Fully symmetric and non-fractionalized Mott insulators at fractional site-filling

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EQPCM @ ISSP

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PRL 2013 (kagome), 1207.0498...[PNAS] (honeycomb)

Collaborators

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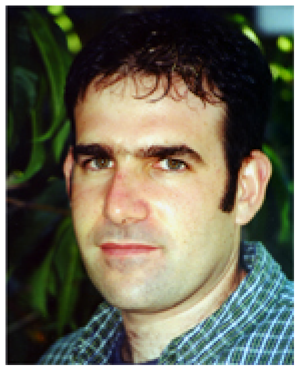
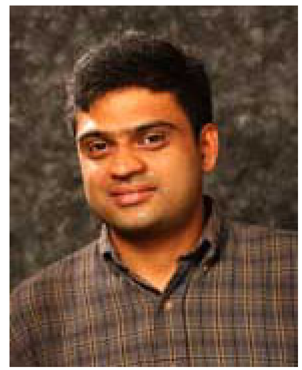
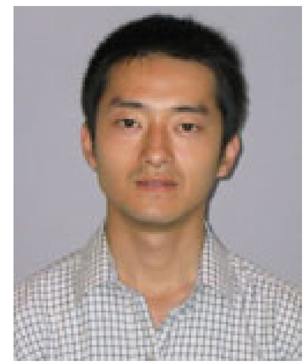


Dan Stamper-Kurn



Ashvin Vishwanath



Fa Wang

Ari Turner

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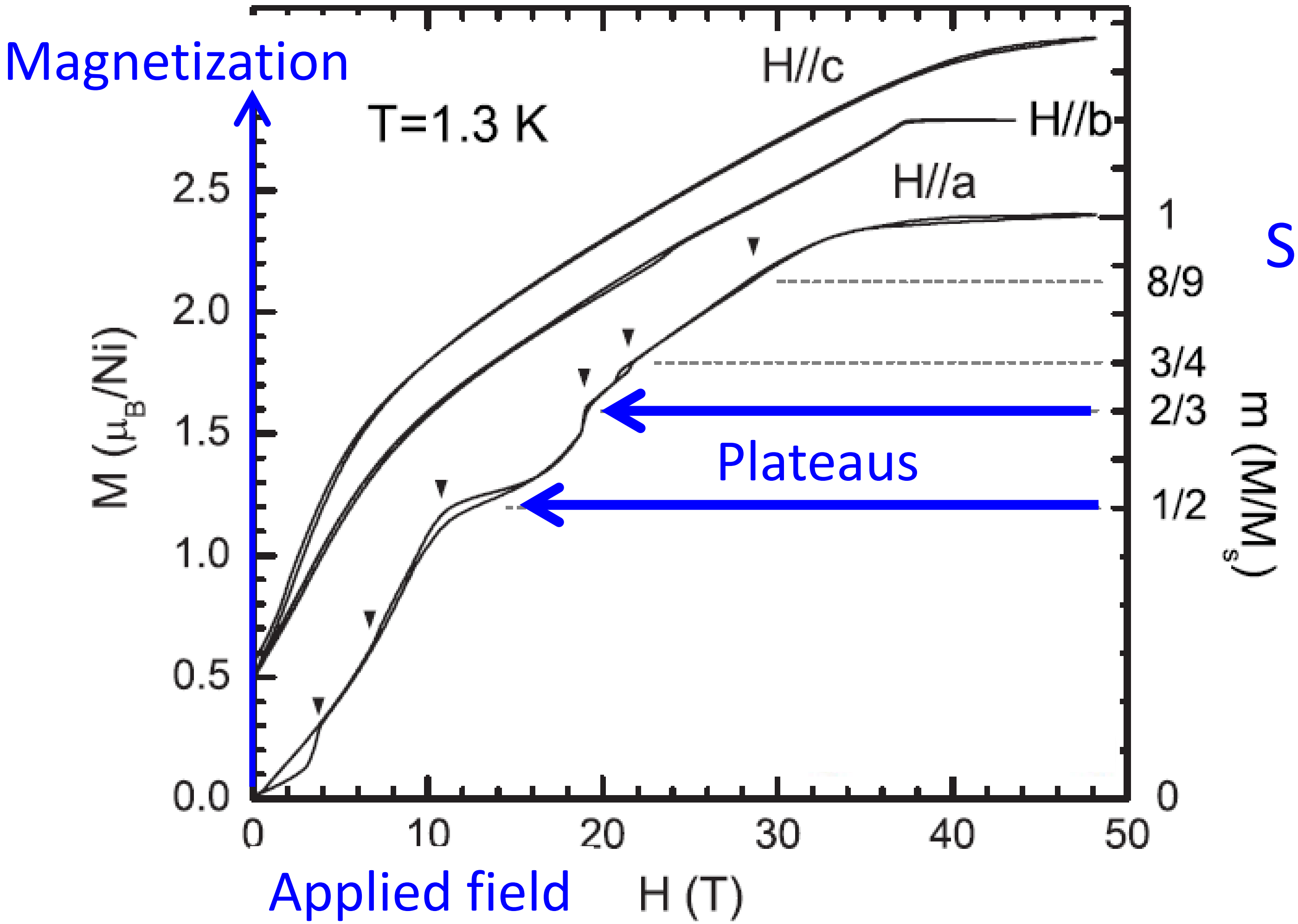
PRL 2013 (kagome), 1207.0498...[PNAS] (honeycomb)

Kagome magnetization plateaus



$Ni_3V_2O_8$

S=1 on kagome

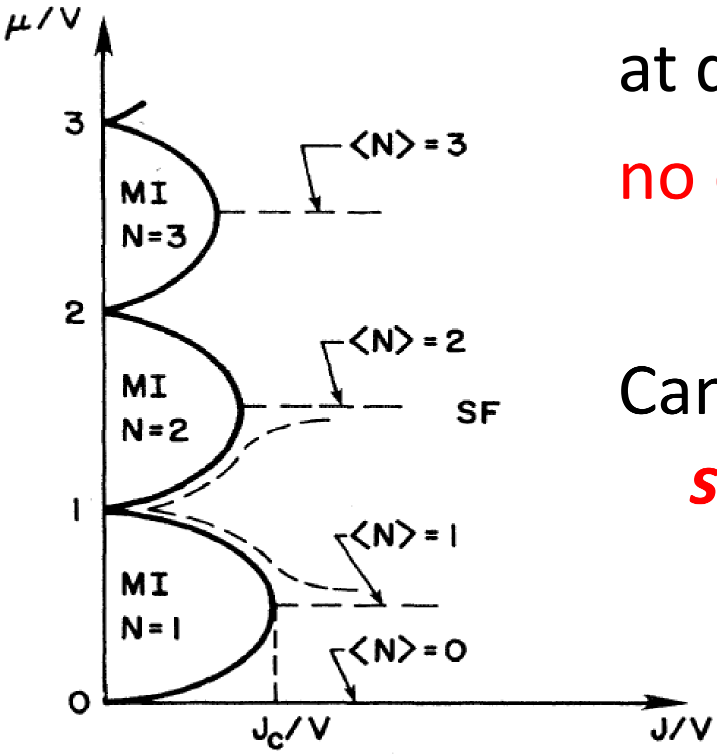
2/3 plateau

Wang, Tokunaga, He, Yamaura, Matsuo, Kindo, Phys. Rev. B 84, 220407(R) (2011)

Magnetization plateau
⟷ boson Mott insulator

Describe plateaus via bosons:

2/3 $S^z=1$ magnetization ⟷ 1/3 boson occupancy



Fisher, Weichman, Grinstein, Fisher (1989)

