Entanglement, quantum spin liquids, and DMRG

Leon Balents, KITP

$$$: NSF, DOE

Plan

- Introduction
- Topological entanglement entropy
- Calculations of TEE with DMRG, and minimally entangled states
- Comparison of scaling of Renyi and von Neumann entropies
- Application of MES to less exotic problems

People

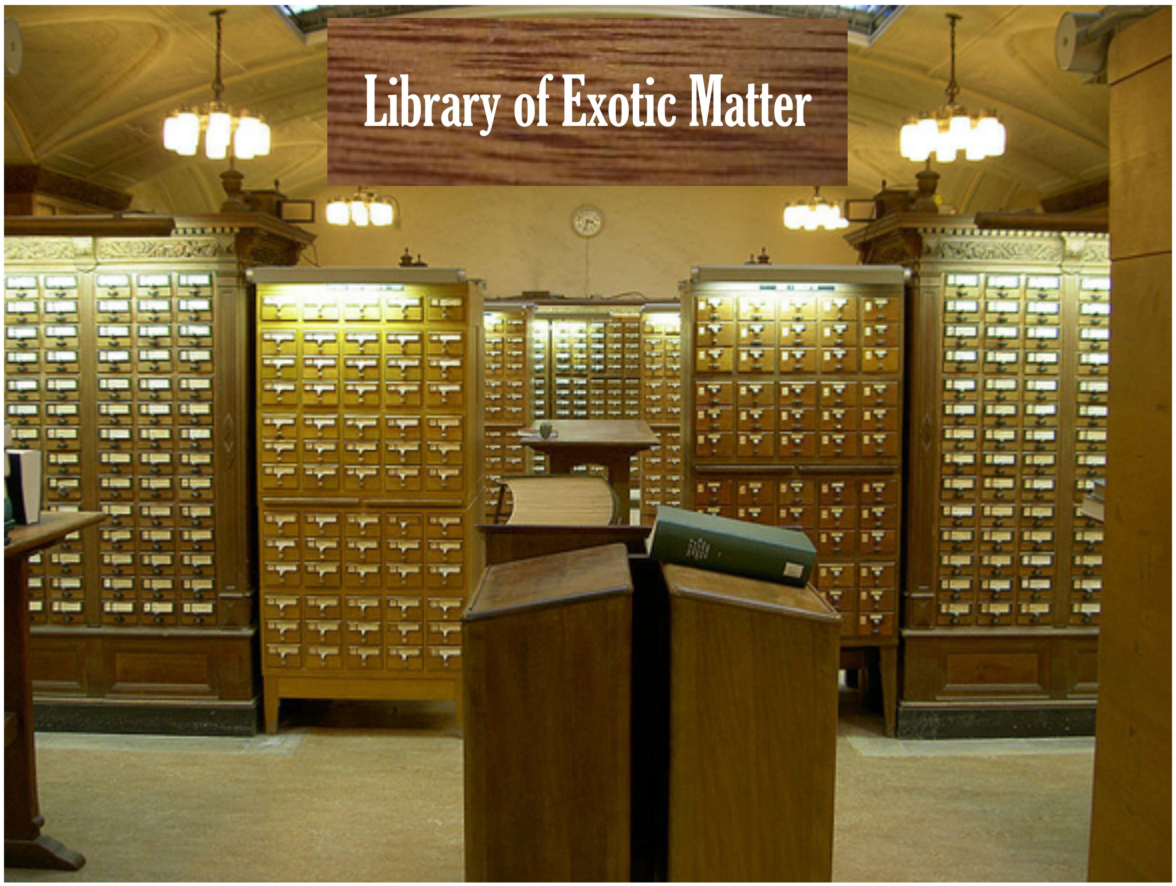
Hong-Chen Jiang
KITP

Rajiv Singh
UC Davis

Zhenghan Wang
Microsoft Q

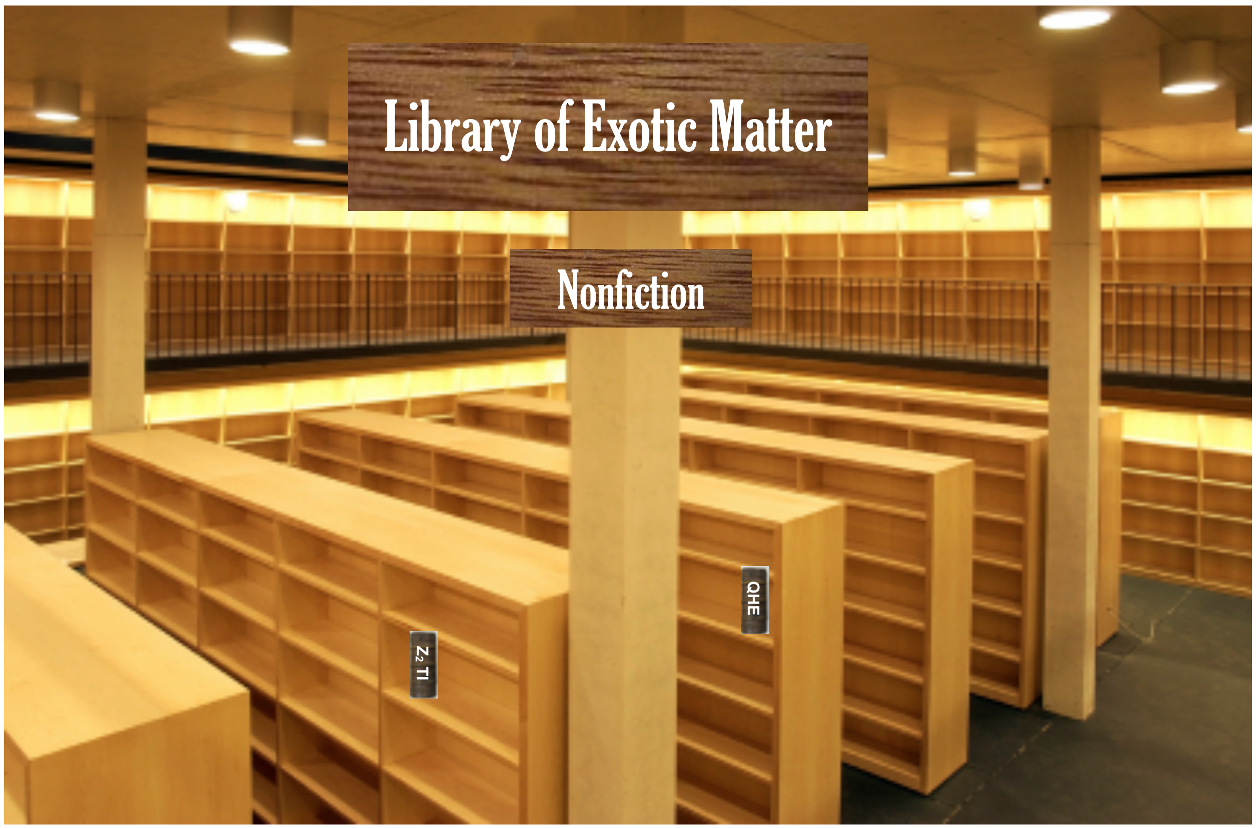
Hong Yao
Tsingua

Classification



cohomology	K-matrix	General Subject
000, 040, 080	AC	Z_2 TI
010, 020, 090	Z	fibonacci
030	AE	IQHE
050	AP	ASL
060	AS	E8
070	PN	$SO(6)_3$
100	B-BJ	Philosophy (Gen.)
110-120	BD	Speculative Philosophy
130, 150	BF	Psychology
140, 180, 190	B	Philosophy (Gen.)
160	BC	Logic
170	BJ	Ethics
200, 210, 290	BL	Religions. Mythology
220	BS	The Bible
230	BT	Doctrinal Theology
240, 250	BV	Practical Theology
260, 270	BR	Christianity
280	BX	Christian Denominations
300	H	Soc. Sci. (General)
310	HA	Statistics
320	J	Gen. Legislative papers
330	HB	Economic Theory
340	K	Law
350	JF-JS	Political Institutions
360	HN, HV	Social History, Soc. Pathology
370	L	Education (General)
380	HD	Industries. Land Use. Labor
390	GT	Manners and customs

Classification



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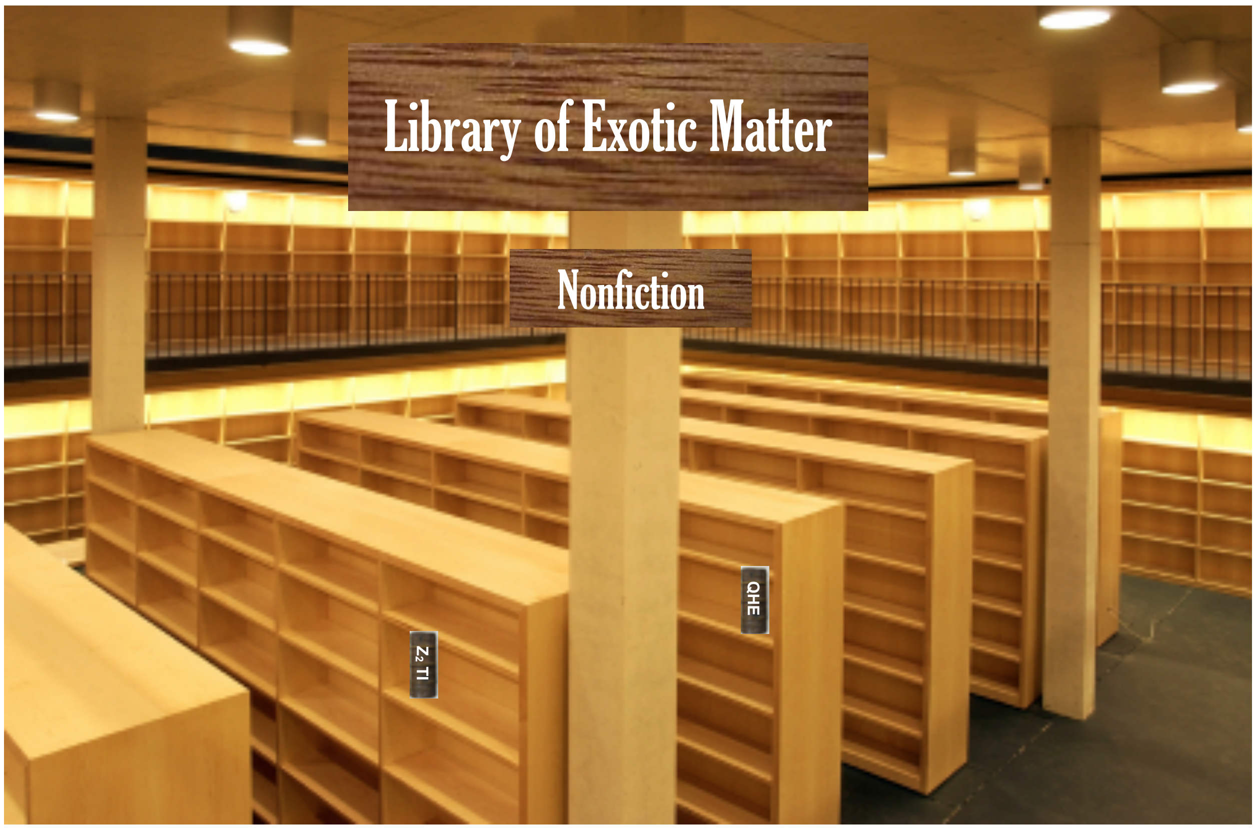

I'd like to find some books

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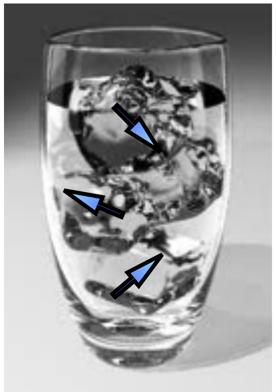
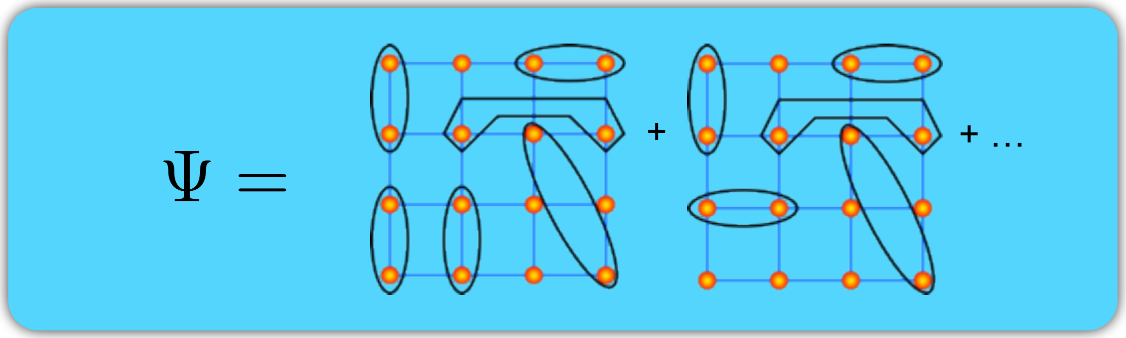
Quantum spin liquids

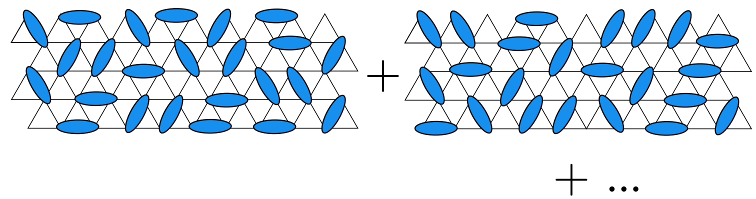


- Long sought non-magnetic ground states of quantum spin systems
- Interesting because of high degree of entanglement
  - Anderson's RVB



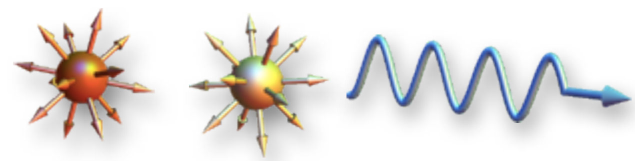
Classes of QSLs

- Topological QSLs
  - full gap



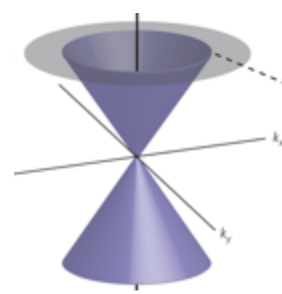
TQFT

- U(1) QSL
  - gapless emergent “photon”



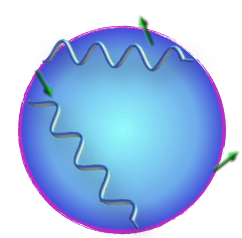
compact U(1) gauge theory

- Algebraic QSLs
  - Relativistic CFT (power-laws)



QED_3

- Spinon Fermi surface QSL



QED_3 w/ $\mu>0$

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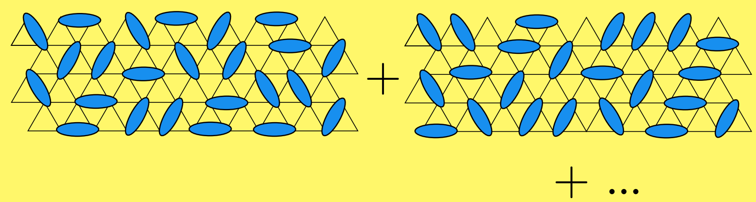

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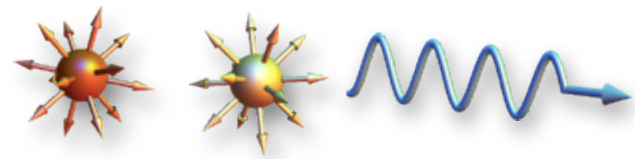

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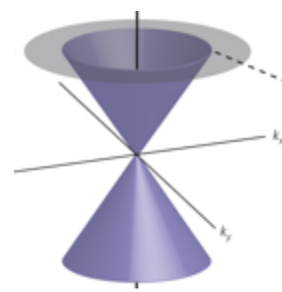

QED_3

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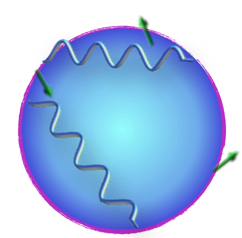
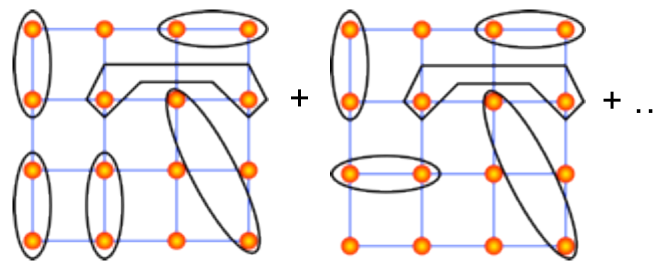


