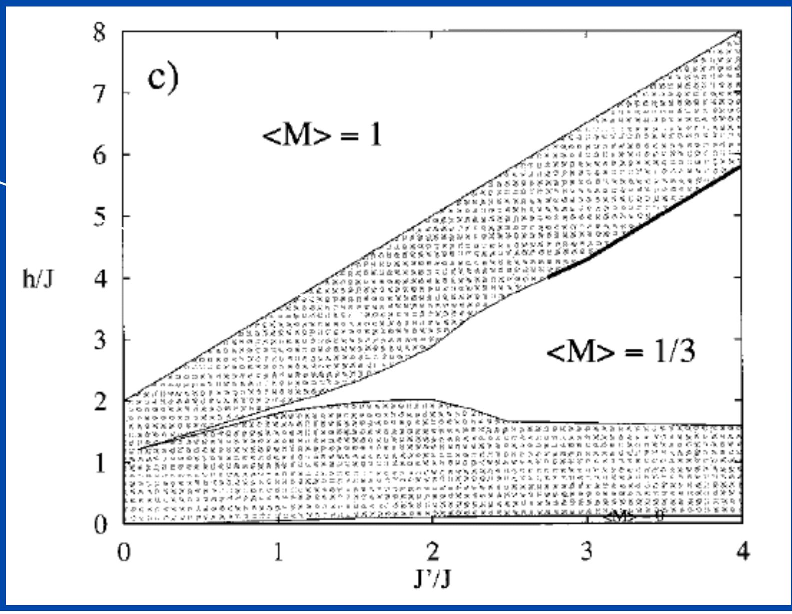
$$\mathcal{H} = \sum_{l,j} \left[ J\vec{S}_{l,j} \cdot \vec{S}_{l,j+1} + J_{\perp}\vec{S}_{l,j} \cdot \vec{S}_{l+1,j} - HS^z_{l,j} \right].$$

One of the simplest 1D magnets with geometrical frustration




Phase diagram of the S=1/2 tube (Exact Diagonalization)

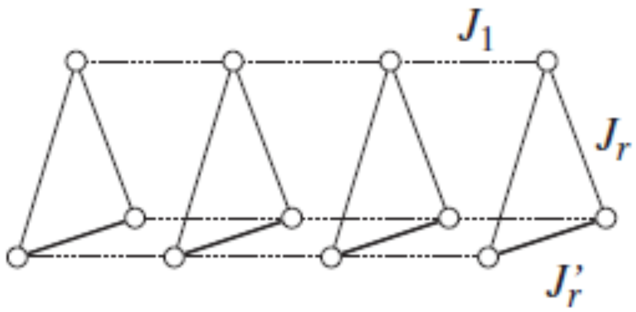
critical




M=1/6 plateau

*Cabra, Honecker and Pujol, PRL79, 5126 (1997); PRB58, 6241 (1998).*

# $S = 1/2$ Three-Leg Spin Nanotube




- Isosceles case

  all the spin-couplings are of $\vec{S} \cdot \vec{S}$ type (isotropic)

  $J_r'/J_r \equiv \alpha$

  $J_r = 1$, unit energy

  $\alpha = 0$ : 3-leg ladder

  $\alpha = 1$ : regular triangle tube

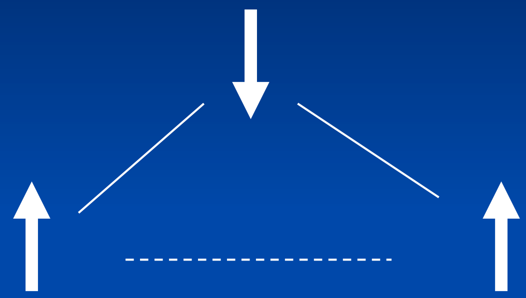
  $\alpha \to \infty$ : 2-leg ladder + single chain

- $M = M_s/3$ state ($M_s$ : saturation magnetization)

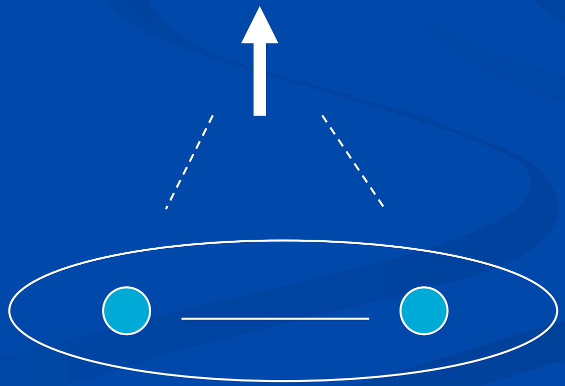
  we are interested in the $M = M_s/3$ state when $J_1 \ll 1$ (maybe plateau)

# Two mechanisms of 1/3 plateau

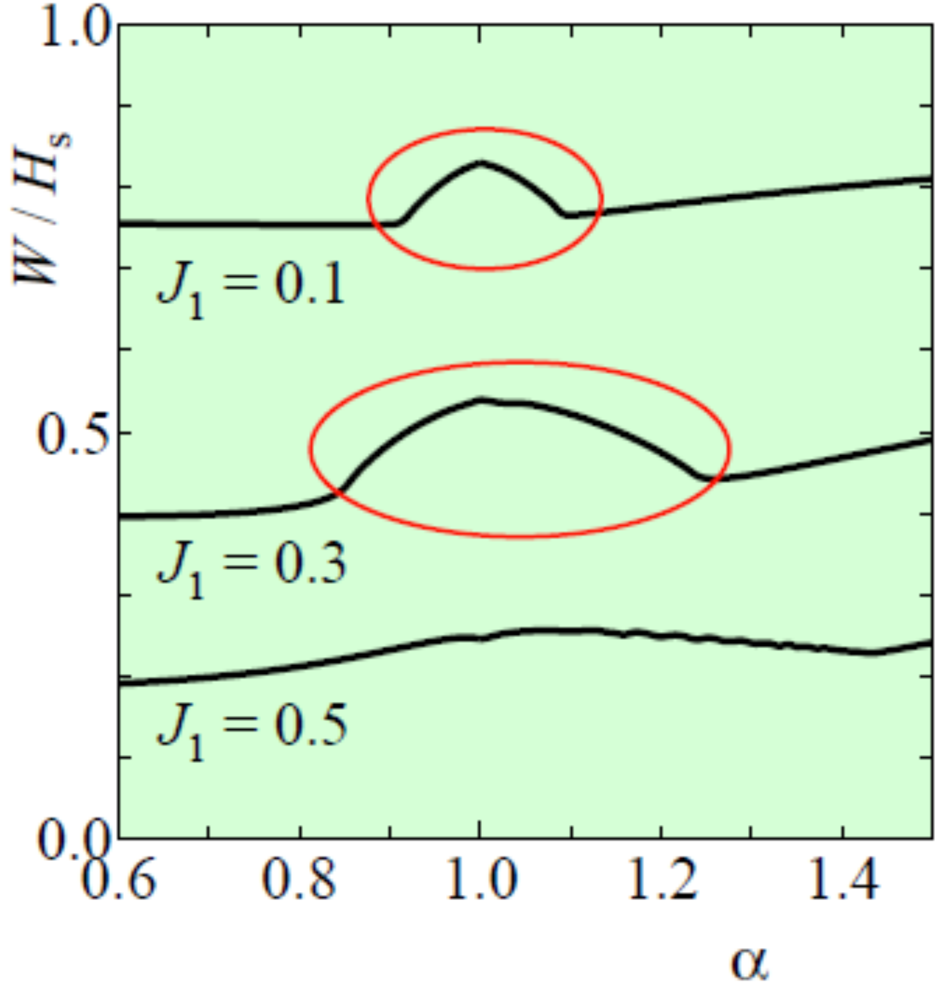
- 3-leg ladder
  uud plateau



- Dimer-monomer
  plateau



- Plateau width $W$ near $\alpha = 1$ and $J_1 \ll 1$ (by DMRG) normalized by the saturation field $H_s$

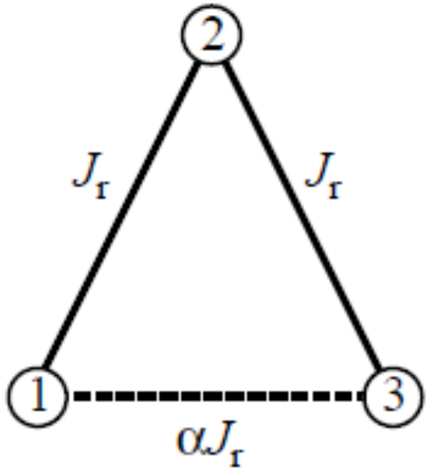



anomalous behavior of the plateau width near $\alpha = 1$

new and exiotic!!