at density n=1/3 boson/site,

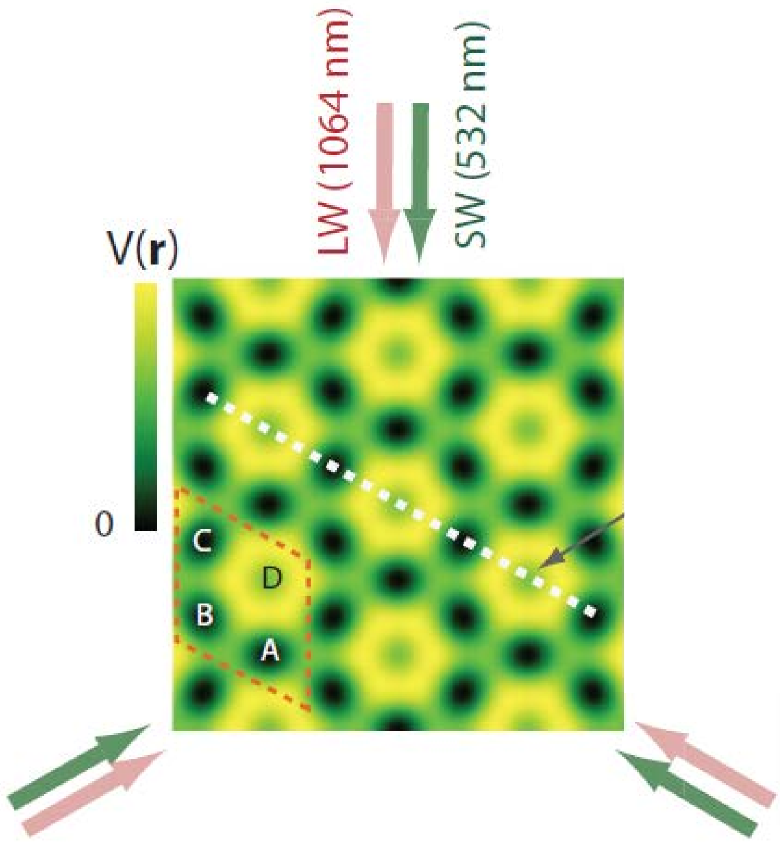
no definite-occupancy Mott insulator

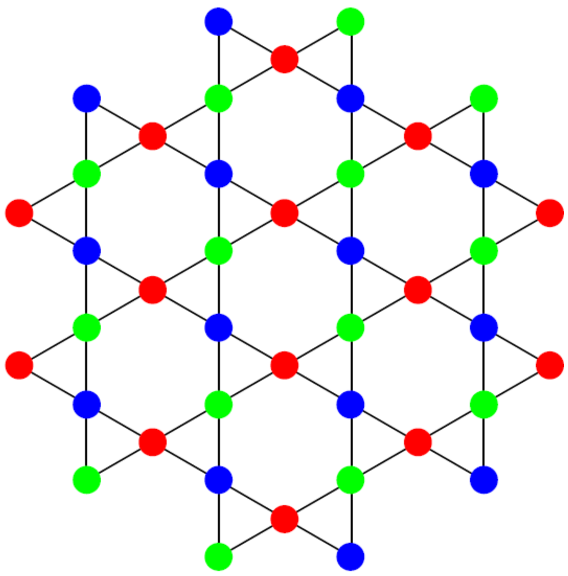
Can make n=1/3 insulator by ***breaking symmetry***

any alternative?

Ultracold atoms on kagome

Must n=1/3 bosonic insulators *break symmetry*?





Gyu-Boong Jo, Phys. Rev. Lett. 108, 045305 (2012)

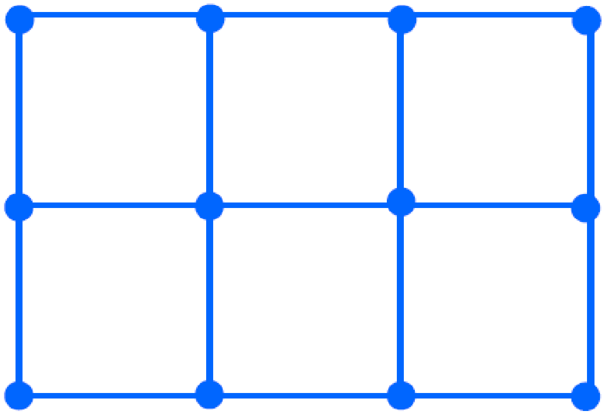
Free fermions, square lattice

Metallic or insulating? <u>Band theory:</u>

$\boldsymbol{f}$ ≡ number of particles per unit cell

= number of filled bands

Non-integer $\boldsymbol{f}$ → partially-filled bands → metal



Free fermions, square lattice

Metallic or insulating? Band theory:

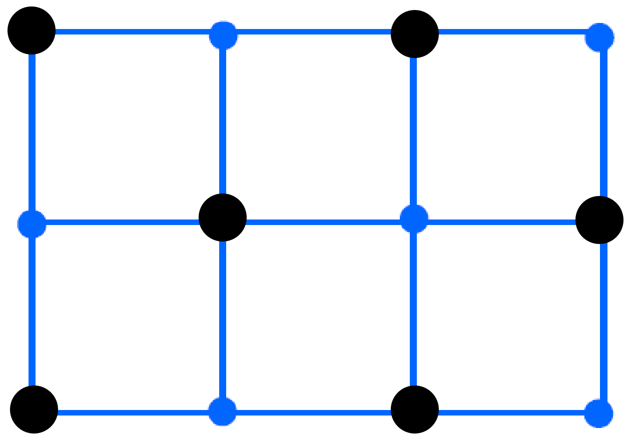
$\boldsymbol{f}$ ≡ number of particles per unit cell

= number of filled bands

Non-integer $\boldsymbol{f}$ → partially-filled bands → metal

Half site-filled square lattice:

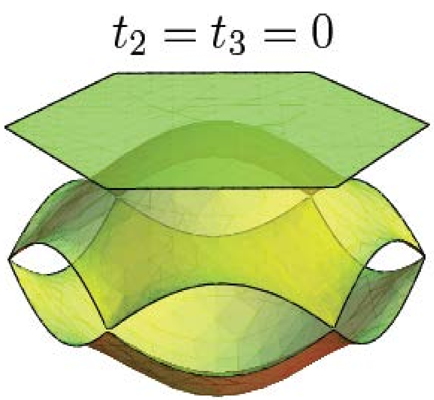
$\boldsymbol{f}$ =1/2 → always metallic

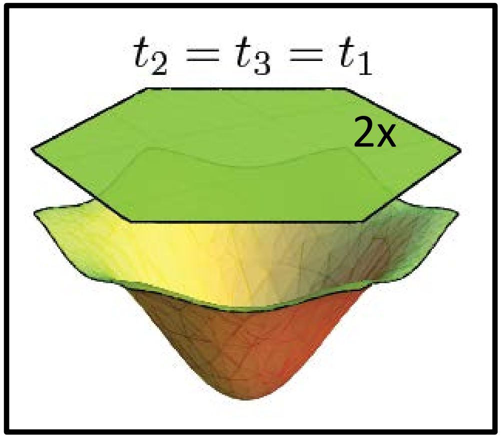


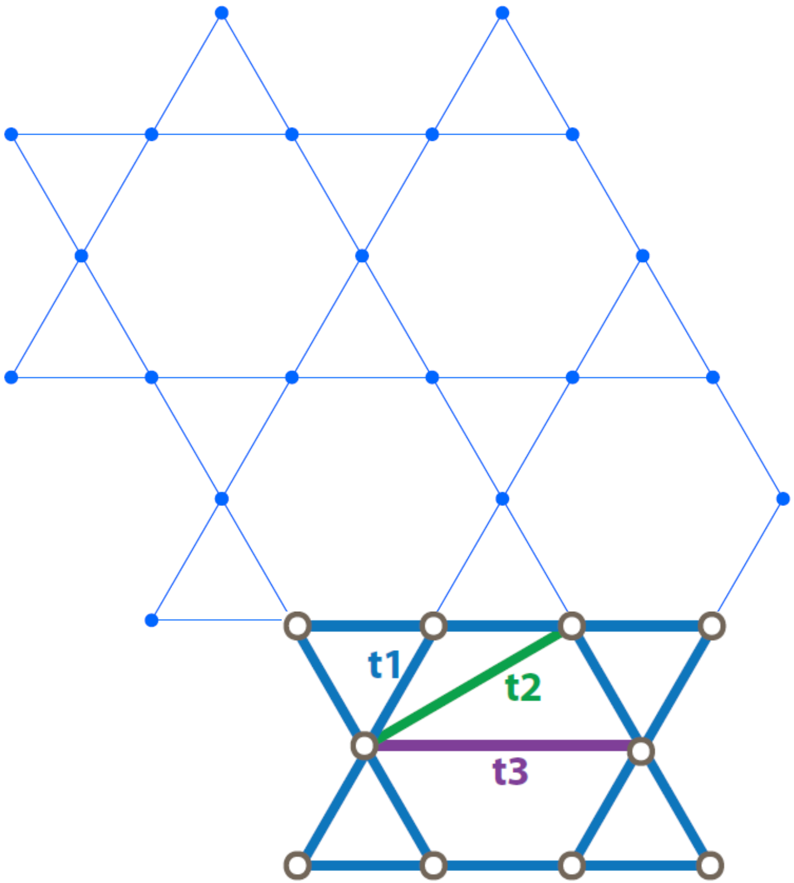
Free fermions, kagome lattice

Nearest-neighbor tight-binding:

Filled bands touch -> metal





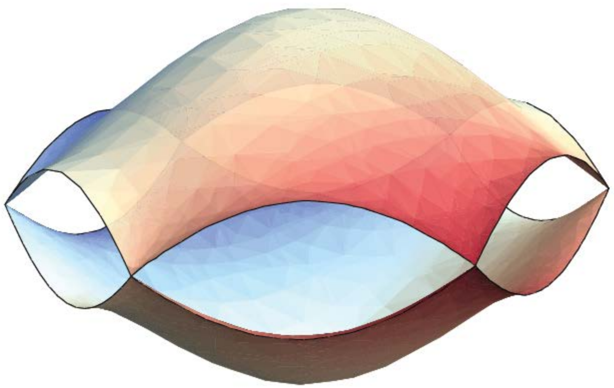


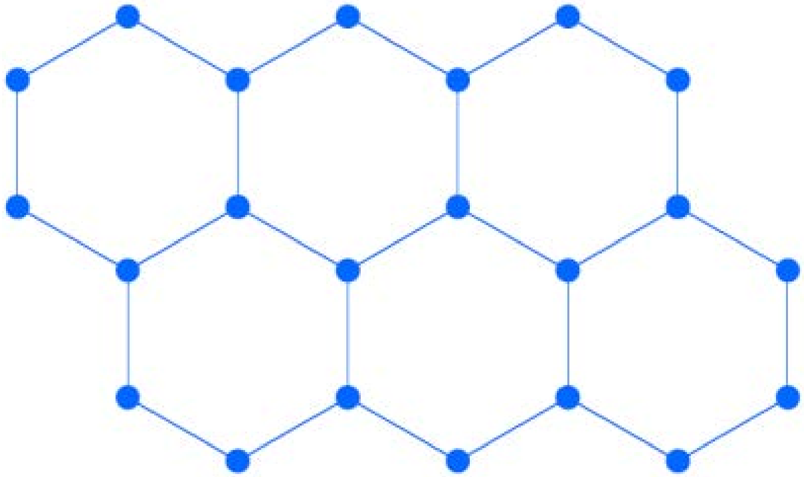
But t_2, t_3 open gap:

1/3 site-filling (***f***=1) can be insulating

Free fermions, honeycomb (graphene) lattice

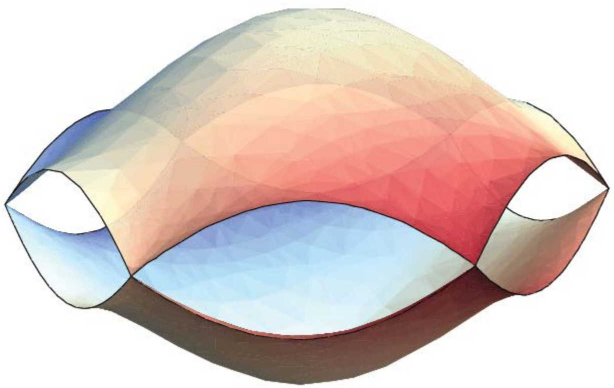
Graphene: gapless Dirac cones

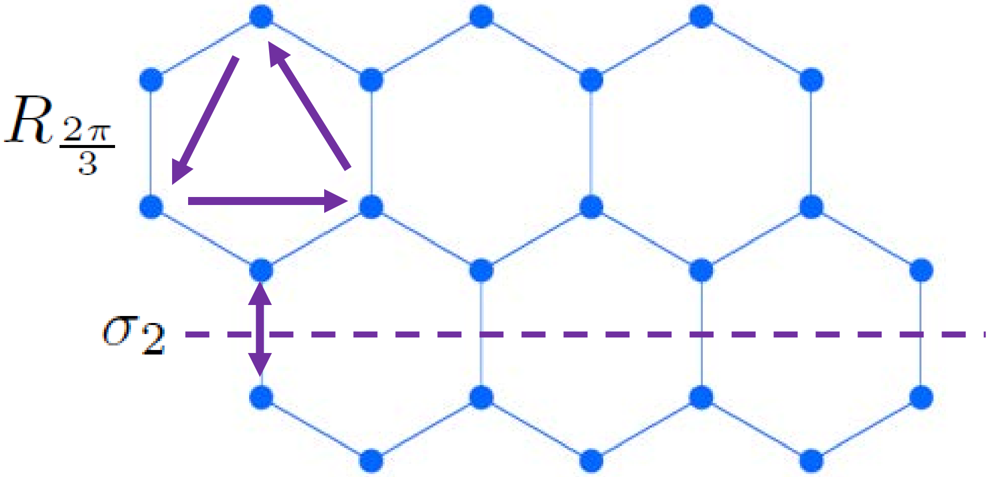




Free fermions, honeycomb (graphene) lattice

Graphene: gapless Dirac cones





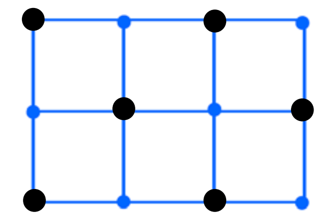
protected by lattice symmetry:

Half site-filled honeycomb lattice ($\boldsymbol{f}$=1)

always metallic

Interacting bosons / fermions

Lieb-Schultz-Mattis-Hastings-Oshikawa theorems:



Non-integer *f*:

always gapless / degenerate

insulators trigger conventional or topological order

Integer *f* = 1:

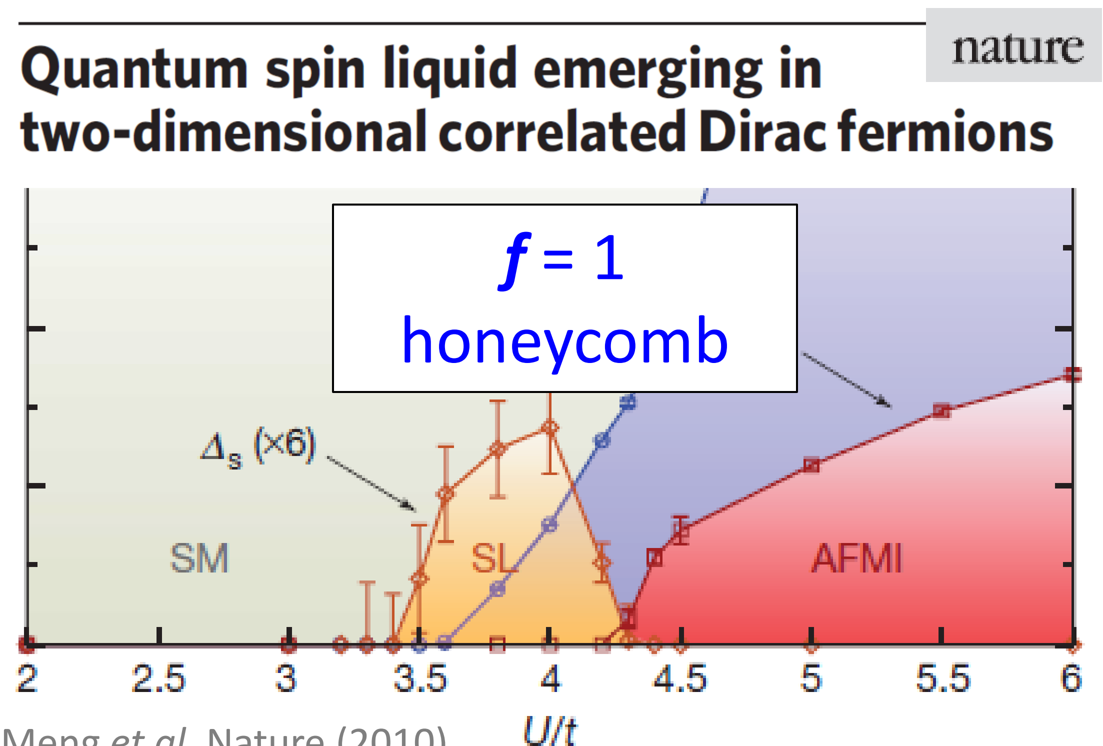
	kagome:	honeycomb:
free fermions:	insulator	always metallic
interacting particles:	?	?

interacting featureless insulators?

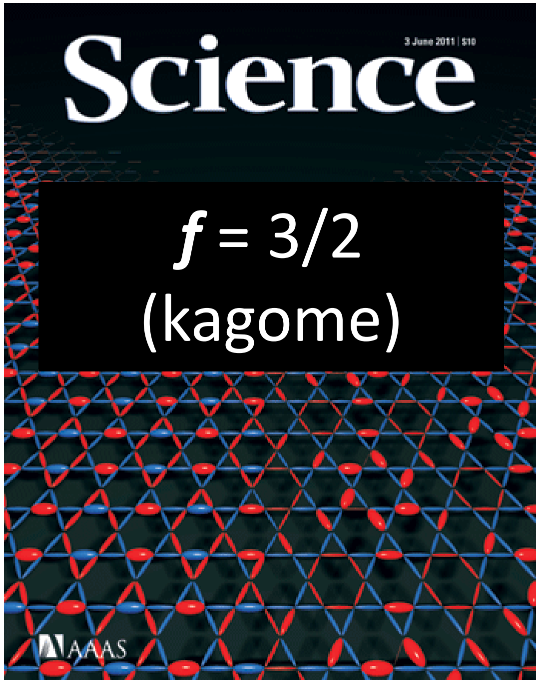
How to identify fractionalization?

Half-filled (f=1) honeycomb:

Gapped + fully-symmetric $\overset{?}{\Longrightarrow}$ Fractionalization



Meng *et al*, Nature (2010)



Yan *et al*, Science (2011)

How to identify fractionalization?

Half-filled (*f*=1) honeycomb:

Gapped + fully-symmetric $\overset{?}{\Longrightarrow}$ Fractionalization

implication holds when
featureless insulators forbidden

e.g. non-integer ***f***

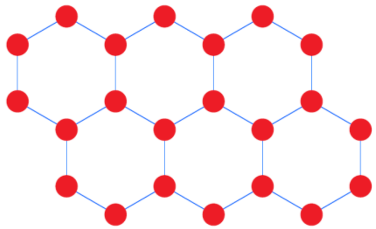
f = 1 kagome, honeycomb:

featureless insulators?

