Efimov effect in quantum magnets

Yusuke Nishida (Tokyo Tech)

ISSP International Workshop on
"Emergent Quantum Phases in
Condensed Matter" June 18 (2013)

Plan of this talk

1. Universality in physics
2. What is the Efimov effect?

   Keywords: universality, discrete scale invariance, RG limit cycle

3. Efimov effect in quantum magnets*
4. New progress: Super Efimov effect

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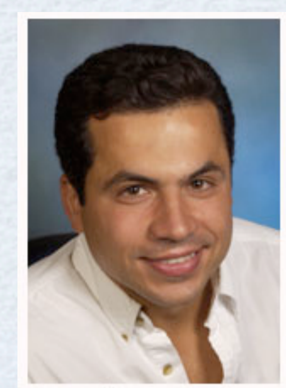
Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista

Physics is said to be universal when it emerges regardless of the underlying microscopic details. A prominent example is



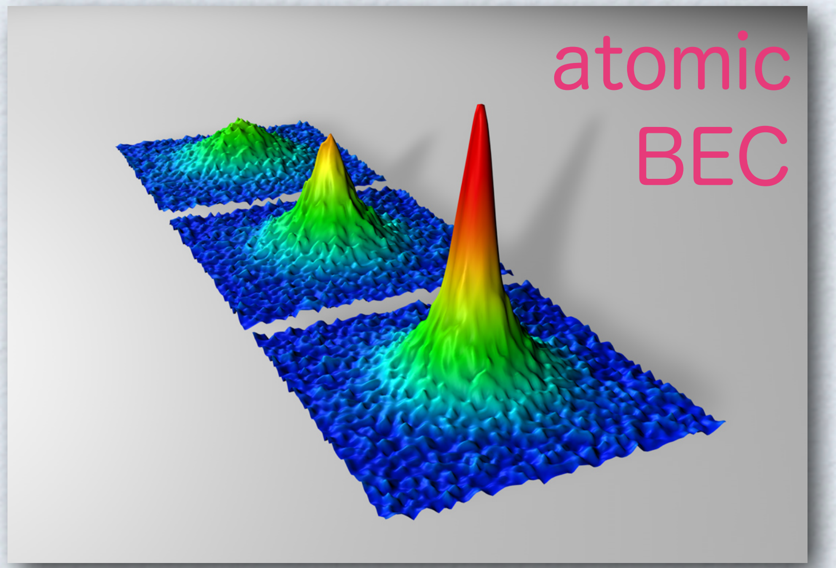
Cristian Batista

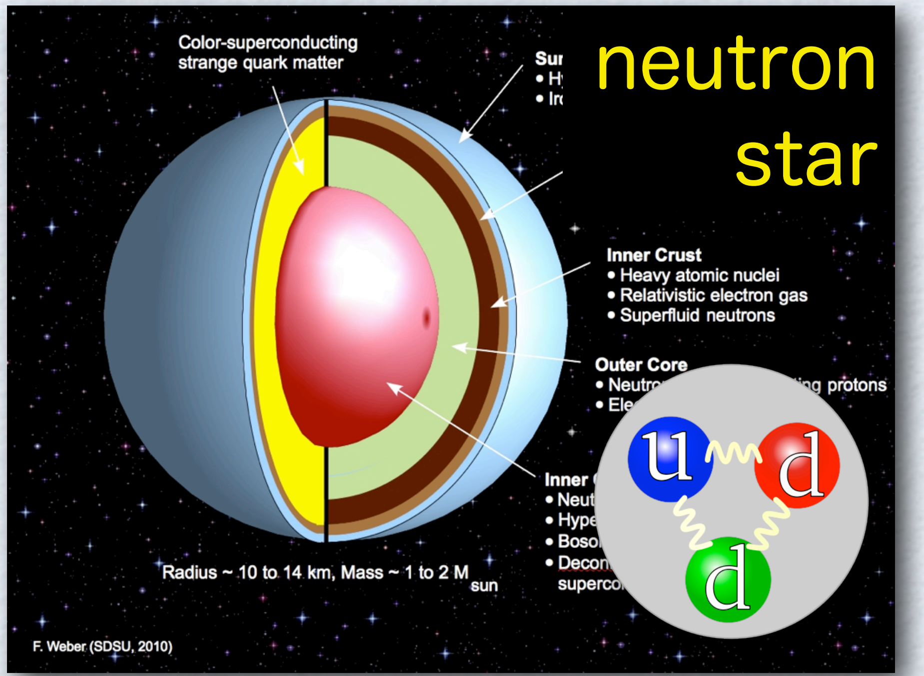
Introduction

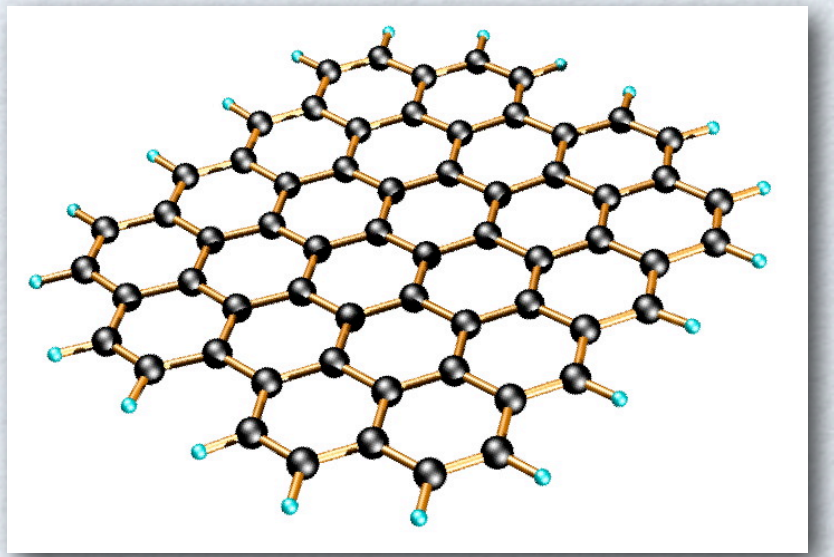
1. **Universality in physics**
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. New progress: Super Efimov effect

(ultimate) Goal of research

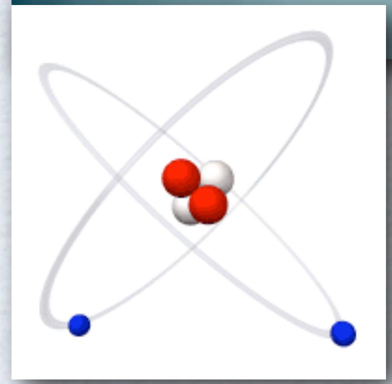
Understand physics of few and many particles governed by quantum mechanics