QED_3 w/ $\mu>0$

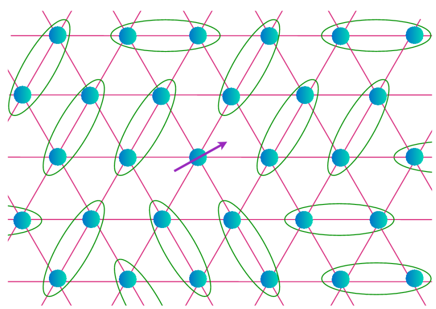
Z_2 spin liquid

- Cartoon

$\Psi =$ 

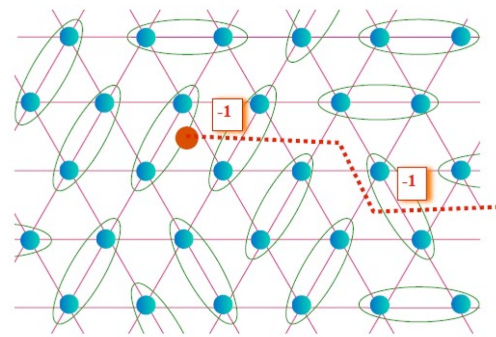
spinon

(e)



vison

(m)



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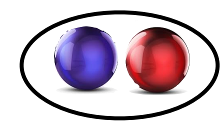
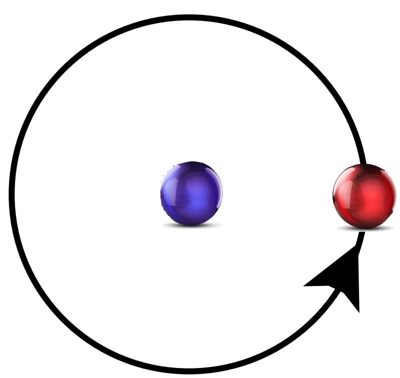


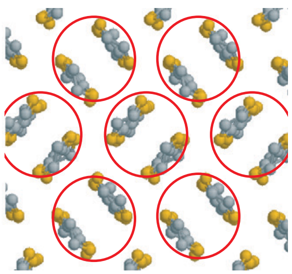
(ε)

$\Psi \to -\Psi$

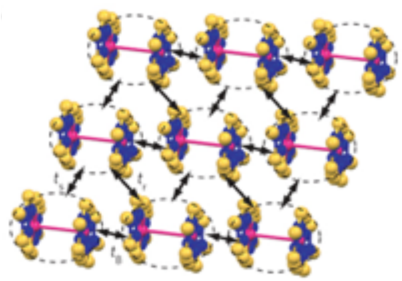


anyons characterized by fusion rules and mutual statistics

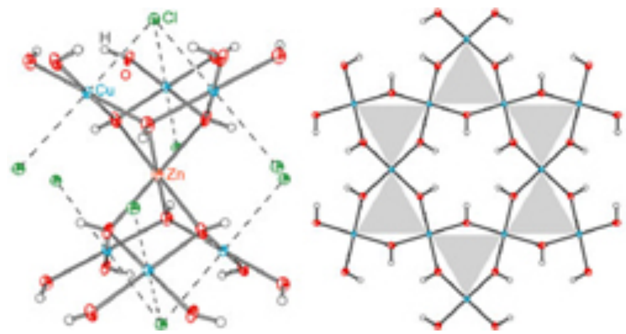
Materials



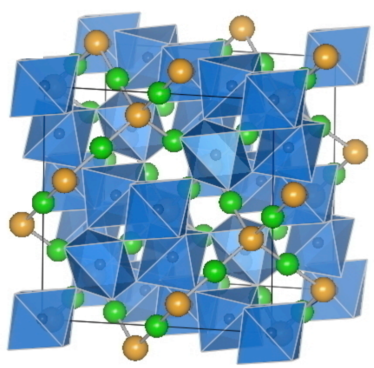
κ-$(ET)_2X$



β’-$Pd(dmit)_2$

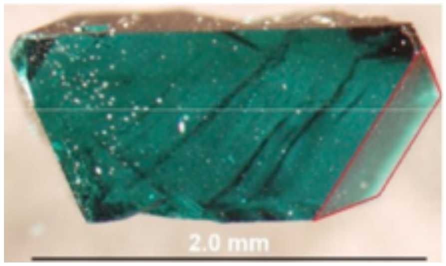


$ZnCu_3(OH)_6Cl_2$



$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$

Herbertsmithite



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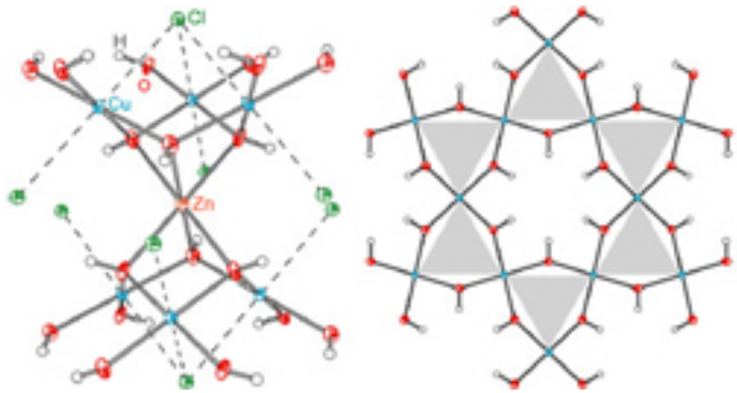
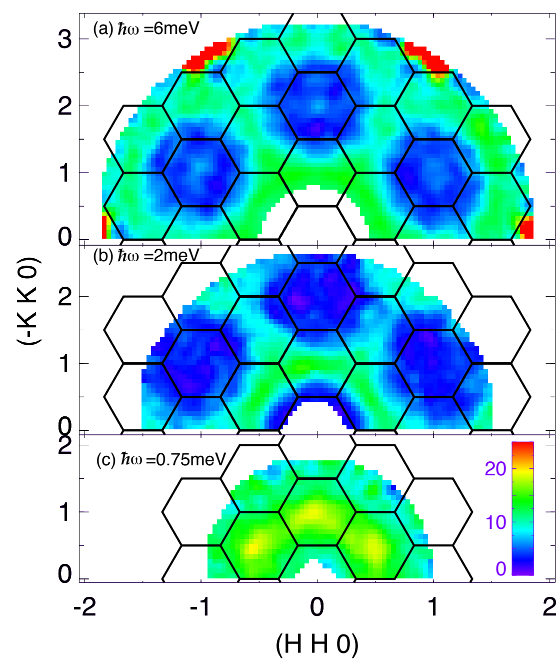
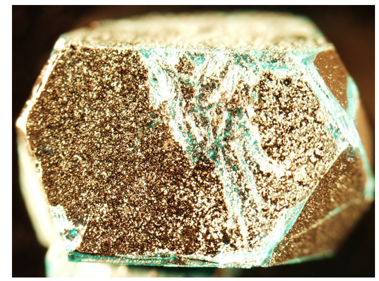


Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han, **Joel S. Helton**, **Shaoyan Chu**, **Daniel G. Nocera**, **Jose A. Rodriguez-Rivera**, **Collin Broholm** & **Young S. Lee**



The search is over?



MIT researchers discover a new kind of magnetism

Experiments demonstrate 'quantum spin liquid,' which could have applications in new computer memory storage.

Quantum spin liquid

From Wikipedia, the free encyclopedia

> This article **may be too technical for most readers to understand**. Please help improve this article to make it understandable to non-experts, without removing the technical details. The talk page may contain suggestions. *(December 2012)*

In condensed matter physics, **quantum spin liquid** is a state that can be achieved in a system of interacting quantum spins. The state is referred to as a "liquid" as it is a disordered state in comparison to a ferromagnetic spin state,[1] much in the way liquid water is in a disordered state compared to crystalline ice. However, unlike other disordered states, a quantum spin liquid state preserves its disorder to very low temperatures.[2]

The quantum spin liquid state was first proposed by physicist Phil Anderson in 1973 as the ground state for a system of spins on a triangular lattice that interact with their nearest neighbors via the so-called antiferromagnetic interaction. Quantum spin liquids generated further interest when in 1987 Anderson proposed a theory that described high temperature superconductivity in terms of a disordered spin-liquid state.[3] A quantum spin liquid state was first realized experimentally in crystalline herbertsmithite by Young Lee and his group at the Massachusetts Institute of Technology in December 2012.[4]

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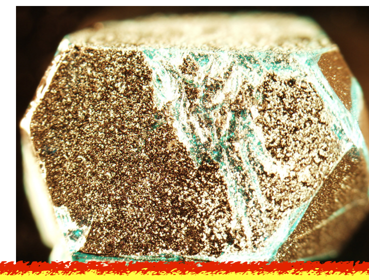

Before we use nearly featureless data to declare victory, we should have some better understanding

Quan

From Wikip

This article **may be too technical for most readers to understand**. Please help improve this article to make it understandable to non-experts, without removing the technical details. The talk page may contain suggestions. *(December 2012)*

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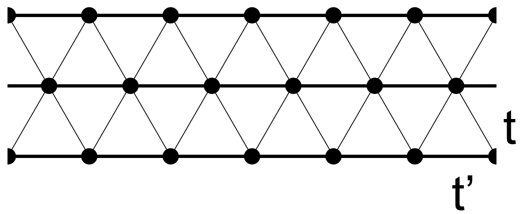
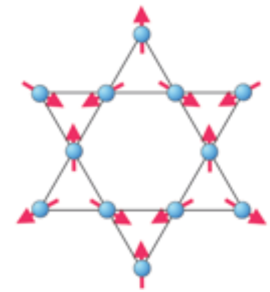
Models



$$H = -\sum_{ij} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



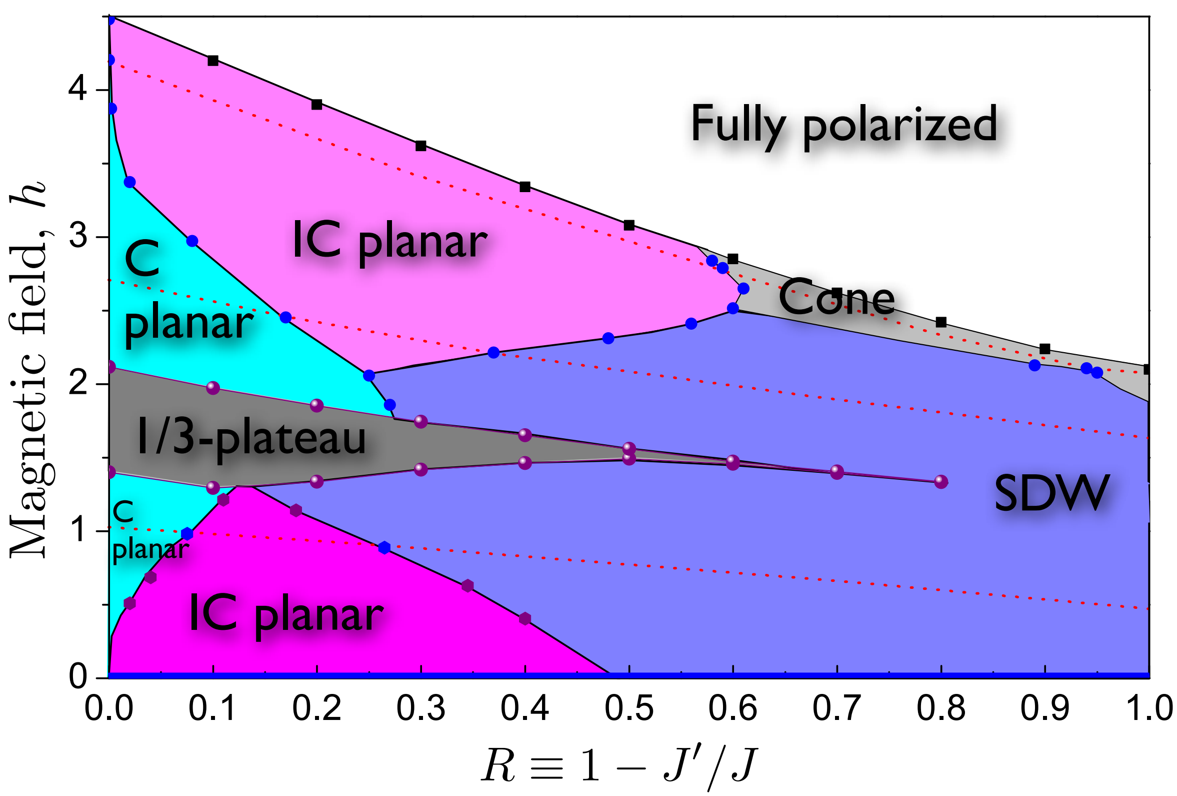
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \cdots$$

$Yb_2Ti_2O_7$
$Pr_2Zr_2O_7$

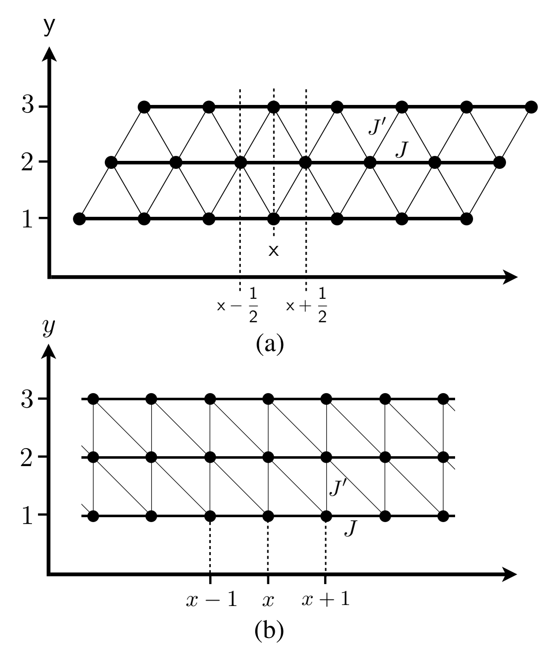
$$\begin{aligned} H \;=\; & J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \\ & -J_{\pm} \sum_{\langle i,j \rangle} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) \\ & + J_{z\pm} \sum_{\langle i,j \rangle} \left[S_i^z \left(\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^- \right) + i \leftrightarrow j \right] \\ & + J_{\pm\pm} \sum_{\langle i,j \rangle} \left(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^- \right) \end{aligned}$$

Phase diagrams

- DMRG gives powerful access to ground states of 1d strips of reasonable width

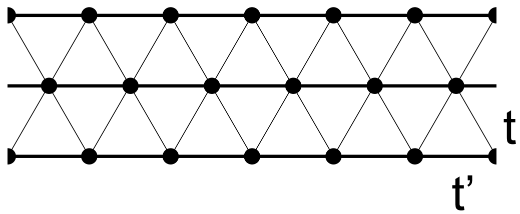


Heisenberg



Ru Chen *et al*, 2013

Models



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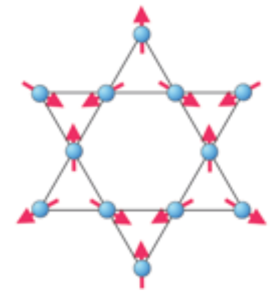

$$H = J \sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \cdots$$

$Yb_2Ti_2O_7$

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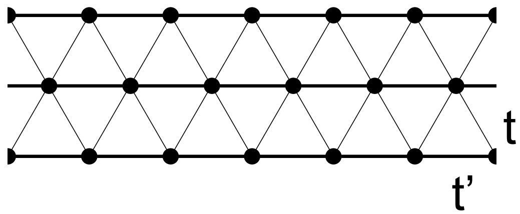
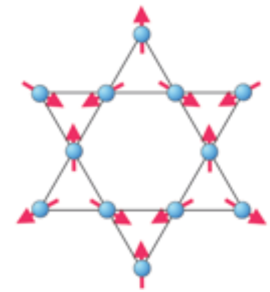




No “obvious” gauge structure/soluble limit.
Diagnostics for QSL states in general models?

