Spin transport via gauge/gravity duality

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collaboration with
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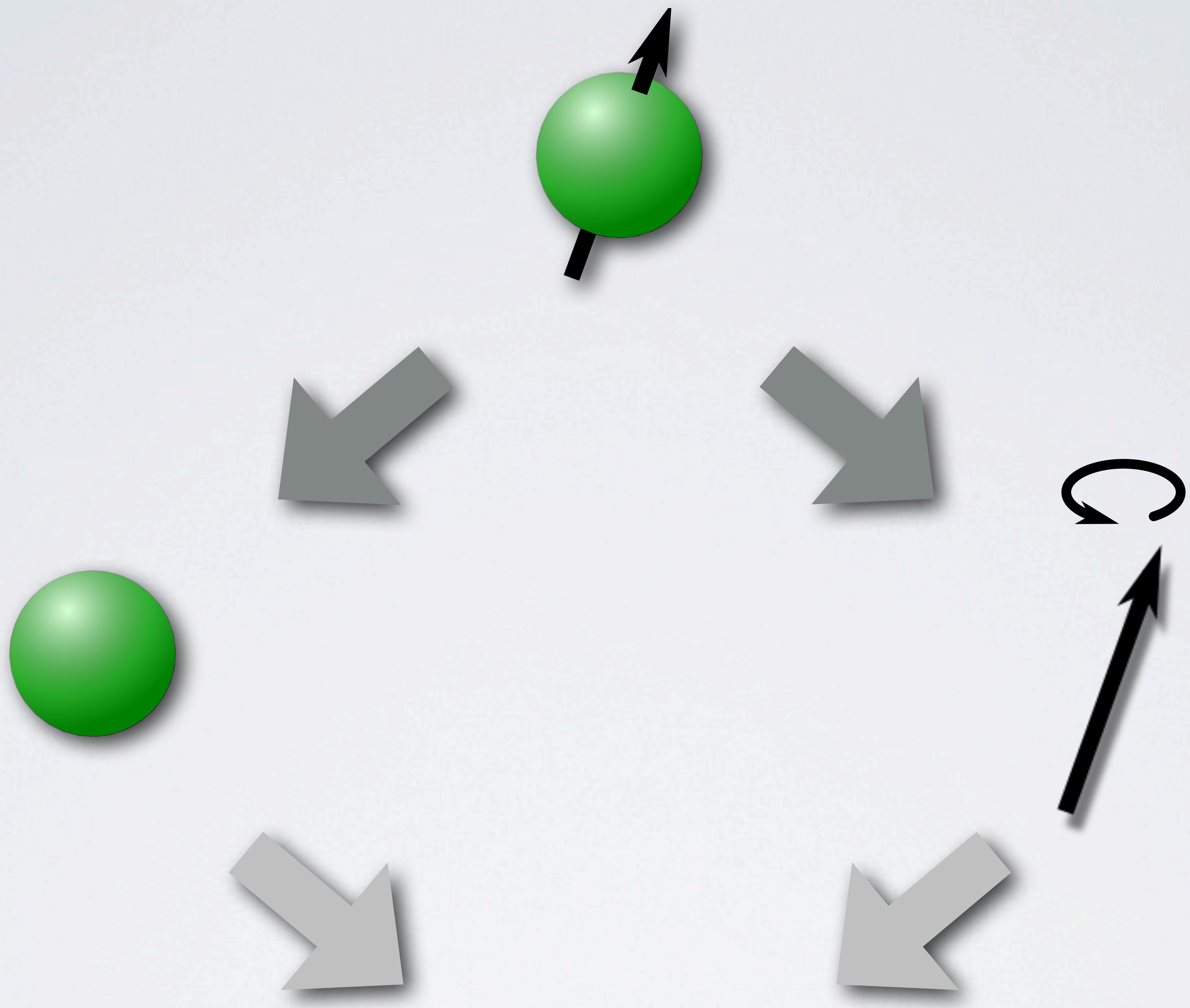
based on arXiv:1304.3126 [hep-th]

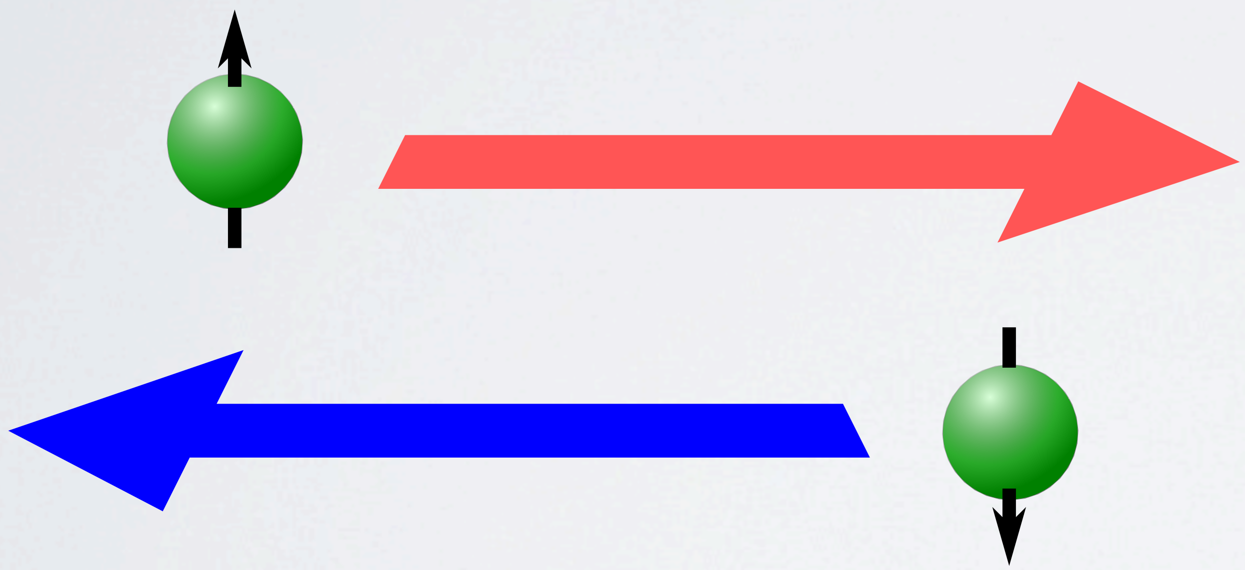
Why spin?