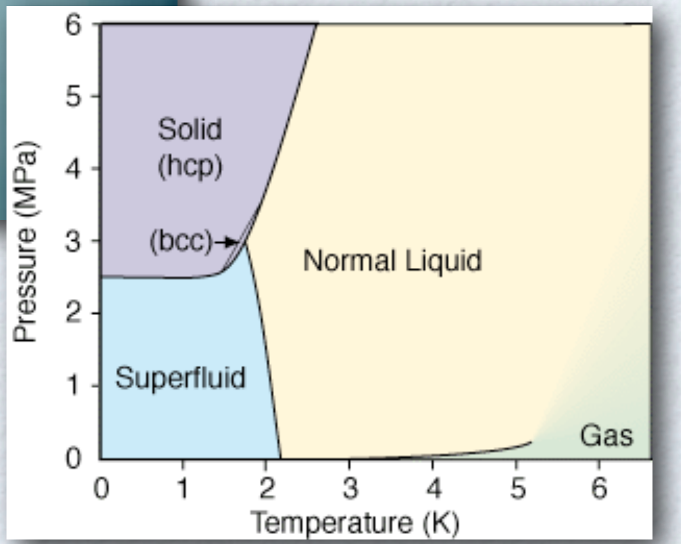




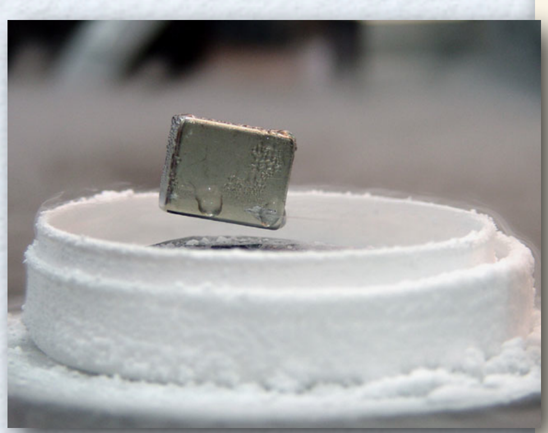


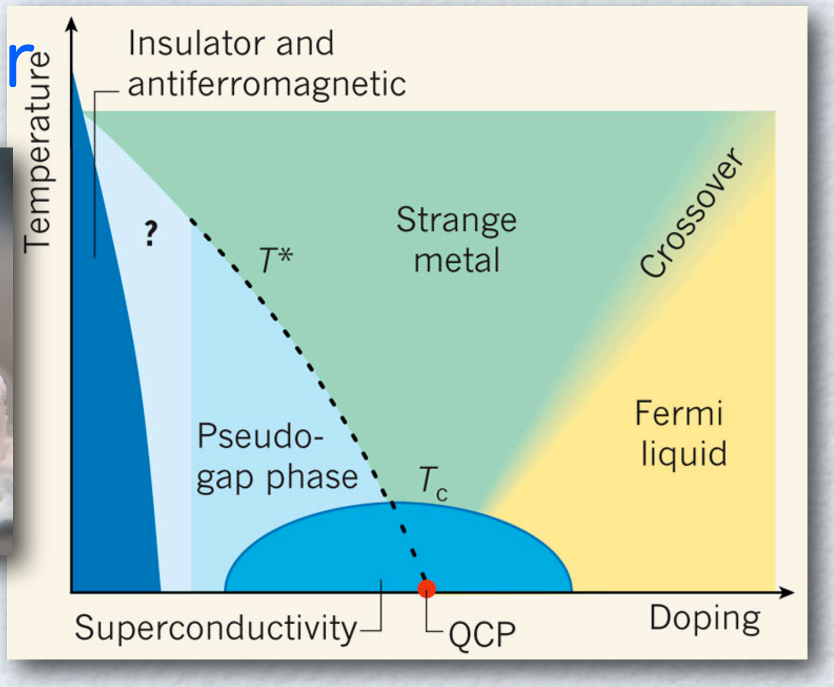
graphene





superconductor

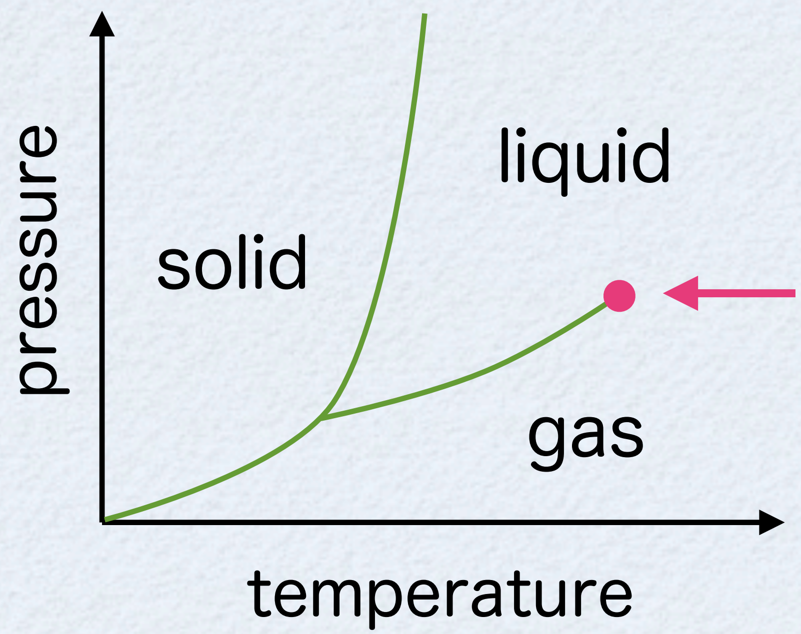


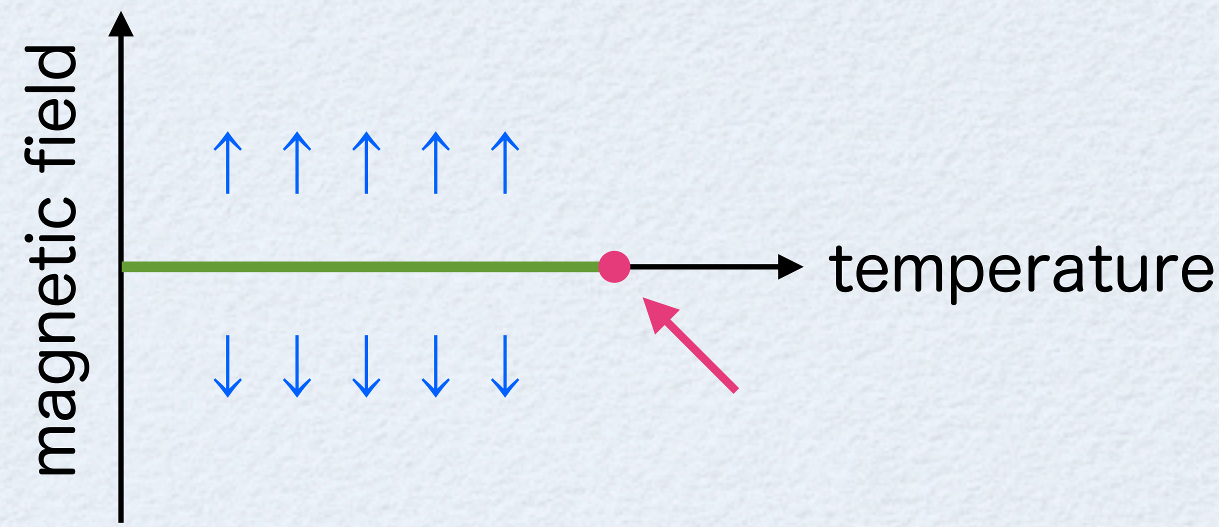


When physics is universal ?

A1. Continuous phase transitions $\Leftrightarrow$ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 