# Spin States of Isosceles Triangles

- Eigenstates of isosceles triangles at $S^z_{\rm tot} = 1/2$

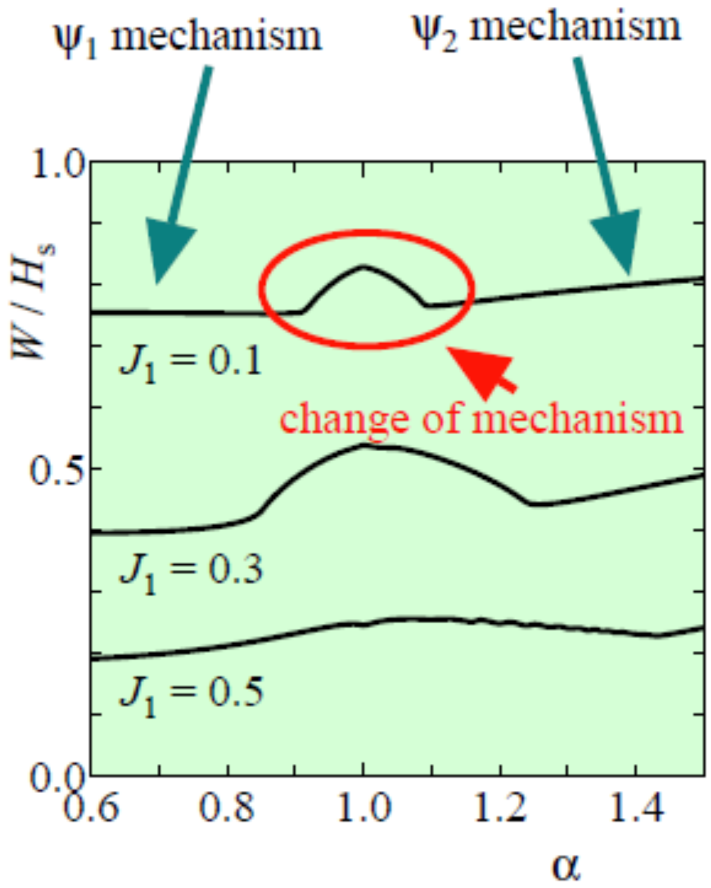



| state | energy | $S_{\rm tot}$ | $P$ |
|---|---|---|---|
| $\psi_1 = \frac{1}{\sqrt{6}}(\lvert\uparrow\uparrow\downarrow\rangle - 2\lvert\uparrow\downarrow\uparrow\rangle + \lvert\downarrow\uparrow\uparrow\rangle)$ | $-1+\alpha/4$, ($\alpha < 1$, GS) | $1/2$ | $+1$ |
| $\psi_2 = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\uparrow\rangle) = \lvert\uparrow\rangle_2[1,3]$ | $-3\alpha/4$, ($\alpha > 1$, GS) | $1/2$ | $-1$ |
| $\psi_3 = \frac{1}{\sqrt{3}}(\lvert\uparrow\uparrow\downarrow\rangle + \lvert\uparrow\downarrow\uparrow\rangle + \lvert\downarrow\uparrow\uparrow\rangle)$ | $1/2+\alpha/4$ | $3/2$ | $+1$ |

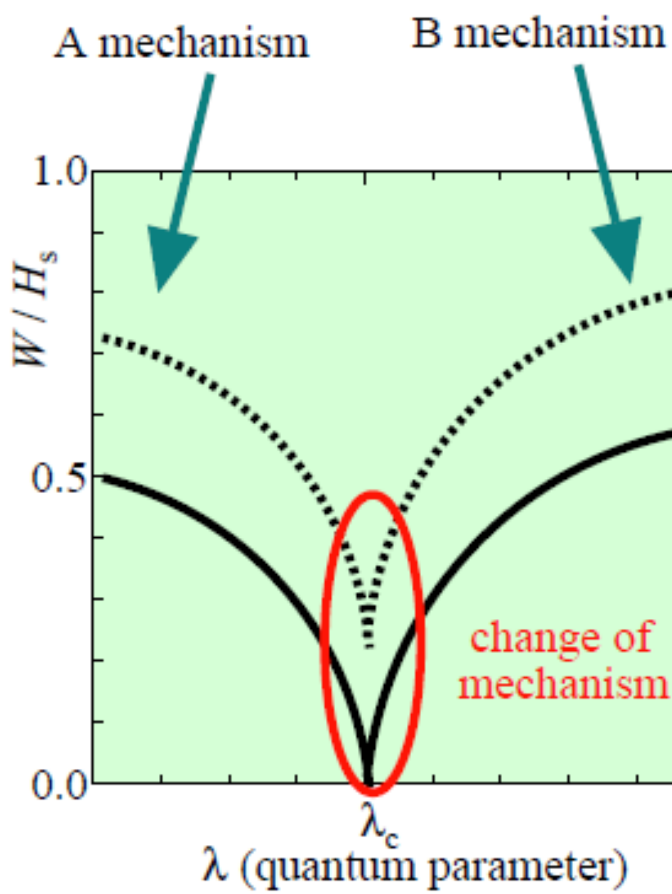
$P$ : $1 \Leftrightarrow 3$ parity

wave functions do not depend on $\alpha$

Behavior of plateau width near the mechanism-changing point




present case




usual cases

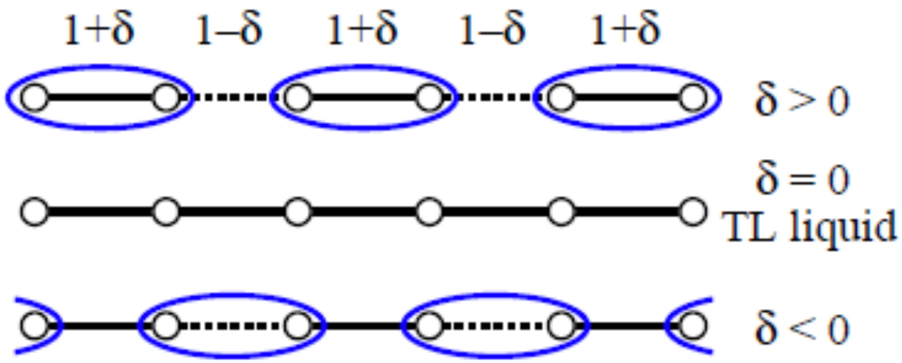
- Usual cases

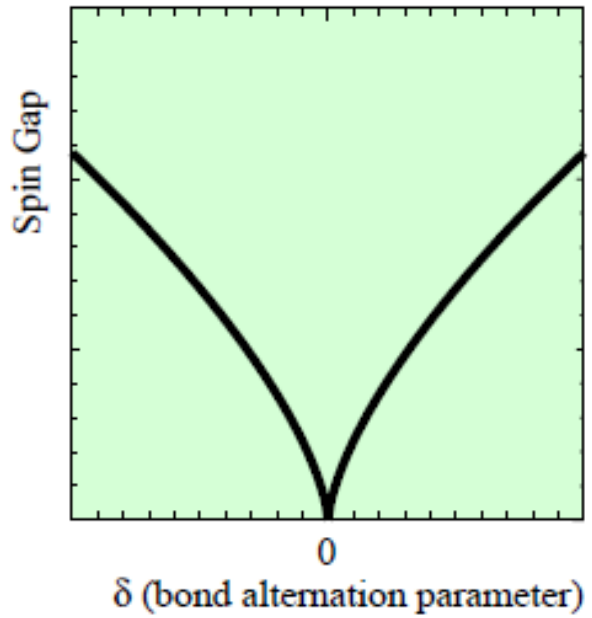
$W$ decreases near $\lambda_c$

reconstruction of the unit cell occurs at $\lambda_c$

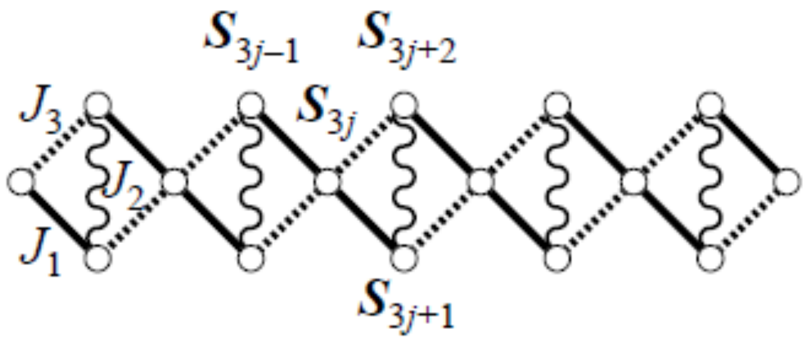
example 1 : $S = 1/2$ bond-alteranting model at $M = 0$,