- ground state degeneracy
- topological entanglement entropy

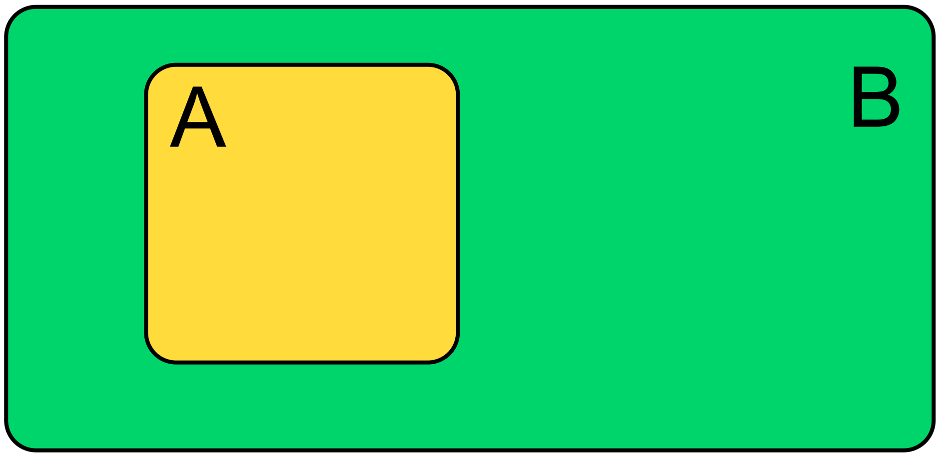
$Yb_2Ti_2O_7$

$$-J_{\pm}\sum_{\langle i,j\rangle}\left(S_i^+S_j^- + S_i^-S_j^+\right)$$

$$+\,J_{z\pm}\sum_{\langle i,j\rangle}\left[S_i^z\left(\zeta_{ij}S_j^+ + \zeta_{ij}^*S_j^-\right) + i\leftrightarrow j\right]$$

$$+\,J_{\pm\pm}\sum_{\langle i,j\rangle}\left(\gamma_{ij}S_i^+S_j^+ + \gamma_{ij}^*S_i^-S_j^-\right)$$

Entanglement entropy

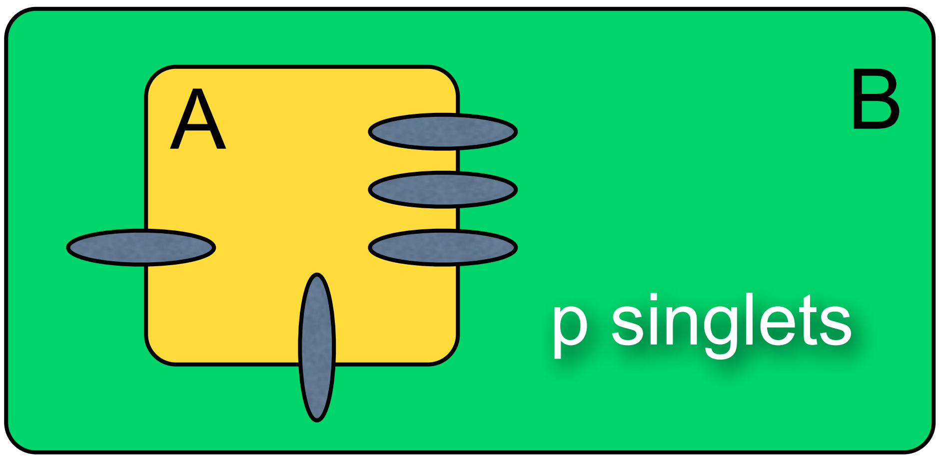


$$\rho_A = \mathrm{Tr}_B \, |\psi\rangle\langle\psi|$$

von Neumann $$S_{vN}(A) = -\mathrm{Tr}\left[\rho_A \ln \rho_A\right]$$

Renyi $$S_\alpha(A) = -\frac{1}{1-\alpha} \ln \mathrm{Tr}\, \rho_A^\alpha$$

Entanglement entropy



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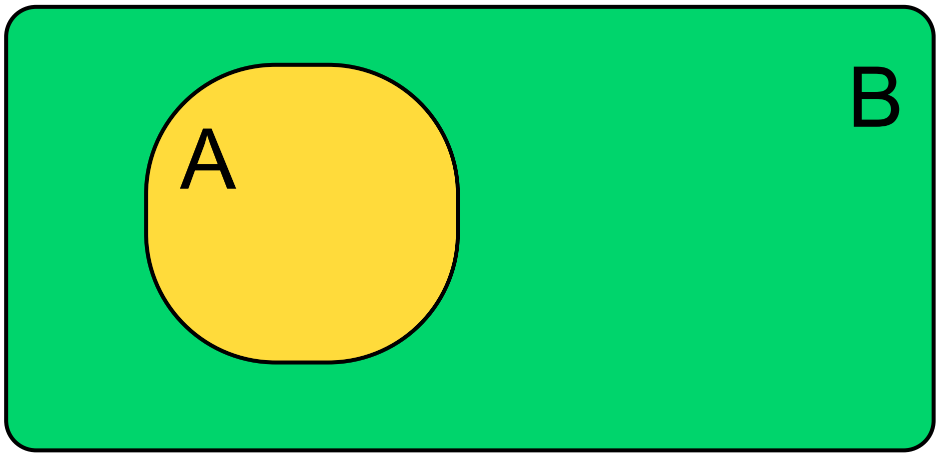
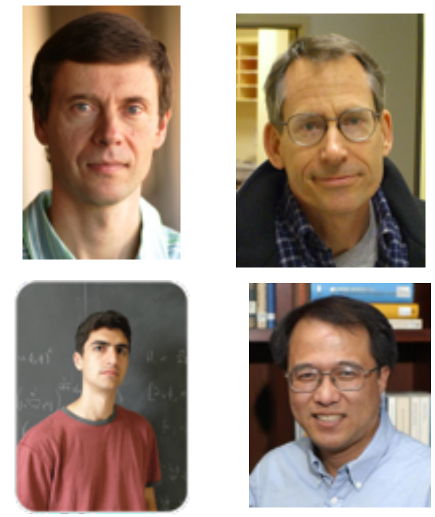
$$= p \ln 2$$

Renyi $\quad S_\alpha(A) = -\frac{1}{1-\alpha} \ln \mathrm{Tr}\, \rho_A^\alpha$

Topological EE





2006

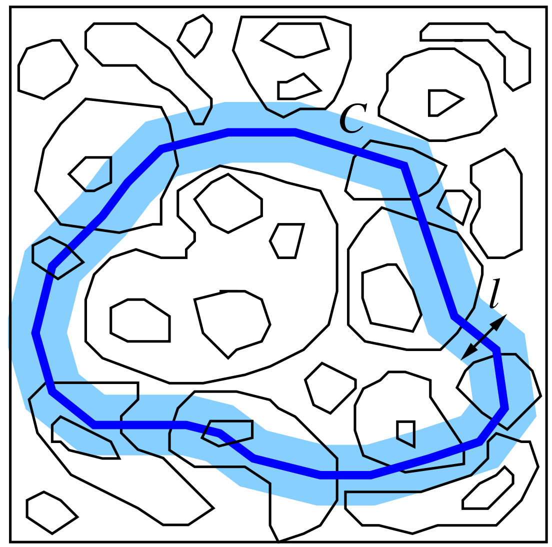
- TQFT result for a smooth boundary

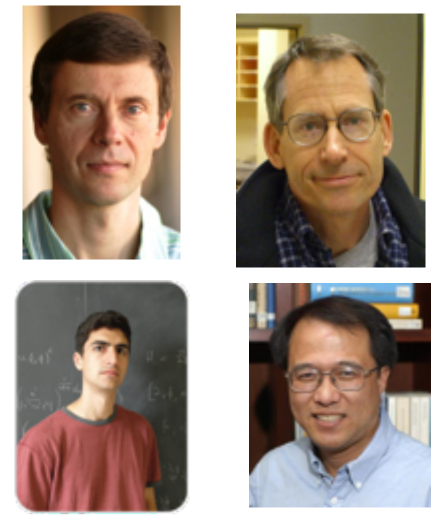
$$S_\alpha(A) \sim c_\alpha L - \gamma$$

$$\gamma = \ln \mathcal{D}$$ total quantum dimension

Topological EE

loops cross boundary even number of times





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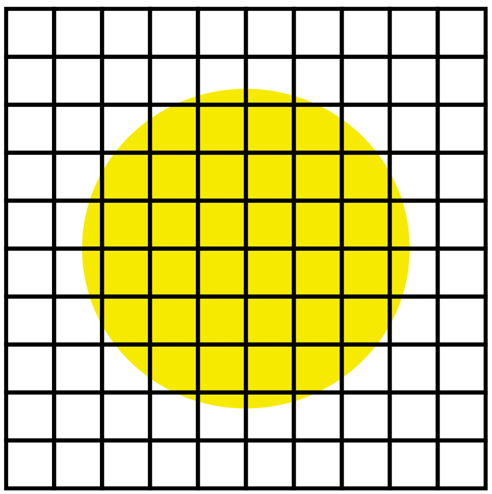
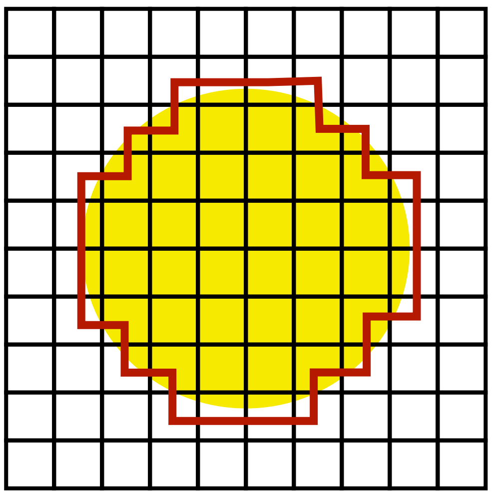
Z_2 spin liquid

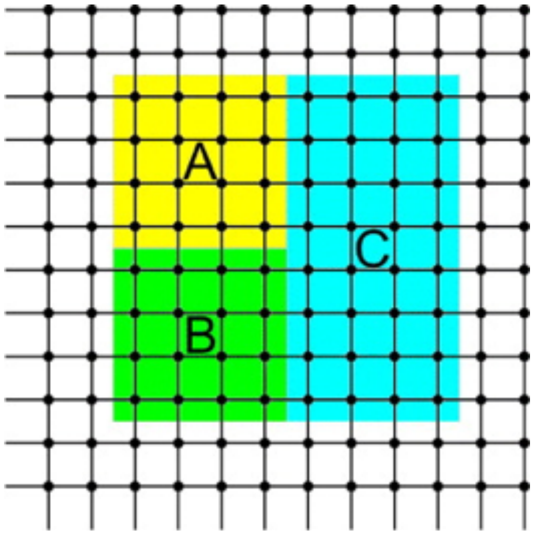
Smooth boundary?



Smooth boundary?

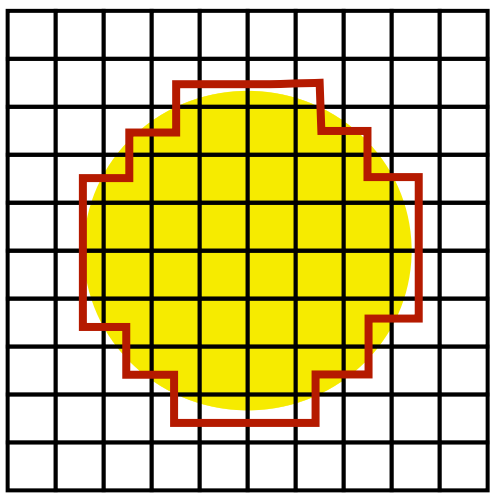


cannot be achieved for disc-like region on a lattice

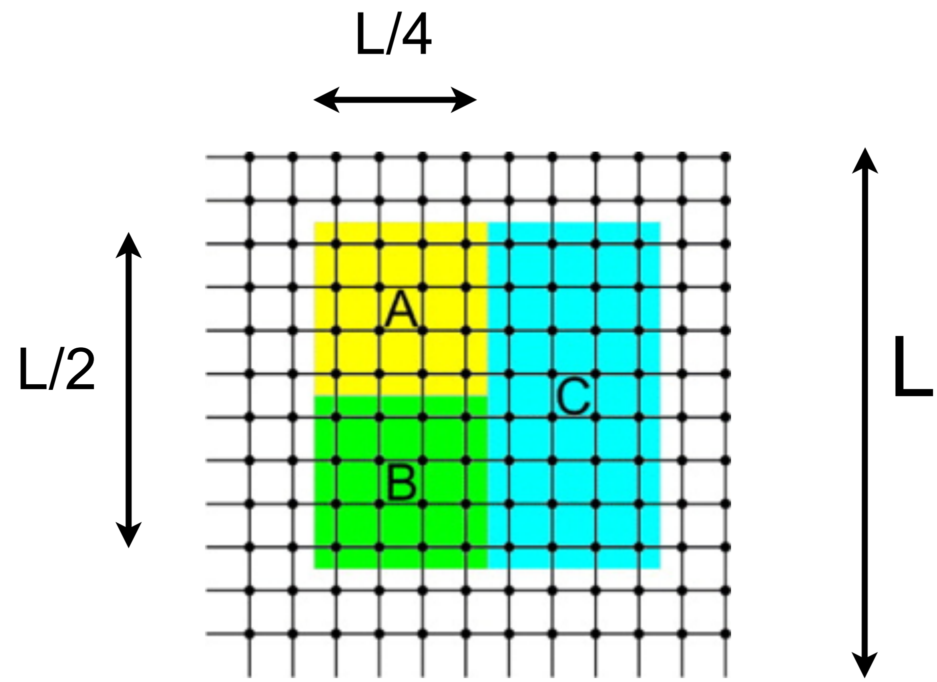


$$-\gamma = S_A + S_B + S_C$$
$$-S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

Smooth boundary?



cannot be achieved for disc-like region on a lattice

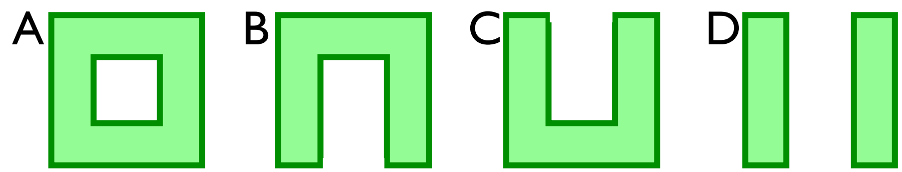


$$-\gamma = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

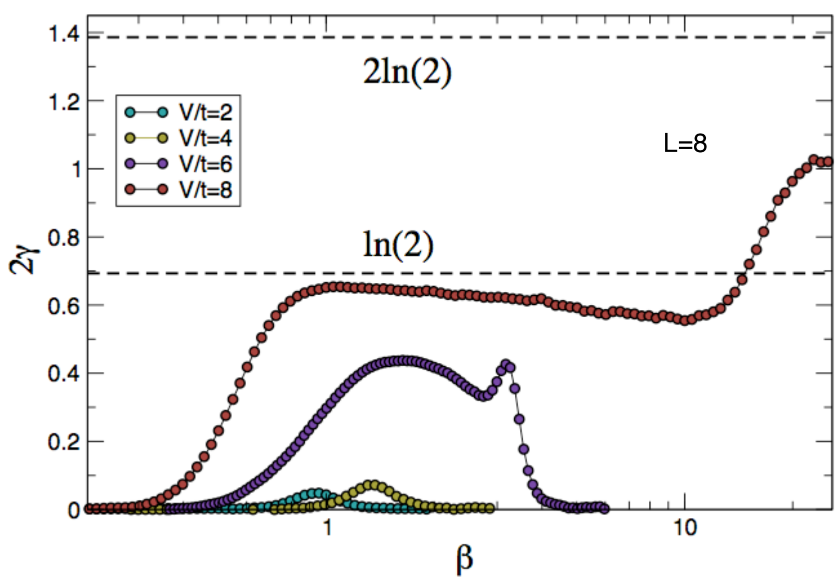
challenging due to cancellation of large numbers, and smaller length scales

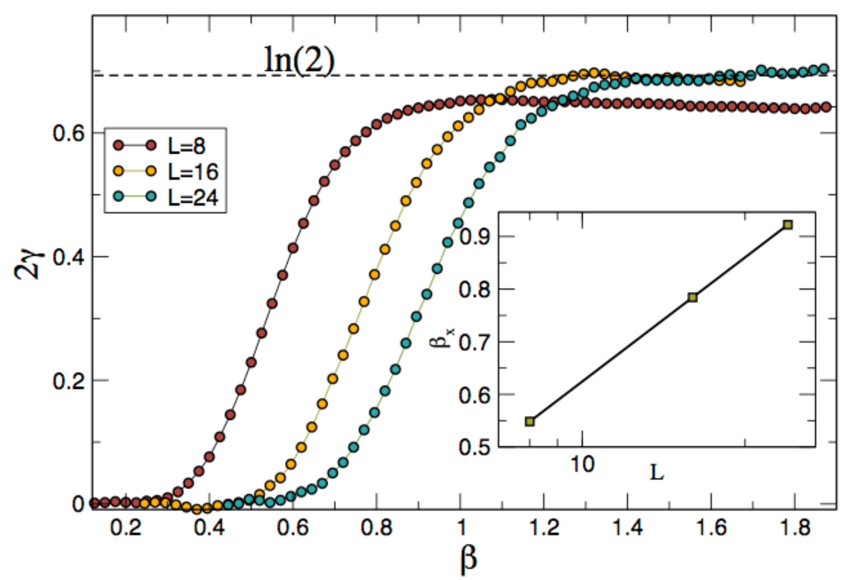
QMC results

Isakov, Hastings, RGM Nature Physics 7, 772 (2011)