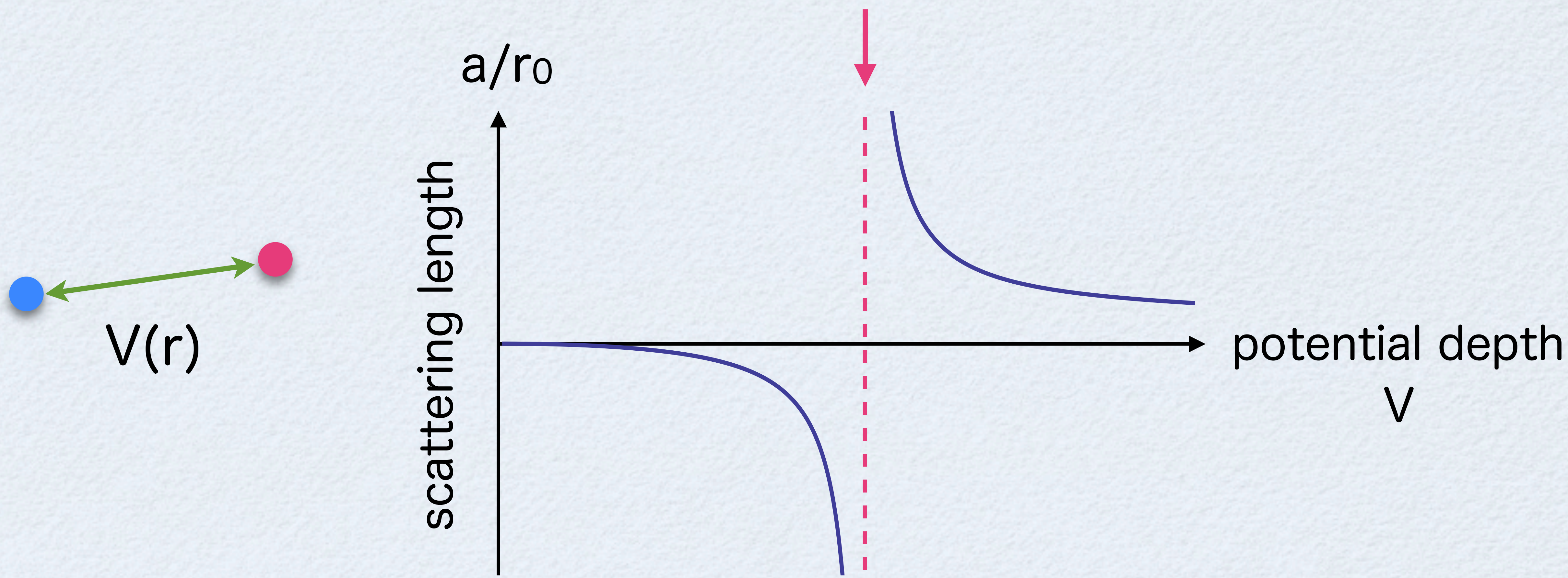


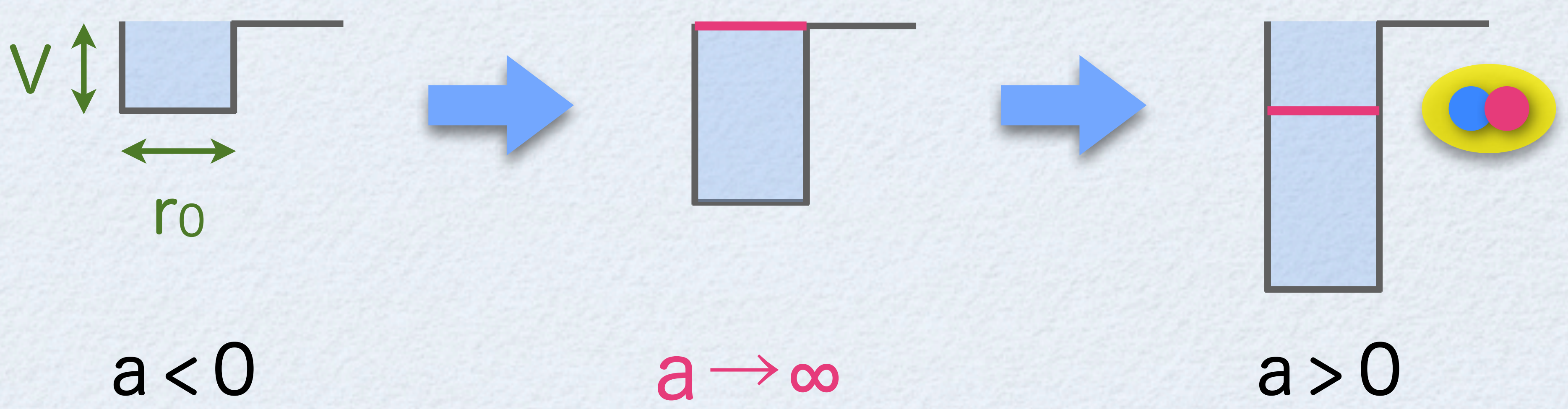
Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\rm liq} - \rho_{\rm gas} \sim (T_{\rm c} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\rm c} - T)^{\beta}$$

When physics is universal ?

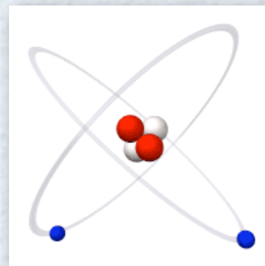
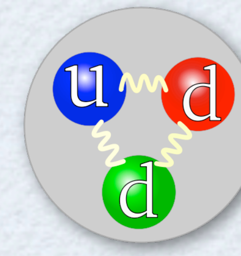
A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$





When physics is universal ?

A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$

E.g. ^{4}He atoms  vs. proton/neutron 

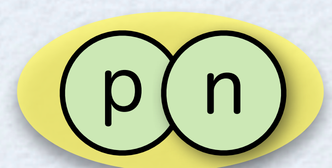
van der Waals force:

$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$

$E_{binding} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{binding} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\text{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

Physics only depends on the scattering length "a"

Efimov effect

1. Universality in physics
2. **What is the Efimov effect?**
3. Efimov effect in quantum magnets
4. New progress: Super Efimov effect

Efimov effect

Efimov (1970)

Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

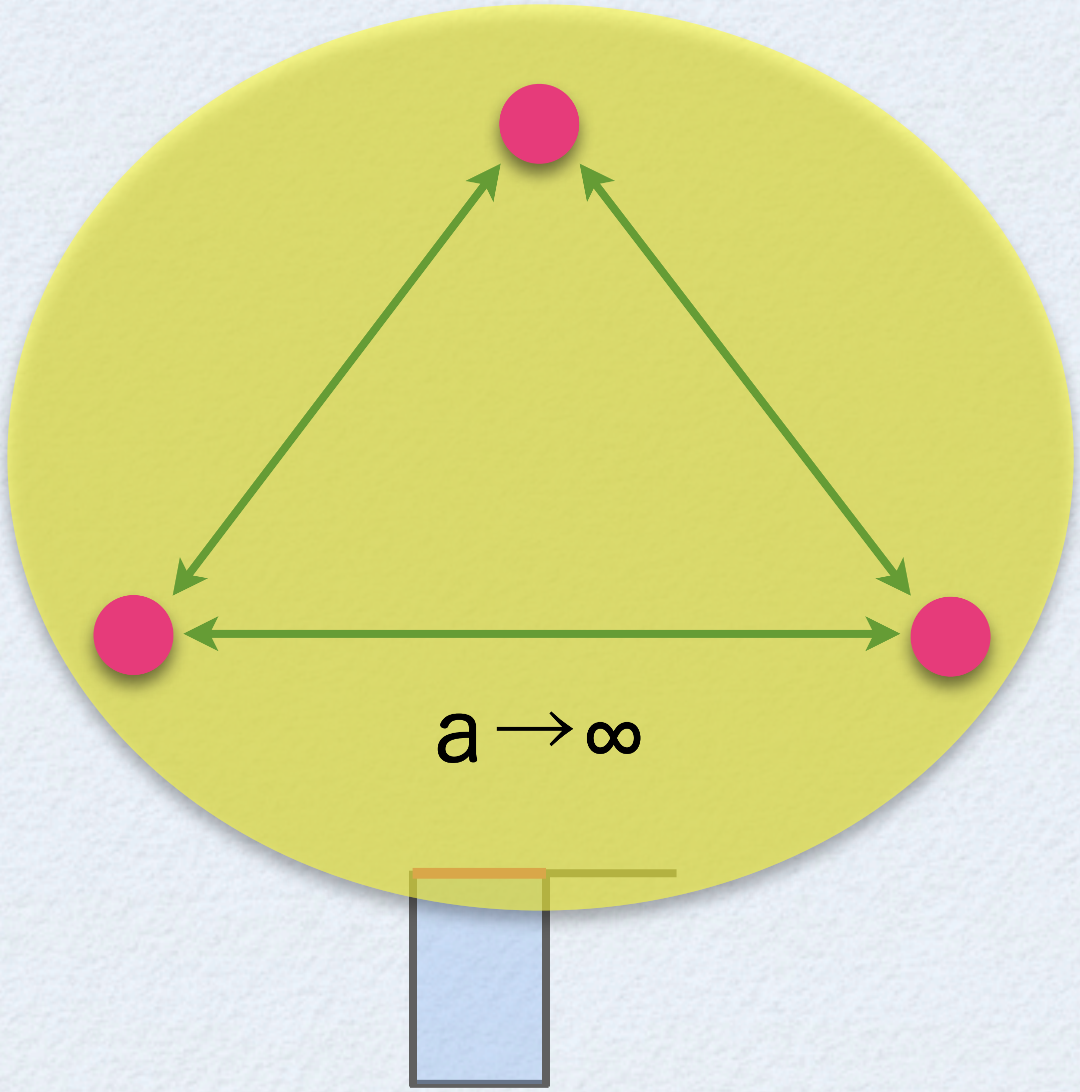
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