electronics + spin = spintronics

Spin transport

- Spin-depend conduction



$$\vec{J}_{\hat{z}} = \vec{J}_{\uparrow} - \vec{J}_{\downarrow}$$

cf. electric current :

$$\vec{J} = \vec{J}_{\uparrow} + \vec{J}_{\downarrow}$$

WARNING!!

This spin current is not conserved.

$$\partial_t S_{\hat{z}} + \vec{\nabla} \cdot \vec{J}_{\hat{z}} = \mathcal{T}_{\hat{z}}$$

1. approximately conserved situation
2. improve the definition of the current

Contents

Spin current definition

- Electric current

$$J^\mu = \frac{\delta S}{\delta A_\mu} \quad \Rightarrow \quad \partial_\mu J^\mu = 0$$

$\bar{\psi}\gamma^\mu\psi,\ i\partial^\mu\phi^*\phi - i\phi^*\partial^\mu\phi$

U(1) gauge symmetry

- Stress tensor

$$T^{\mu\nu} = \frac{\delta S}{\delta g_{\mu\nu}} \quad \Rightarrow \quad \partial_\mu T^{\mu\nu} = 0$$

Translation symmetry

- Spin current

cf. [Wen-Zee '92] [Froehlich-Studer '93]

spin connection

$$J^{\mu}{}_{\hat{a}\hat{b}} = \frac{\delta S}{\delta \boxed{\omega_{\mu}{}^{\hat{a}\hat{b}}}}$$

$$\Downarrow$$

$$\partial_{\mu} J^{\mu}{}_{\hat{a}\hat{b}} = 0$$

Local Lorentz symmetry
(rotation symmetry)

$$\bar{\psi}\left[i\gamma^{\mu}\left(\partial_{\mu}-\frac{i}{2}\omega_{\mu}{}^{\hat{a}\hat{b}}\Sigma_{\hat{a}\hat{b}}\right)-m\right]\psi$$

Covariant derivative : $\nabla = \partial - \Gamma$

Local Lorentz generator : $\Sigma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b]$

$$J^{\mu}{}_{\hat{a}\hat{b}} = \frac{1}{2}\bar{\psi}\gamma^{\mu}\Sigma_{\hat{a}\hat{b}}\psi$$

$$J^{\mu}{}_{\hat{a}} \equiv \epsilon_{\hat{0}\hat{a}\hat{b}\hat{c}}\, J^{\mu\,\hat{b}\hat{c}}$$

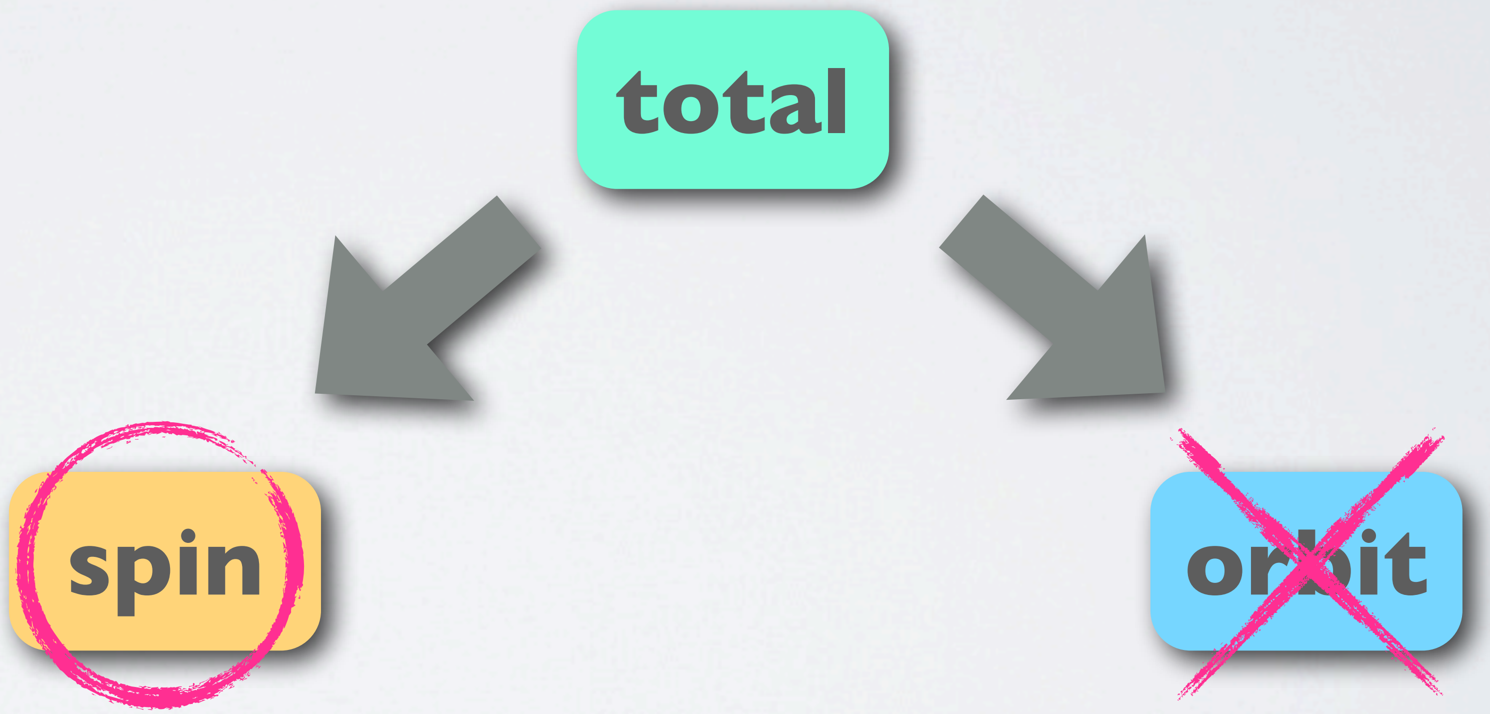
$$= \boxed{\frac{1}{2}\bar{\psi}\gamma^{\mu}\left(\sigma_{\hat{a}}\otimes \mathbb{1}\right)\psi}$$

