Quantum interaction quench in the presence of a long-range order

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This talk is based on:

NT, Werner, arXiv:1306.0307.

NT, Eckstein, Werner, Phys. Rev. Lett. **110**, 136404 (2013).

Werner, NT, Eckstein, Phys. Rev. B **86**, 205101 (2012).

Outline

1. Introduction to the quantum interaction quench problem.
   How does an isolated quantum system thermalize?
   Is there any intermediate nonthermal quasi-stationary state?

2. Interaction quench in the presence of a long-range order.
   Antiferromagnetic phase in the fermionic Hubbard model studied by the nonequilibrium dynamical mean-field theory (DMFT).

3. Nonthermal criticality.
   A long-lived quasi-stationary state with effective $T > T_c$, showing a Higgs amplitude mode characterized by a nonthermal critical point.

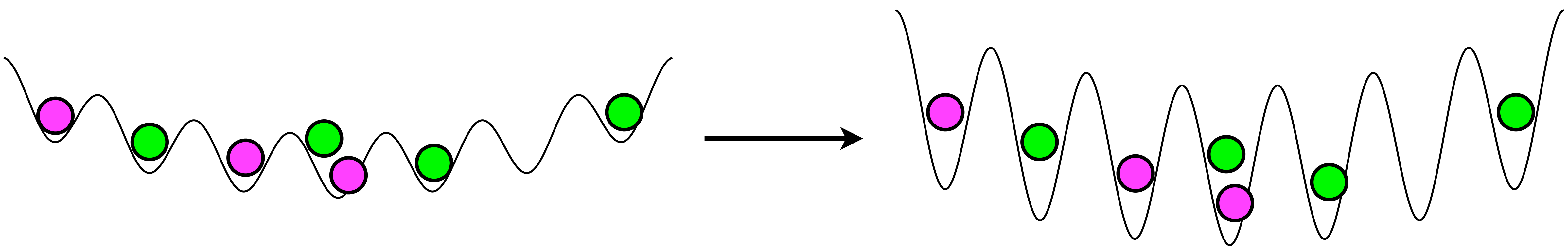
4. Nonthermal universality class.
   Distinct from the conventional Ginzburg-Landau universality.

Interaction quench problem

An abrupt change of the interaction parameter in an isolated quantum system generates a nonequilibrium dynamics of interest.

Fermionic Hubbard model:

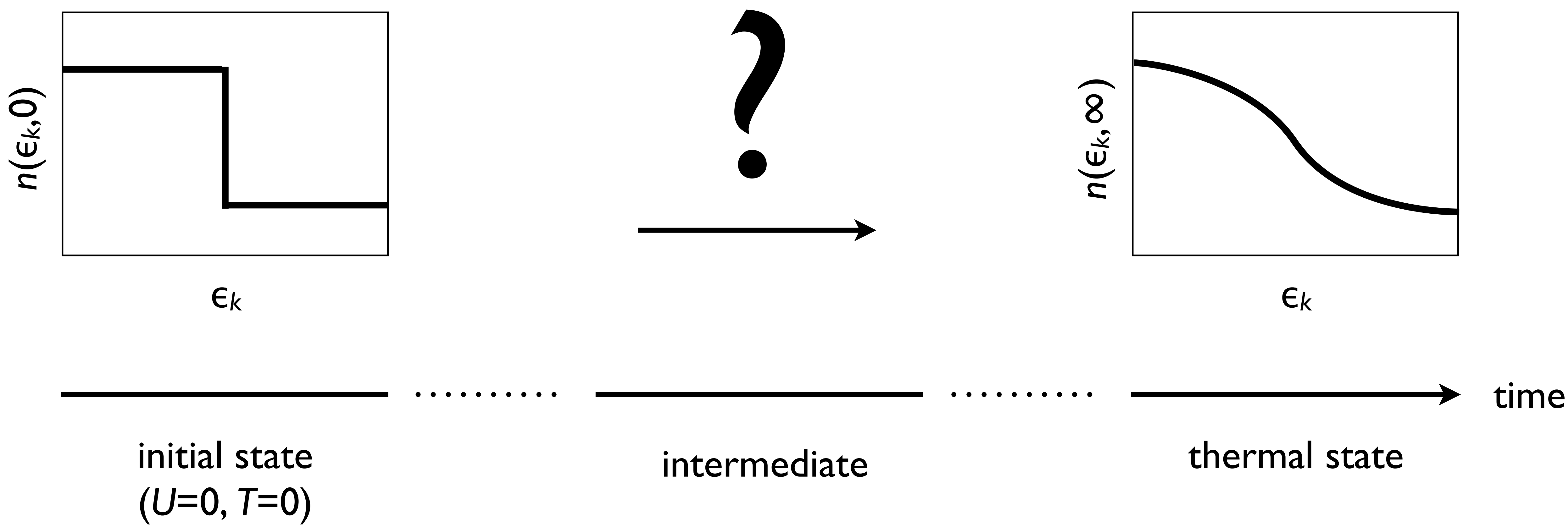
$$H(t) = \sum_{ij,\sigma} v_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U(t) \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$



The interaction quench can be experimentally implemented in cold-atom systems trapped in an optical lattice by changing the lattice potential depth or using Feshbach resonance. Greiner, et al., Nature '02, Bloch, Dalibard, Zwerger, RMP '08.

Why study interaction quench?

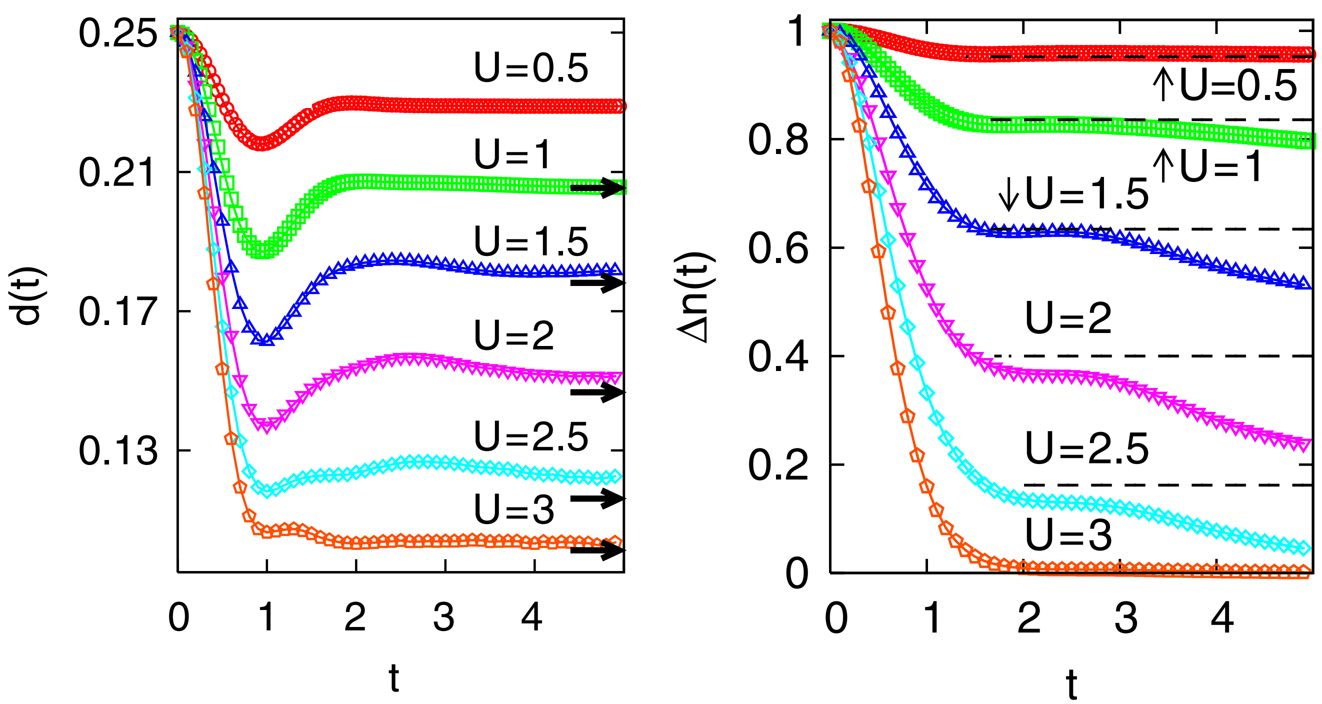
It provides a lot of fundamental theoretical questions:
Does an isolated quantum system thermalize after quench? If so, how does it thermalize? Is there any intermediate nonthermal quasi-stationary state (nonthermal fixed point)?



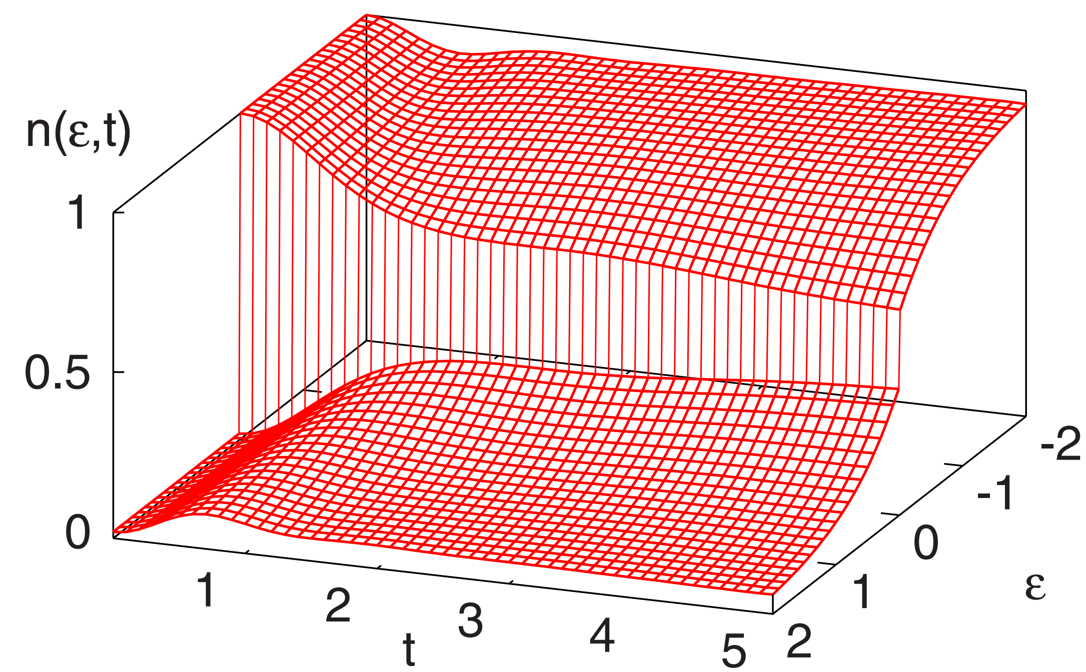
$n(\epsilon_k,t)=\langle c_k^\dagger(t)c_k(t)\rangle$: momentum distribution function

Prethermalization

The weakly correlated system evolves to an intermediate "prethermalized" state, in which an integrated quantity such as the double occupancy $d = \langle n_\uparrow n_\downarrow \rangle$ thermalizes much earlier than the momentum distribution $n(\epsilon_k, t)$.