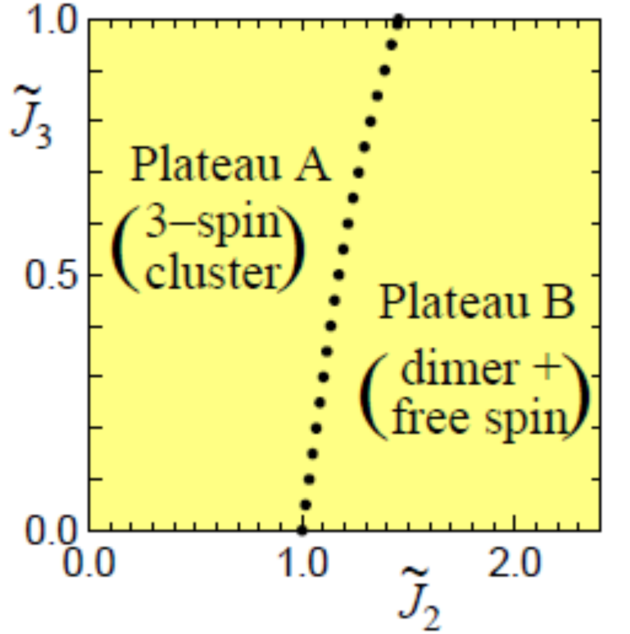
spingap $\propto |\delta|^{2/3}/\sqrt{|\log|\delta||}$

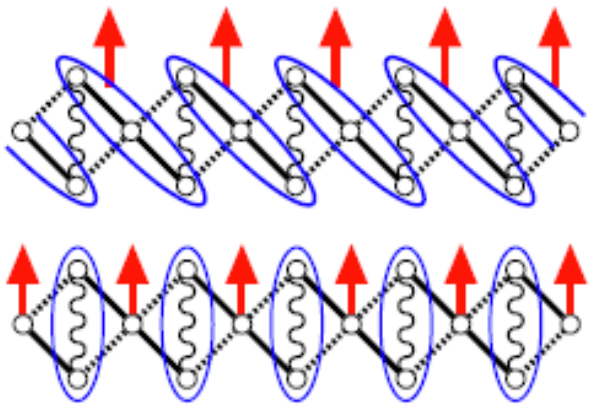


example 2 : $S=1/2$ distorted diamond chain model at $M=M_s/3$

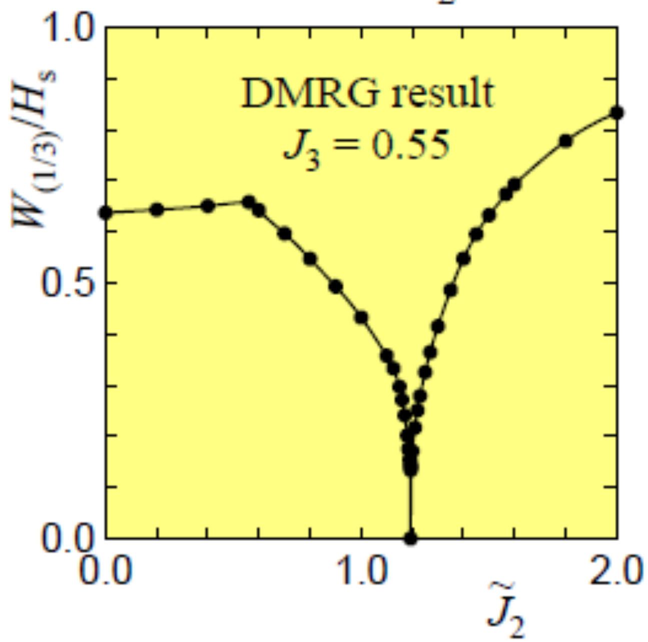


Plateau A

Plateau B




also, reconstruction of the unit cell

- present case (note that $J_r \gg J_1$)

  reconstruction of the unit cell does not occur at $\alpha = 1$

  unit cell is always a triangle for $\alpha \lesseqqgtr 1$

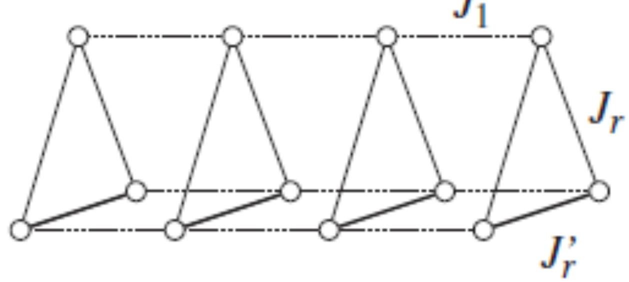
  GS of a triangle is changed at $\alpha = 1$

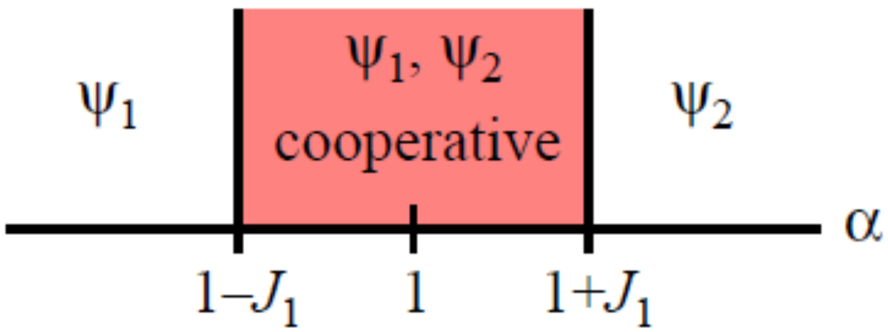
  energy difference between $\psi_1$ and $\psi_2$ states, $E_2 - E_1 = 1 - \alpha$

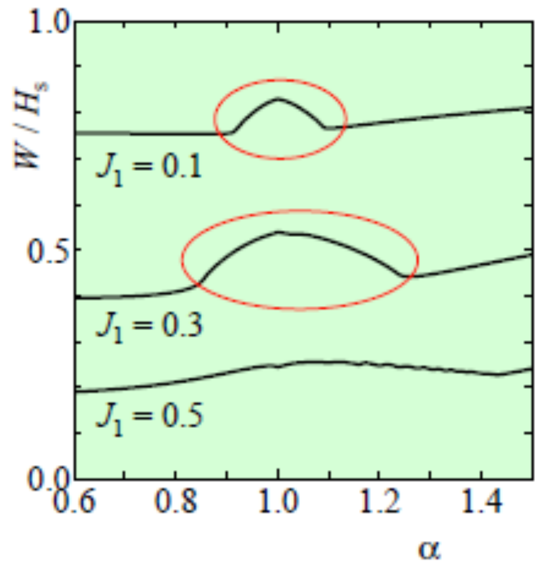
  pertubation by $J_1$, both of $\psi_1$ and $\psi_2$ states are relevant when $J_1 \gtrsim |1 - \alpha|$

  thus, in the $1 - J_1 \lesssim \alpha \lesssim 1 + J_1$ region, interesting phenomena will occur

  in fact, the increase of $W$ is observed in this region



# Perturbation Theory from the $J_1 \ll 1$ Limit

- Eigenstates of isosceles triangles at $S^z_{\rm tot} = 1/2$

| state | energy | $S_{\rm tot}$ | $P$ |
|---|---|---|---|
| $\psi_1 = \frac{1}{\sqrt{6}}(\lvert\uparrow\uparrow\downarrow\rangle - 2\lvert\uparrow\downarrow\uparrow\rangle + \lvert\downarrow\uparrow\uparrow\rangle)$ | $-1+\alpha/4$, ($\alpha < 1$, GS) | $1/2$ | $+1$ |
| $\psi_2 = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\uparrow\rangle) = \lvert\uparrow\rangle_2[1,3]$ | $-3\alpha/4$, ($\alpha > 1$, GS) | $1/2$ | $-1$ |
| $\psi_3 = \frac{1}{\sqrt{3}}(\lvert\uparrow\uparrow\downarrow\rangle + \lvert\uparrow\downarrow\uparrow\rangle + \lvert\downarrow\uparrow\uparrow\rangle)$ | $1/2+\alpha/4$ | $3/2$ | $+1$ |