- Remark

“spin current”
= total angular momentum current

Relativistic :

Non-relativistic :

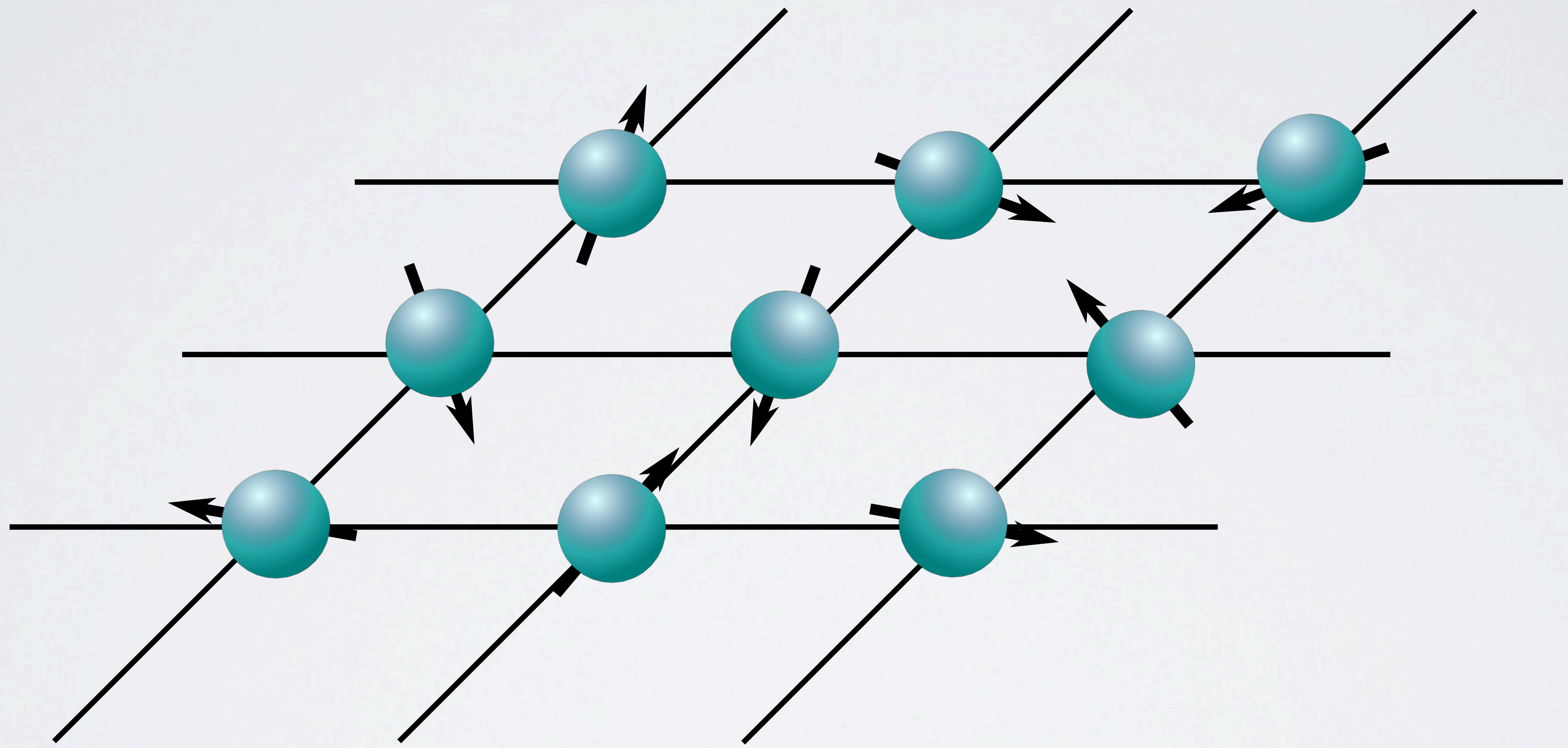


Summary

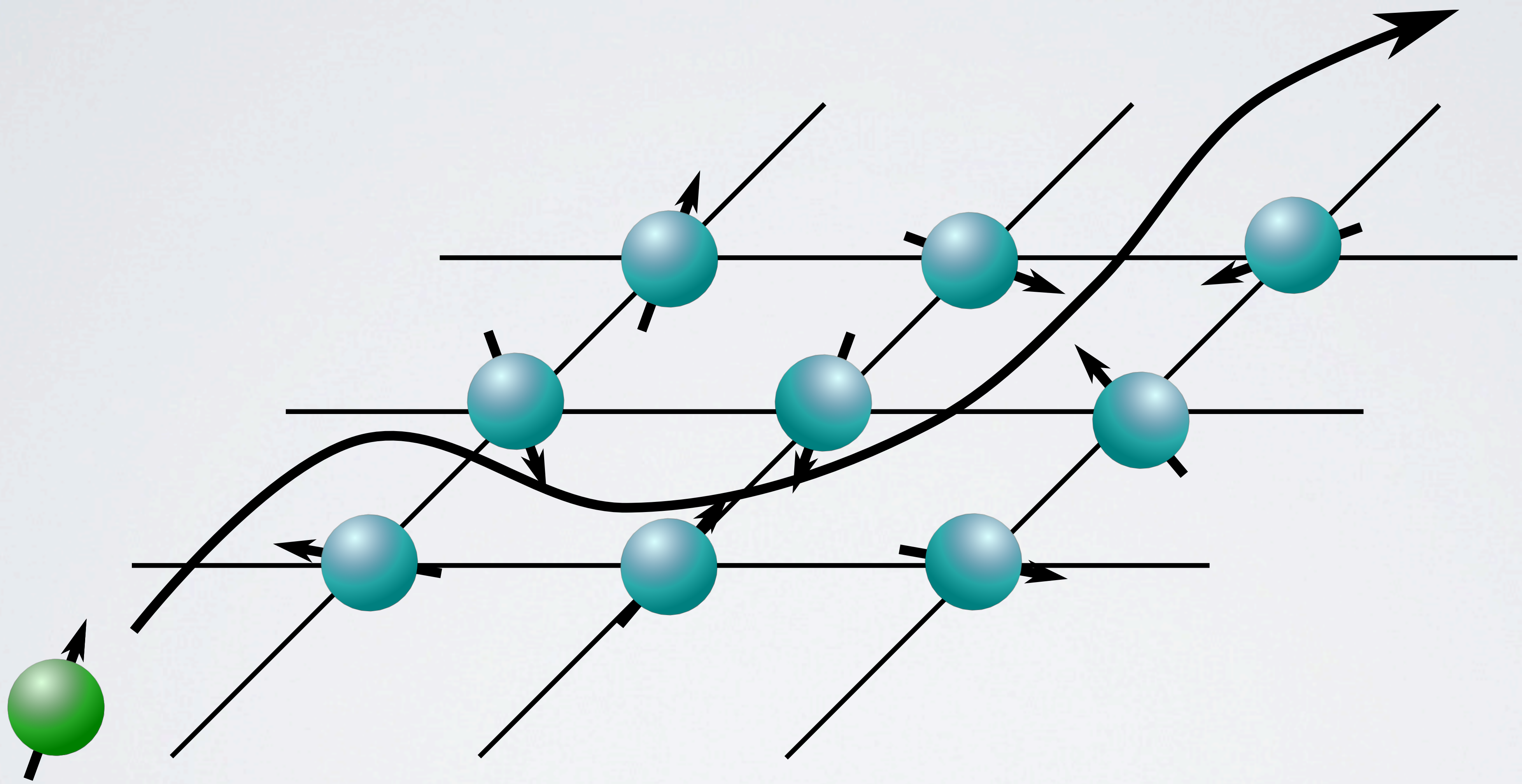
- Spin current definition
  - **spin connection** associated with local Lorentz symmetry
  - fermionic contribution → spin-depend current
  - a proper non-relativistic limit required

Contents

Strongly correlated spin transport



Strongly correlated spin transport



Strongly correlated spin transport

- A method to approach such a difficult system

Gauge/gravity duality

also AdS/CFT, holography,..