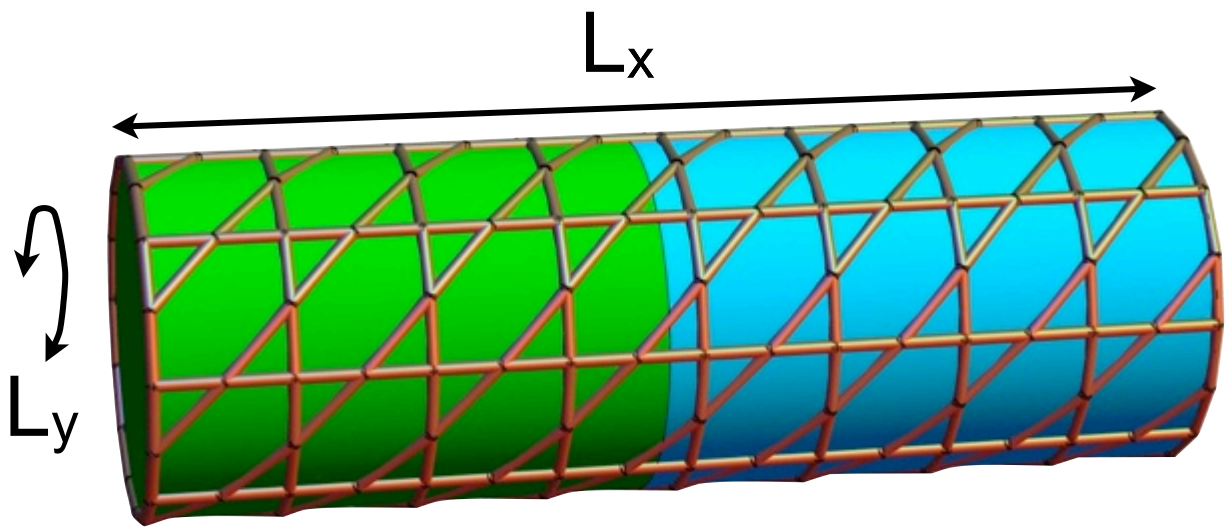
$$2\gamma = -S_n^A + S_n^B + S_n^C - S_n^D$$





heroic efforts get you halfway there

Cylinder



Easily implemented on lattice

- Only scales are circumference L_y, length L_x, and lattice spacing

$$S_\alpha \sim c_\alpha L_y - \gamma \qquad L_x \gtrsim L_y \gg 1$$

$$+O(e^{-L_y/\xi_\alpha})$$

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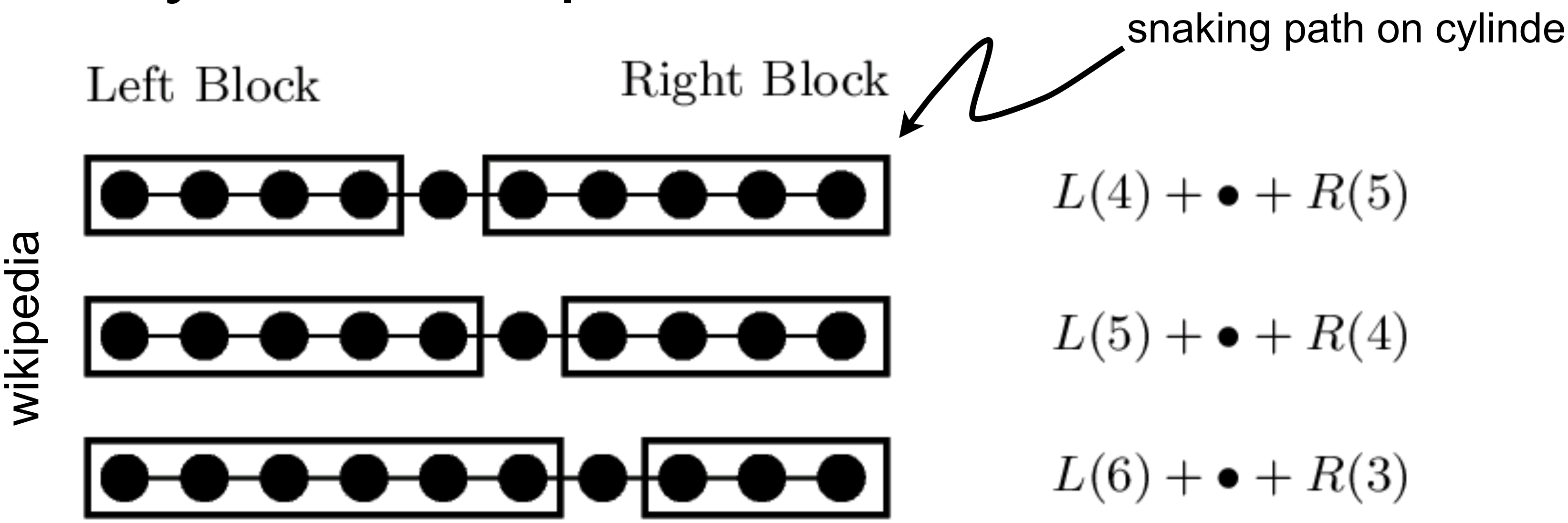
- Only scales are circumference L_y, length L_x, and lattice spacing

$$S_\alpha \sim c_\alpha L_y - \gamma \qquad\qquad L_x \gtrsim L_y \gg 1$$

$$+O(e^{-L_y/\xi_\alpha})$$

DMRG

- Systems is split into “blocks”



$L(4) + \bullet + R(5)$

$L(5) + \bullet + R(4)$

$L(6) + \bullet + R(3)$

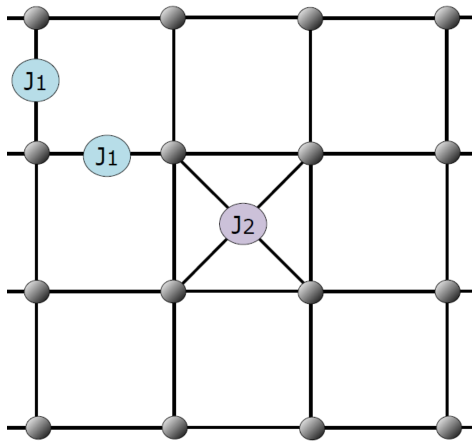
- Really a Schmidt decomposition

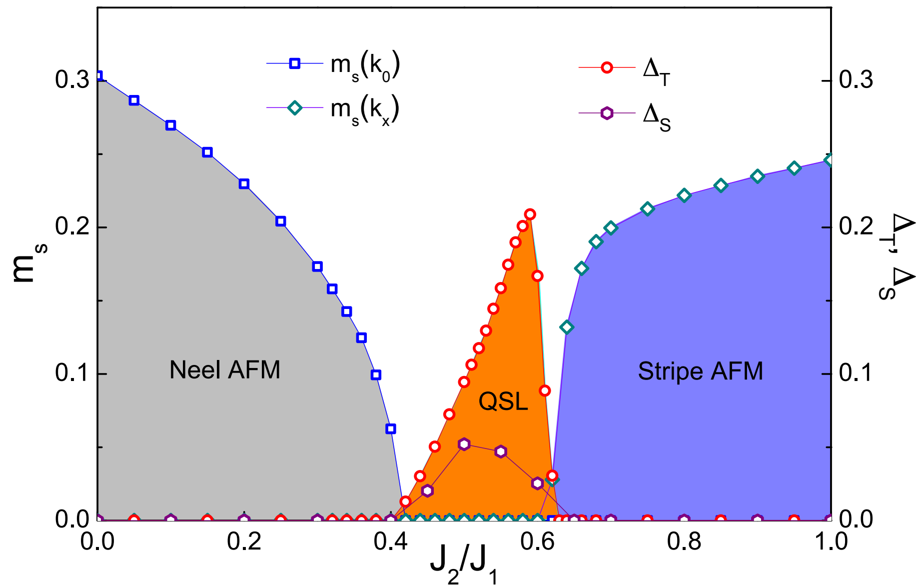
$$|\Psi\rangle = \sum_i c_i |\Psi\rangle_L \otimes |\Psi\rangle_R$$

Full entanglement spectrum and all S_α readily available

Example 1: J_1-J_2 model

$$H = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

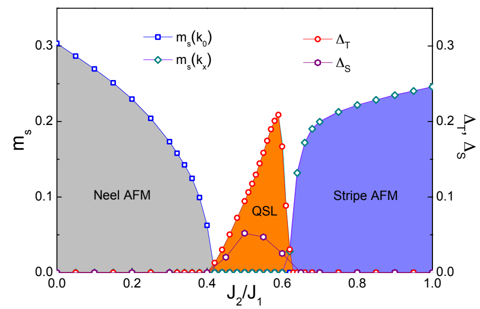




Hongchen Jiang, Hong Yao, LB (2012)

decades of work shows that intermediate phase has a gap and no magnetic order

Example 1: J_1-J_2 model



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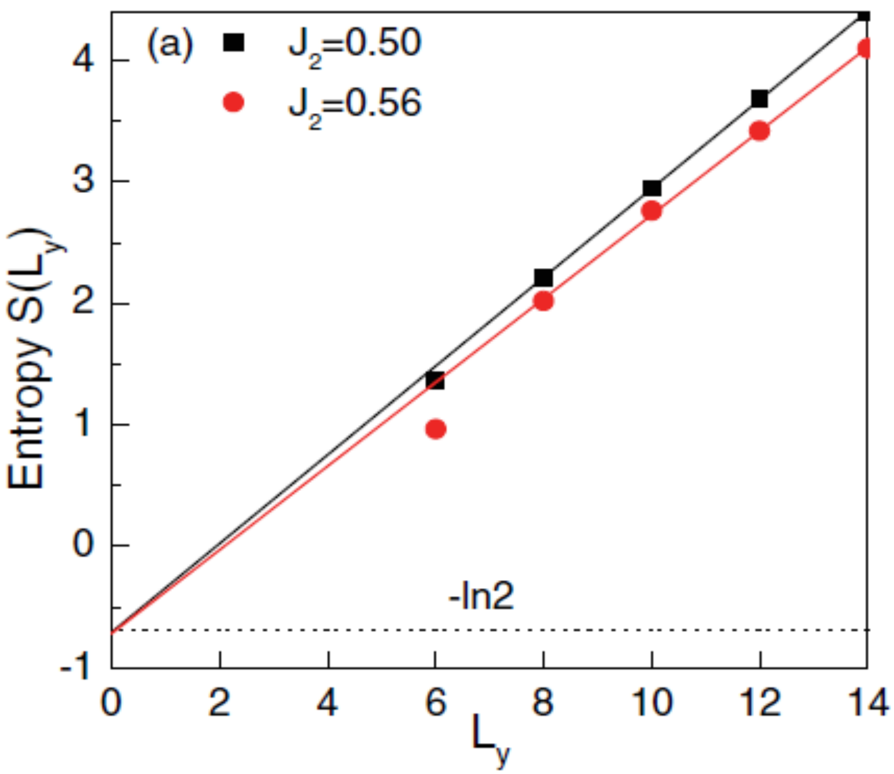

γ = 0.70 ± 0.02 $\quad$ J_2=0.50

= 0.72 ± 0.04 $\quad$ J_2=0.56

match to $\ln(2) = 0.69\cdots$

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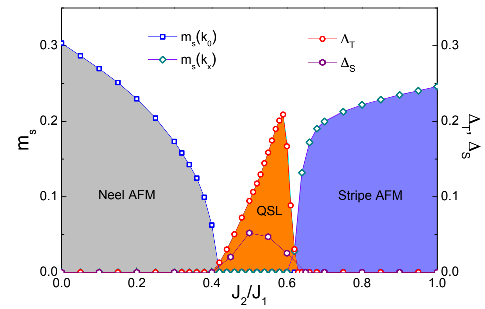
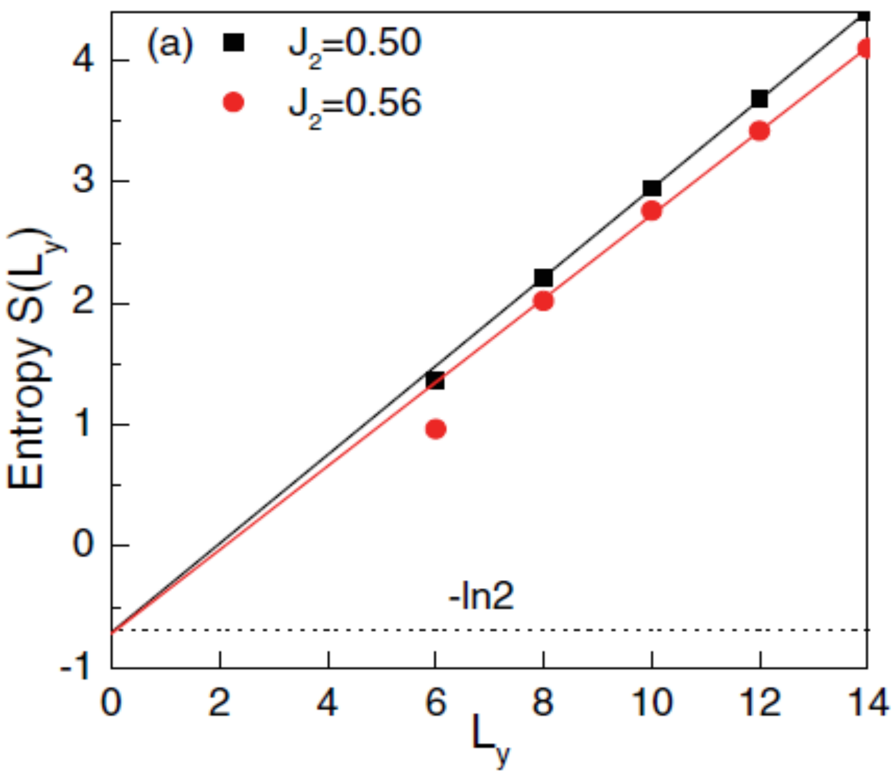




γ = 0.70 ± 0.02 J_2=0.50

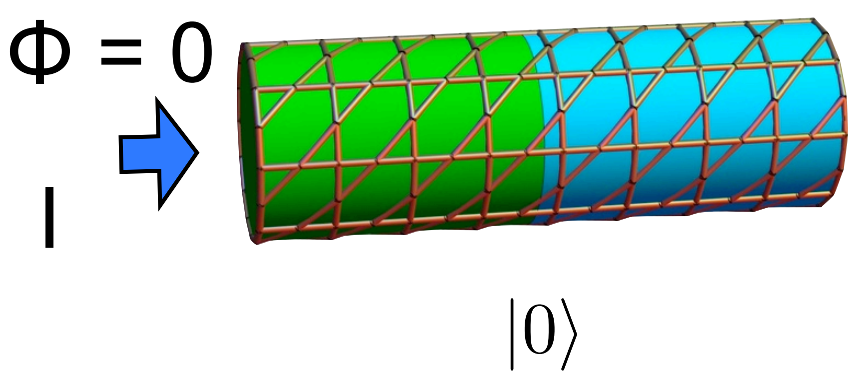
= 0.72 ± 0.04 J_2=0.56

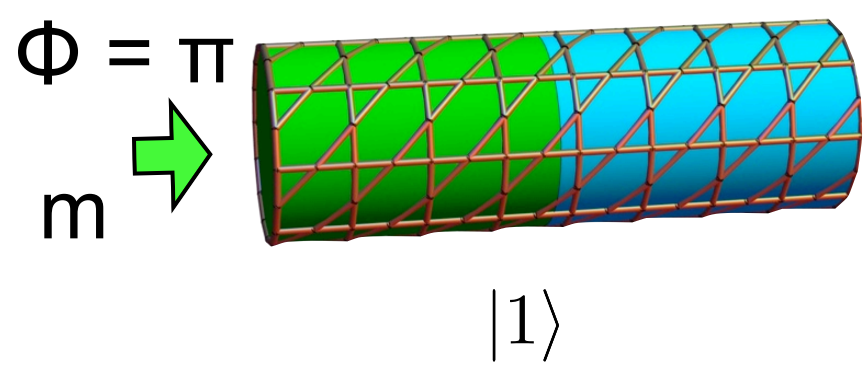
match to ln(2) = 0.69$\cdots$

seems good...look more carefully

State dependence

- A basic characteristic of topological phases is ground state degeneracy
- For the Z_2 case (toric code), it is two-fold on the cylinder





State dependence

- The EE depends upon the state

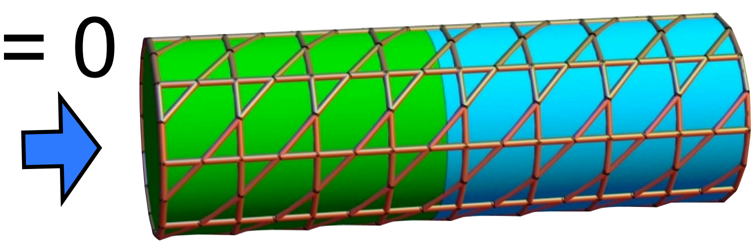
c.f. Yi Zhang *et al,* 2011

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right) \qquad 0 \leq \gamma \leq \ln 2 \qquad |0\rangle, |1\rangle$$