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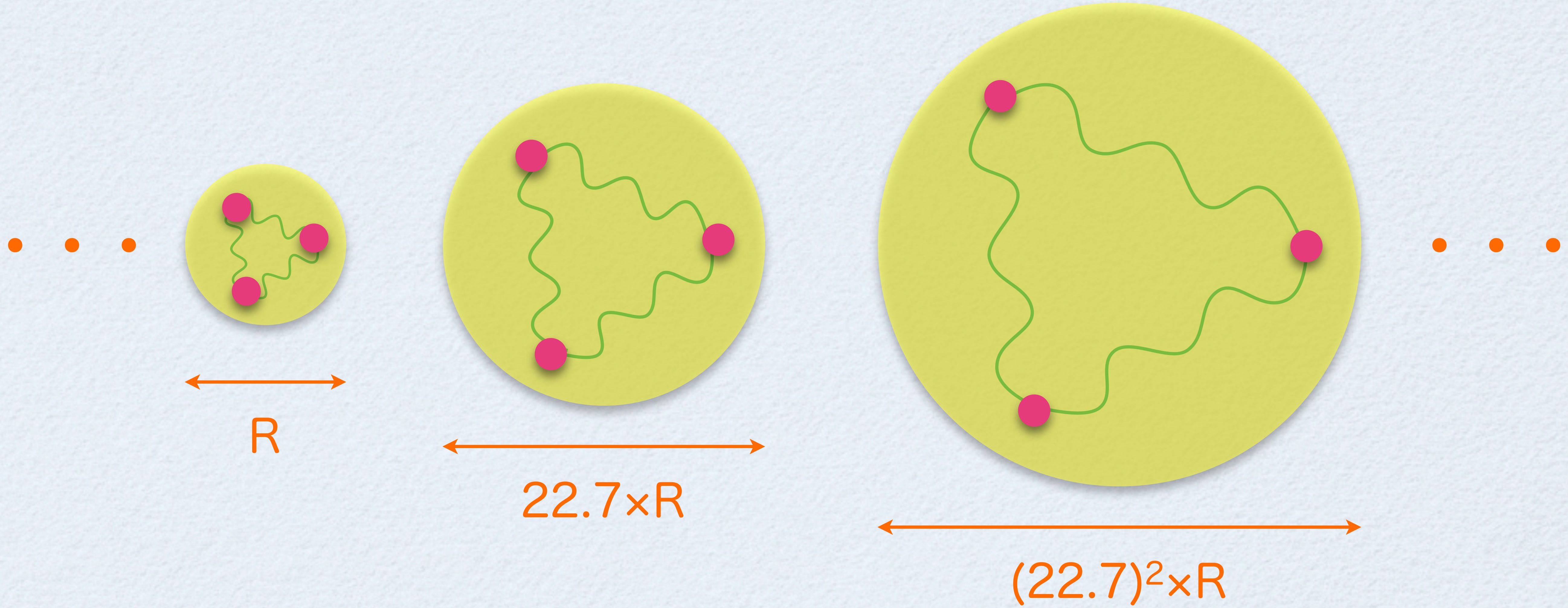


Discrete scaling symmetry

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)

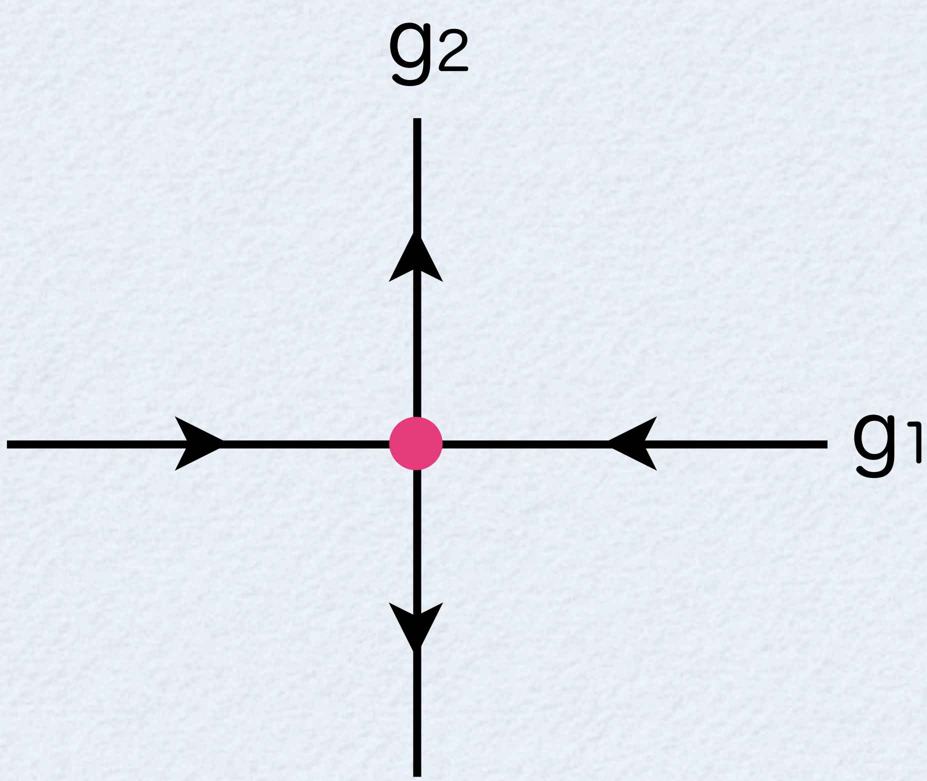
. . .

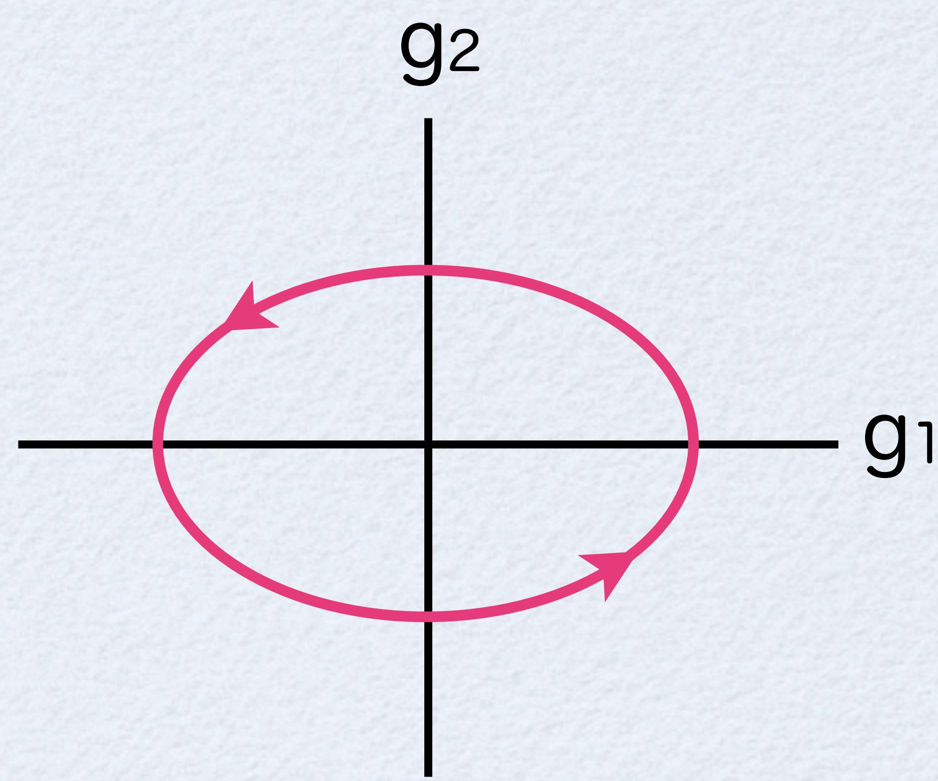
. . .

Discrete scaling symmetry

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

RG limit cycle

⇒ Discrete scale invariance

E.g. Efimov effect

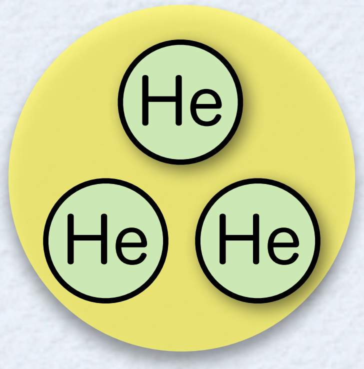
Rare manifestation in physics!

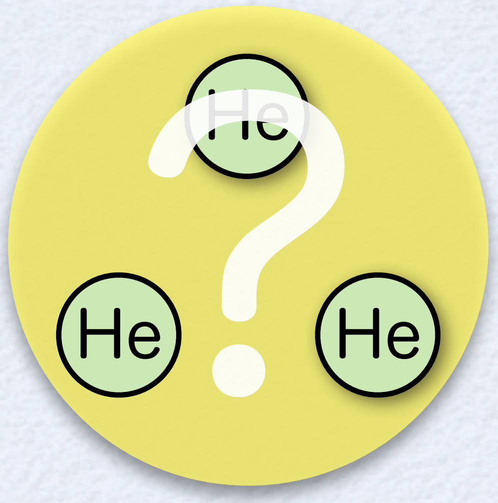
Where Efimov effect appears ?