- Perturbation theory by use of pseudospin representation

pseudospin $\vec{T}$ $(T=1/2)$, $\psi_2 \Leftrightarrow \lvert\Uparrow\rangle$, $\psi_1 \Leftrightarrow \lvert\Downarrow\rangle$

note that $S^z_{\rm tot} = 1/2$ for both of $\lvert\Uparrow\rangle$ and $\lvert\Downarrow\rangle$

lowest order in $J_1$

$$\mathcal{H}_{\rm eff} = J_1 \sum_j \left(T^x_j T^x_{j+1} + T^z_j T^z_{j+1}\right) - (\alpha - 1)\sum_j T^z_j$$

transverse field $XZ$ (or equivaelently, $XY$) model

## Transverse field $XZ$ model

transverse field XXZ model ($\Delta \equiv J_z/J_{xy}$)

field $h$ along the $x$-direction

(Dmitriev et al, PRB **65**, 172409 (2002);

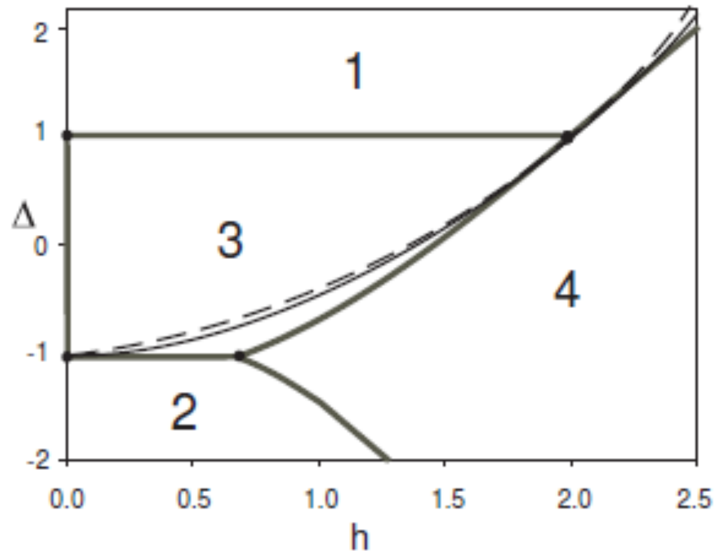
JETP **95**, 538 (2002))

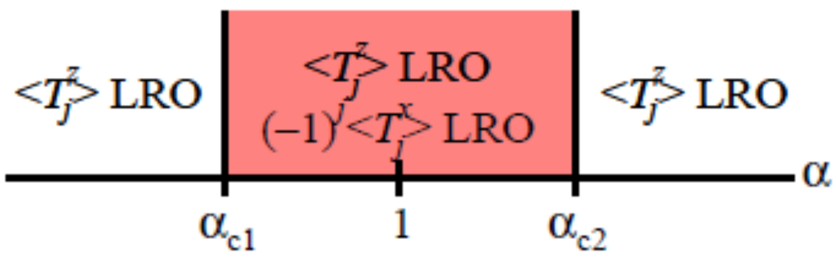
1: $z$-Néel, $(-1)^j\langle S_j^z\rangle$ LRO, (of course $\langle S_j^x\rangle$ LRO)

2: $z$-Ferro, $\langle S_j^z\rangle$ LRO, (of course $\langle S_j^x\rangle$ LRO)

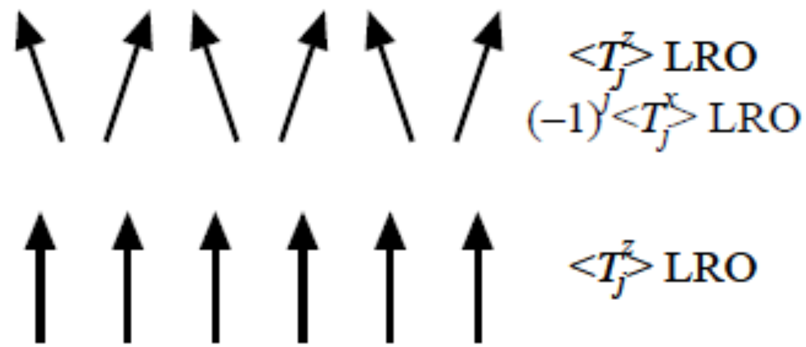
3: $y$-Néel, $(-1)^j\langle S_j^y\rangle$ LRO, (of course $\langle S_j^x\rangle$ LRO)

4: No LRO, (of course $\langle S_j^x\rangle$ LRO)




their results read for our case




semiclassical picture

suppose the pseudo spin $\overrightarrow{T}$ turn to the $+x$ direction

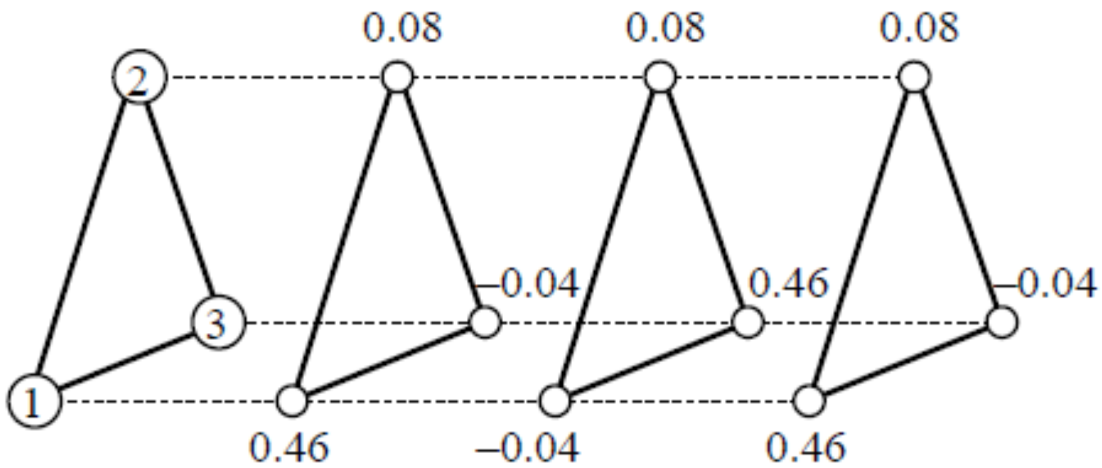
$$|T_{+x}\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\rangle + |\Downarrow\rangle) = \left(\frac{1}{2} + \frac{1}{4\sqrt{3}}\right)|\uparrow\uparrow\downarrow\rangle - \frac{1}{\sqrt{3}}|\uparrow\downarrow\uparrow\rangle + \left(\frac{1}{2} - \frac{1}{4\sqrt{3}}\right)|\downarrow\uparrow\uparrow\rangle$$

$\langle S_1^z\rangle \equiv \langle T_{+x}|S_1^z|T_{+x}\rangle = \frac{1}{2\sqrt{3}} + \frac{1}{6} = 0.455$

$\langle S_2^z\rangle \equiv \langle T_{+x}|S_2^z|T_{+x}\rangle = \frac{1}{6} = 0.167$

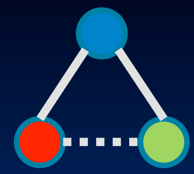
$\langle S_3^z\rangle \equiv \langle T_{+x}|S_3^z|T_{+x}\rangle = \frac{1}{2\sqrt{3}} - \frac{1}{6} = -0.122$