MES

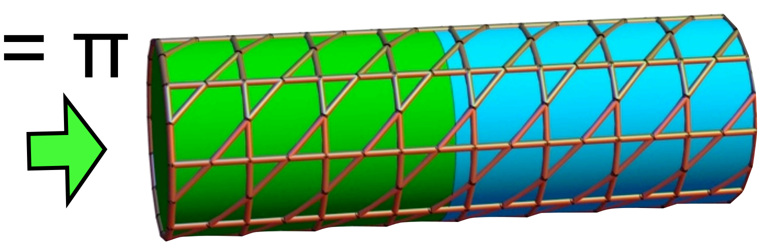
"minimally entangled states",
"minimum entropy states"

$\Phi = 0$



$|0\rangle$

$\Phi = \pi$



$|1\rangle$

State dependence

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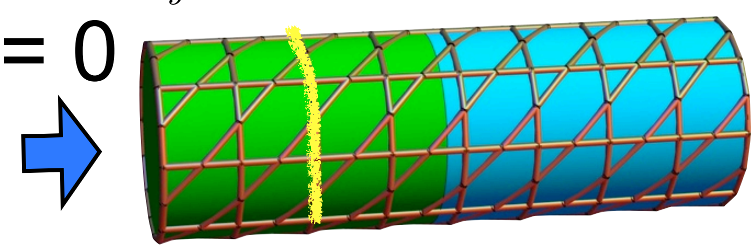
MES

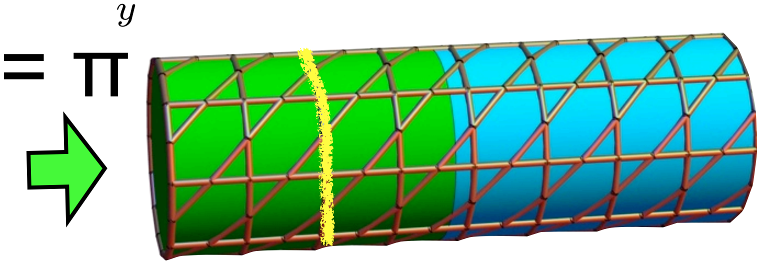
$\prod_y \sigma^z_{r,r+y} = +1$ $\qquad$ $\prod_y \sigma^z_{r,r+y} = -1$

Φ = 0



$|0\rangle$

Φ = π



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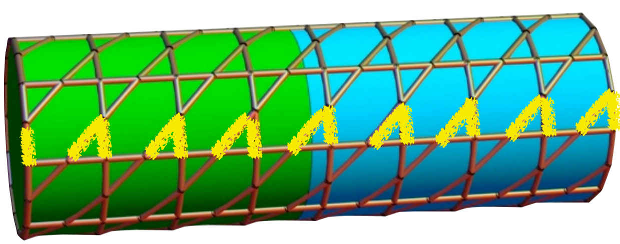
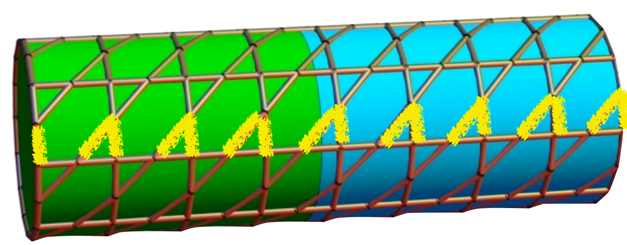


$|+\rangle$



$|-\rangle$

definite parity of loops around cylinder

State dependence

- The EE depends upon the state

c.f. Yi Zhang *et al,* 2011

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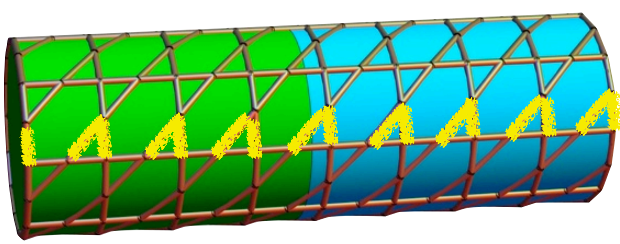
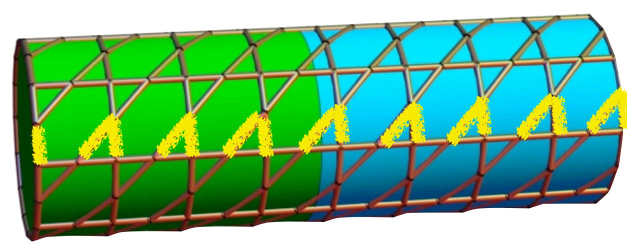


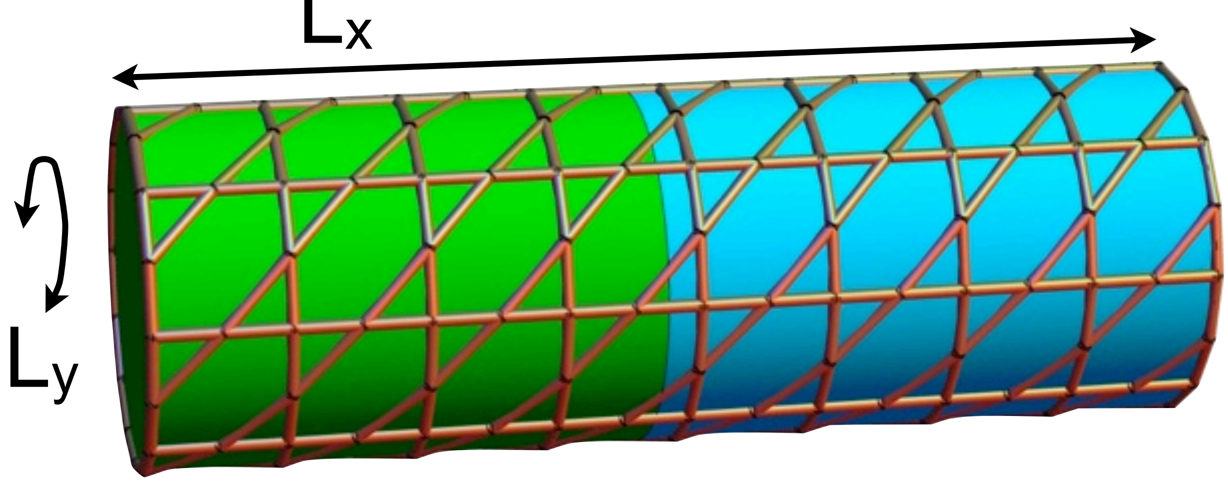
$|+\rangle$



$|-\rangle$

$$S(|\pm\rangle) = S(|0,1\rangle) + \ln 2$$

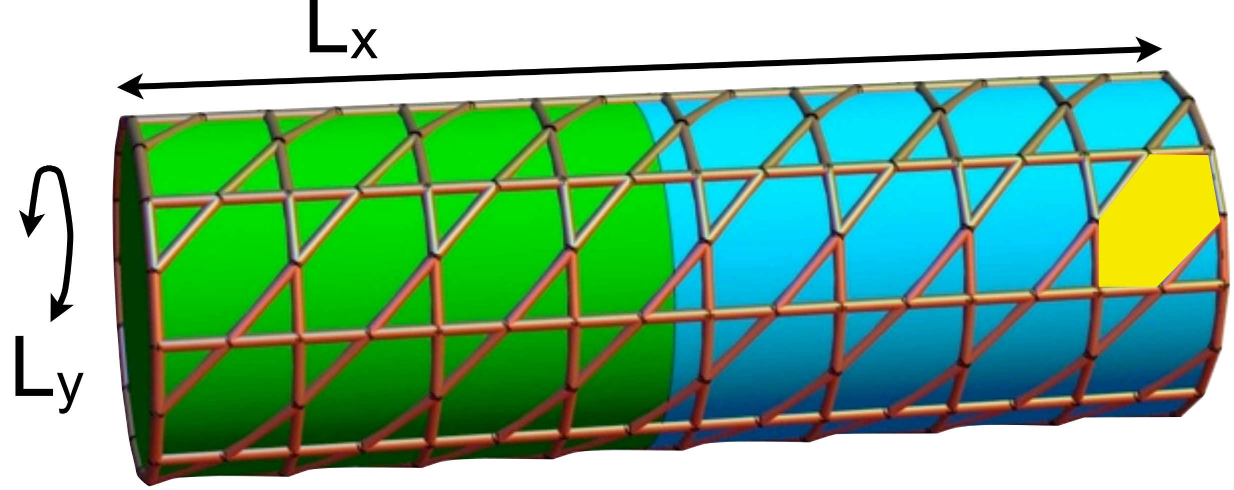
Quasi-degeneracy



- Hamiltonian

$$H_{\text{eff}} = -h\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) - h'\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)$$

Quasi-degeneracy



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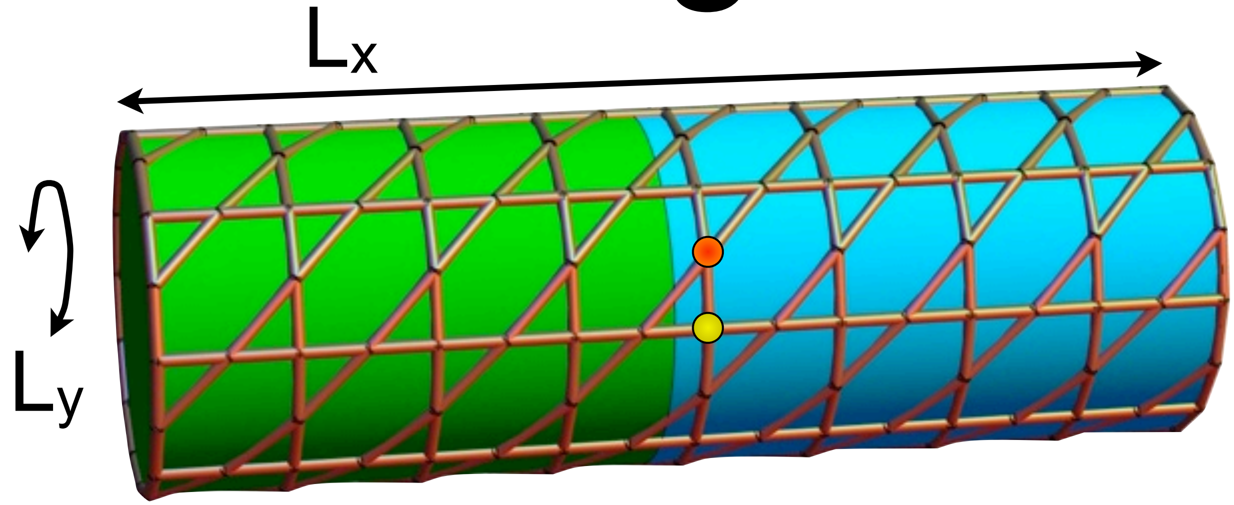
vison hopping $\qquad h \sim h_0 L_y e^{-L_x/\xi}$

Quasi-degeneracy



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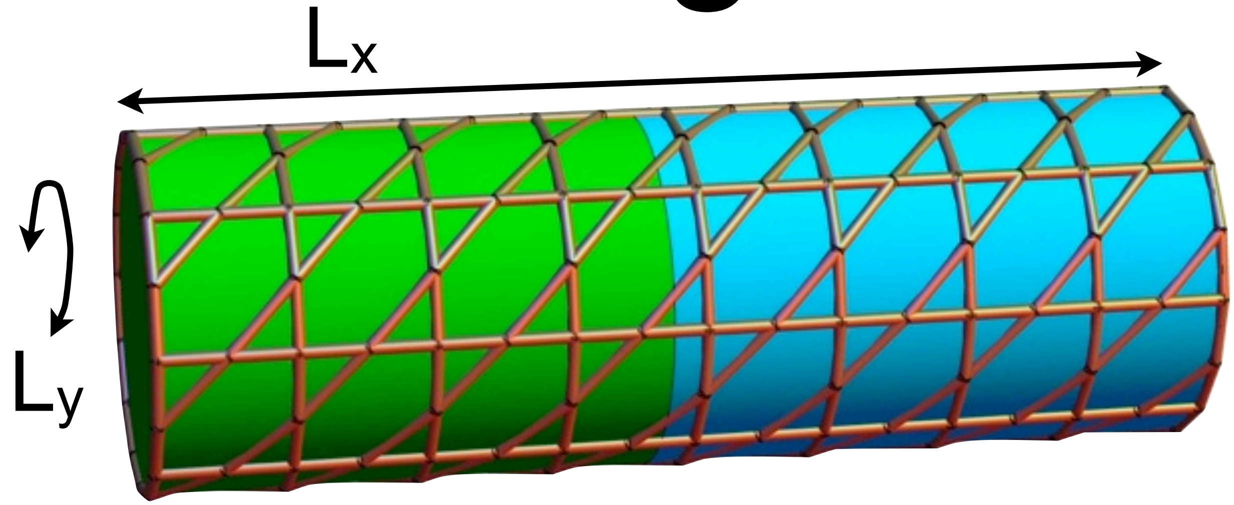

- Hamiltonian

GS is MES for L_x large

$$H_{\text{eff}} = -h\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) - h'\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)$$

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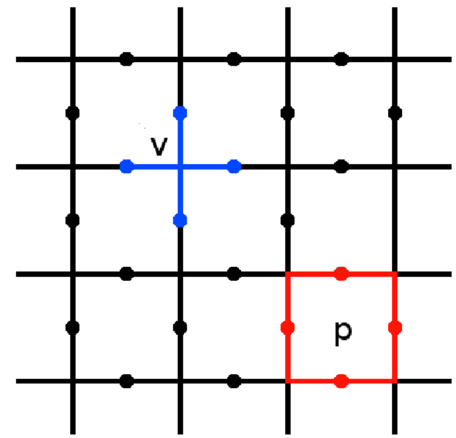
Toric Code model

- Hamiltonian

$$H_{TC} = -\sum_s A_s - \sum_p B_p$$



$$A_s = \prod_\mu \sigma^x_{s+\mu}$$

$$B_p = \prod_\mu \sigma^z_{p+\mu}$$

Toric Code model

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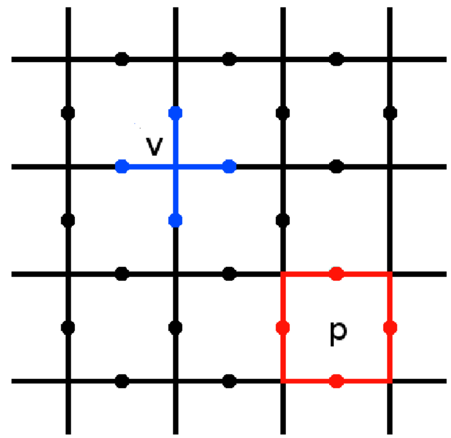

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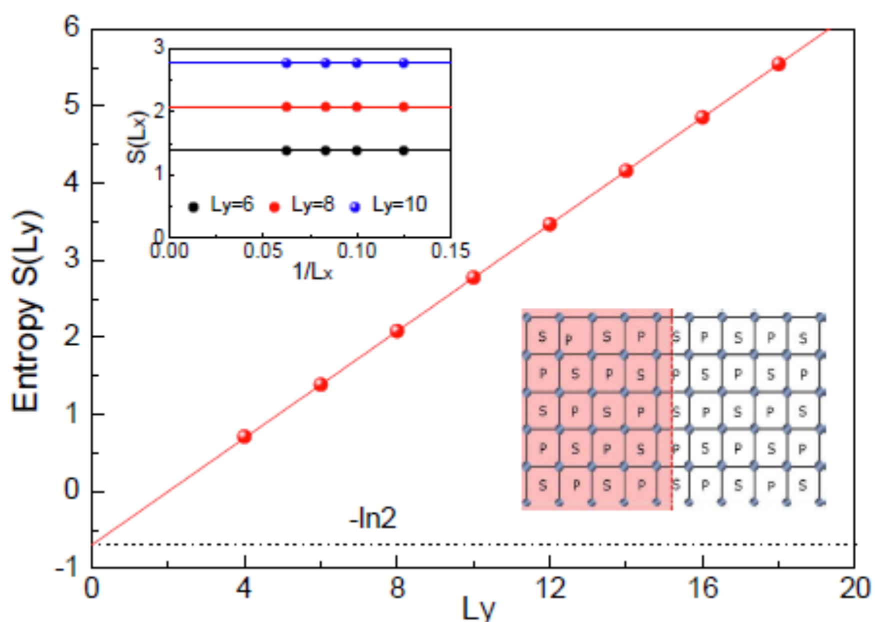


$$S = \frac{\ln 2}{2} L_y - \ln 2$$

Hongchen Jiang, Zhenghan Wang, LB (2012)