Eckstein, Kollar, Werner, PRL '09.

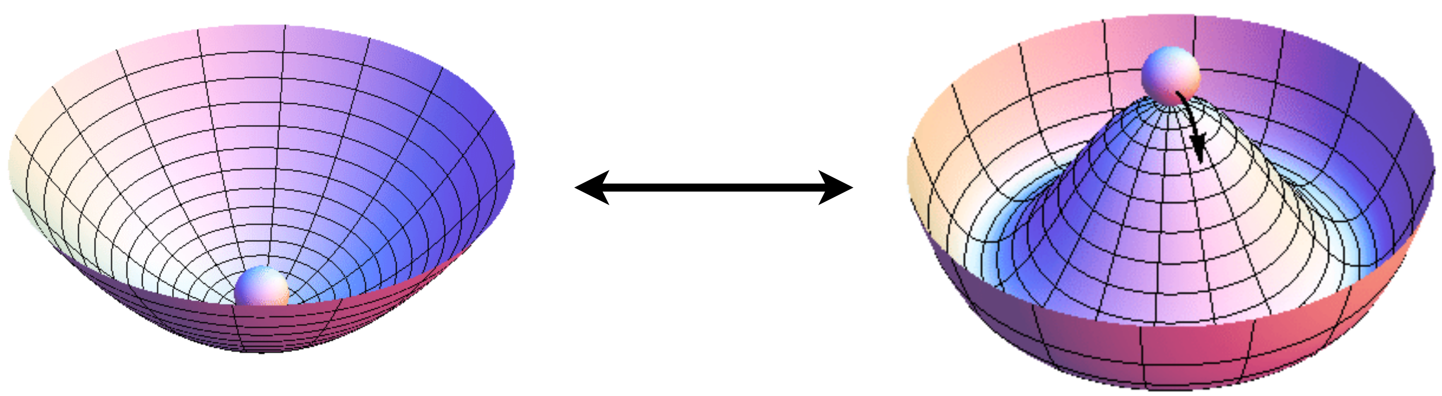


Eckstein, Kollar, Werner, PRB '10.

Prethermalization can be understood from a unitary perturbation theory (Moeckel, Kehrein, PRL '08) and the generalized Gibbs ensemble (Kollar, Wolf, Eckstein, PRB '11).

Interaction quench w/ long-range order

How does the fermionic system thermalize after the interaction quench in the presence of a long-range order? When one goes across the thermal phase transition point, how does the order parameter relax?



The problem of dynamical phase transition has been discussed with a macroscopic (phenomenological) Ginzburg-Landau equation,

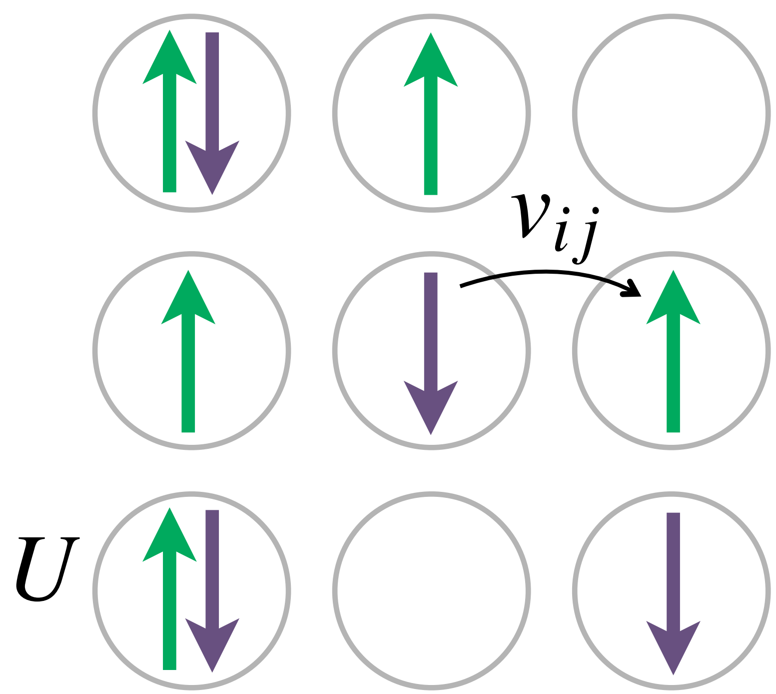
$$-M\frac{\partial^2 m}{\partial t^2} - \Gamma\frac{\partial m}{\partial t} = \frac{\delta \mathcal{F}_{GL}}{\delta m^*} = am + b|m|^2 m - c\nabla^2 m$$

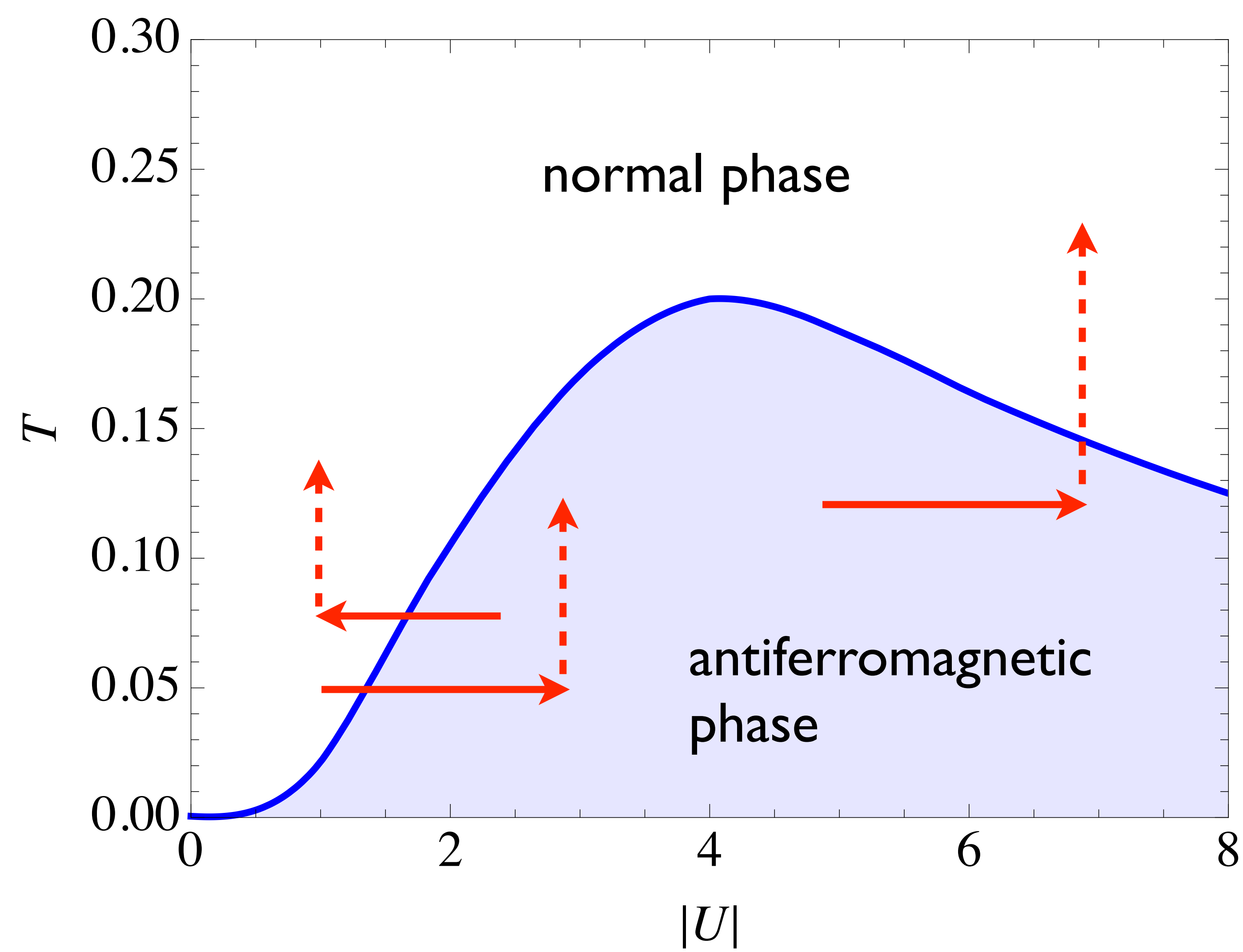
which is valid only when the order parameter varies sufficiently slowly in time. In the quench problem, we have to go beyond GL Eq.