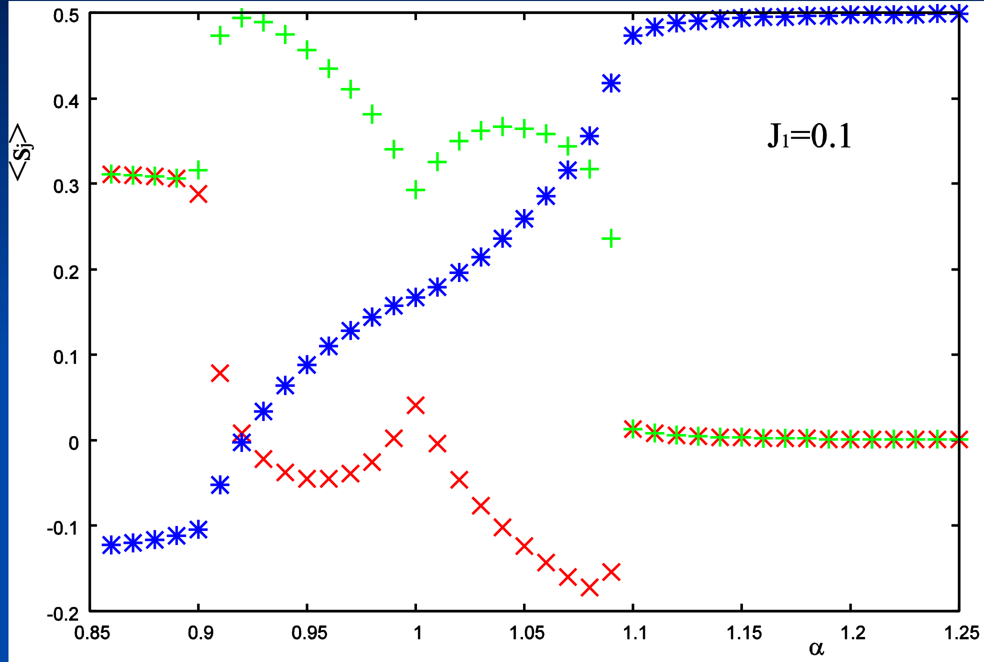
these expectation values well explain the DMRG results



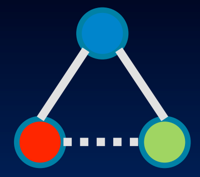

$J_1 = 0.1,\ \alpha = 0.95$

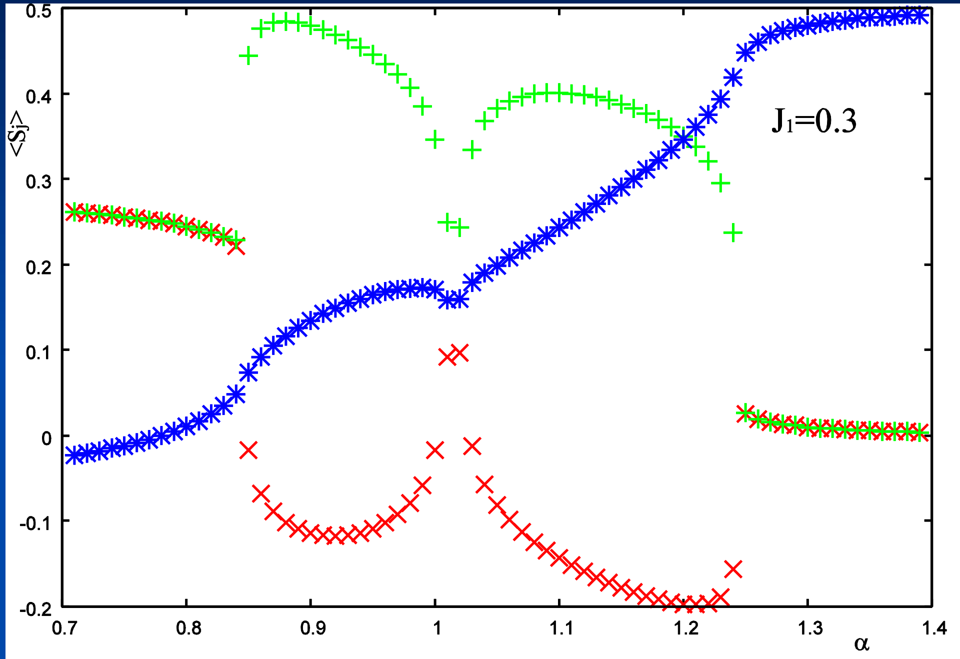
Expectation value of each spin by DMRG for small $J_1$



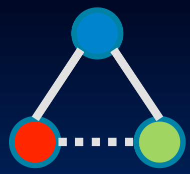


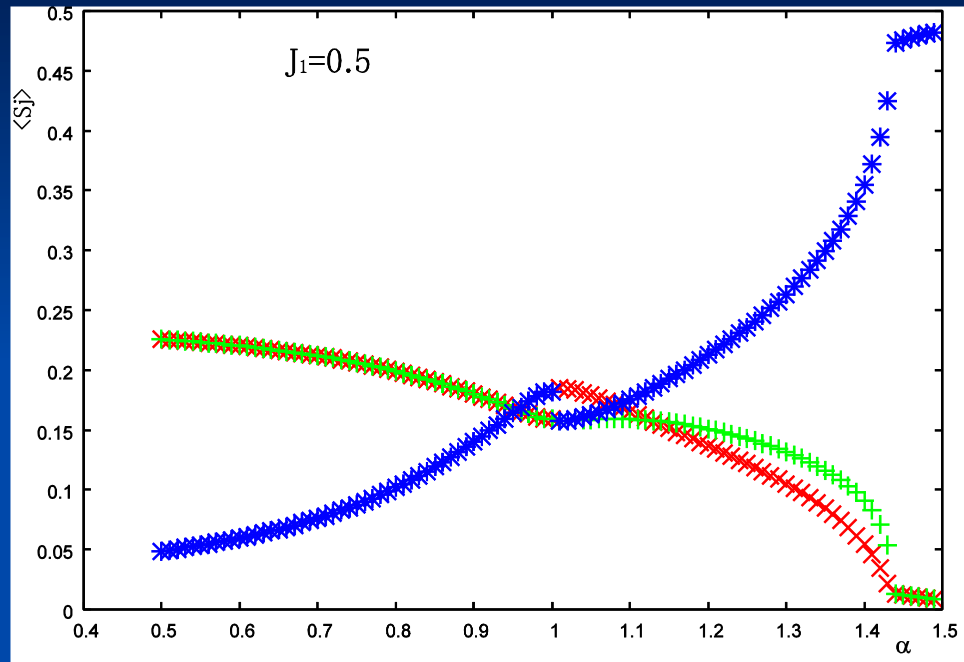
Expectation value of each spin by DMRG
for intermediate $J_1$