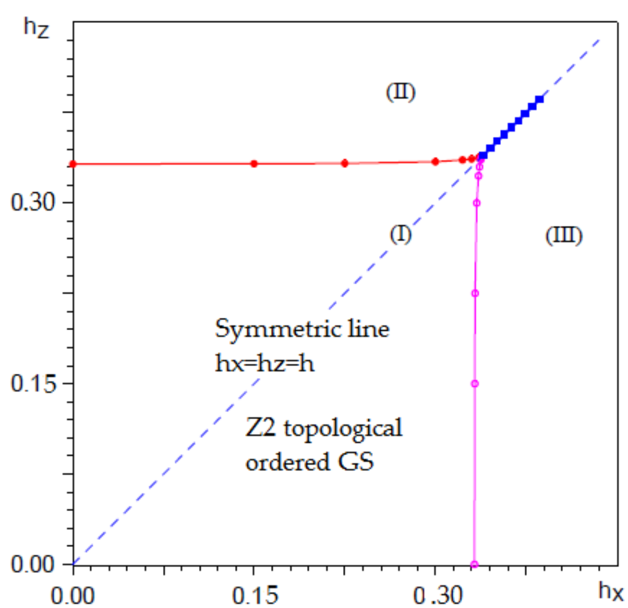
Perturbed Toric Code

- Hamiltonian:

$$H = H_{TC} - \sum_i \left[h_x \sigma_i^x + h_z \sigma_i^z\right]$$

- Phase diagram:

Tupitsyn *et al*, 2010



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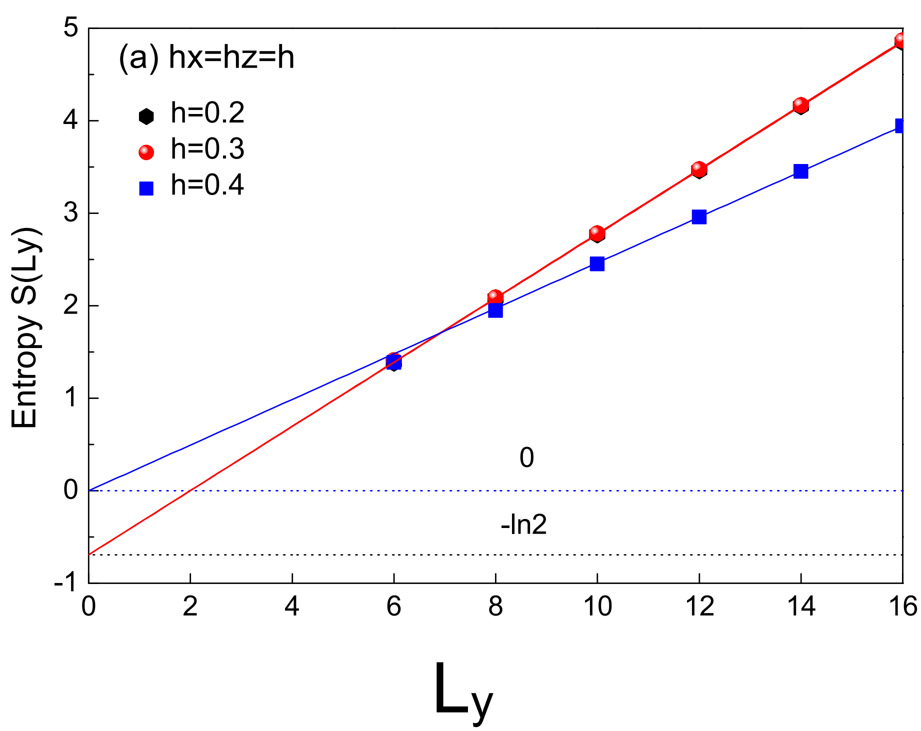
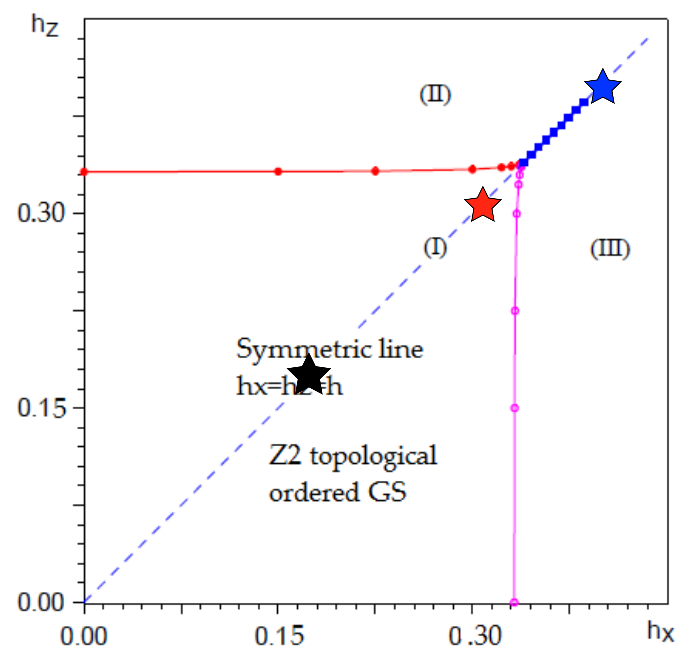
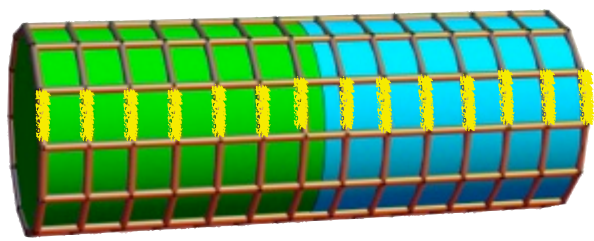




Perturbed Toric Code

- State dependence: $h_z = 0$

$$G = |0\rangle\langle 1| + |1\rangle\langle 0| = \prod_x \sigma^x_{i=(x,y)} \qquad [G, H] = 0$$



- The absolute ground state is a *G* eigenstate, $|\pm\rangle$, *not* a MES

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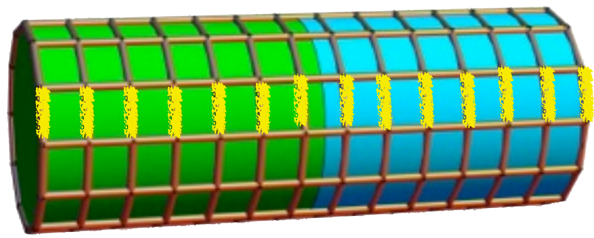

- The absolute ground state is a *G* eigenstate, $|\pm\rangle$, *not* a MES

$$H_{\text{eff}} = -h\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) - \underbrace{h'\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)}_{=0}$$

Perturbed Toric Code

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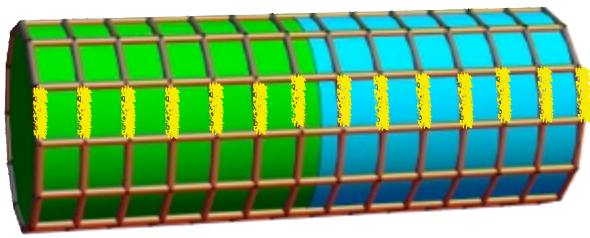
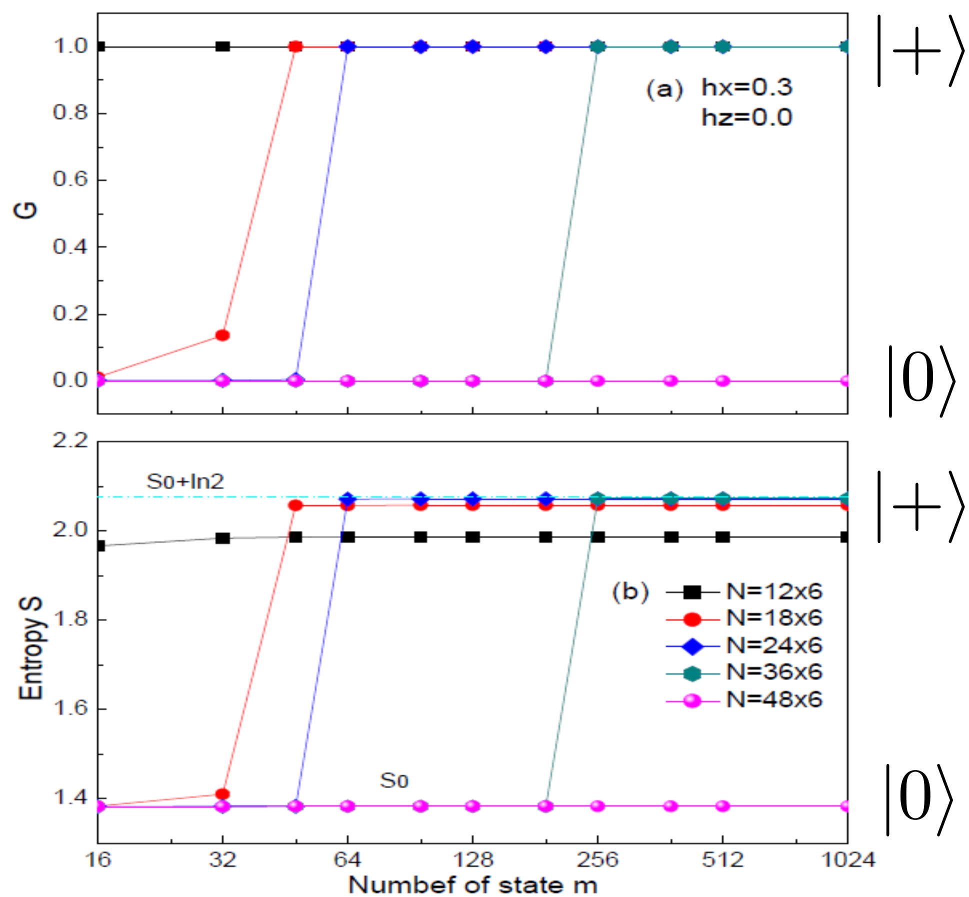


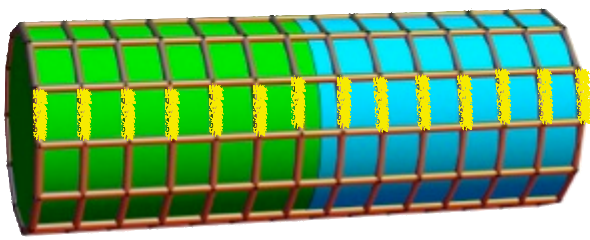
$$G|\pm\rangle = \pm|\pm\rangle$$



Perturbed Toric Code

- State dependence: $h_z = 0$

m$_c$



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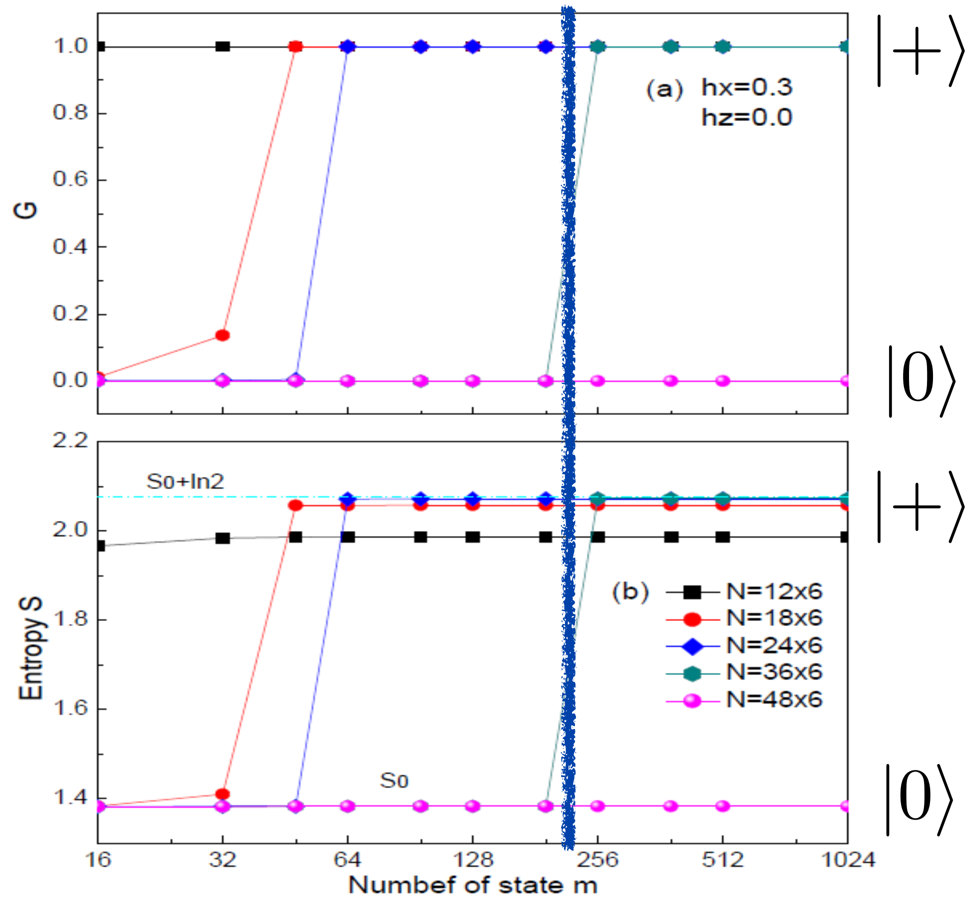
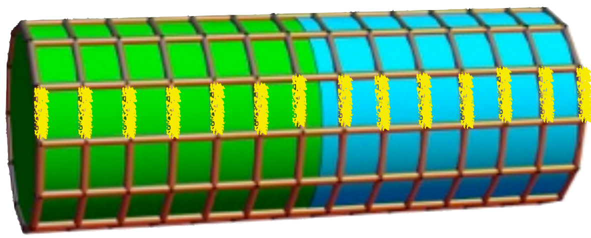


DMRG
favors MES

Perturbed Toric Code

- State dependence: $h_z = 0$



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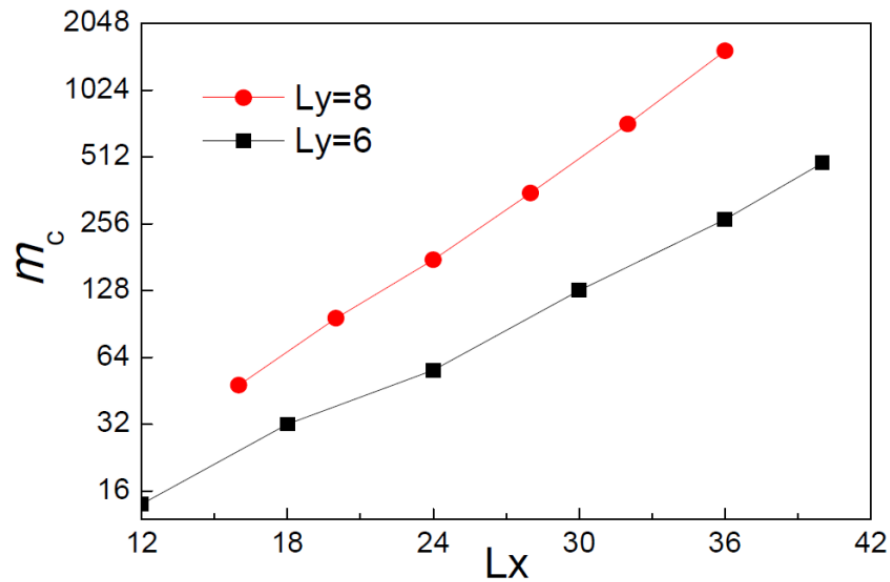
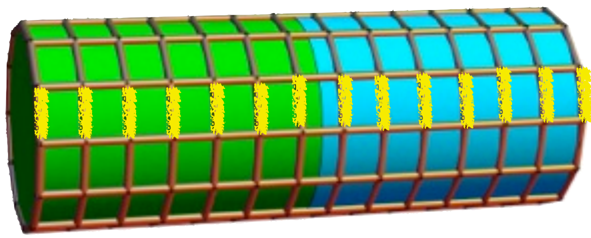


$$m_c \sim m_0 e^{cL_x}$$

last qubit of entanglement takes exponential effort

Perturbed Toric Code

- State dependence: $h_z = 0$



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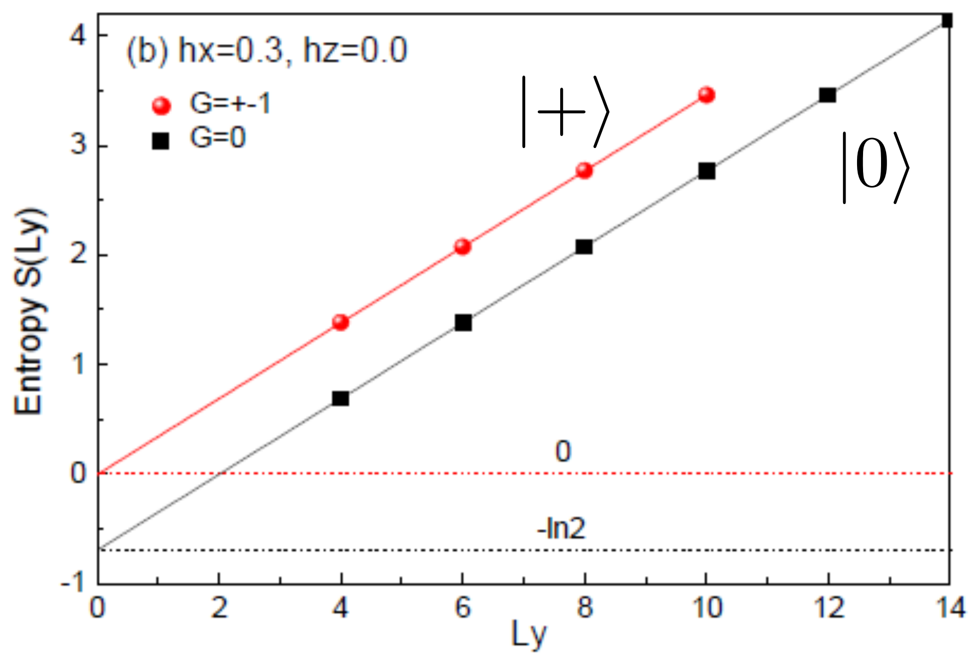
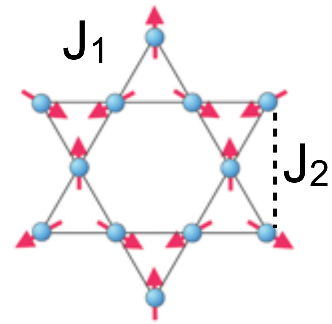