Strongly correlated QFT ⟷ **Classical gravity**

- Generating function

$$Z_{\mathrm{QFT}} = \int \mathcal{D}\phi \, e^{-S[\phi] + \int dx \, J\phi}$$

GKP-Witten relation

$$Z_{\mathrm{SUGRA}} = e^{-S_{\mathrm{gravity}}[\phi_0]}$$

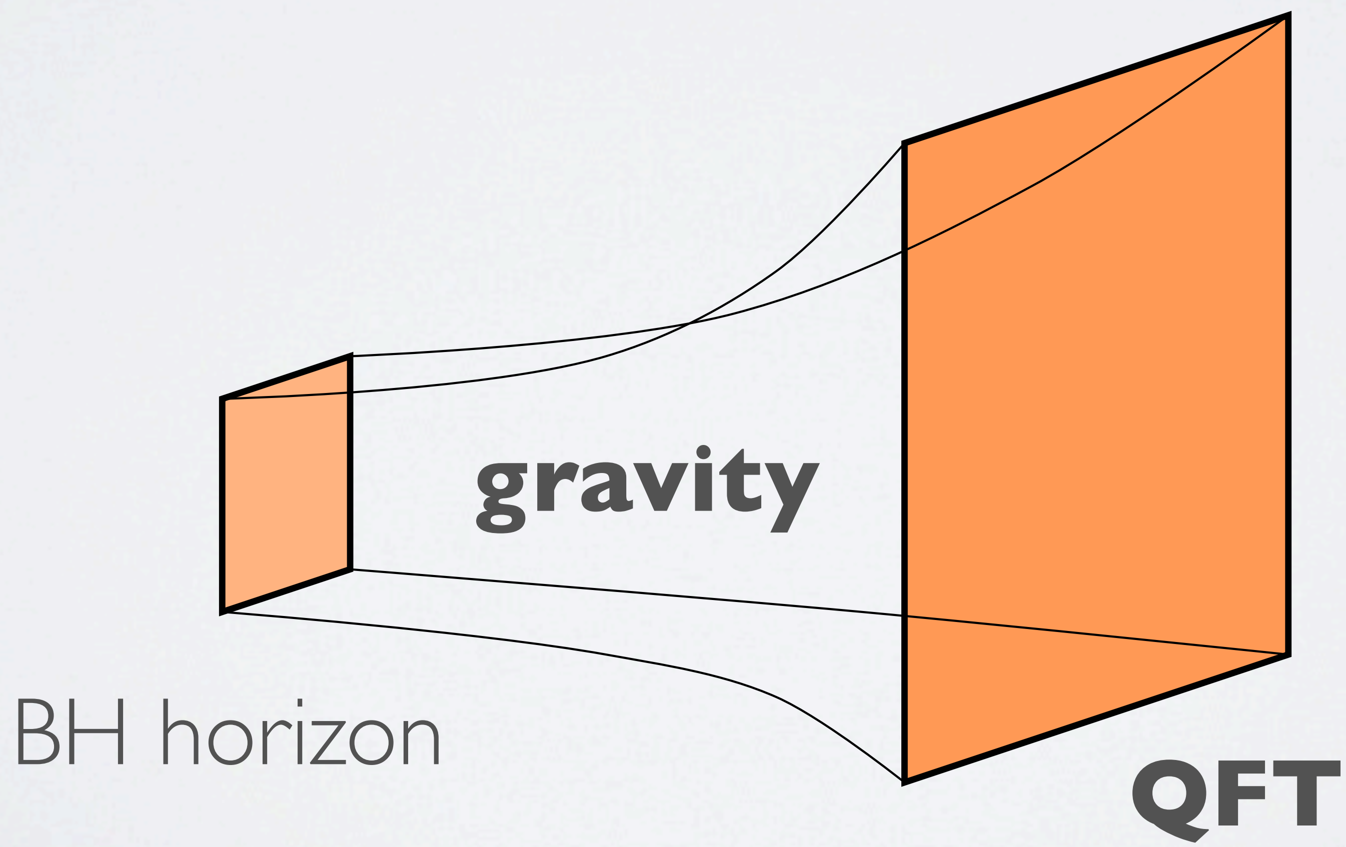
not quantum anymore, but classical

just solve Einstein equation

- Source term for QFT

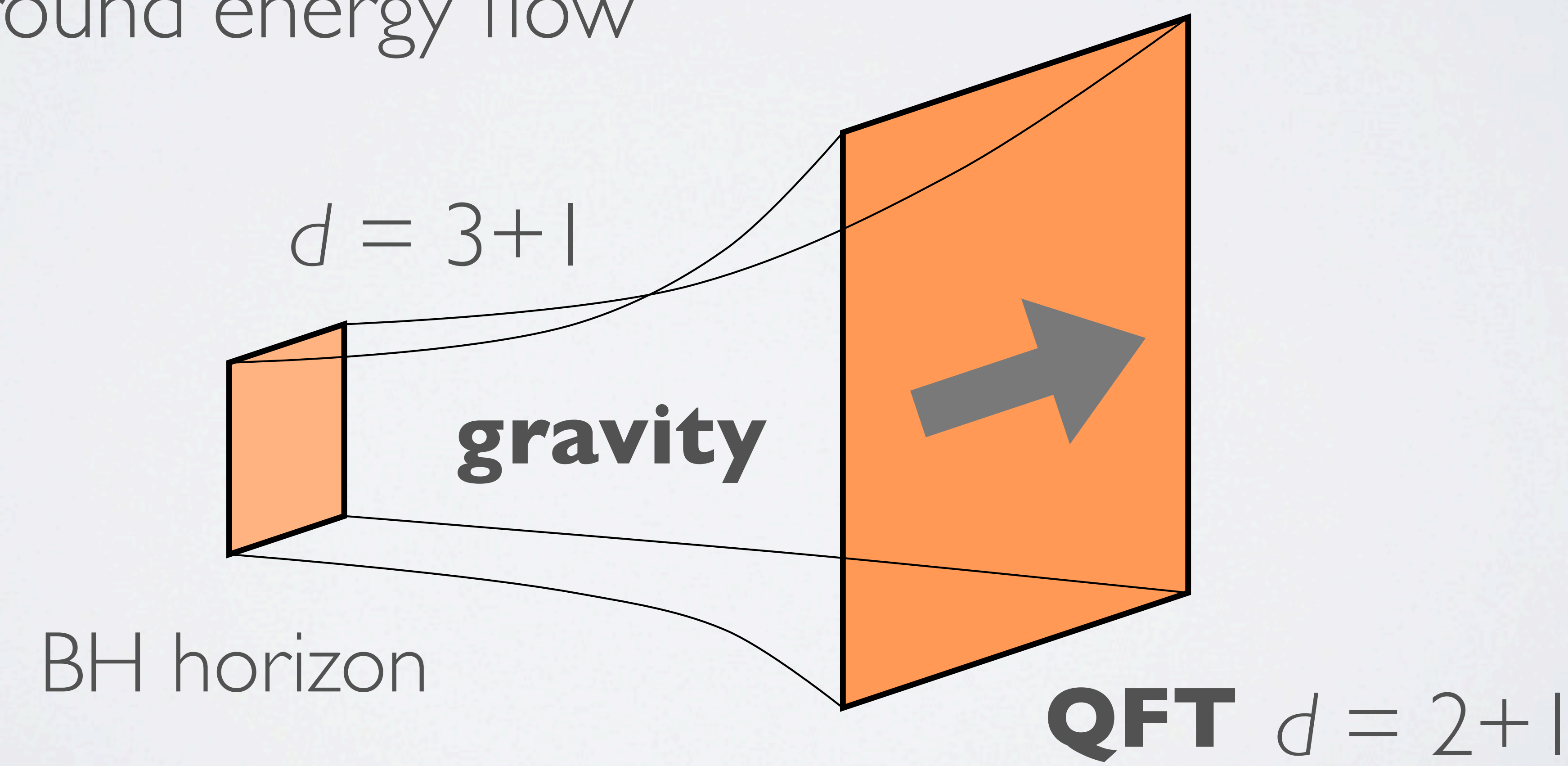
$$\mathcal{L}_{\text{source}} = A_\mu J^\mu,\ \ g_{\mu\nu} T^{\mu\nu},\ \ \omega_\mu{}^{\hat{a}\hat{b}} J^\mu{}_{\hat{a}\hat{b}}$$