× Originally, Efimov considered
^{3}H nucleus (≈3n) and ^{12}C nucleus (≈3α)

△ ^{4}He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted
1 was observed (1994)



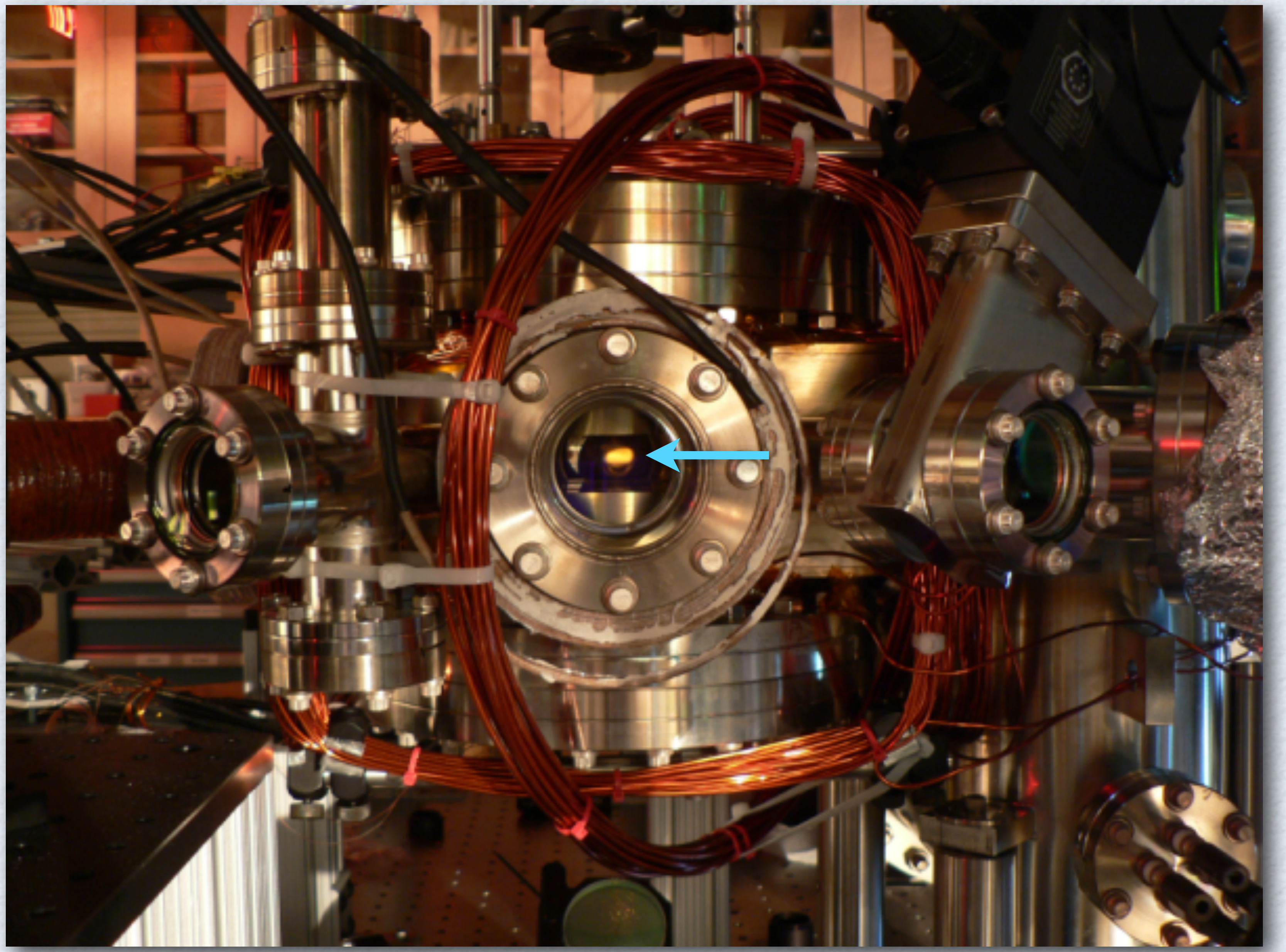


E_b = 125.8 mK   (E_b = 2.28 mK)

→ Ultracold atoms !

Ultracold atom experiments

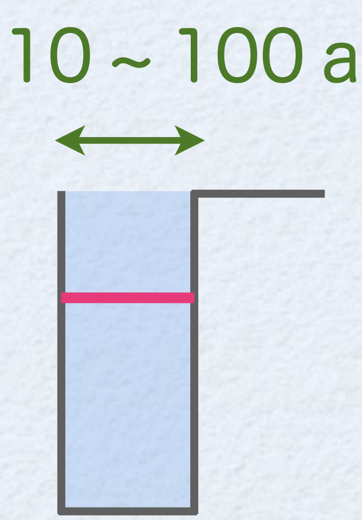
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

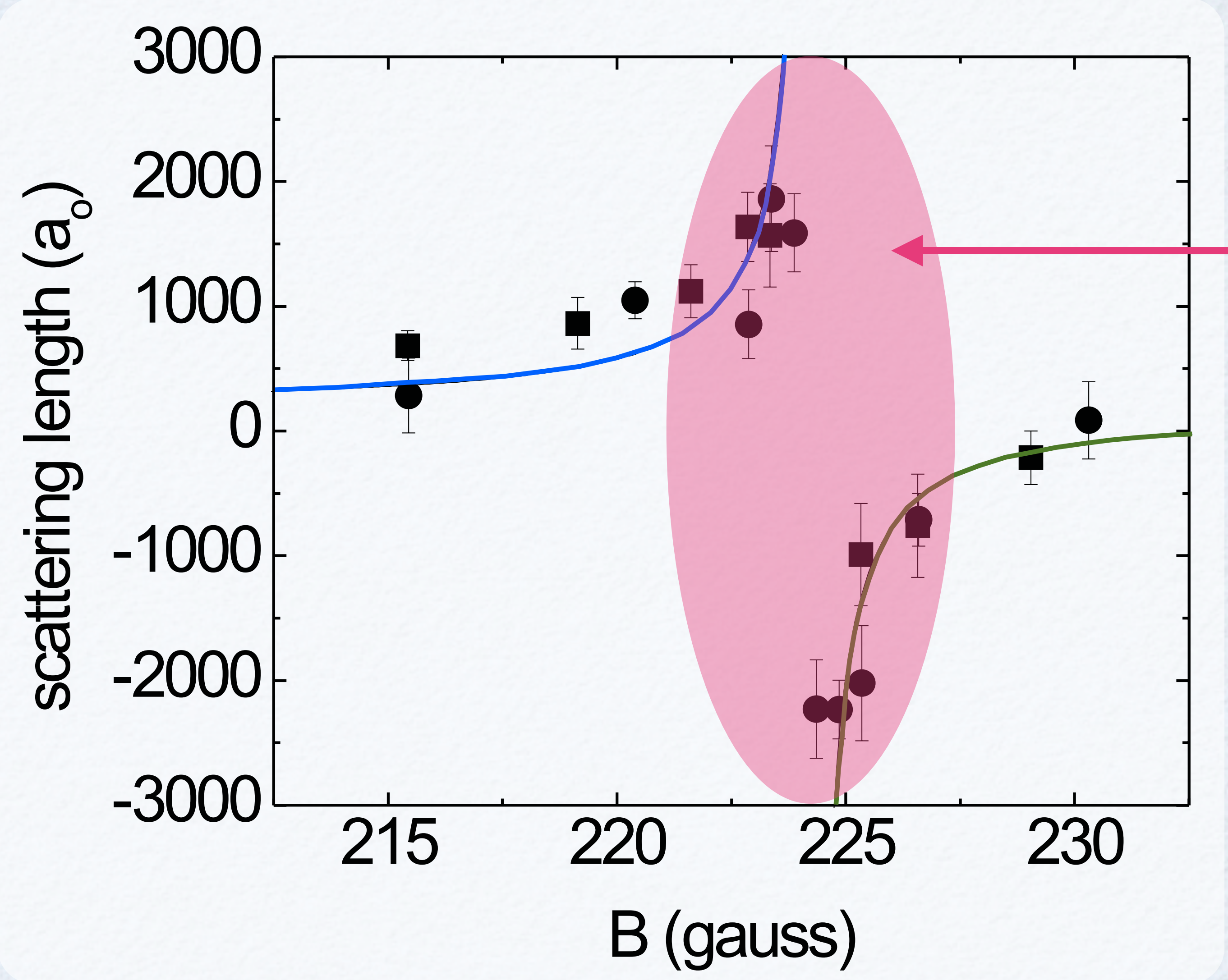


Ultracold atom experiments

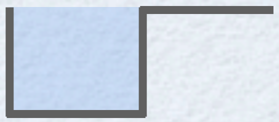
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances