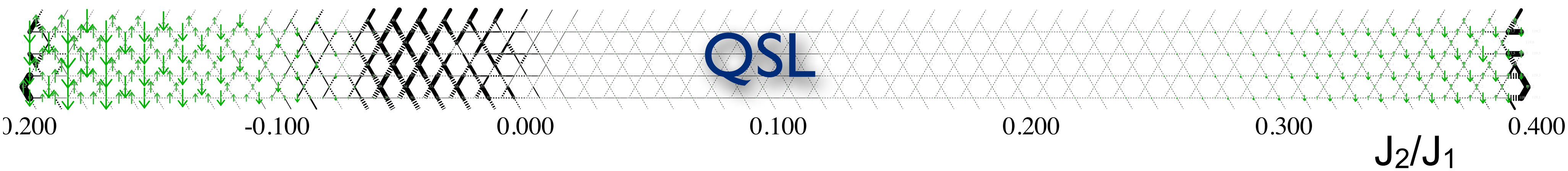
Kagome

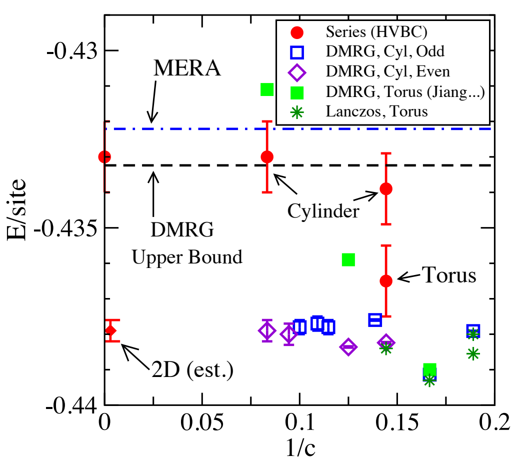
- Hamiltonian

$$H = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



- Courtesy of Steve White:

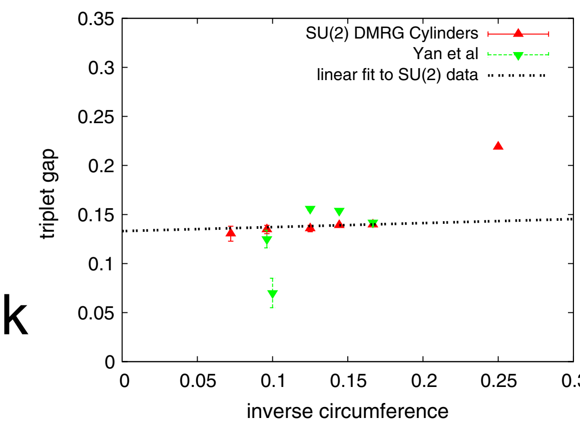




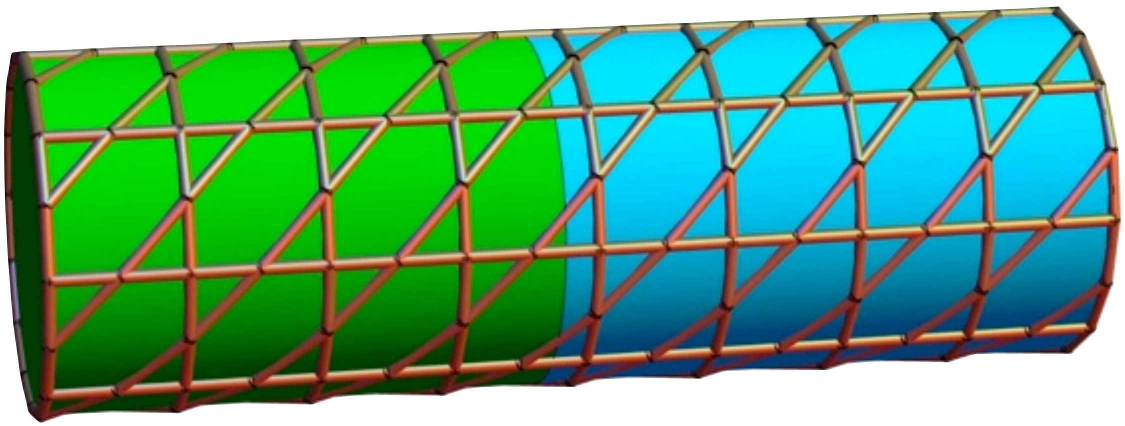
S. Yan *et al*, 2010

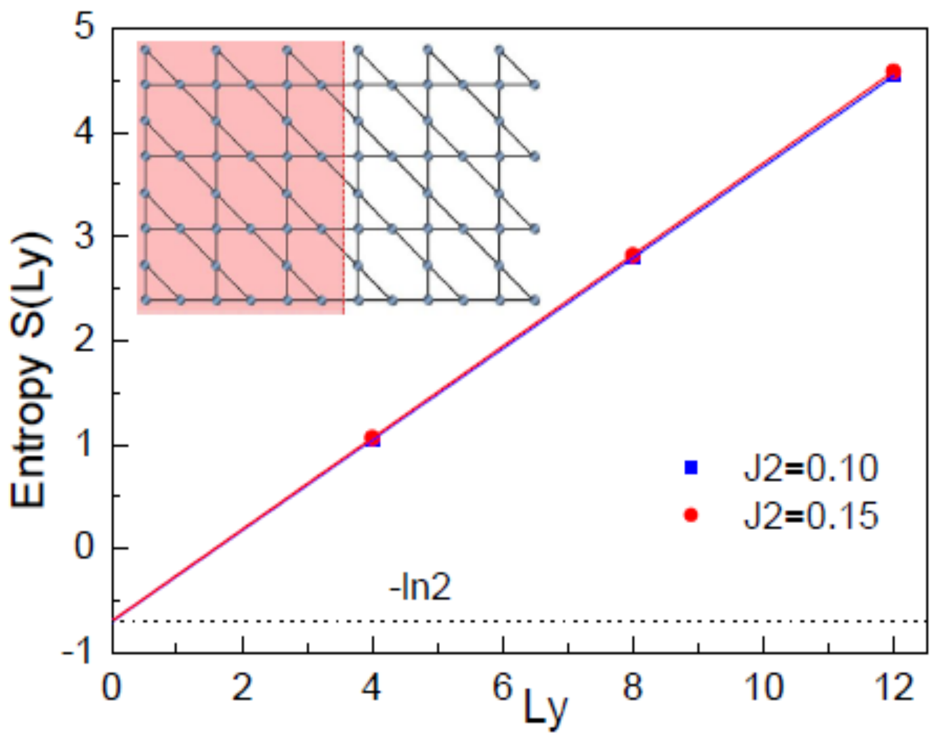
a Z_2 state?

S. Depenbrock *et al*, 2012



Kagome



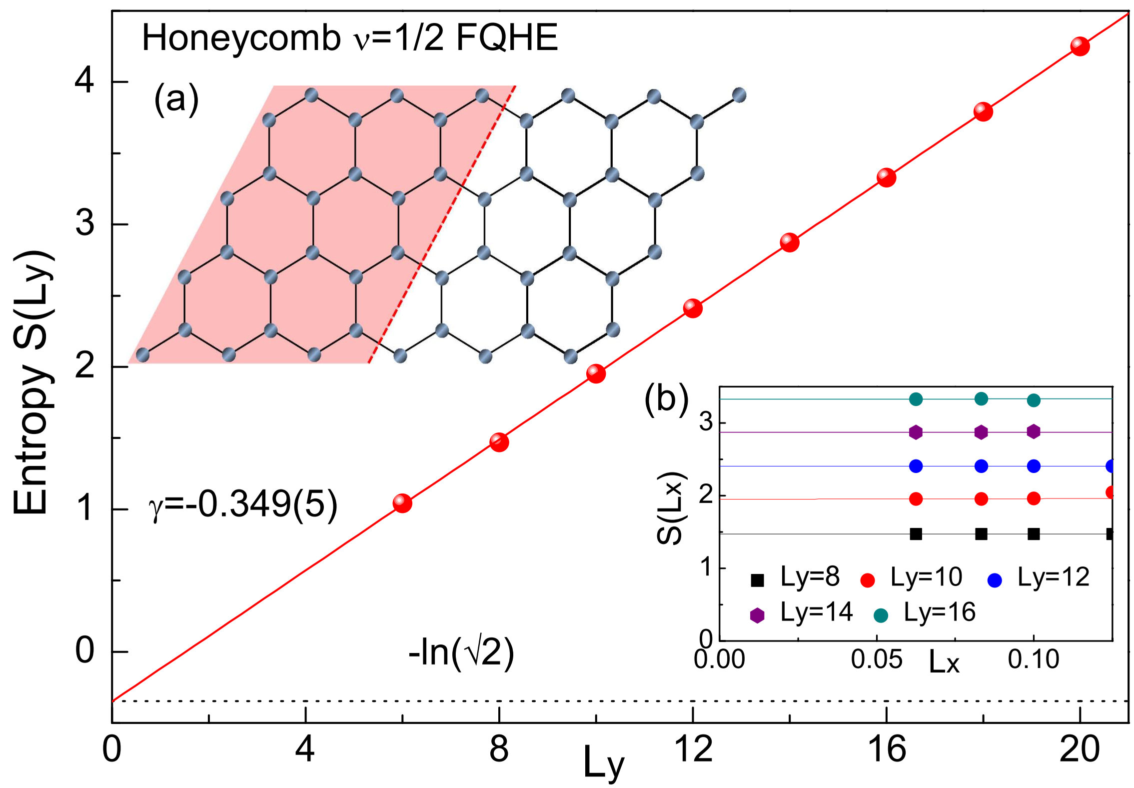


$$\gamma = \begin{cases} 0.698(8) & J_2 = 0.10 \\ 0.694(6) & J_2 = 0.15 \end{cases}$$

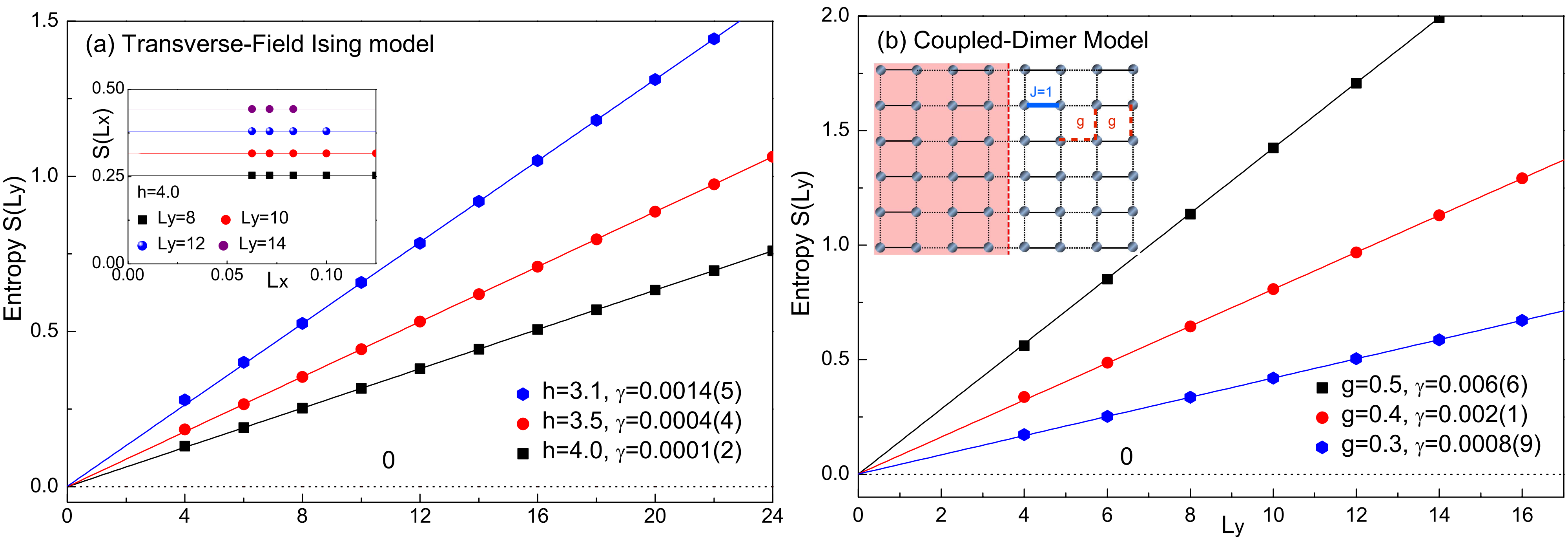
$[\ \ln(2) = 0.693\cdots]$

ν=1/2 FQHE

$$H = -t' \sum_{\langle\langle rr' \rangle\rangle} \left[b_{r'}^{\dagger} b_r e^{i\phi_{r'r}} + \mathrm{H.c.} \right] - t \sum_{\langle rr' \rangle} \left[b_{r'}^{\dagger} b_r + \mathrm{H.c.} \right] - t'' \sum_{\langle\langle\langle rr' \rangle\rangle\rangle} \left[b_{r'}^{\dagger} b_r + \mathrm{H.c.} \right],$$