- "Dictionary" between QFT and gravity



Example :
AdS_4 Schwarzschild geometry

- "Boosted" AdS_4 Schwarzschild black brane solution
  - Strongly correlated QFT (Conformal Field Theory)
  - $d = 2+1$
  - Finite temperature
  - Background energy flow



- "Boosted" AdS4 Schwarzschild black brane solution

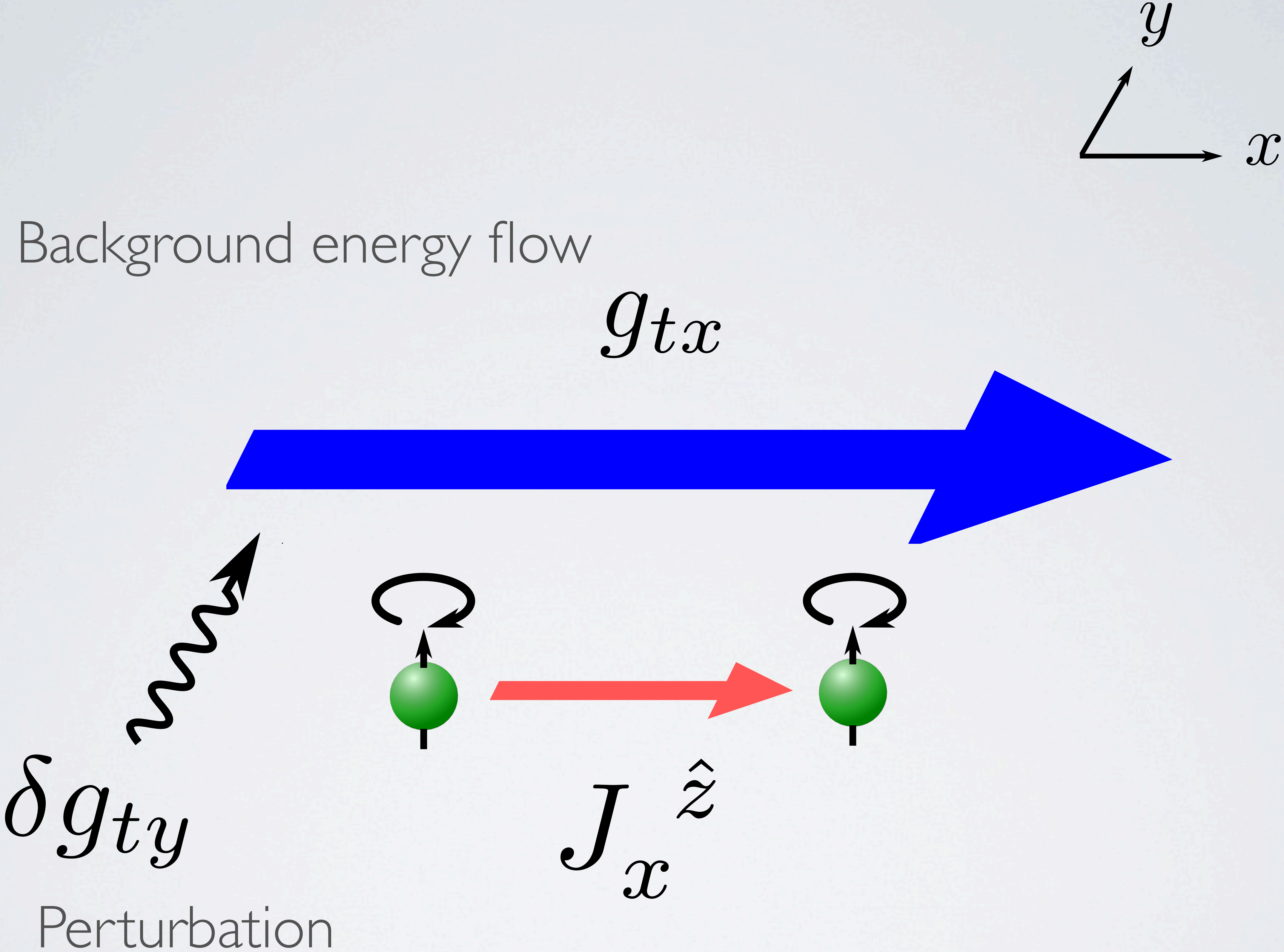
$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + r^2dy^2$$