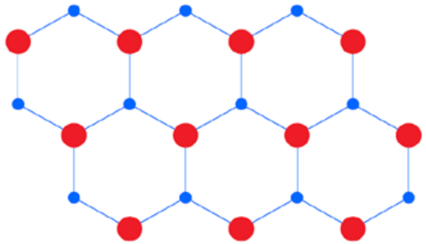
Bosons (cold atoms, magnetized spins, paired electrons)

Trying to make a featureless Bose insulator

Honeycomb at half site-filling:

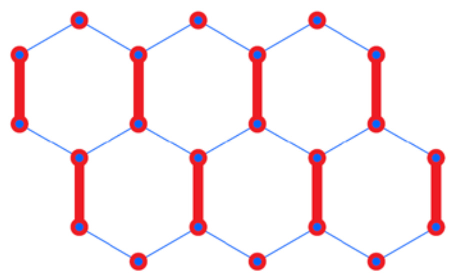


- Trivial n=1 Mott insulator -> wrong filling



- Put 1 boson in some sites

or



- Smear bosons across unit cell

-> breaks lattice symmetry

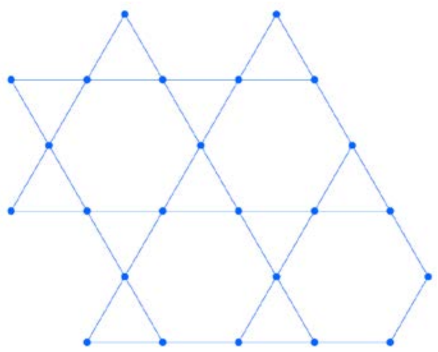
- Smear bosons across overlapping regions -> likely superfluid

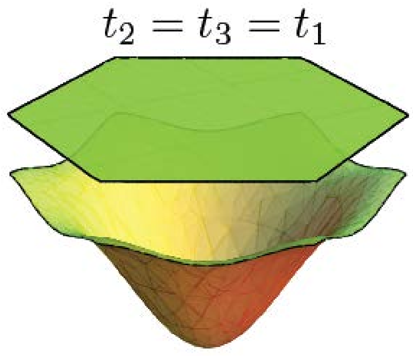
- Put 1 fractionalized 1/2-quasiparticle per site -> fractionalization

Route #1 (kagome):

Band to *Mott*

given a band insulator



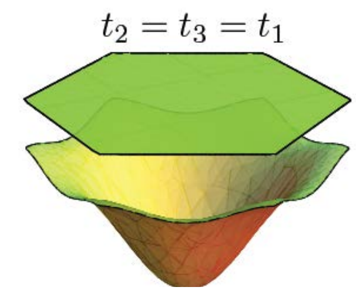


can construct an analogous

featureless boson insulator

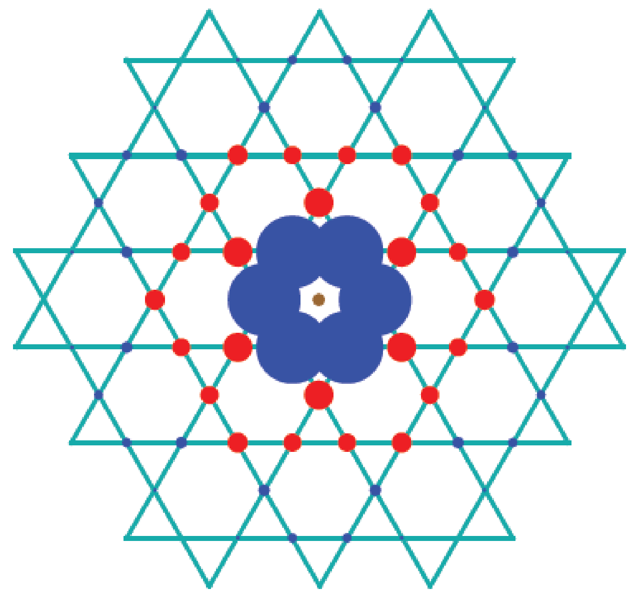
Wannier orbital permanent wavefunction

Band insulator H_0 , gapped lowest band



Bloch states

$$\int_{\mathrm{BZ}} e^{i\phi(q)} \psi_q \quad \Longrightarrow$$



symmetric, localized, orthogonal

Wannier orbitals $w^\dagger_{\mathbf{R}}$

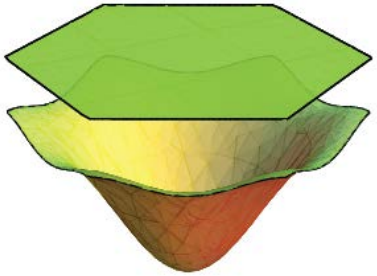
Wannier permanent $|\Psi_W\rangle = \prod_{\mathbf{R}} w^\dagger_{\mathbf{R}}|0\rangle$

insulating, symmetric, ***f***=1 (1/3 site-filling)

Featureless Insulators?

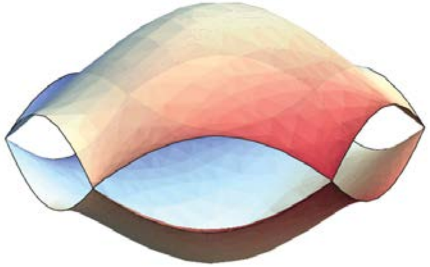
- ***f***=1 kagome: yes -- Wannier Permanent.

  PRL 2013

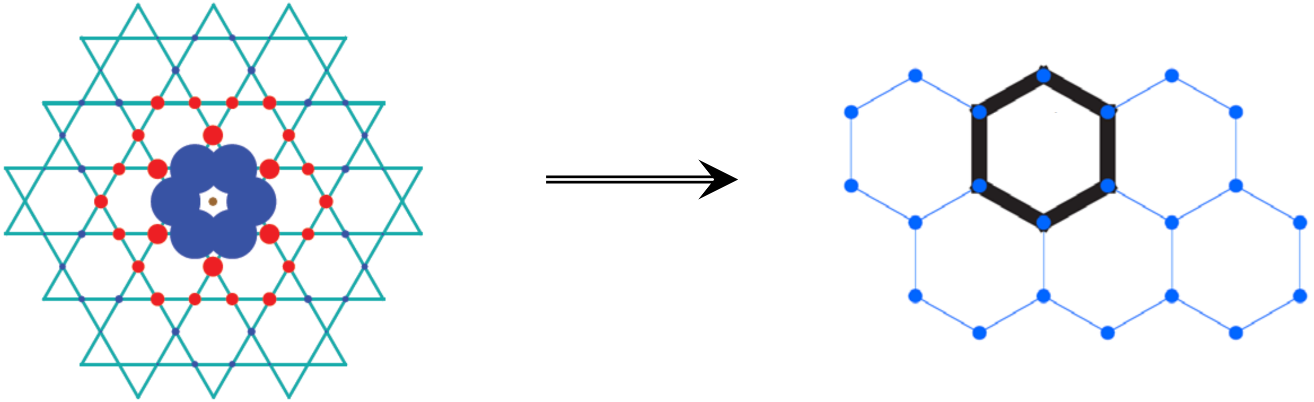


- ***f***=1 honeycomb: band insulator prohibited

  → no Wanniers



- Route #2: use a localized symmetric orbital

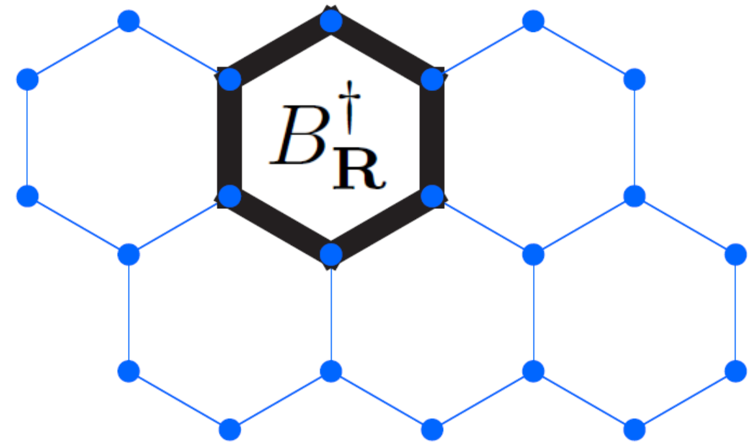


Candidate featureless Boson insulator

Hexagon-orbital operators

$$B_{\mathbf{R}}^{\dagger} \equiv \frac{1}{\sqrt{6}} \sum_{j \in \hexagon_{\mathbf{R}}} b_j^{\dagger}$$

($\mathbf{R}$ on Bravais lattice)



$$|\Psi_{\hexagon}\rangle = \prod_{\mathbf{R}} B_{\mathbf{R}}^{\dagger} |0\rangle$$

site-filling of
1/2 on honeycomb
1/3 on kagome

f=1

Establishing $|\Psi_{\hexagon}\rangle$ is featureless, insulating

$$|\Psi_{\hexagon}\rangle = \prod B^{\dagger}_{\mathbf{R}}|0\rangle$$ highly entangled state

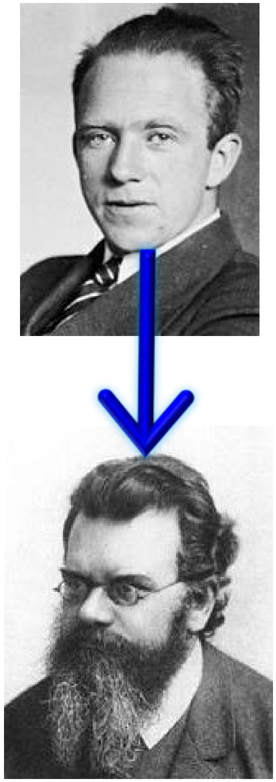
must explicitly check for superfluidity, symmetry-breaking, fractionalization

Compute within classical statistical mechanics model that maps to $|\Psi_{\hexagon}\rangle$

Like *plasma analogy* for Laughlin's 1/m FQHE states:

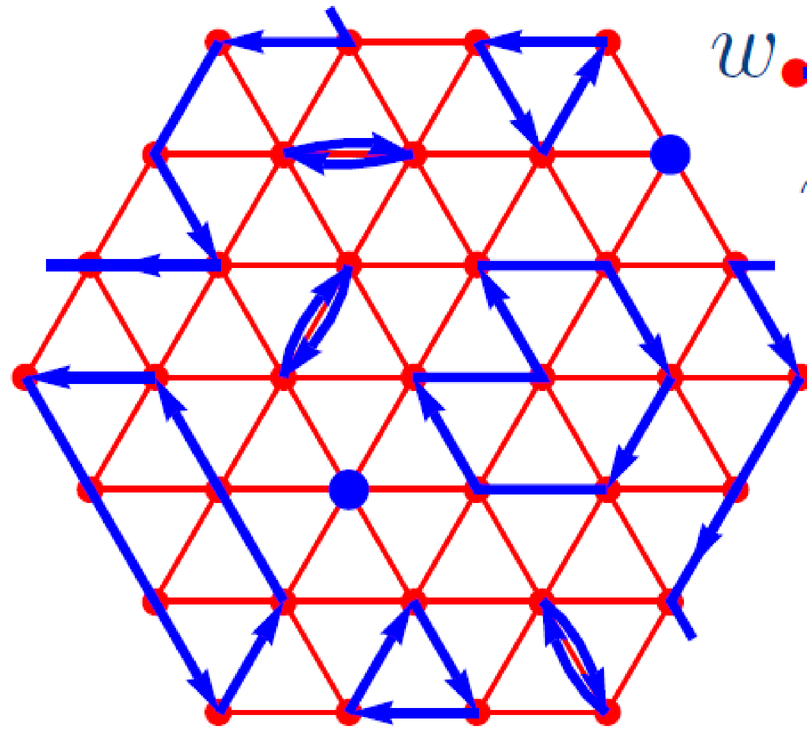
$|\Psi_{\hexagon}\rangle$ defines **2D** classical partition function, of

self-avoiding closed loops on Bravais triangular lattice



$$\mathbf{Z} \equiv \langle \Psi_{\hexagon} | \Psi_{\hexagon} \rangle$$

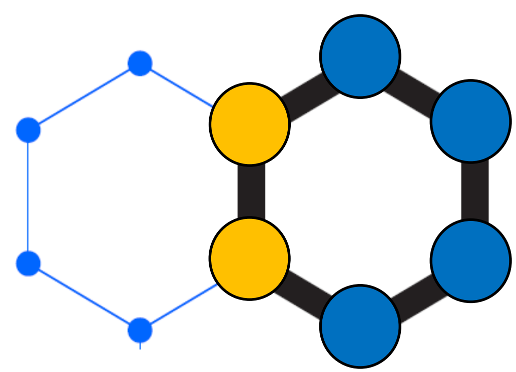
$$\boxed{= \sum_{\text{loop configs}} m^{L_{\text{total}}}}$$



$$w_{\bullet\!\to\!\bullet} = m$$

$$w_{\bullet} = 1$$

m = 1/3



$$\mathbf{Z} \equiv \langle \Psi_{\hexagon} | \Psi_{\hexagon} \rangle$$

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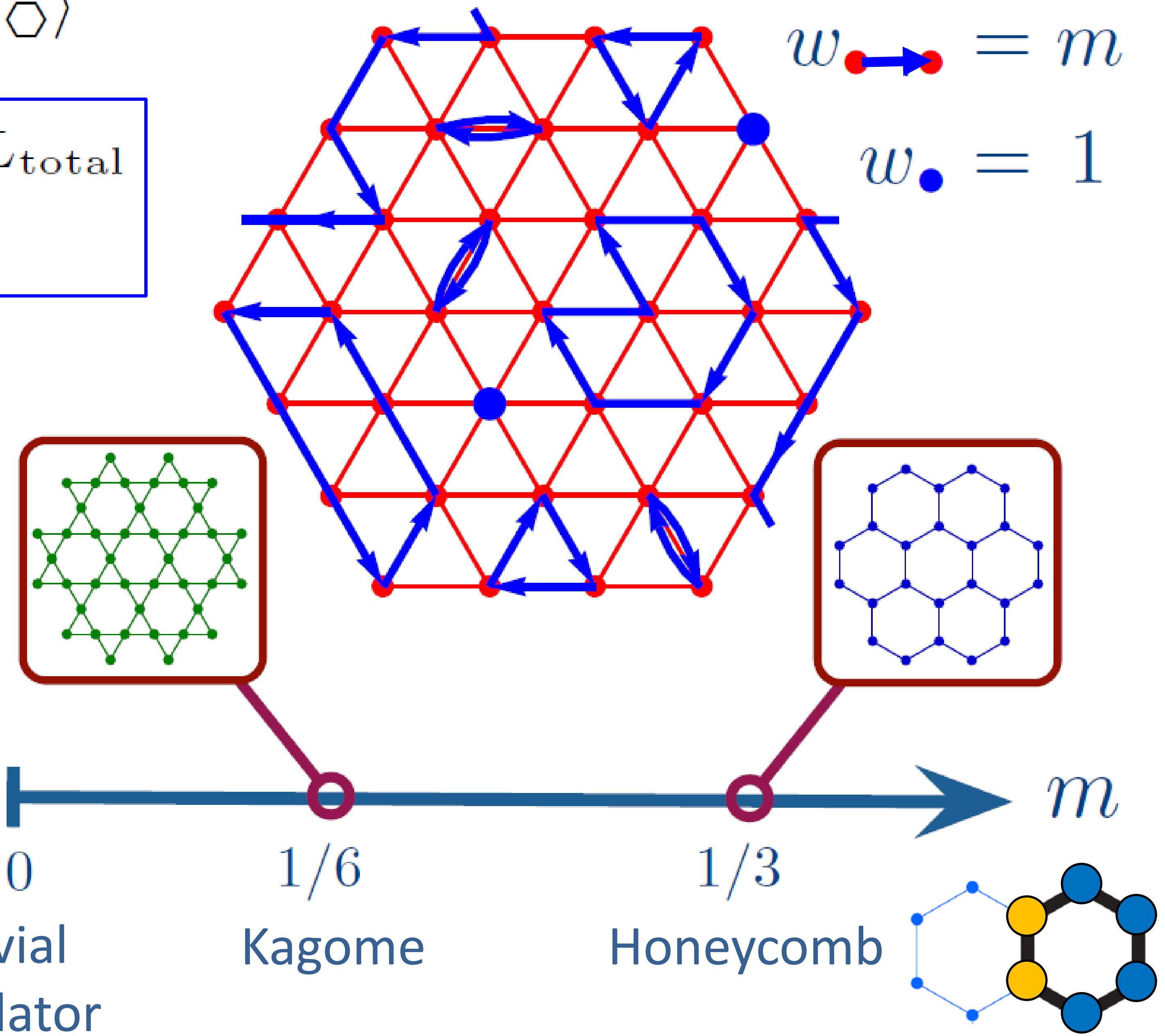
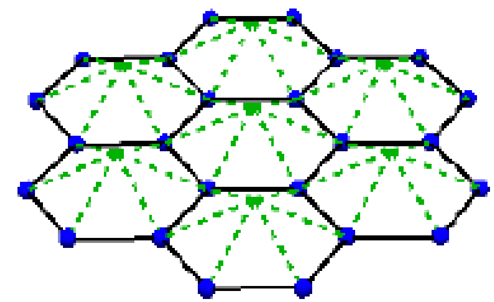