Fermionic Hubbard model

We study interaction quenches in the repulsive (attractive) fermionic Hubbard model:

$$H(t) = \sum_{ij,\sigma} v_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U(t) \sum_{i} \left(\hat{n}_{i\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{i\downarrow} - \frac{1}{2}\right)$$



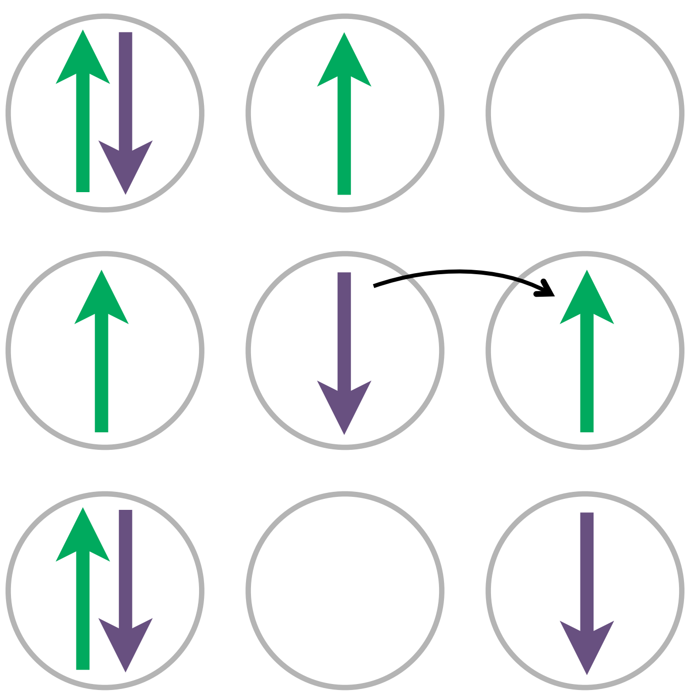


Nonequilibrium DMFT

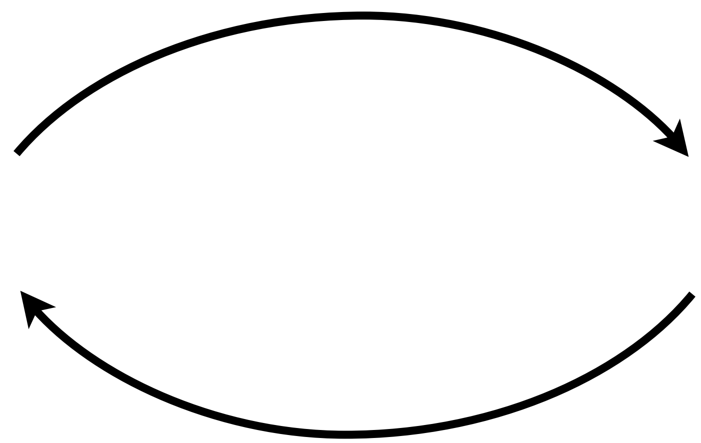
Schmidt, Monien ‘02; Freericks, Turkowski, Zlatić, PRL ‘06.

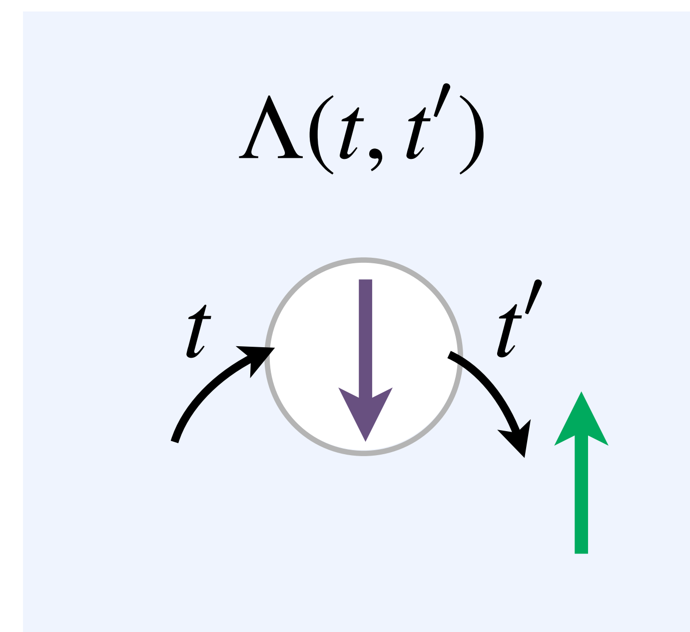
Nonequilibrium lattice problem is mapped to a single-site impurity problem embedded in an effective dynamical mean field $\Lambda(t,t')$.

$$G^{lat}_{ii}(t,t') = G^{imp}[\Lambda](t,t')$$



Lattice model



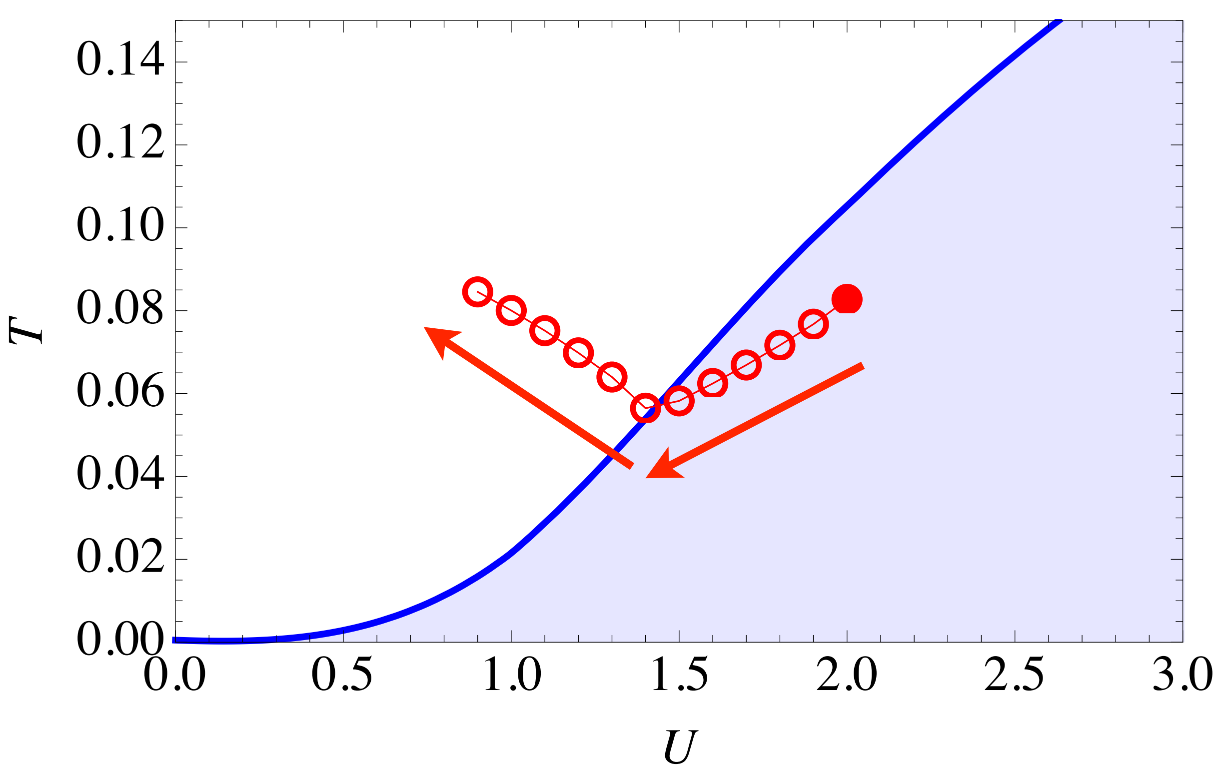


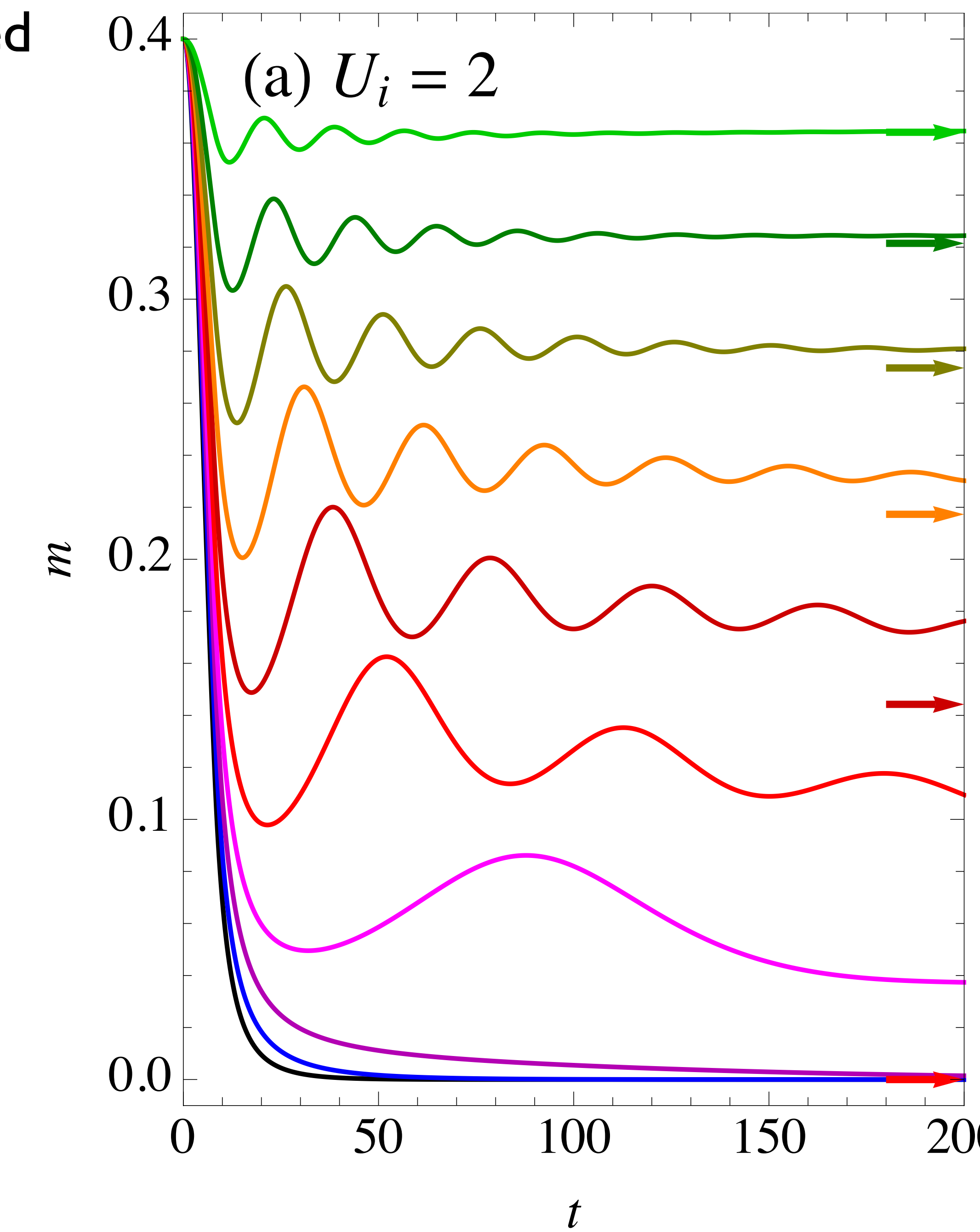
Impurity model

$$\Sigma^{lat}_{ij}(t,t') = \delta_{ij}\Sigma^{imp}(t,t')$$

Quench: ordered → normal

We calculate the time evolution of the staggered magnetization $m=\langle|n_\uparrow - n_\downarrow|\rangle$ for quenches $U_i \to U_f$ ($U_i > U_f$). U_i is fixed, while U_f is systematically changed to go across the phase transition line.