off-diagonal

$$+\left(r^2 - a^2U(r)\right)dx^2 - 2aU(r)dtdx$$

with $U(r) = \dfrac{r^3 - r_0^3}{r}$

- Temperature : $T = \dfrac{3r_0}{4\pi}$
- "boost" : $t \longrightarrow t + at$



y
x
Background energy flow
g_{tx}
δg_{ty}
Perturbation
$J_x^{\hat{z}}$

- Gravity action

$$S_{\text{gravity}} = \int d^4x \sqrt{-g}\,\left(R[g] - 2\Lambda\right) + \int d^3x \sqrt{-\gamma}\,\Theta$$

- Fluctuations :

$$\delta g_{xy} = \epsilon\, e^{-i\omega t} r^2 h(r)$$
$$\delta g_{ty} = \epsilon\, e^{-i\omega t} r^2 f(r)$$

- Einstein equations (e.o.m.)

$$ry\text{-comp.}: \quad f'(r) = \left(a + \frac{1}{a}\frac{r^3}{r_0^3 - r^3}\right)^{-1} h'(r)$$

$$ty\text{-comp.}: \quad h(r) + \frac{1}{\omega^2}\frac{r^3 - r_0^3}{r^3}\frac{d}{dr}\left[\frac{r^3 - r_0^3}{(1-a^2)r^3 + a^2 r_0^3} r^4 \frac{d}{dr} h(r)\right] = 0$$

- Einstein equations (e.o.m.)

$$ry\text{-comp.}: \quad f'(r) = \left(a + \frac{1}{a}\frac{r^3}{r_0^3 - r^3}\right)^{-1} h'(r)$$

$$ty\text{-comp.}: \frac{\omega^2}{r_0^2} h(x) = x^2(x^3 - 1)\frac{d}{dx}\left[\frac{1 - x^3}{x^2(1 - a^2 + a^2 x^3)}\frac{d}{dx}h(x)\right]$$

dimensionless ratio : $\dfrac{\omega}{r_0} = \dfrac{3}{4\pi}\dfrac{\omega}{T}$ with $x = \dfrac{r_0}{r}$

Boundary conditions

- Near the horizon $x \sim 1$

$$h(x) \sim \exp\left(-\frac{i}{3}\frac{\omega}{r_0}\log(1-x)\right)$$

in-going condition

- Near the boundary $x \sim 0$
    - Two independent solutions

$$h(x) \sim h_0 + \mathcal{O}(x^2)$$

Non-normalizable mode

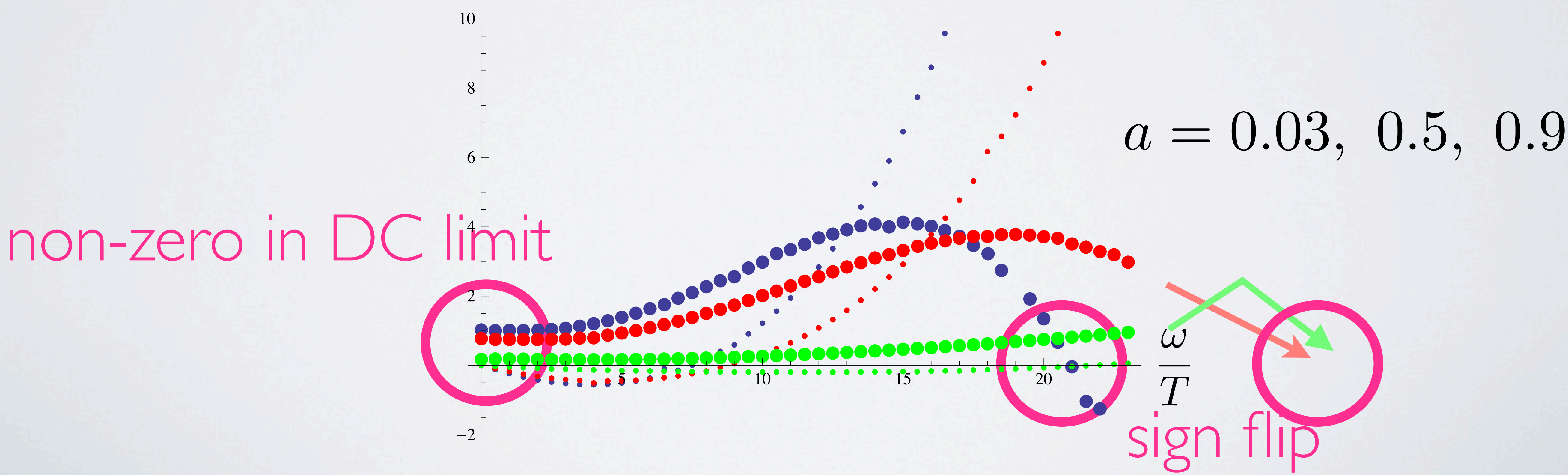
$$h(x) \sim h_3\, x^3 + \mathcal{O}(x^5)$$