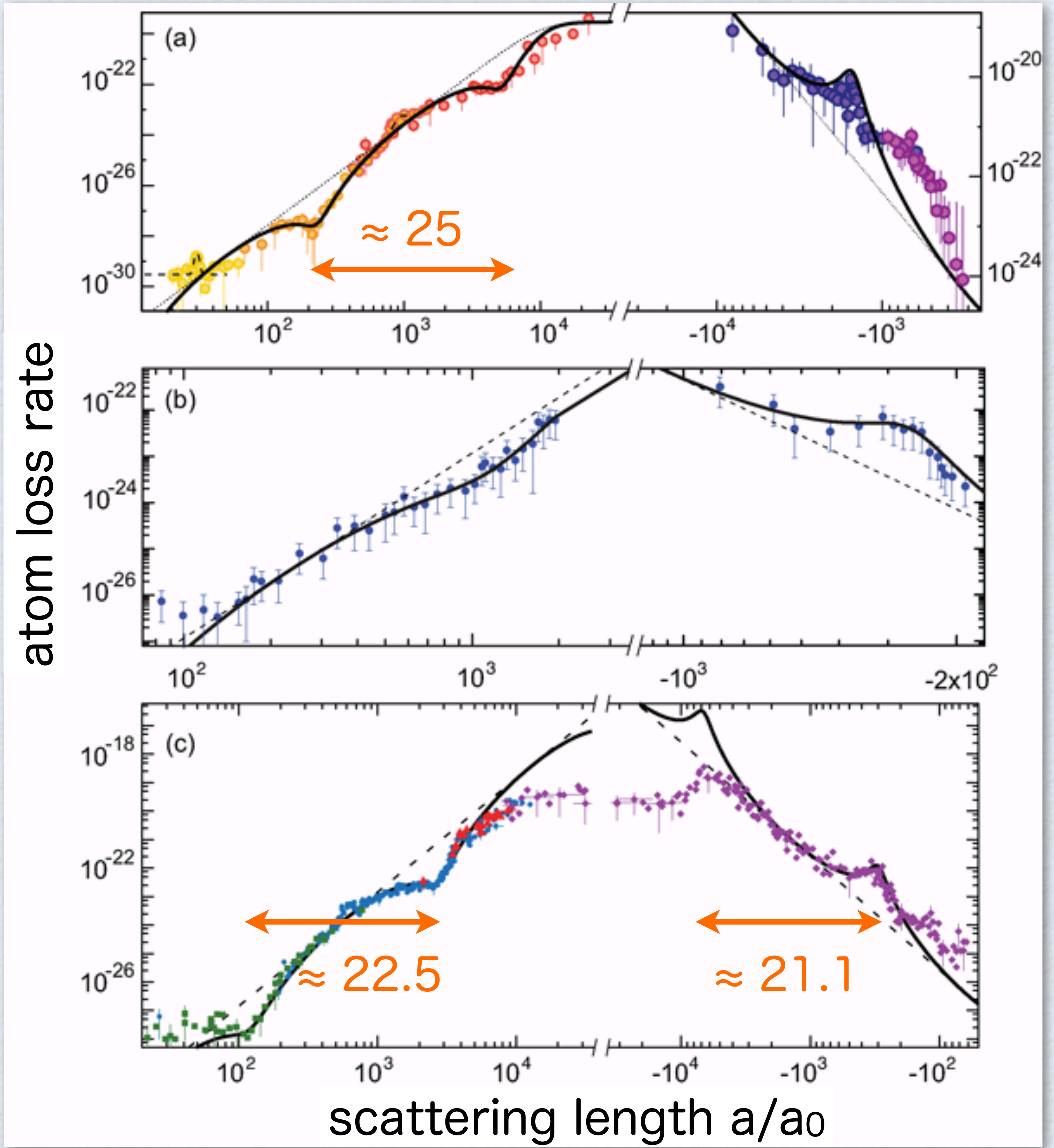


Universal regime



C.A. Regal & D.S. Jin PRL90 (2003)

Ultracold atom experiments



Florence group for ^{39}K (2009)

Bar-Ilan University for ^{7}Li (2009)

Rice University for ^{7}Li (2009)

Discrete scaling & Universality !

Efimov effect is "universal" ?

- Efimov effect is "universal"

  = appears regardless of microscopic details

  (physics technical term)

- Efimov effect is not "universal"

  universal = present or occurring everywhere

  (Merriam-Webster Online)

→ Can we find the Efimov effect in other physical systems ?

Efimov effect in quantum magnets

1. Universality in physics
2. What is the Efimov effect?
3. **Efimov effect in quantum magnets**
4. New progress: Super Efimov effect

Quantum magnet

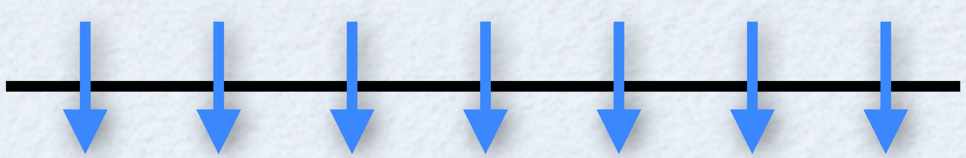
Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\right]$$

exchange anisotropy

single-ion anisotropy

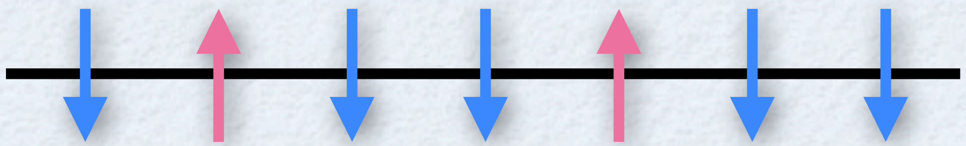
Spin-boson correspondence



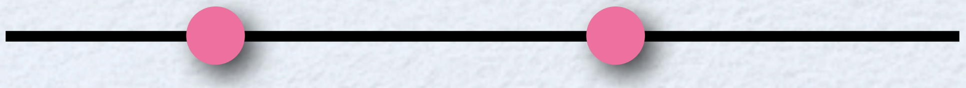
$\Leftrightarrow$

fully polarized state (B→∞)

No boson = vacuum



$\Leftrightarrow$



N spin-flips

N bosons = magnons

Quantum magnet

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xy-exchange coupling

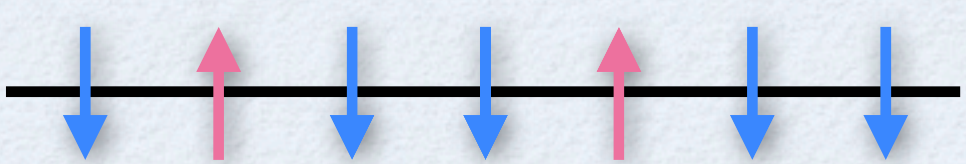
⇔ hopping

single-ion anisotropy

⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction



⇔

N spin-flips

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Quantum magnet

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xy-exchange coupling

⇔ hopping

single-ion anisotropy

⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction

Tune these couplings to induce

scattering resonance between two magnons

⇒ Three magnons show the Efimov effect

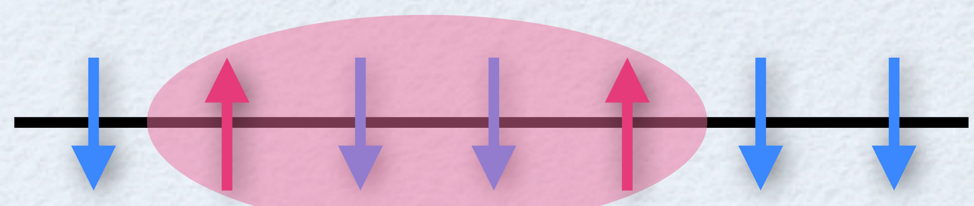
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

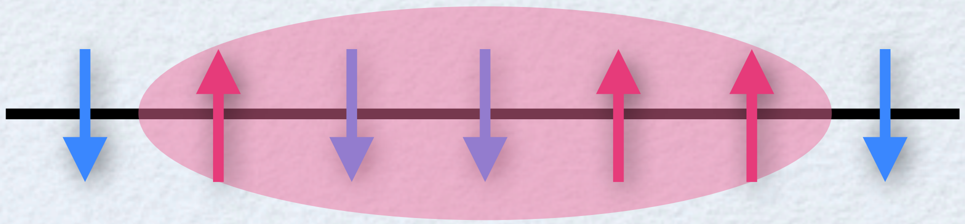
Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- **Spin-1, J_z=J>0**