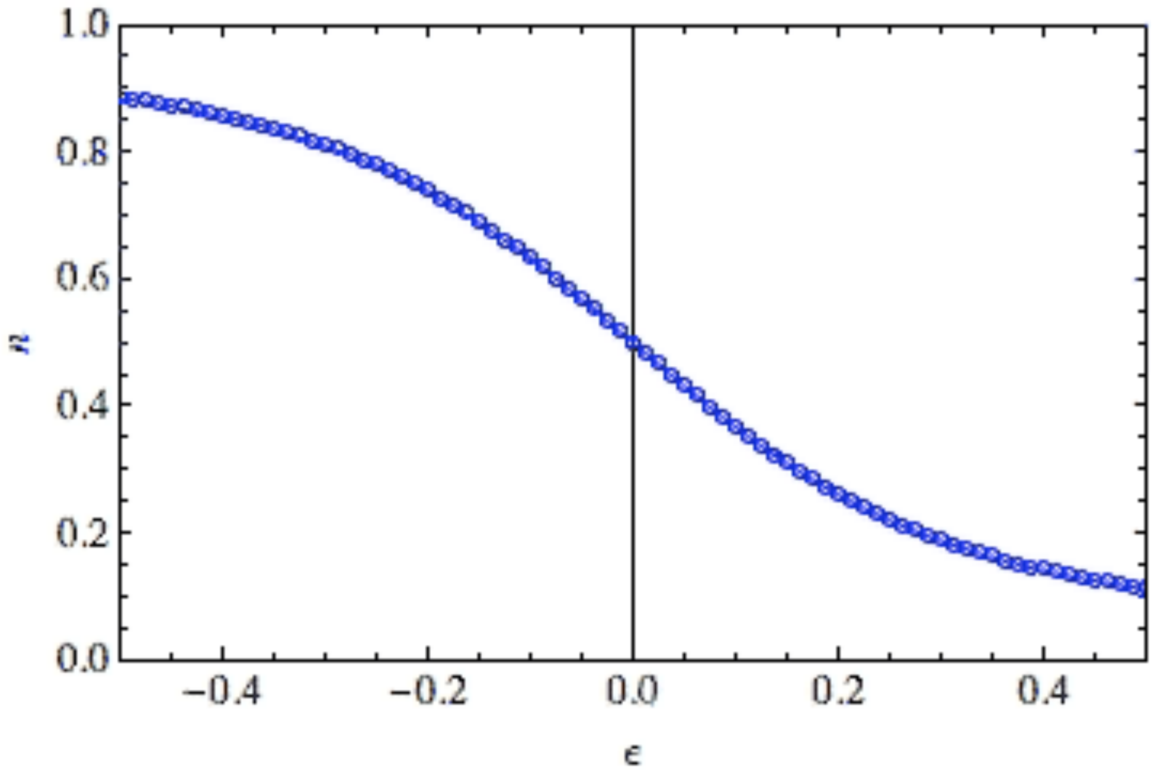


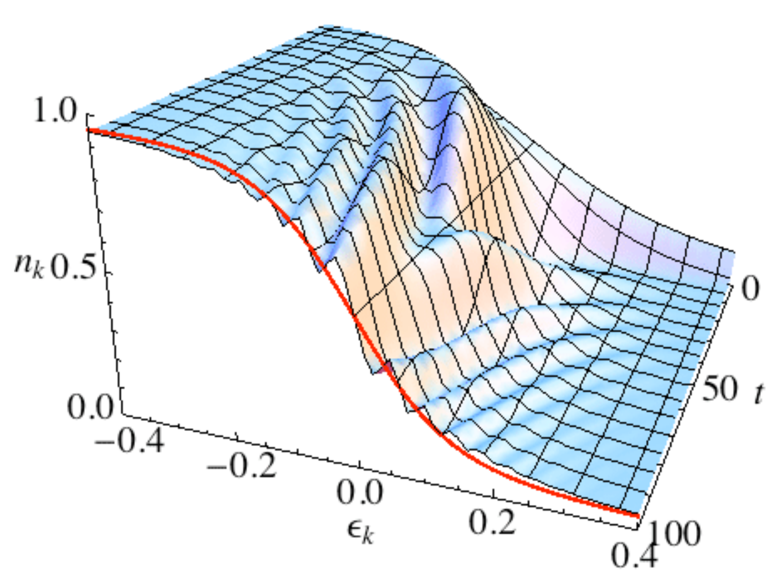
$U_f = 1.0, 1.1, \ldots, 1.9$

Momentum distribution

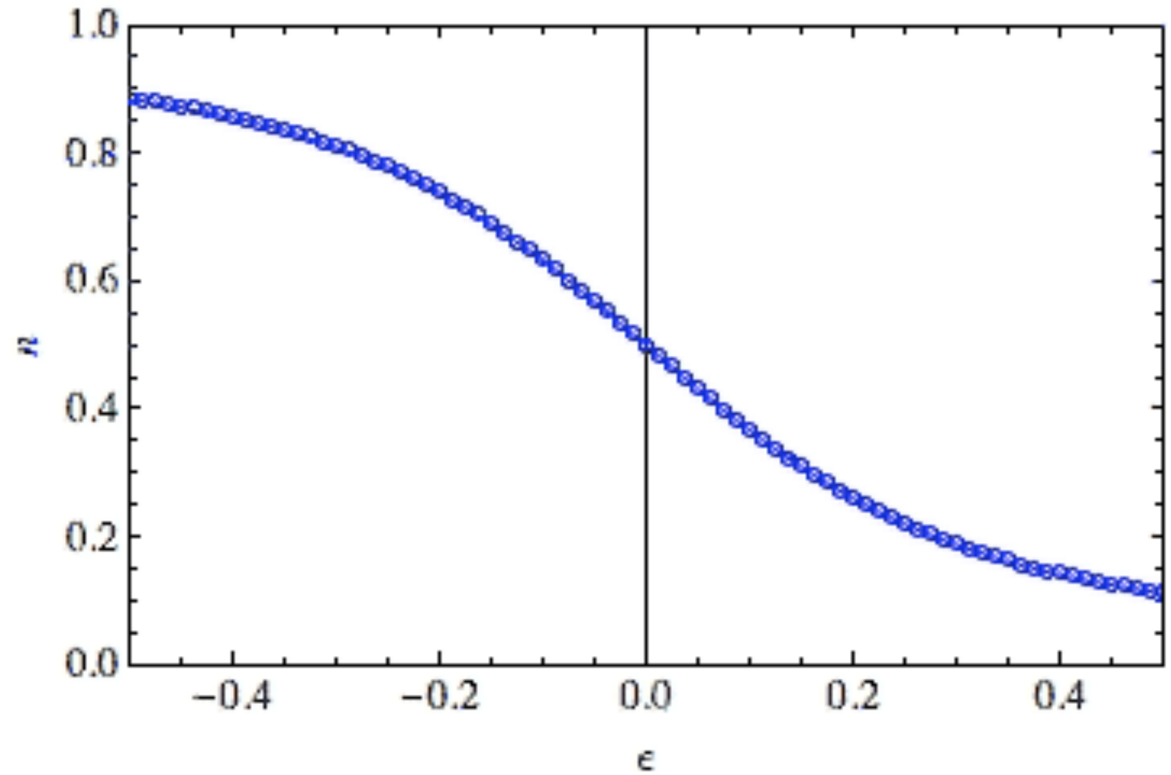
$$n_{\boldsymbol{k}}(t) = N^{-1} \sum_{ij} e^{i\boldsymbol{k}\cdot(\boldsymbol{R}_i - \boldsymbol{R}_j)} \langle c_{i\sigma}^\dagger(t) c_{j\sigma}(t) \rangle$$