Normalizable mode

- Thermal spin Hall conductivity

$$J_x{}^{\hat{z}} = -\kappa_{\mathrm{sH}} \nabla_y T$$

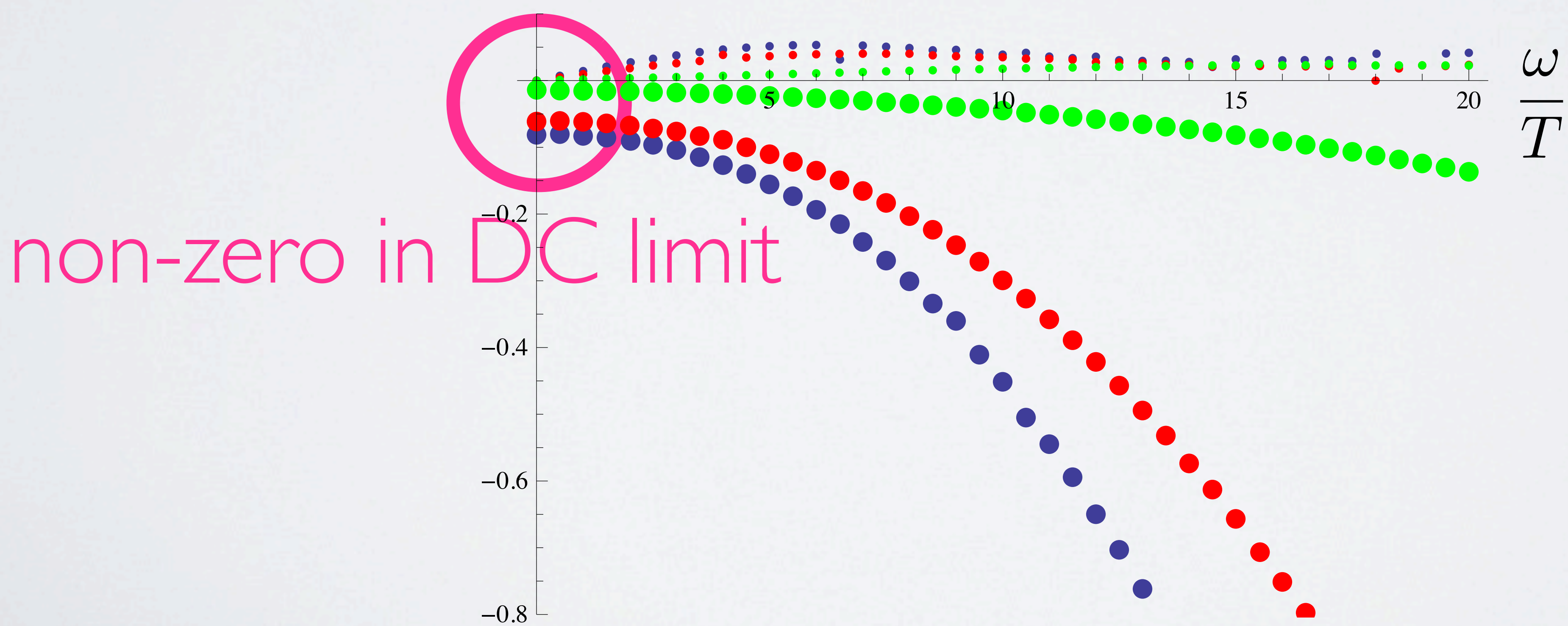
$$\Rightarrow \quad \kappa_{\mathrm{sH}} = -\frac{J_x{}^{\hat{z}}}{\nabla_y T} = \frac{\omega_x{}^{\hat{x}\hat{y}\,(\mathrm{N})}}{i\omega T g_{ty}^{(\mathrm{NN})}} \propto \frac{h_3}{f_0}$$



- Another spin transport coefficient

$$J_x^{\hat{z}} = -\alpha_{\rm s}\, \nabla_x \mu^{\hat{z}}$$

$$\alpha_{\rm s} = -\frac{J_x^{\hat{z}}}{\nabla_x \mu^{\hat{z}}} = \frac{\omega_x{}^{\hat{x}\hat{y}\,(\mathrm{N})}}{i\omega\, \omega_x{}^{\hat{x}\hat{y}\,(\mathrm{NN})}} \propto \frac{h_3}{h_0}$$



Summary

- Spin transport via gauge/gravity duality
  - Example : pure gravity on "boosted" AdS_4 Schwarzschild
  - ω-dependence of thermal spin Hall conductivity and more

Summary

Summary

- Definition of the spin current as a conserved one
    - spin connection associated with local Lorentz symmetry
- Spin transport via gauge/gravity duality
    - AC conductivities estimated

Summary

- TO DO
  - Non-relativistic limit
  - Analysis with U(1) EM field