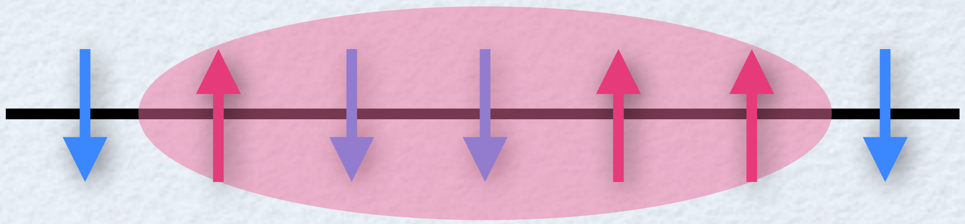
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- **Spin-1, J_z=J<0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



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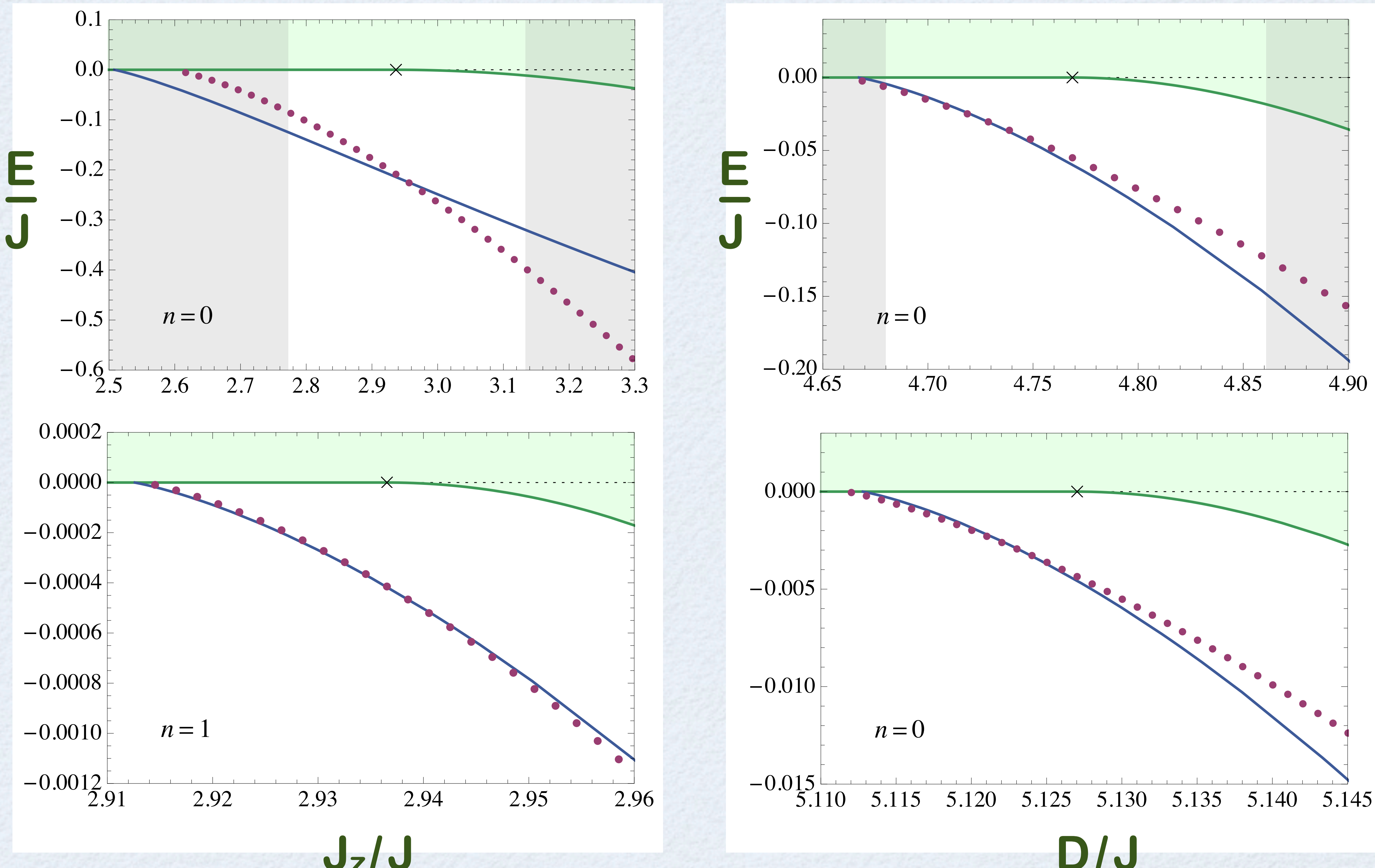
Universal scaling law by ~ 22.7 confirms they are Efimov states !

Three-magnon spectrum

- Spin-1/2



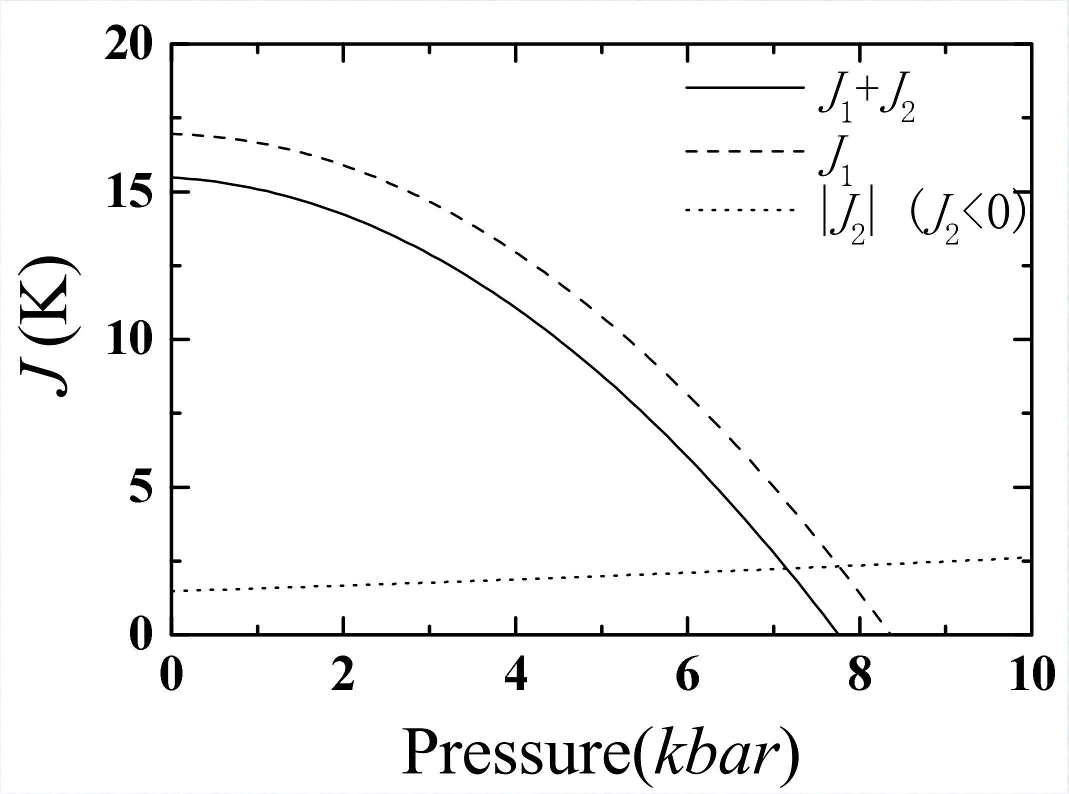
- S=1 , J_z=J>0
- S=1 , J_z=J<0

Agree with universal prediction : $E_n = -\lambda^{-2n} \dfrac{\kappa_*^2}{m} F\!\left(\dfrac{\lambda^n}{\kappa_* a_s}\right)$

Toward experimental realization

1. Find a good compound
   whose anisotropy is close to the critical value
   E.g. Ni-based organic ferromagnet with D/J~3 (critical 4.8)
   R. Koch et al., Phys. Rev. B 67, 094407 (2003)

C.f. TDAE-C_{60}



T. Kawamoto et al, JPSJ (2001)

2. Tune the exchange coupling
   with pressure to induce
   the two-magnon resonance

3. Observe the Efimov states
   of three magnons with

- absorption spectroscopy
- inelastic neutron scattering
- electron spin resonance

(see Y.N., arXiv:1302.5908)