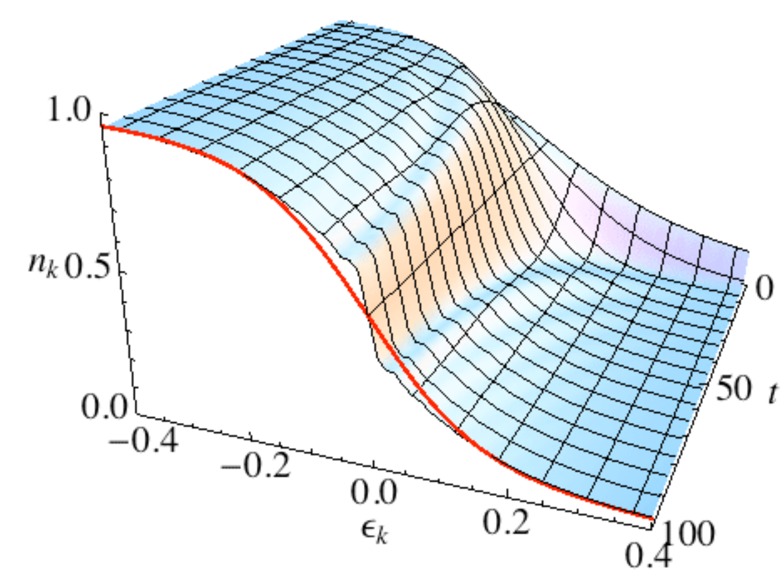
$U_i = 2 \rightarrow U_f = 1.4$



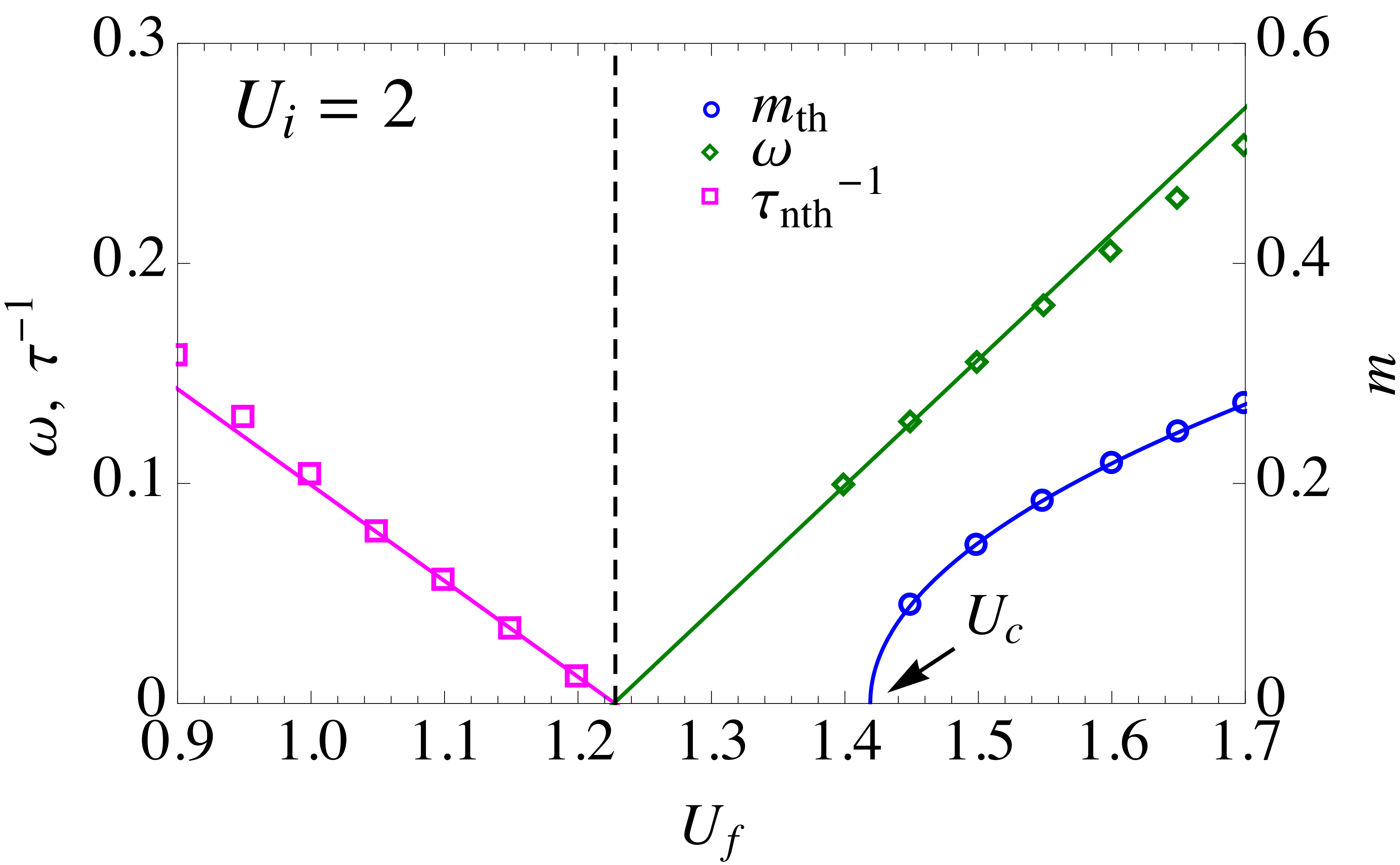


$U_i = 2 \rightarrow U_f = 1.2$





Nonthermal criticality

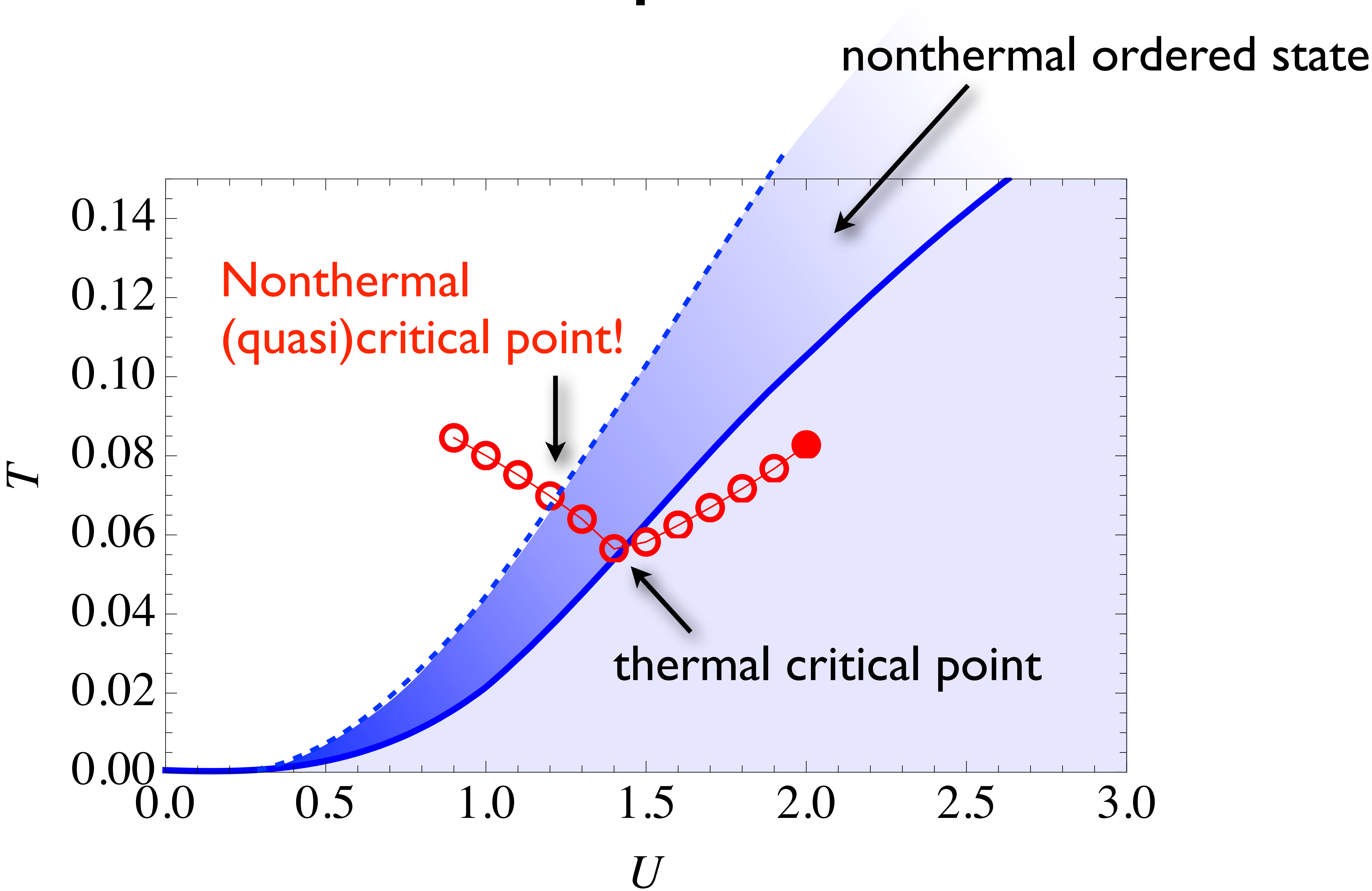


m_{th} : Thermal values of order parameter reached in the long-time limit.

ω : Frequency of the amplitude mode ("Higgs mode").