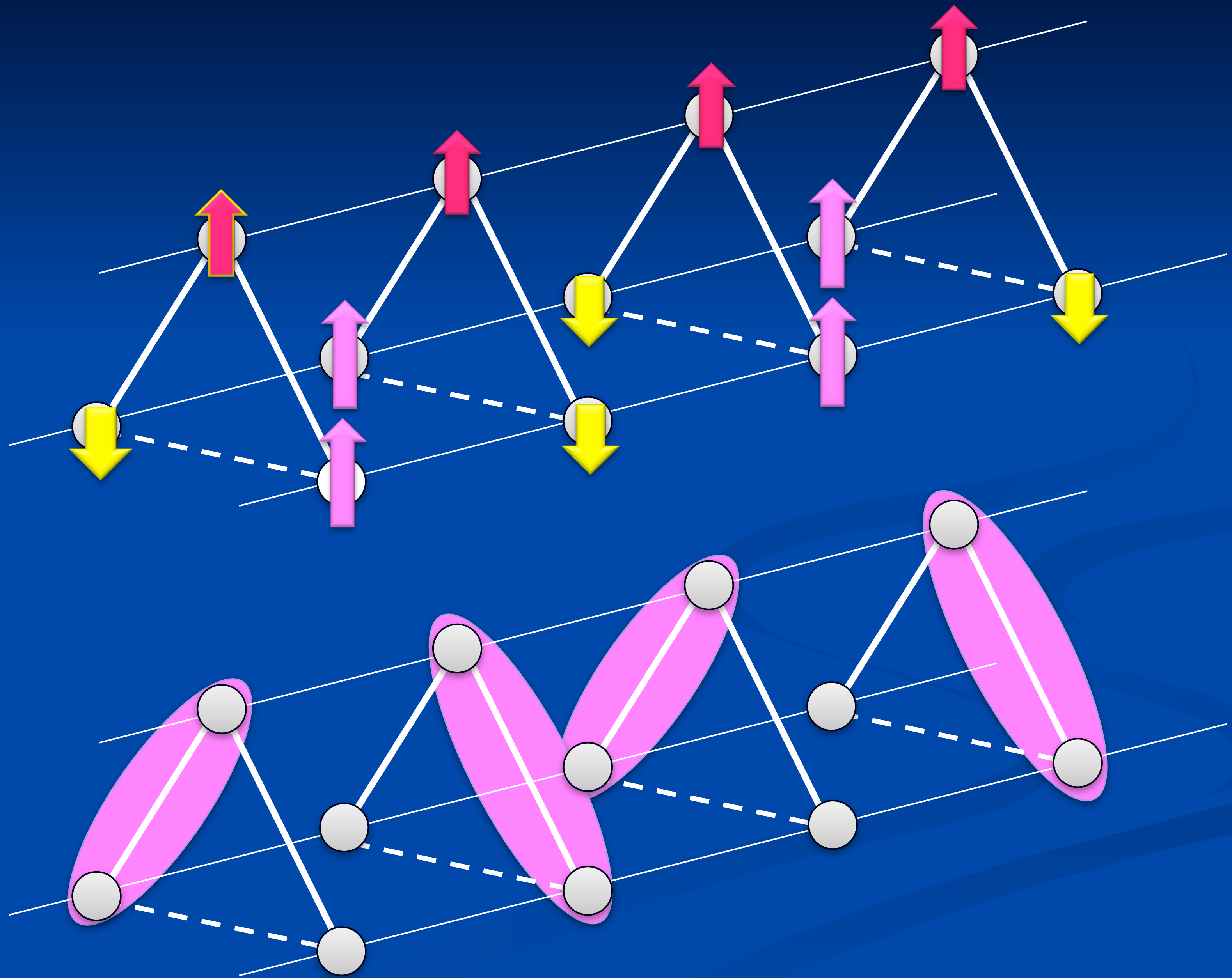


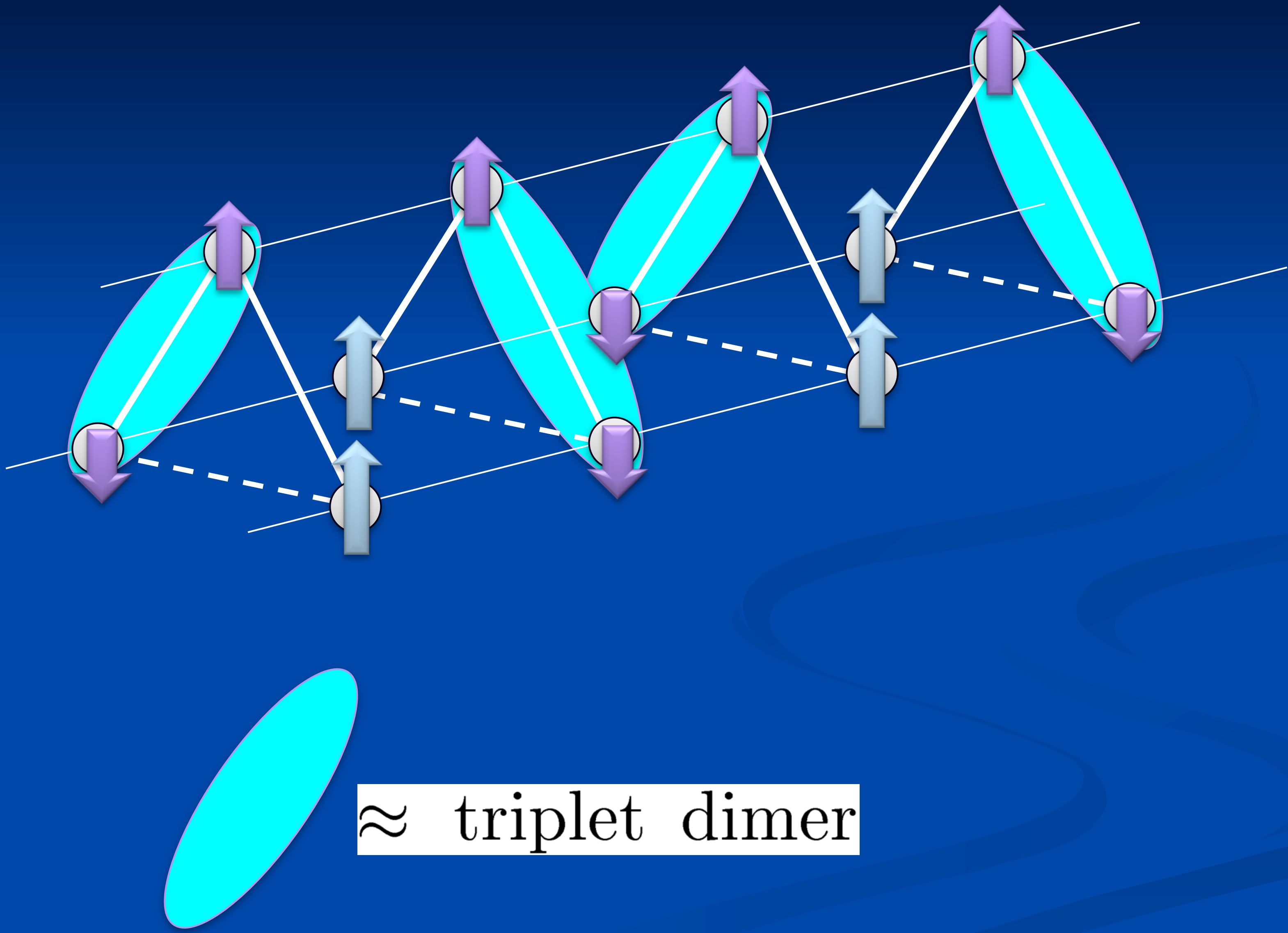
# Expectation value of each spin by DMRG for large $J_1$