Toward experimental realization

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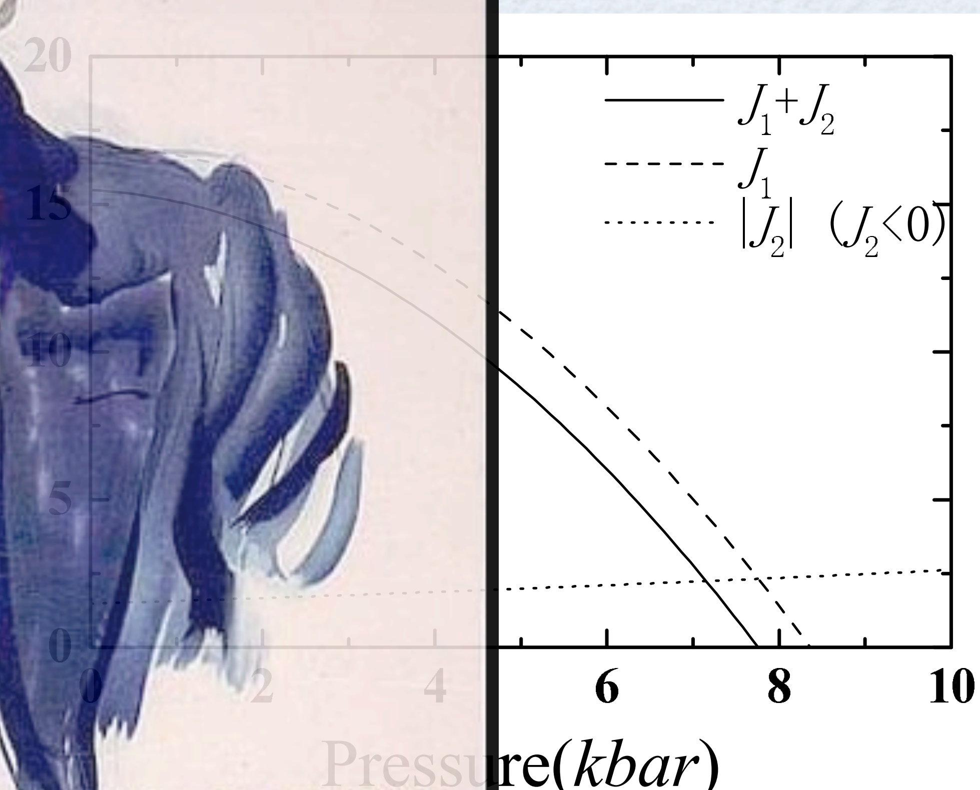

T. Kawamoto et al, JPSJ (2001)

3. Observe the Efimov states
   of three magnons with
   - absorption spectroscopy • electron spin resonance
   - inelastic neutron scattering (see Y.N., arXiv:1302.5908)

Find interested experimentalists !

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

Efimov effect in quantum magnets induced by

- exchange anisotropy
- single-ion anisotropy
- spatial anisotropy
- fructration

[Y.N., Y.K, C.D.B, Nature Physics 9, 93-97 (2013)]

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

Atomic BEC (1995 )

Efimov effect (2006 )

Magnon BEC (1999 )

Efimov effect (201? ?)

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

condensed matter

Atomic BEC (1995 )

Magnon BEC (1999 )

Efimov effect (2006 )

Efimov effect (201? ?)

New link between atomic and magnetic systems

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

condensed matter

Few-body physics

Many-body physics

- magnetism
- superconductivity
- superfluidity
- ...

How interplay ?

New progress

1. Universality in physics
2. What is the Efimov effect ?
3. Efimov effect in quantum magnets
4. **New progress: Super Efimov effect**

Super Efimov effect

PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

Yusuke Nishida,[1] Sergej Moroz,[2] and Dam Thanh Son[3]

[1] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2] *Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[3] *Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA*

(Received 18 January 2013; published 4 June 2013)

We study a system of spinless fermions in two dimensions with a short-range interaction fine-tuned to a p-wave resonance. We show that three such fermions form an infinite tower of bound states of orbital angular momentum $\ell = \pm 1$ and their binding energies obey a universal doubly exponential scaling $E_3^{(n)} \propto \exp(-2e^{3\pi n/4+\theta})$ at large n. This "super Efimov effect" is found by a renormalization group analysis and confirmed by solving the bound state problem. We also provide an indication that there are $\ell = \pm 2$ four-body resonances associated with every three-body bound state at $E_4^{(n)} \propto \exp(-2e^{3\pi n/4+\theta-0.188})$. These universal few-body states may be observed in ultracold atom experiments and should be taken into account in future many-body studies of the system.

DOI: 10.1103/PhysRevLett.110.235301 PACS numbers: 67.85.Lm, 03.65.Ge, 05.30.Fk, 11.10.Hi

Introduction.—Recently topological superconductors have attracted great interest across many subfields in physics [1,2]. This is partially because vortices in topological superconductors bind zero-energy Majorana fermions and obey non-Abelian statistics, which can be of potential use for fault-tolerance topological quantum computation [3,4]. A canonical example of such topological superconductors is a p-wave paired state of spinless fermions in two dimensions [5], which is believed to be realized in Sr_2RuO_4 [6]. Previous mean-field studies revealed that a topological quantum phase transition takes place across a

of resonantly interacting fermions in two dimensions should be taken into account in future many-body studies beyond the mean-field approximation.

Renormalization group analysis.—The above predictions can be derived most conveniently by a renormalization group (RG) analysis. The most general Lagrangian density that includes up to marginal couplings consistent with rotation and parity symmetries is

$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a$$

Super Efimov effect

superconductors bind zero-energy Majorana fermions and obey non-Abelian statistics, which can be of potential use for fault-tolerance topological quantum computation [3,4]. A canonical example of such topological superconductors is a p-wave paired state of spinless fermions in two dimensions [5], which is believed to be realized in Sr_2RuO_4 [6]. Previous mean-field studies revealed that a topological quantum phase transition takes place across a p-wave Feshbach resonance [7–9].

In this Letter, we study few-body physics of spinless fermions in two dimensions right at the p-wave resonance. We predict that three such fermions form an infinite tower of bound states of orbital angular momentum $\ell = \pm 1$ and their binding energies obey a universal doubly exponential scaling

$$E_3^{(n)} \propto \exp(-2e^{3\pi n/4+\theta}) \tag{1}$$

at large n. Here θ is a nonuniversal constant defined modulo $3\pi/4$. This novel phenomenon shall be termed a super Efimov effect, because it resembles the Efimov effect in which three spinless bosons in three dimensions right at an s-wave resonance form an infinite tower of $\ell = 0$ bound states whose binding energies obey the universal exponential scaling $E_3^{(n)} \propto e^{-2\pi n/s_0}$ with $s_0 \approx 1.00624$ [10] (see Table I for comparison). While the Efimov effect is possible in other situations [11,12], it does not take place in two dimensions or with p-wave interactions [12–14]. We also provide an indication that there are $\ell = \pm 2$ four-body resonances associated with every three-body bound state at

$$E_4^{(n)} \propto \exp(-2e^{3\pi n/4+\theta-0.188}), \tag{2}$$

which also resembles the pair of four-body resonances in the usual Efimov effect [15,16]. These universal few-body states

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$$\begin{aligned}\mathcal{L} = {} & \psi^\dagger\Big(i\partial_t + \frac{\nabla^2}{2}\Big)\psi + \phi_a^\dagger\Big(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\Big)\phi_a \\ & + g\phi_a^\dagger\psi(-i\nabla_a)\psi + g\psi^\dagger(-i\nabla_{-a})\psi^\dagger\phi_a \\ & + v_3\psi^\dagger\phi_a^\dagger\phi_a\psi + v_4\phi_a^\dagger\phi_{-a}^\dagger\phi_{-a}\phi_a \\ & + v_4'\phi_a^\dagger\phi_a^\dagger\phi_a\phi_a.\end{aligned} \tag{3}$$

Here and below, $\hbar = m = 1$, $\nabla_\pm \equiv \nabla_x \pm i\nabla_y$, and sums over repeated indices $a = \pm$ are assumed. ψ and $\phi_\pm$ fields correspond to a spinless fermion and $\ell = \pm 1$ composite boson, respectively. The p-wave resonance is defined by the divergence of the two-fermion scattering amplitude at zero energy, which is achieved by tuning the bare detuning parameter at $\varepsilon_0 = g^2\Lambda^2/(2\pi)$ with Λ being a momentum cutoff.

TABLE I. Comparison of the Efimov effect versus the super Efimov effect.

Efimov effect	Super Efimov effect
Three bosons	Three fermions
Three dimensions	Two dimensions
s-wave resonance	p-wave resonance
$\ell = 0$	$\ell = \pm 1$
Exponential scaling	Doubly exponential scaling