States for continuous m loop models

$B^{\dagger}_{\mathbf{R}}$

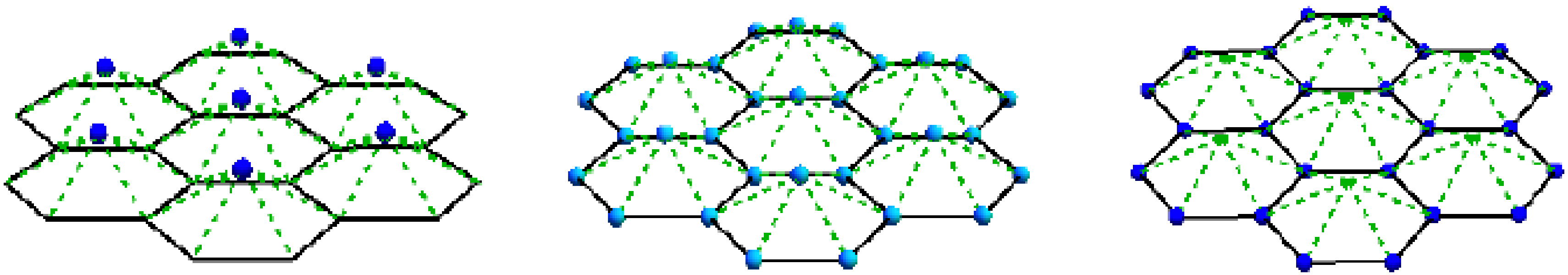


m=1/3

States for continuous m loop models

Shift to hexagon-center site $a^\dagger_{\mathbf{R}}$

$a^\dagger_{\mathbf{R}}$ $B^\dagger_{\mathbf{R}}$

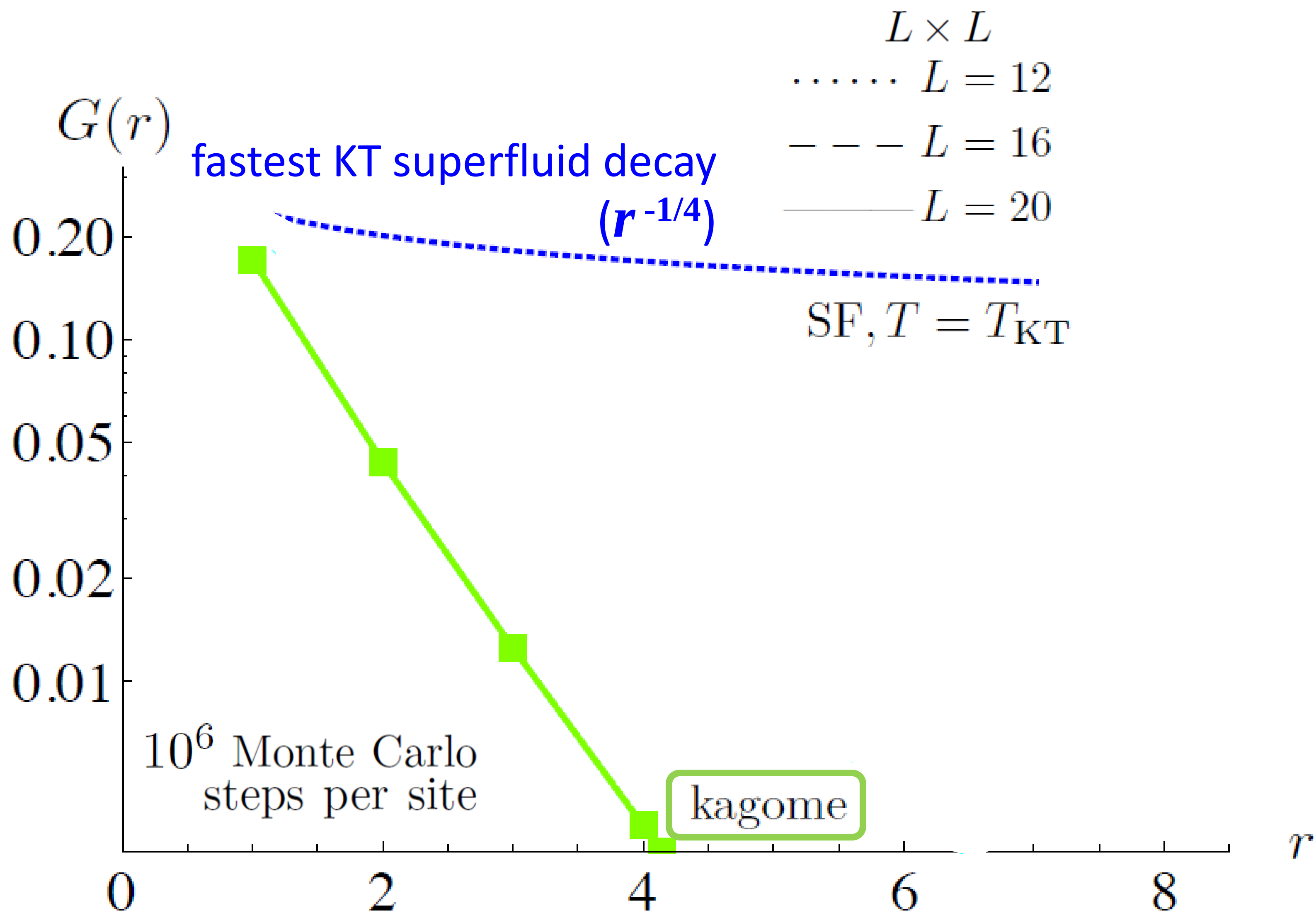


m=0 m=1/3

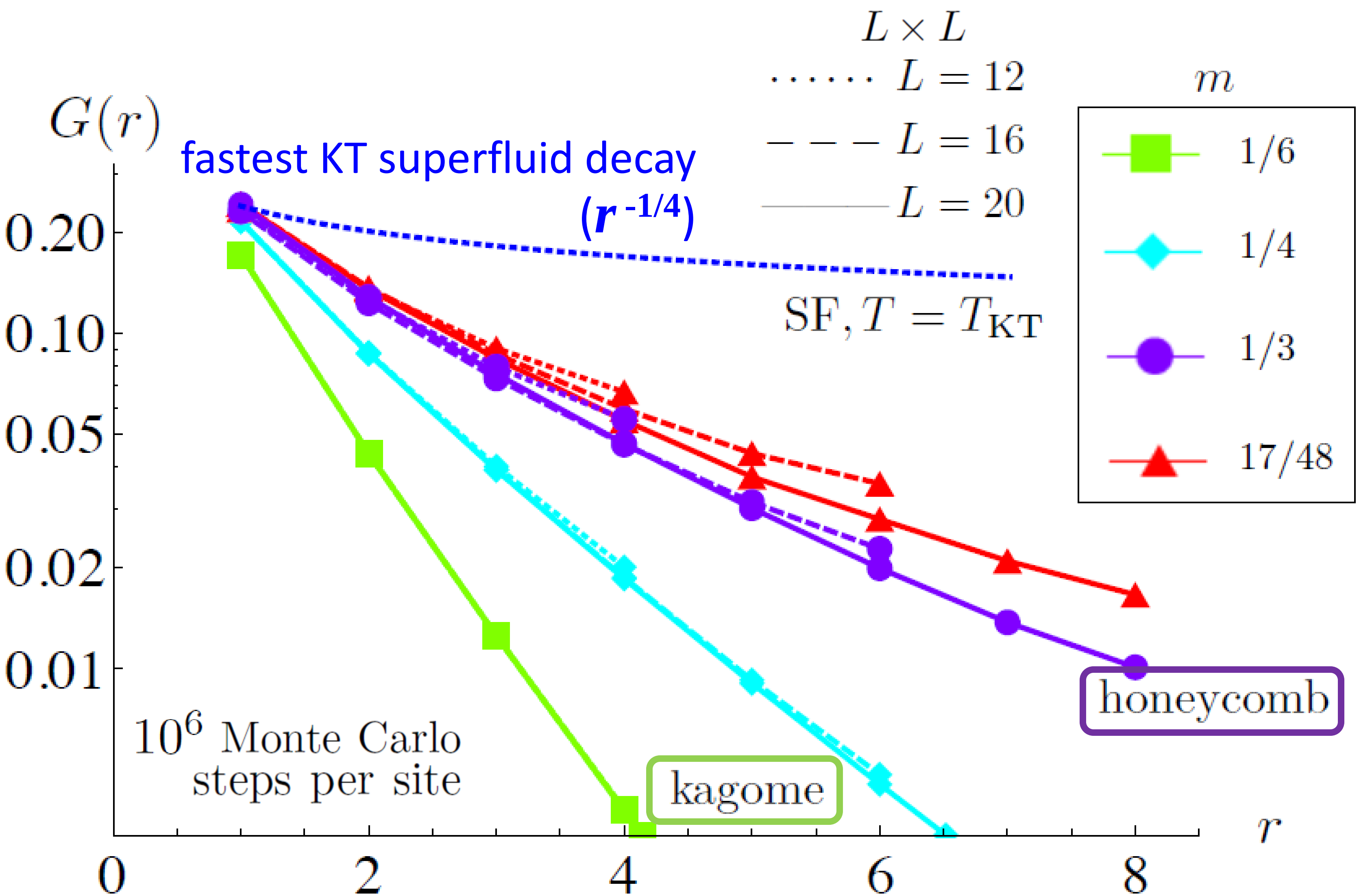
trivial insulator $\longrightarrow$ $|\Psi_{\hexagon}\rangle$

smooth evolution?

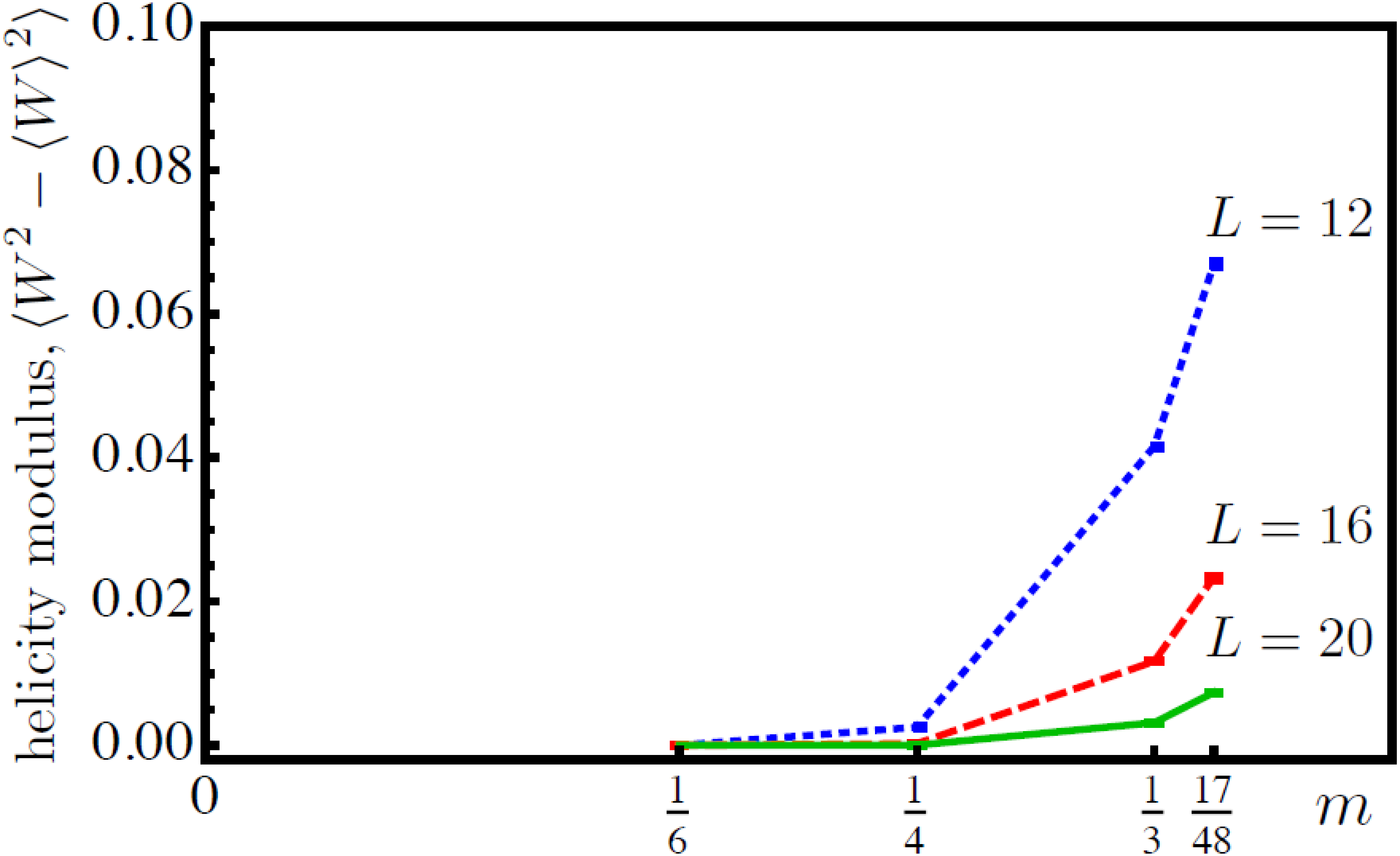
Loop model correlators:



Loop model correlators: decay exponentially



Helicity modulus: vanishes, no transition



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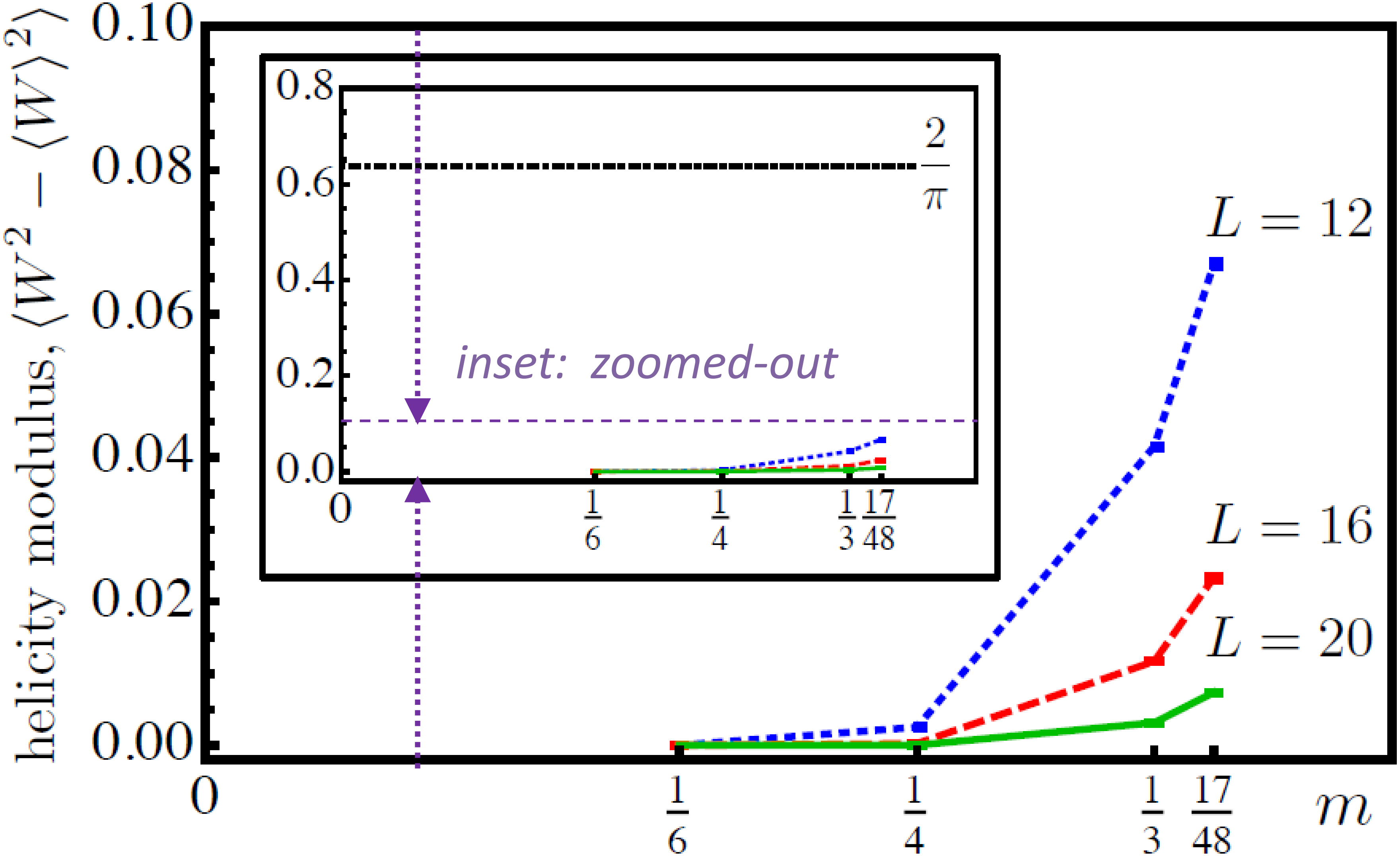
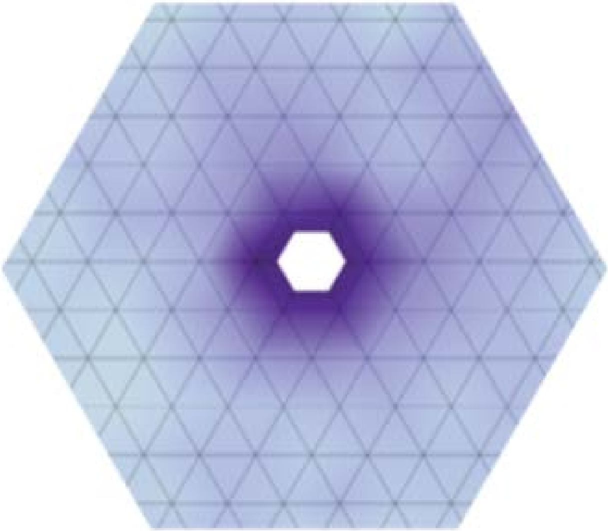


No lattice symmetry breaking



$|\Psi_{\hexagon}\rangle$ is featureless

- Mott insulator with inherent quantum fluctuations

- $|\Psi_{\hexagon}\rangle$ is positive → arises from unfrustrated *H*?
- Construction generalizes to *symmorphic* lattices

fully-symmetric orbital at each unit cell

Symmorphic Lattice Symmetry Groups

Non-symmorphic:

Point group and translations split: no glides or screws

Fully-symmetric orbital for each unit cell

Non-symmorphic lattices at ***f***=1 prohibit featureless insulators

Nat Phys 9, 299 (2013) & arxiv:1212.2944

Hexagon construction & classical Bravais lattice loop model

generalize to all symmorphic lattices

Project out multiple occupancy →

hardcore bosons = onsite paired electrons

$$b_i^\dagger \Longleftrightarrow c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger$$

$$\boxed{|\Psi_{\text{spinful electrons}}\rangle = \prod_R \sum_{j \in \hexagon_R} c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger \, |0\rangle}$$

Featureless insulator?

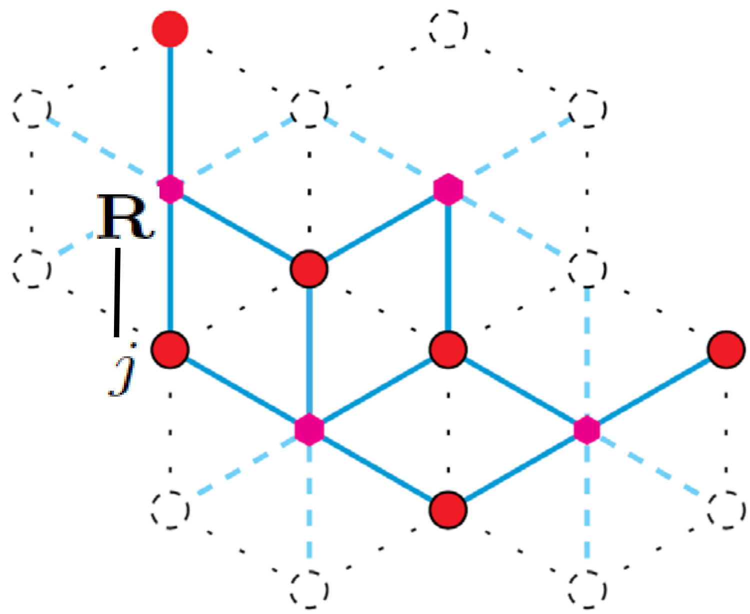
check via dimer model

Density correlator via dimer model mapping

Site occupancy $\{n_i\}$ amplitudes map to classical dimer model on *depleted dice lattice*