Z$_2$-symmetry breaking state
in the 1/3 plateau of asymmetric spin tube

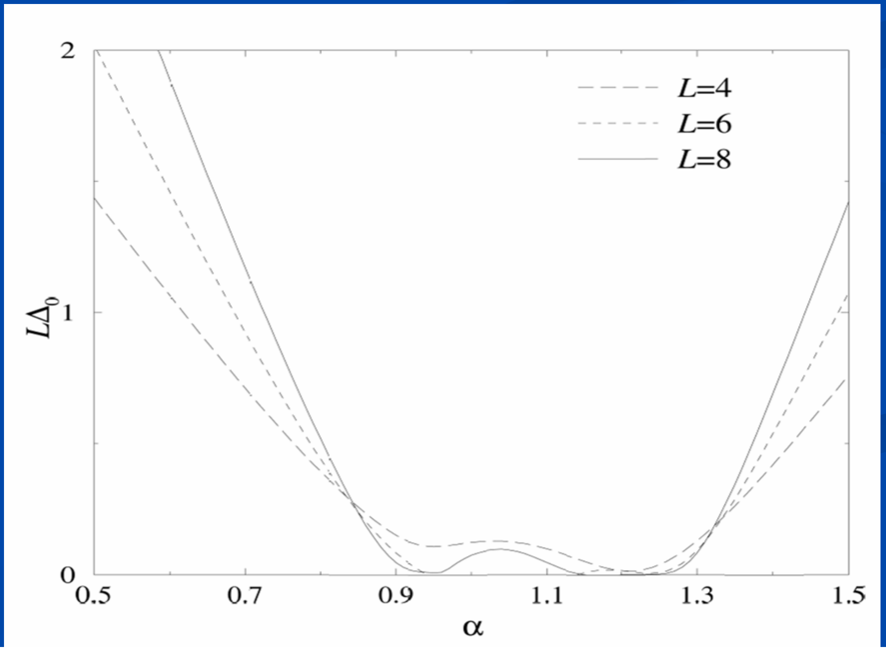



≈ triplet dimer

# Phenomenological renormalization

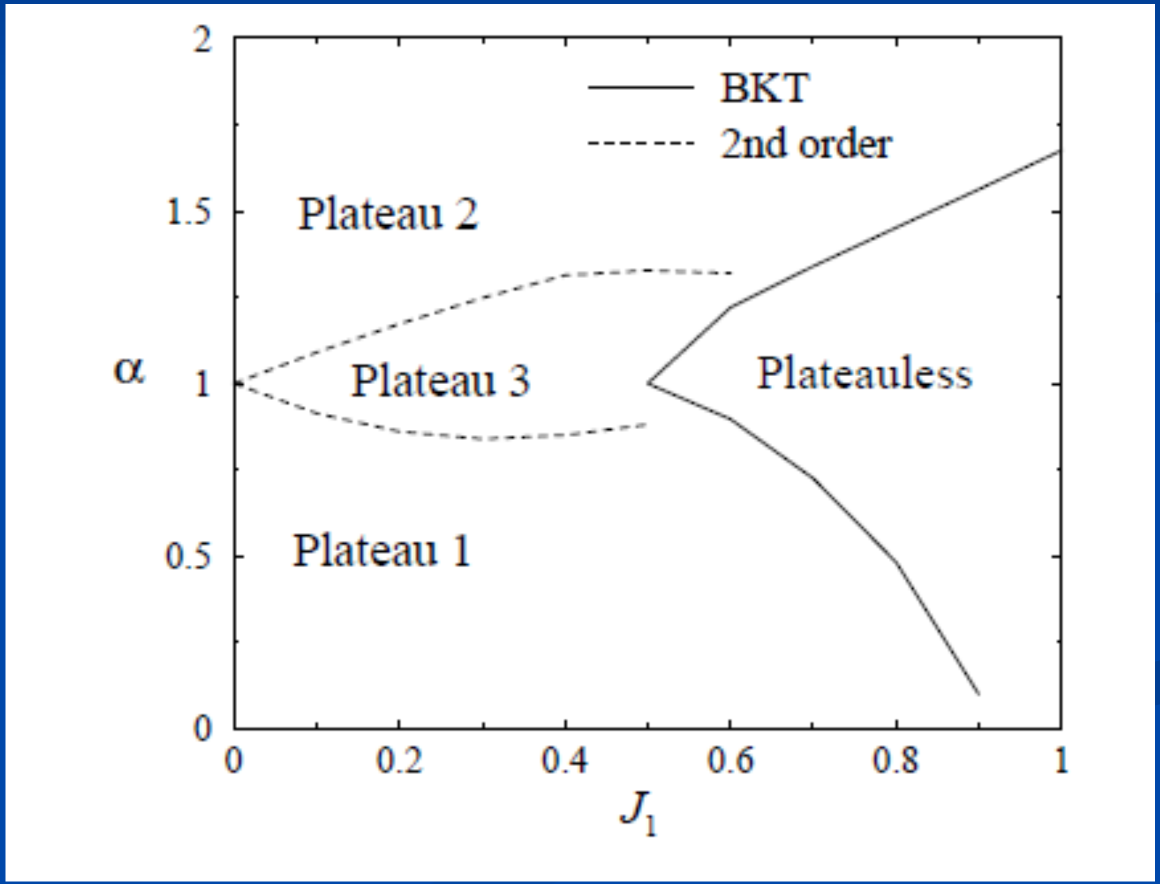
- New phase
  Excitation $\Delta_0$ with k=π, Parity=odd
  is degenerated with the ground state at m/ms=1/3
- Scaled gap $L\Delta_0$
  should be decreasing fn. of L at the new phase.

$J_1$=0.3

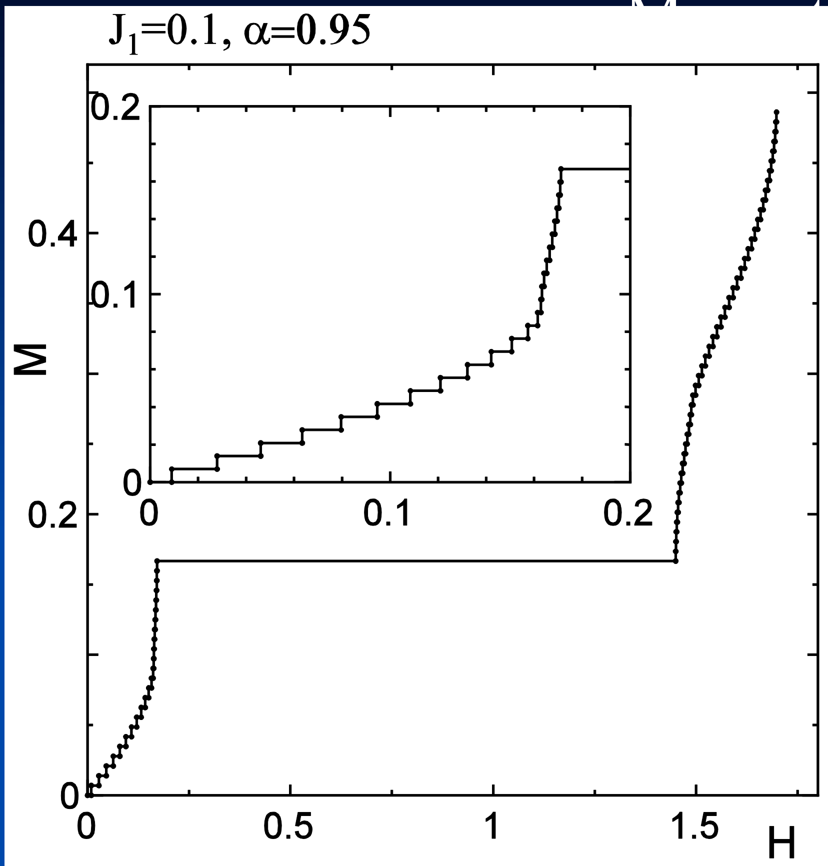


# Phase diagram for m/ms=1/3

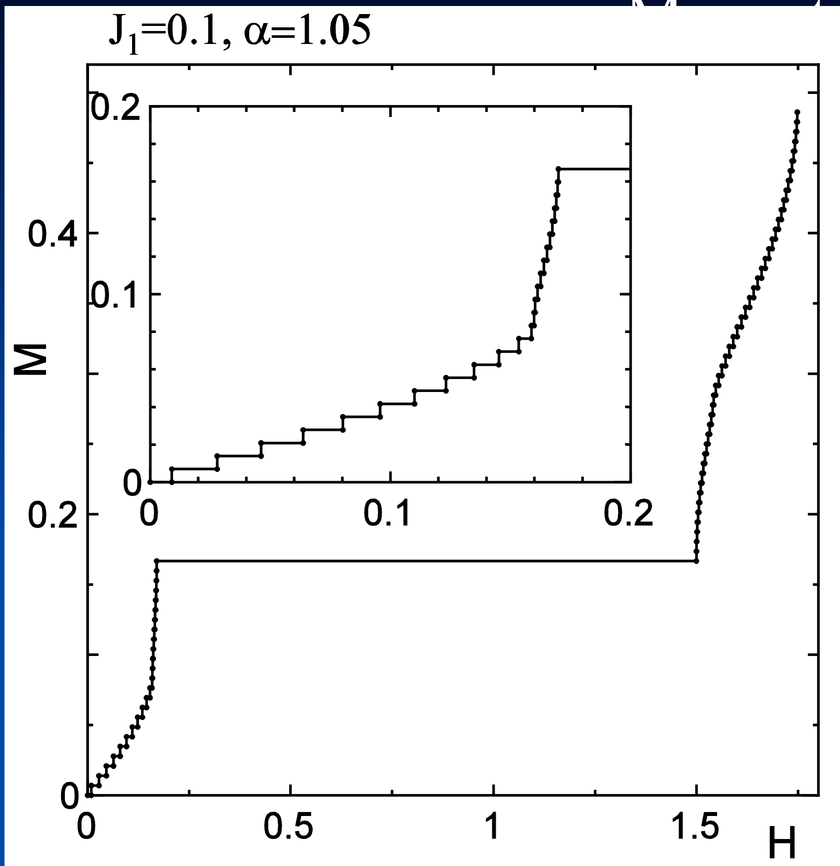
- Dimer-monomer plateau
- New phase Staggered order
- uud plateau



## …y DMRG



# y DMRG



# Summary

- 1/3 magnetization plateau of S=1/2 three-leg spin tube
- New exotic phase between two conventional plateau phases
- New phase:

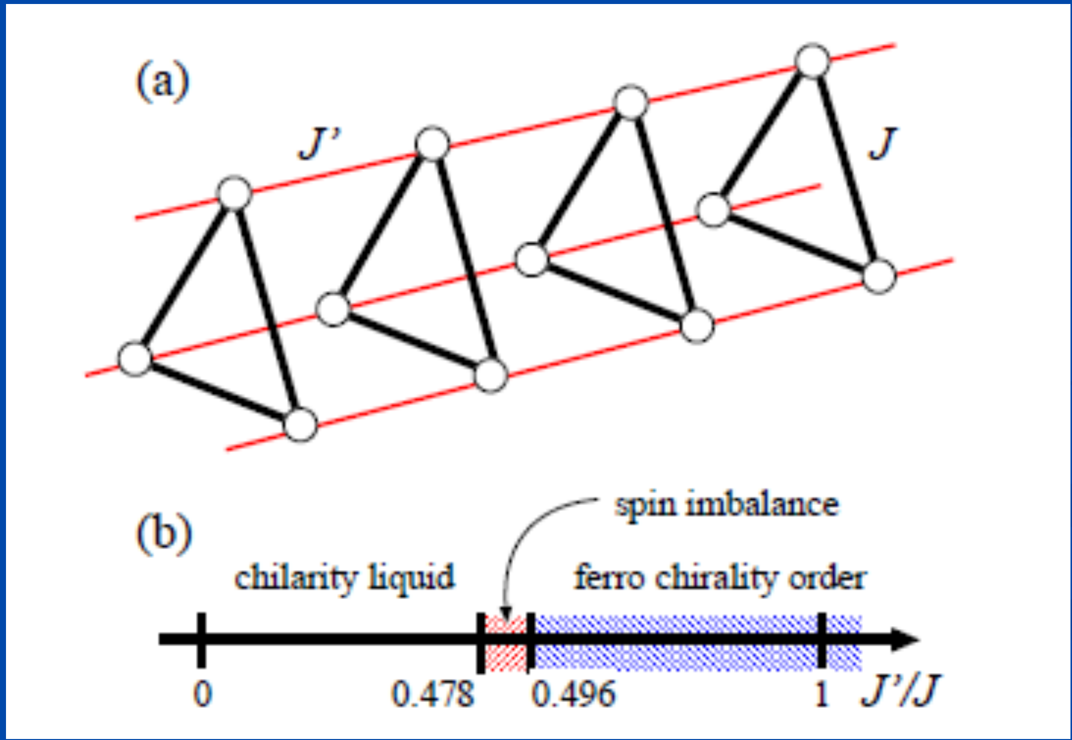
  translational symmetry breaking,

  staggered magnetization, dimer, chial order

$$\left\langle \vec{S}_1 \times \vec{S}_2 \times \vec{S}_3 \right\rangle$$

# Chirality at m=1/3

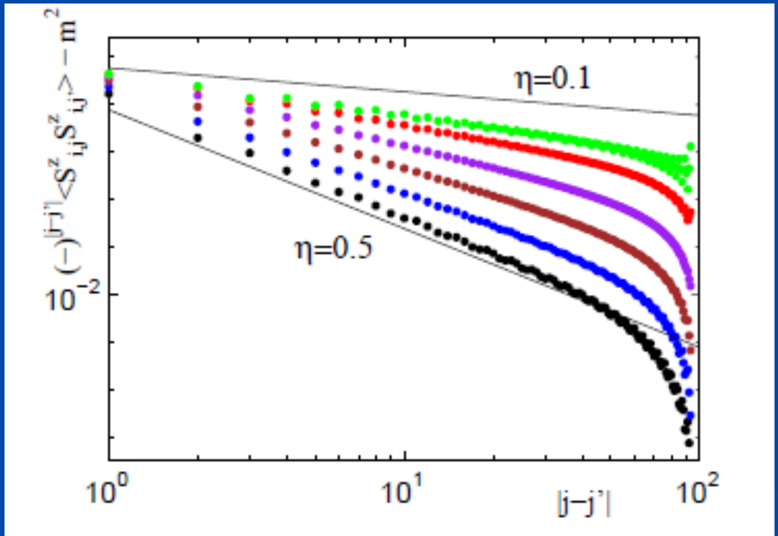
K. Okunishi et al. PRB 2012

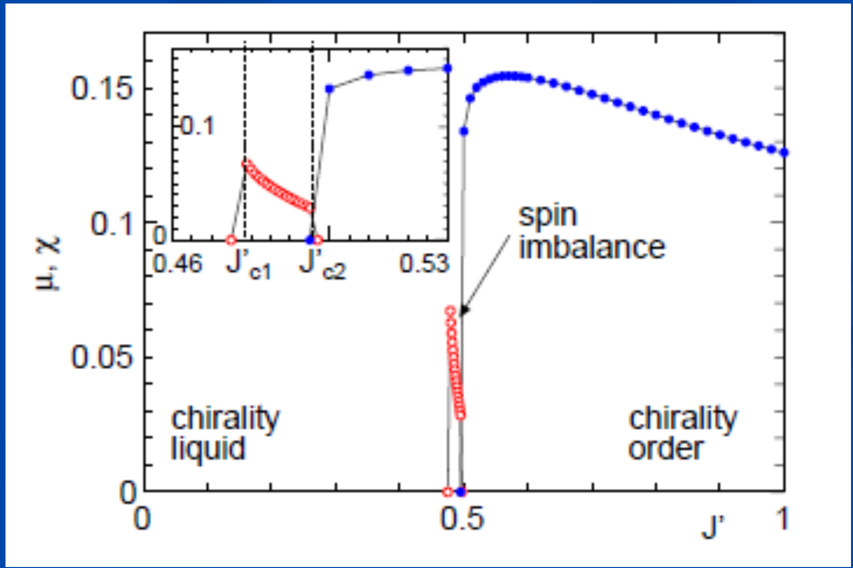


# DMRG calculation (L=144)

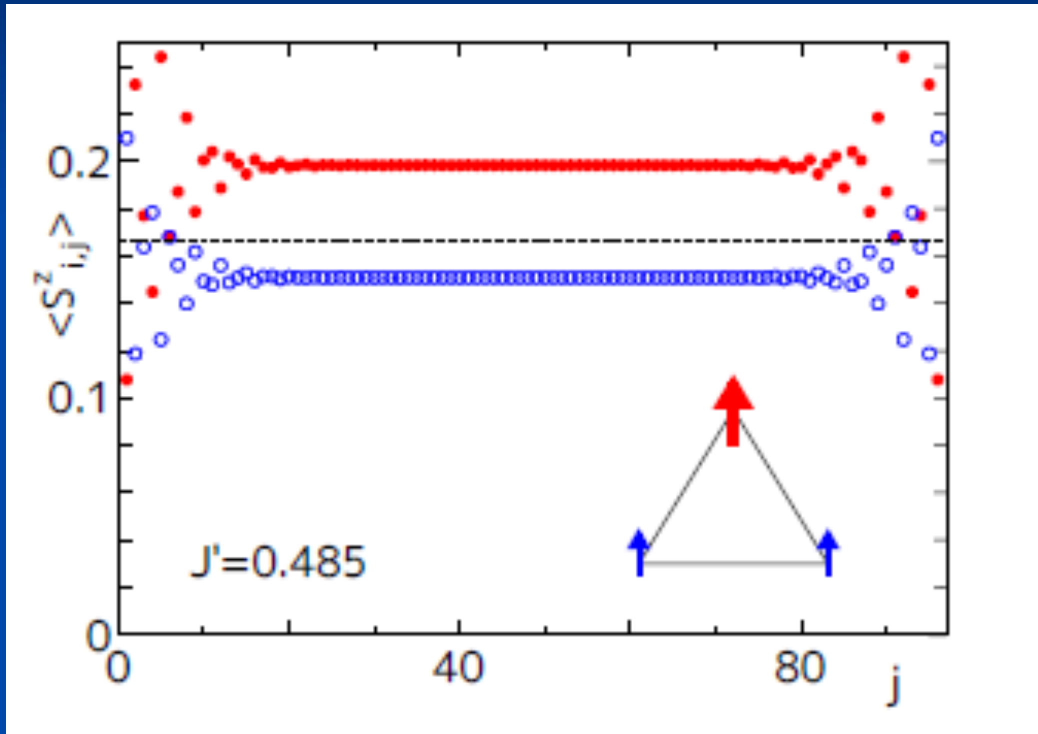
$$\chi_j = \sum_{i=1}^{3} (\boldsymbol{S}_{i,j} \times \boldsymbol{S}_{i+1,j})^z/3,$$

$$\mu_j = S^z_{1,j} - (S^z_{2,j} + S^z_{3,j})/2,$$



# Spin imbalance phase



# Phase diagram