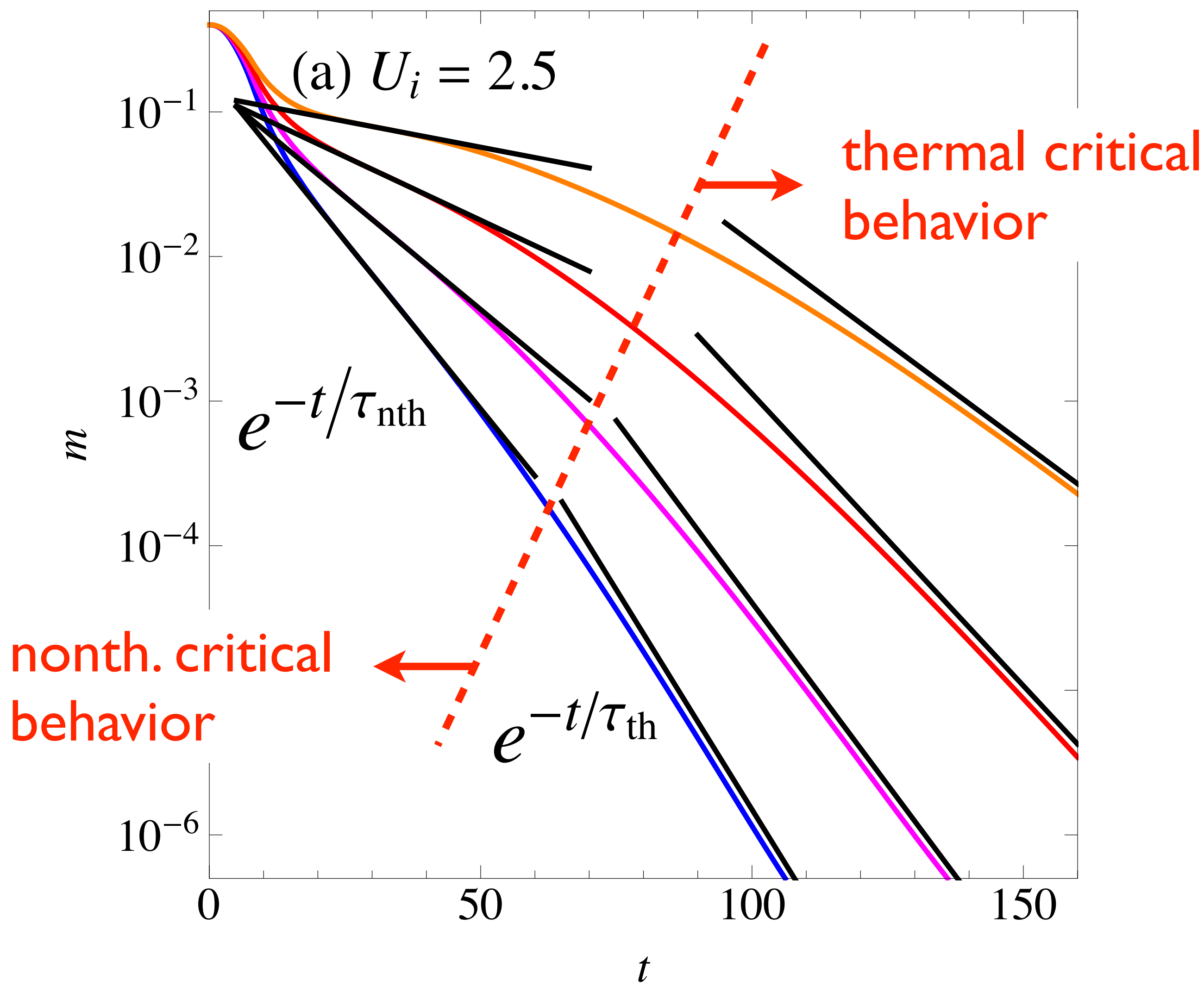
τ_{nth} : Relaxation time of order parameter in the intermediate time scale.

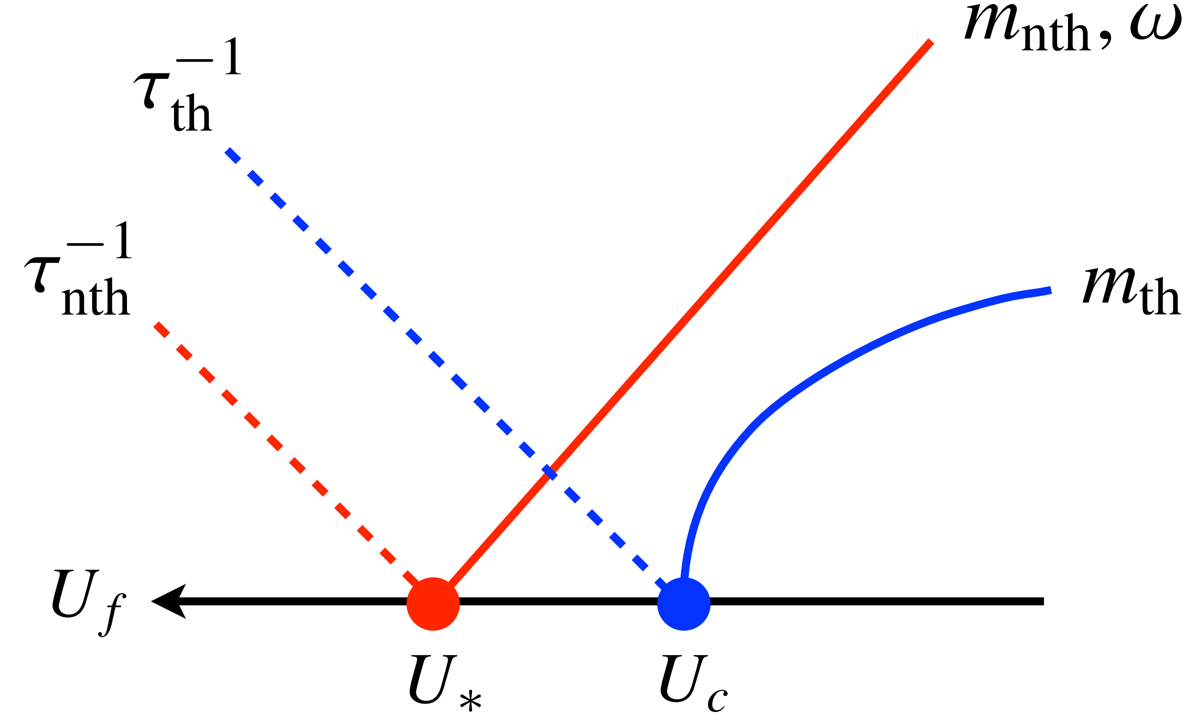
This implies...



Two step relaxation

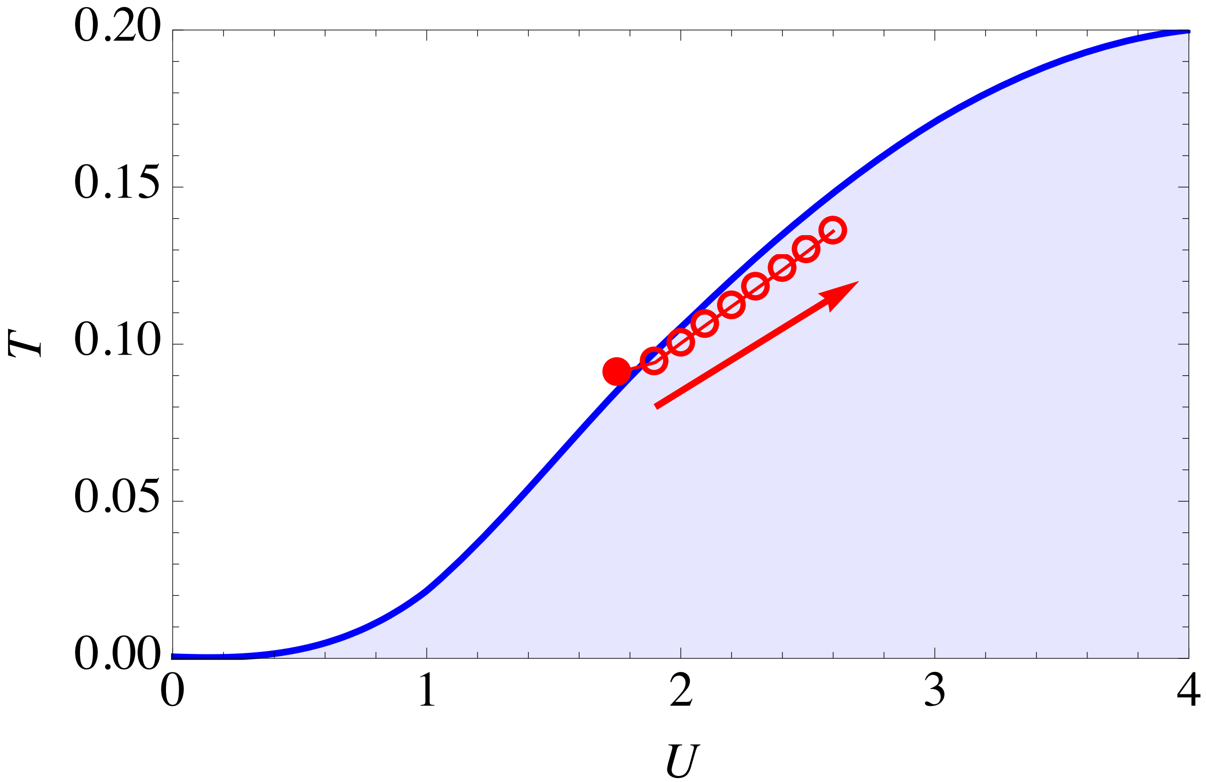
Relaxation crossovers from the nonthermal critical behavior in the intermediate time scale to the thermal critical behavior in the long time scale.