$\mathbf{R}\!-\!j$ dimer selects

$b_j^\dagger$ from hexagon $B_{\mathbf{R}}^\dagger$



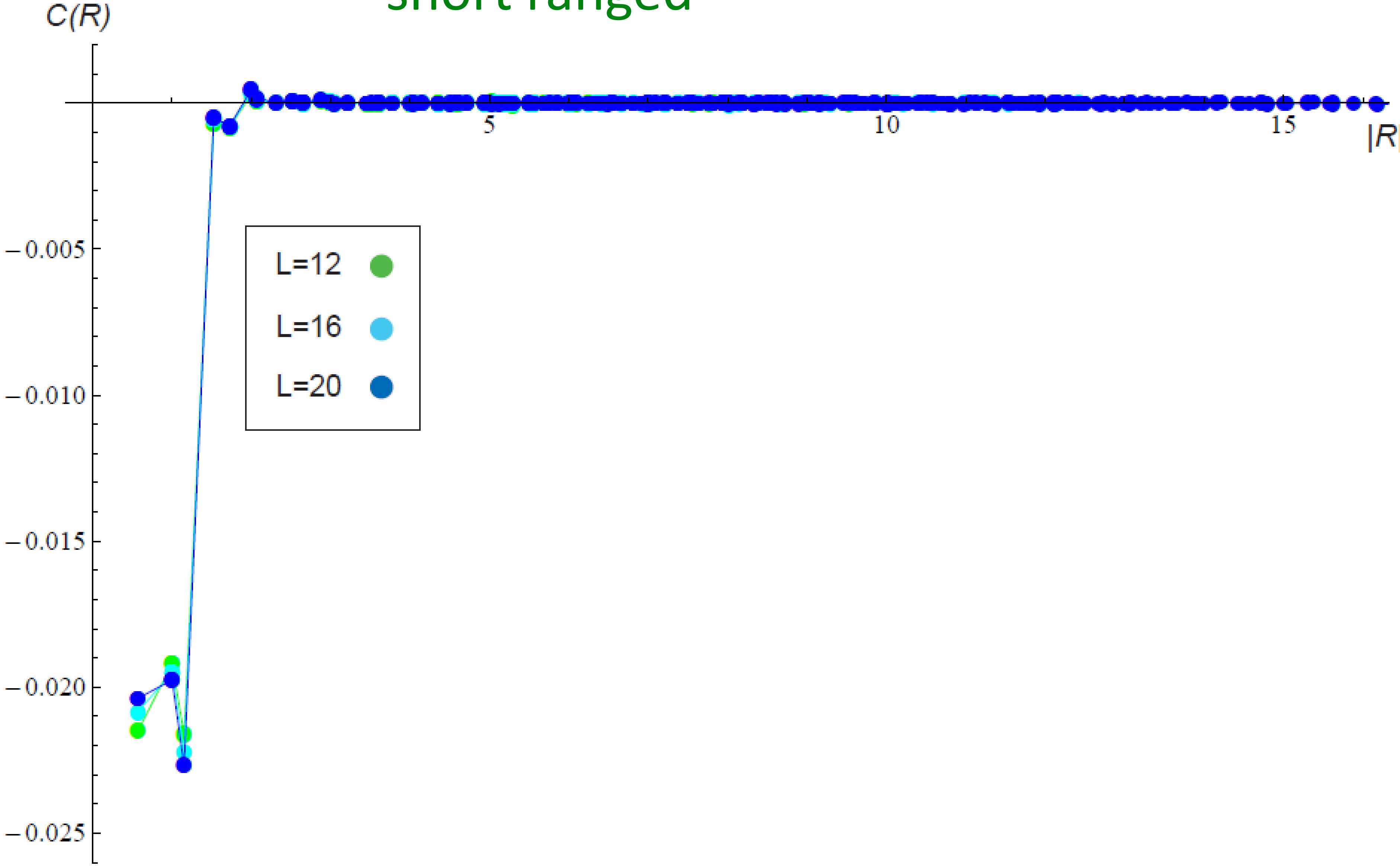
Kasteleyn algorithm:

bipartite dimer coverings

→ determinant (Monte Carlo)

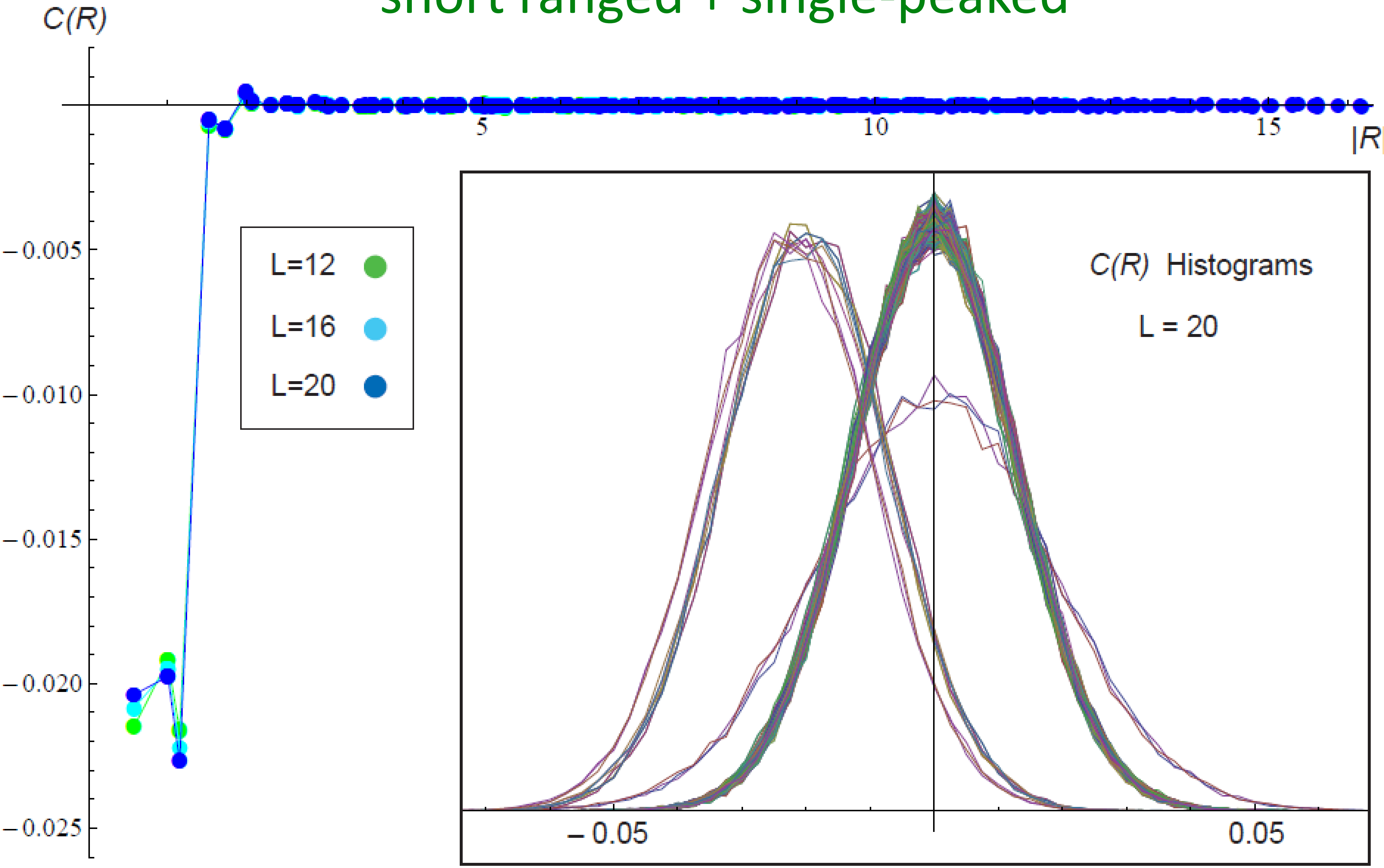
Density correlator

short ranged



Density correlator

short ranged + single-peaked



Spin-singlet electron featureless insulator

$$|\Psi_{\text{spinful electrons}}\rangle = \prod_R \sum_{j \in \hexagon_R} c^\dagger_{j,\uparrow} c^\dagger_{j,\downarrow} |0\rangle$$

1/2 electron of each spin / site: ***f***=1

Pairing is short ranged → insulator
(cooper pairs aren't condensed)

Featureless, SU(2)-symmetric, fully-gapped
spinful electron state

How to identify fractionalization?

Half-filled (f=1) Honeycomb
Hubbard model (generic interactions):

Gapped + fully-symmetric $\overset{?}{\Longrightarrow}$ Fractionalization

How to identify fractionalization?

Half-filled (f=1) Honeycomb
Hubbard model (generic interactions):

Gapped + fully-symmetric $\not\Longrightarrow$ Fractionalization

could be SU(2)-singlet electron featureless insulator

Future directions: SU(2) spin-1/2 paramagnetic state?

Parent Hamiltonian for honeycomb state? Simple one?

Featureless (fully symmetric, non-fractionalized) bosonic Mott insulators:

1. If band gap (kagome): *Band to Mott*

2. On honeycomb at half site-filling (despite no band insulator)

3. Construction generalizes to symmorphic lattices

4. Hard core bosons

→ $f = 1$ featureless SU(2)-singlet **electron** insulator

Then, absence of order does not imply fractionalization

PRL 2013 (kagome), 1207.0498...[PNAS] (honeycomb)