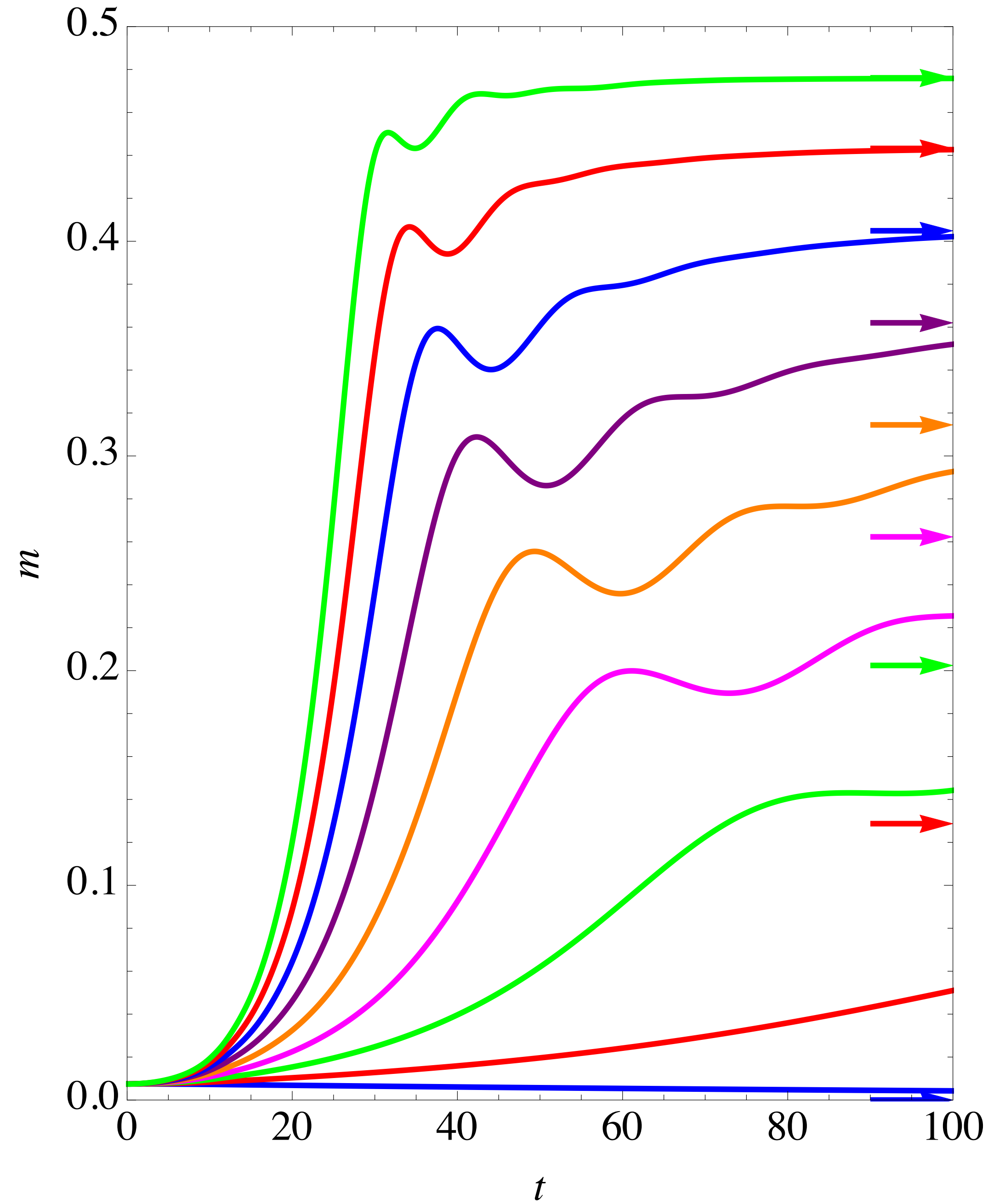




Quench: normal → ordered

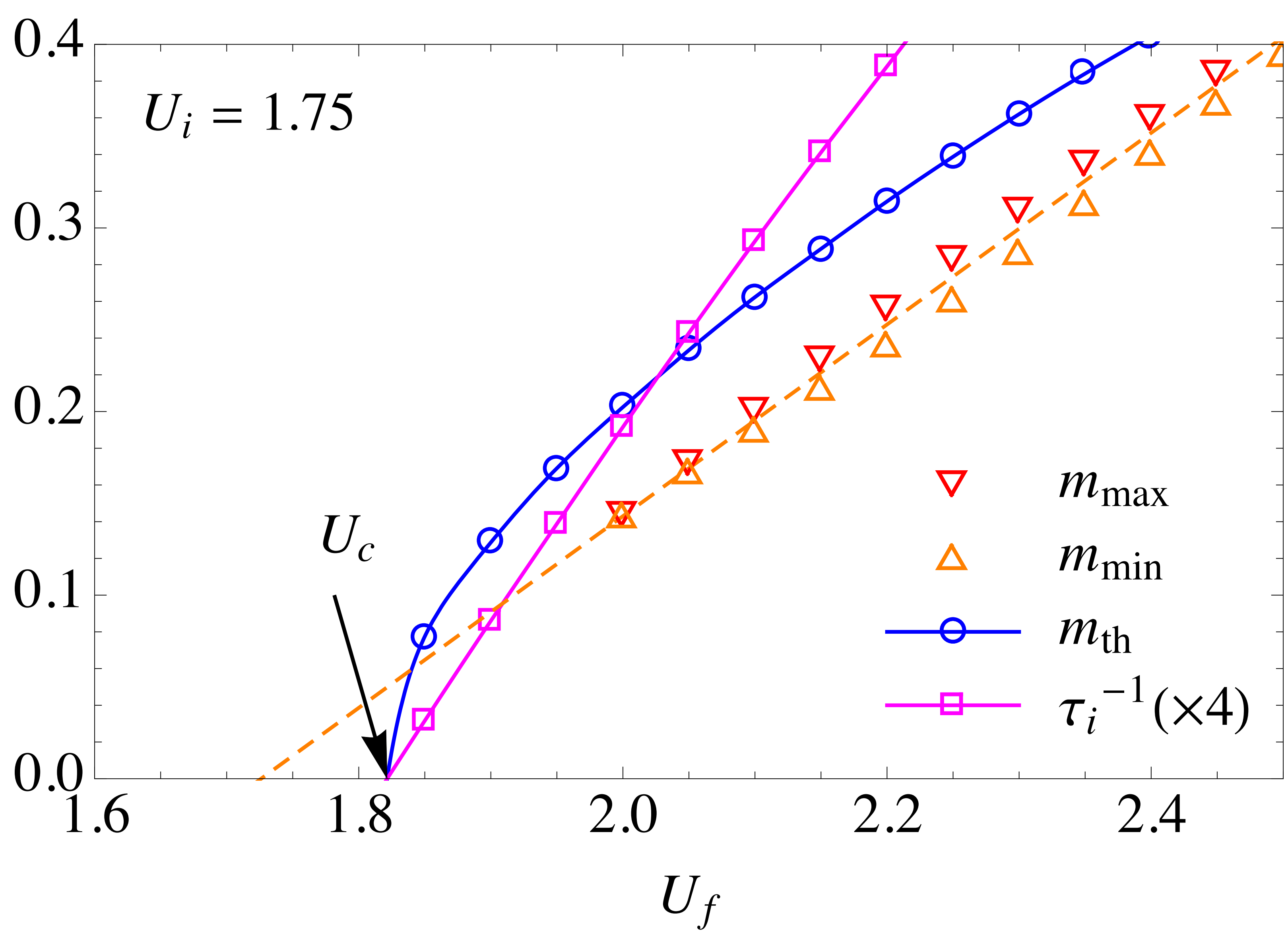
We can study dynamical symmetry breaking from the paramagnetic to antiferromagnetic state with quenches $U_i \to U_f$ ($U_i < U_f$). To trigger symmetry breaking, we introduce small seed magnetic field.





$$U_i = 1.75, U_f = 1.8, 1.9, \ldots, 2.6$$

Nonthermal criticality



$m_{\max}$: Maximum of the first peak in amplitude oscillation.

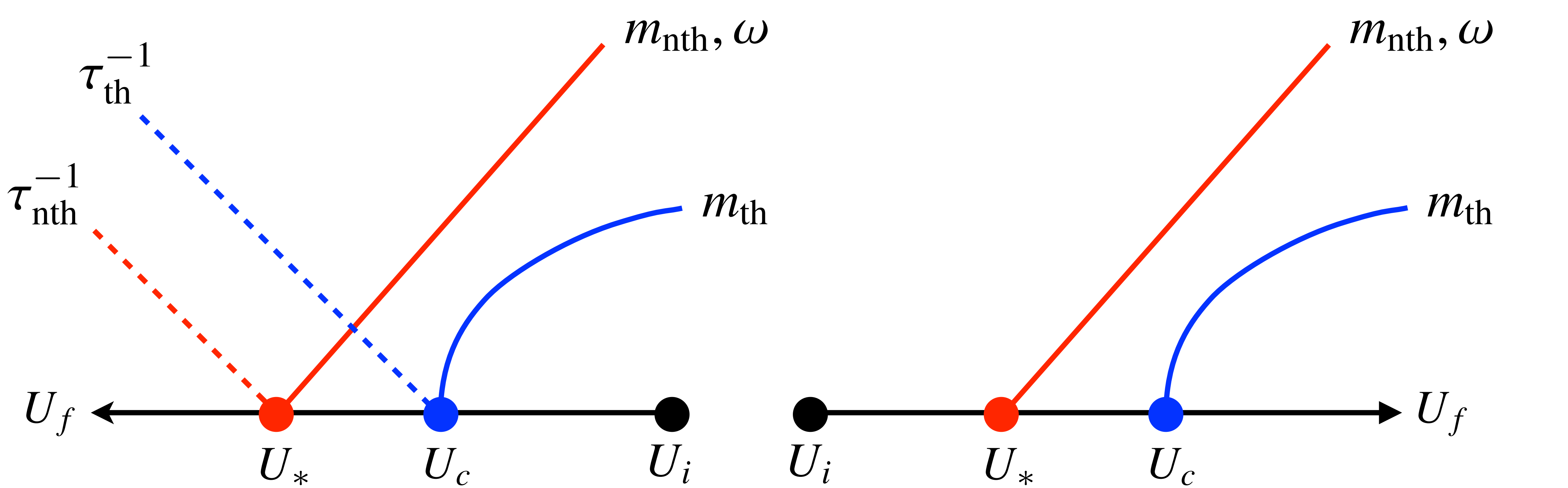
$m_{\min}$: Minimum of the first peak in amplitude oscillation.

m_{th} : Thermal values of order parameter reached in the long-time limit.

τ_i : Rate of the initial exponential growth ($m \propto e^{t/\tau_i}$).

Summary of critical behavior

	intermediate time scale	longer time scale
τ	$\propto \lvert U_f - U_* \rvert^{-1}$	$\propto \lvert U_f - U_c \rvert^{-1}$ ← $z\nu$
m	$\propto \lvert U_f - U_* \rvert^{1}$	$\propto \lvert U_f - U_c \rvert^{1/2}$ ← β
ω	$\propto \lvert U_f - U_* \rvert^{1}$	—



Hartree approximation

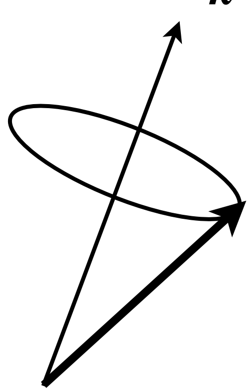
We have seen that the order-parameter dynamics in the intermediate time scale cannot be described by the conventional Ginzburg-Landau theory.

For very small *U*, the time-dependent Hartree approximation is applicable.

$$\Sigma_\sigma(t,t') = U(t) n_{\bar{\sigma}}(t)\delta(t,t')$$

It turns out that equation of motion is reduced to Bloch equation for "spin precession".

$$\frac{\partial}{\partial t}\vec{\sigma}_{\boldsymbol{k}}(t) = \vec{b}_{\boldsymbol{k}}(t) \times \vec{\sigma}_{\boldsymbol{k}}(t) \qquad \vec{b}_{\boldsymbol{k}} = (-2\epsilon_{\boldsymbol{k}}, 0, U(t)m(t))$$



Here we introduce momentum distributions analogous to Anderson's pseudospin representation for superconductors (Anderson, Phys. Rev. '58).

$$\sigma^x_{\boldsymbol{k}} = \frac{1}{2}\sum_\sigma [\langle c^{A\dagger}_{\boldsymbol{k}\sigma} c^B_{\boldsymbol{k}\sigma}\rangle + \langle c^{B\dagger}_{\boldsymbol{k}\sigma} c^A_{\boldsymbol{k}\sigma}\rangle]$$

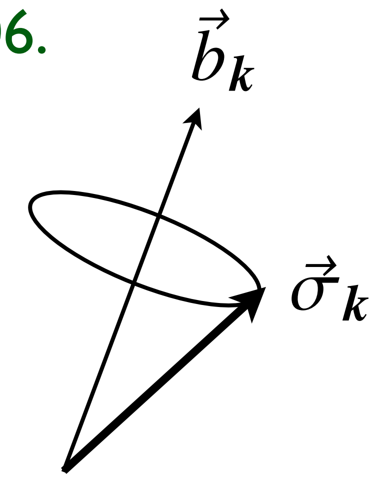
$$\sigma^y_{\boldsymbol{k}} = \frac{i}{2}\sum_\sigma \sigma[\langle c^{A\dagger}_{\boldsymbol{k}\sigma} c^B_{\boldsymbol{k}\sigma}\rangle - \langle c^{B\dagger}_{\boldsymbol{k}\sigma} c^A_{\boldsymbol{k}\sigma}\rangle] \qquad m = \sum_{\boldsymbol{k}} \sigma^z_{\boldsymbol{k}}$$

$$\sigma^z_{\boldsymbol{k}} = \frac{1}{2}\sum_\sigma \sigma[\langle c^{A\dagger}_{\boldsymbol{k}\sigma} c^A_{\boldsymbol{k}\sigma}\rangle - \langle c^{B\dagger}_{\boldsymbol{k}\sigma} c^B_{\boldsymbol{k}\sigma}\rangle]$$

Integrable equation

This equation is mathematically equivalent to time-dependent BCS (or BdG) equation, which is known to be integrable. Barankov, Levitov, Spivak, PRL '04; Yuzbashyan et al. PRB '05; Warner, Leggett, PRB '05; Barankov, Levitov, PRL '06; Yuzbashyan, Dzero, PRL '06.

$$\frac{\partial}{\partial t}\vec{\sigma}_{\boldsymbol{k}}(t) = \vec{b}_{\boldsymbol{k}}(t) \times \vec{\sigma}_{\boldsymbol{k}}(t)$$



We find that this equation defines a universality class distinct from GL.

In the case of dynamical symmetry breaking ($U_i < U_f$), one can show that the order parameter obeys a "GL-like" equation

$$-\frac{\partial^2 m}{\partial t^2} = \frac{\partial \mathcal{F}_{\rm nth}}{\partial m}$$

with a nonthermal potential

$$\mathcal{F}_{\rm nth} = -\frac{1}{2}am^2 + \frac{U_f^2}{8}m^4$$

Nonthermal criticality

$$\mathcal{F}_{\text{nth}} = -\frac{1}{2}am^2 + \frac{U_f^2}{8}m^4$$

The constant *a* satisfies a condition

$$-U_f \sum_{\boldsymbol{k}} \frac{2\epsilon_{\boldsymbol{k}}}{(2\epsilon_{\boldsymbol{k}})^2 + a} f_0(\epsilon_{\boldsymbol{k}}) = 1$$

where $f_0(\epsilon_k)$ is a momentum distribution determined from the initial condition. From this, one can show that

$$a = a_0(U_f - U_*)^2 \qquad a_0 = \left(\frac{8}{\pi\beta U_*^2 D(\epsilon_F)}\right)^2$$

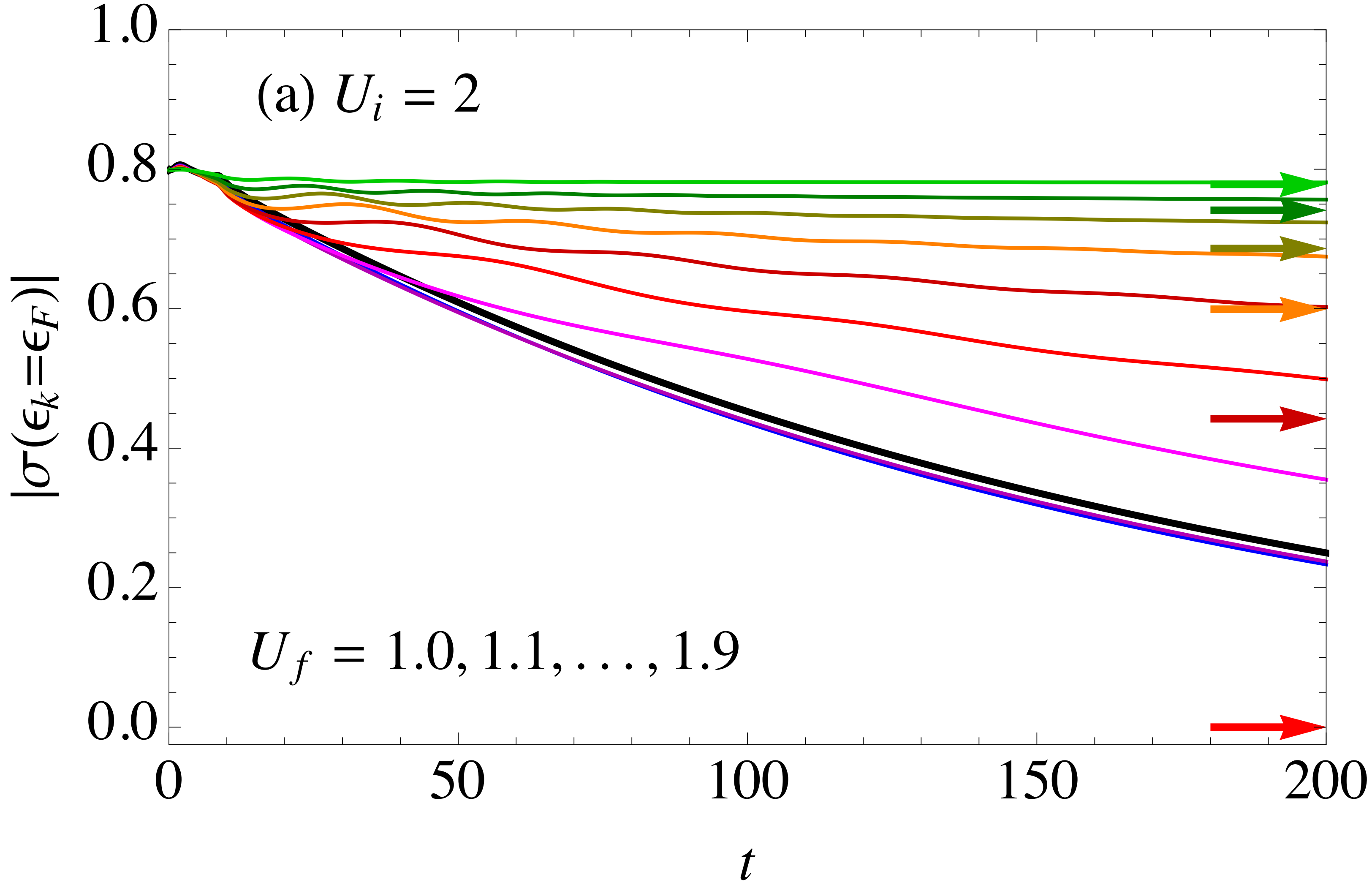
which contrasts with the conventional GL theory,

$$a = a_0(U_f - U_c)$$

This evidences that the nonthermal critical point belongs to a universality class different from the conventional GL.

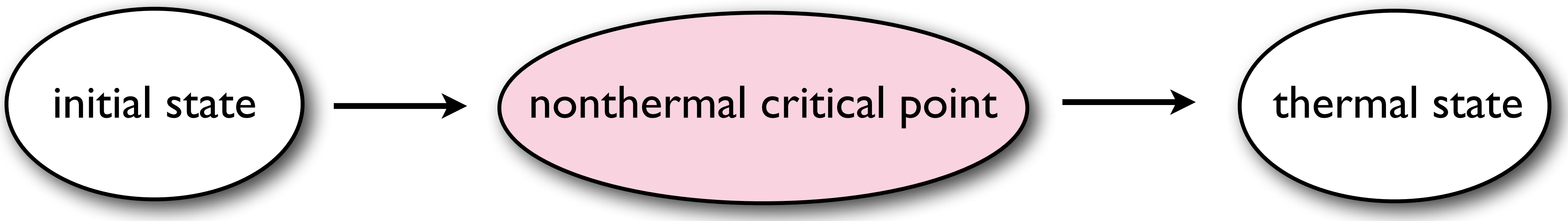
Away from integrable regime

As one increases *U*, integrability is quickly lost. For example, the length of the pseudospin $|\vec{\sigma}_{\boldsymbol{k}}|$ is conserved for each *k*. However, it is already not conserved at *U*~1.



But still qualitative features of the nonthermal critical point are maintained.

Summary



	intermediate time scale	longer time scale
τ	$\propto \lvert U_f - U_* \rvert^{-1}$	$\propto \lvert U_f - U_c \rvert^{-1}$
m	$\propto \lvert U_f - U_* \rvert^{1}$	$\propto \lvert U_f - U_c \rvert^{1/2}$
ω	$\propto \lvert U_f - U_* \rvert^{1}$	—