Not topological



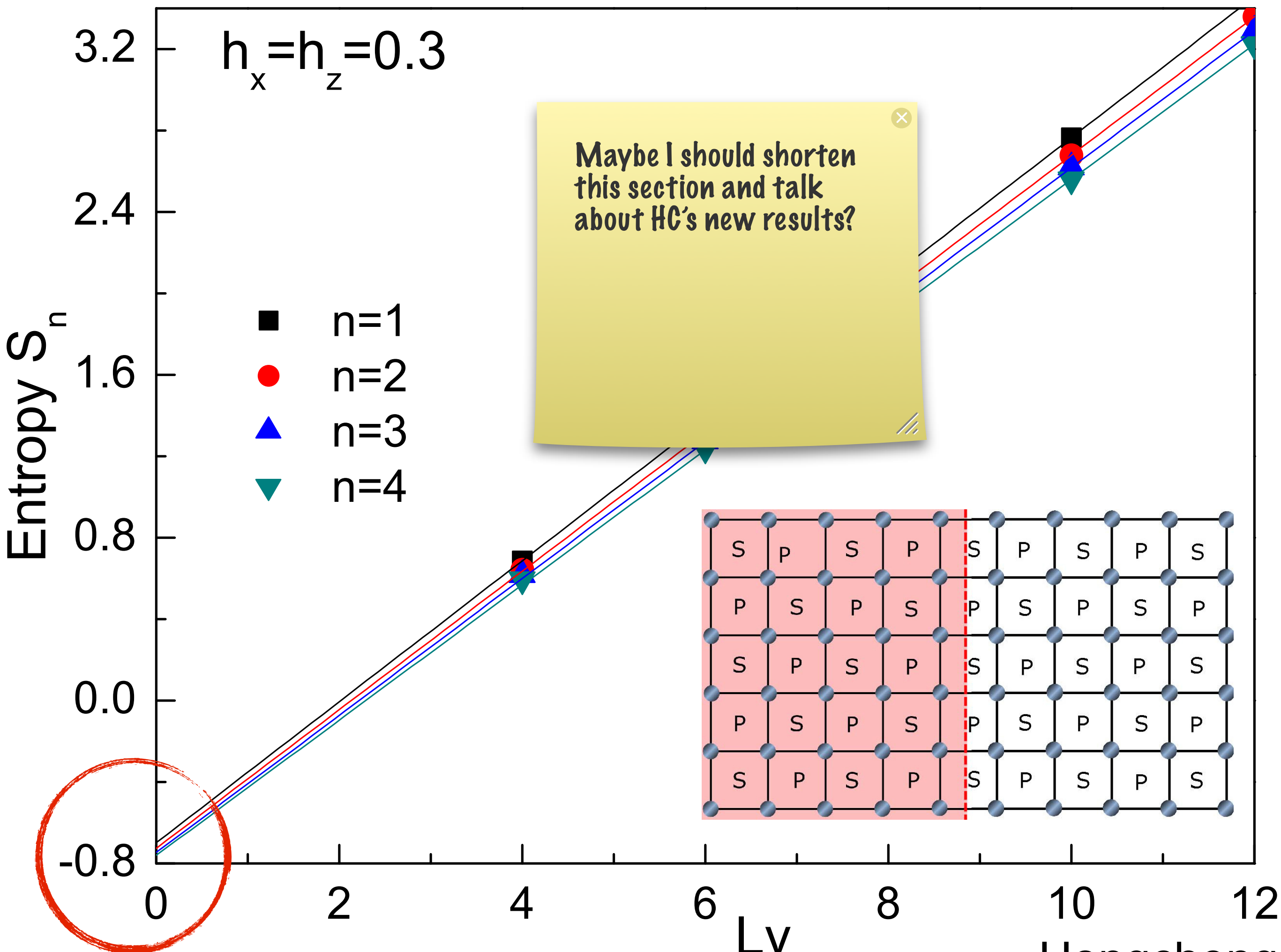
Von Neumann vs. Renyi

Both are expected to give universal TEE

Von Neumann vs. Renyi

Numerical result: Renyi scales *much* worse

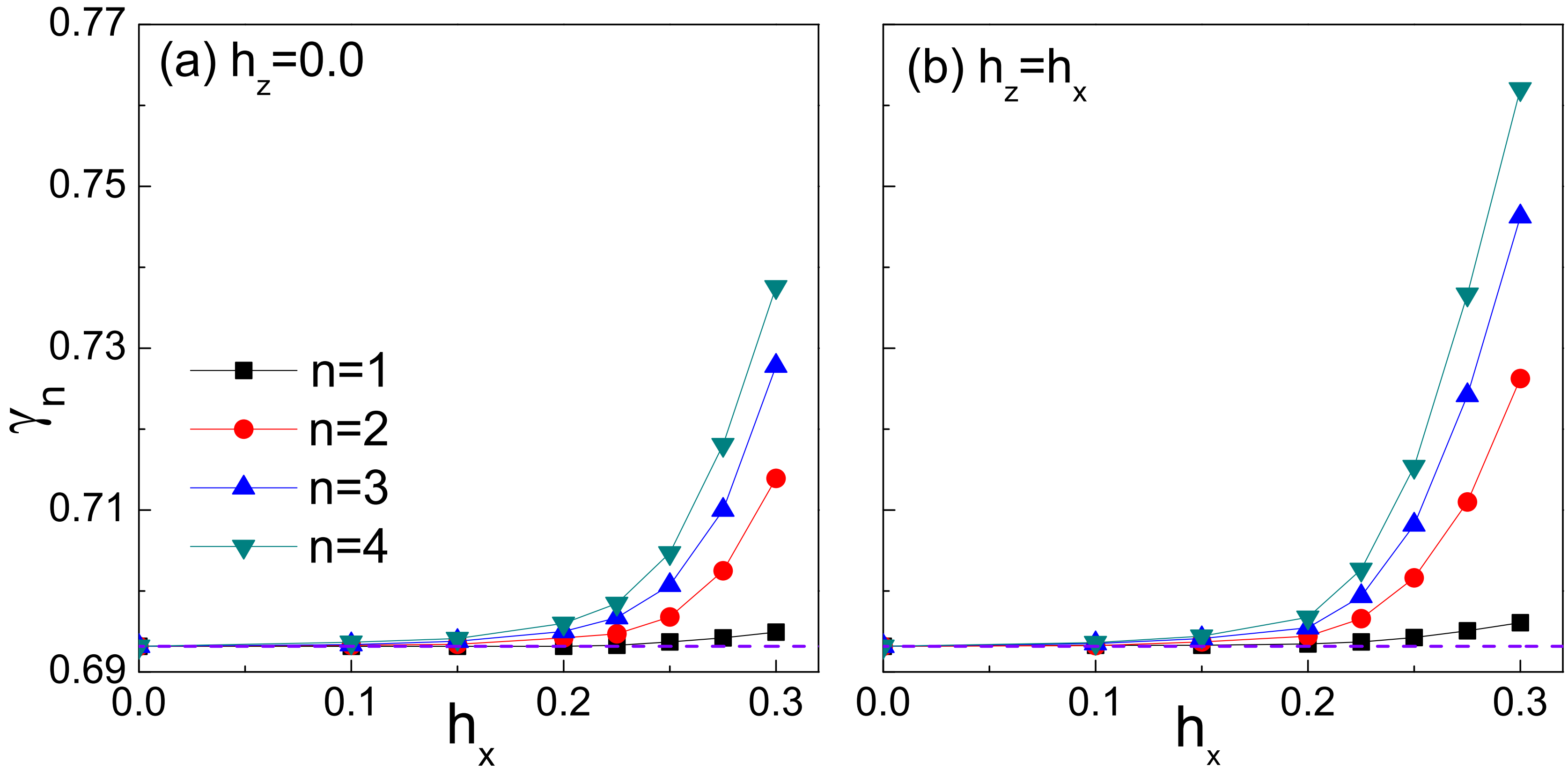
Trouble?



Hongcheng Jiang + LB (2013)

Trouble?

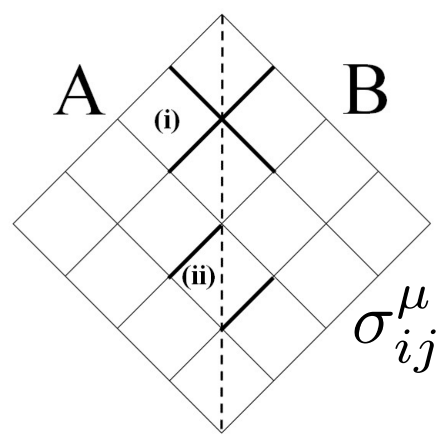
- Naively extrapolated TEE



Series Expansion

- We can check the DMRG, and get some insight, from numerical series expansion
- We use the linked cluster method, where the series can be extracted from a study of exact solutions on small clusters

clusters to $O(h^4)$



toric code variables on links

Series Expansion

- Entanglement entropy is *exactly* linear in L_y until clusters span cylinder width

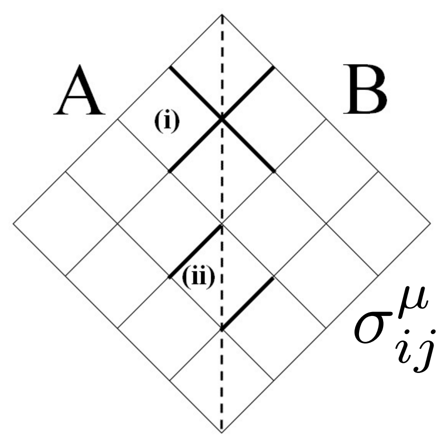
$$S_\alpha = c_\alpha L_y - \ln 2$$

line entropy

$$c_\alpha = \frac{1}{2}\left(\ln 2 - \frac{9\alpha}{32}h^4 + \frac{3\alpha}{\alpha - 1}\frac{h^4}{128}\right)$$

$h_x = h, h_z = 0$

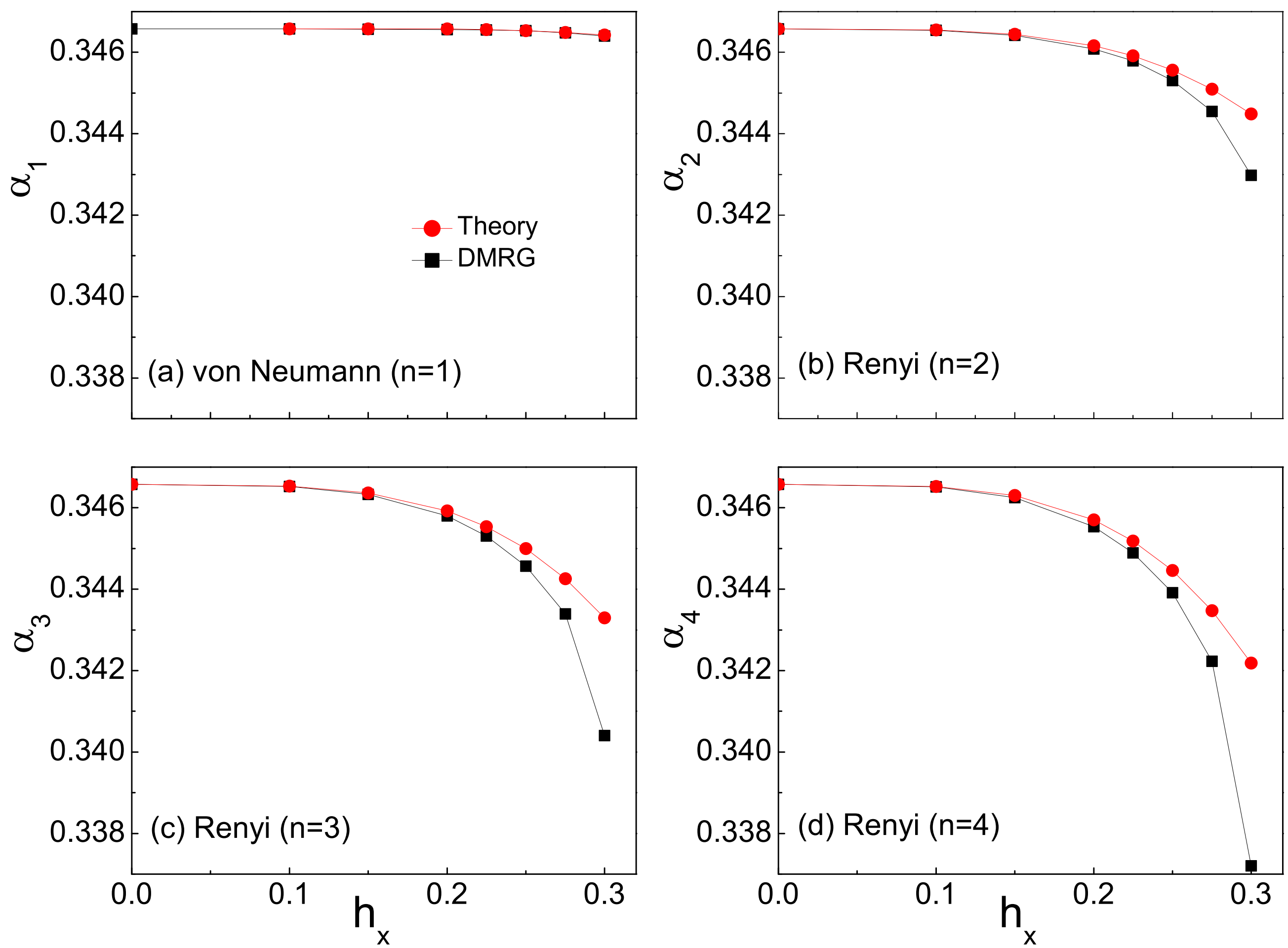
clusters to $O(h^4)$



toric code variables on links

Line Entropy

reasonable agreement for $O(h^4)$ expansion



$$h_x = h, h_z = 0$$

$$h_c \approx 0.33$$

TEE

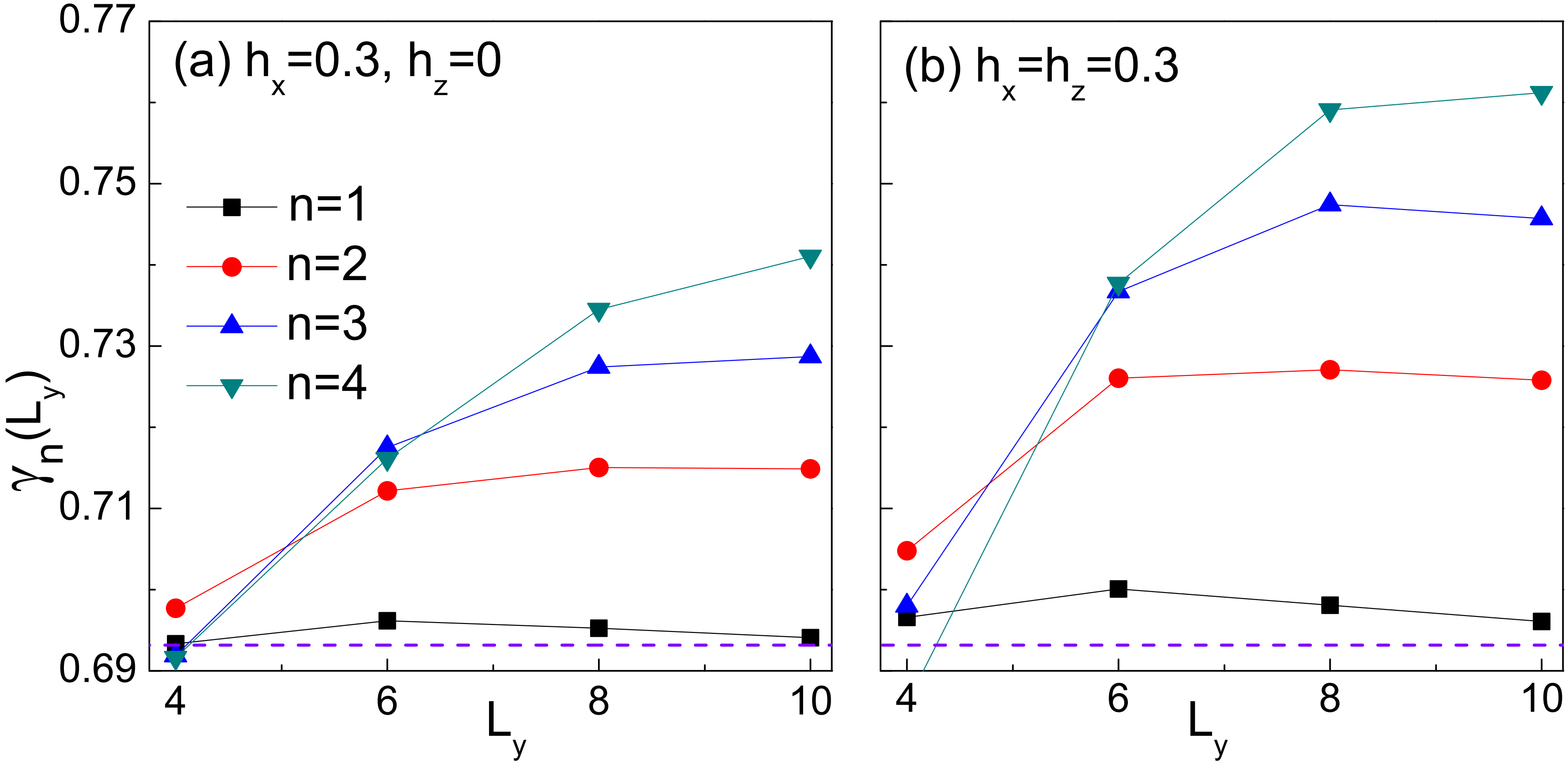
- TEE is *subdominant* term in an *asymptotic* large L_y expansion
  - Reliable extraction is more difficult with increasing line entropy

$$c_\alpha = \frac{1}{2}\left(\ln 2 - \frac{9\alpha}{32}h^4 + \frac{3\alpha}{\alpha - 1}\frac{h^4}{128}\right)$$

large sensitivity of line entropy
with increasing Renyi index:
larger L_y needed to extract TEE?

TEE

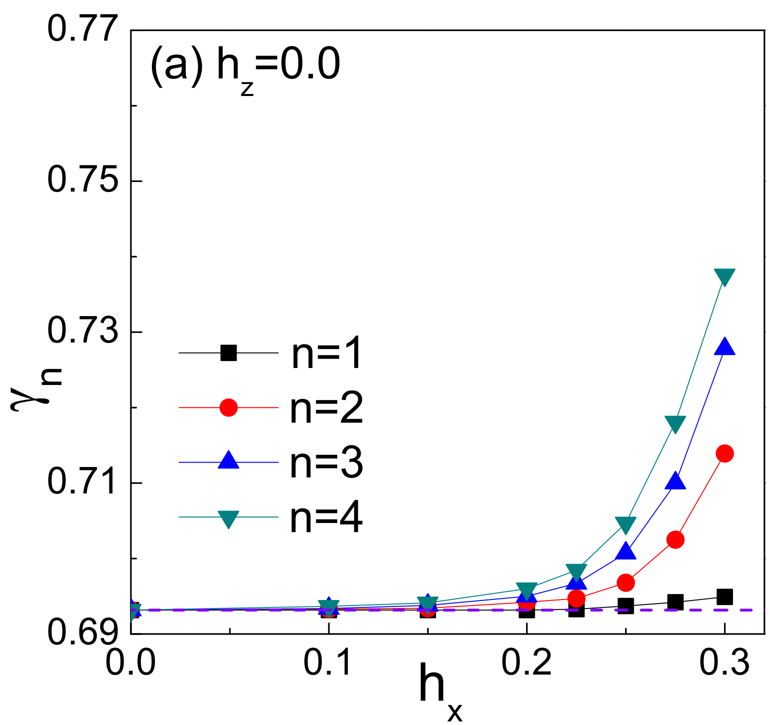
- Running two point fits at L_y and L_{y+2}

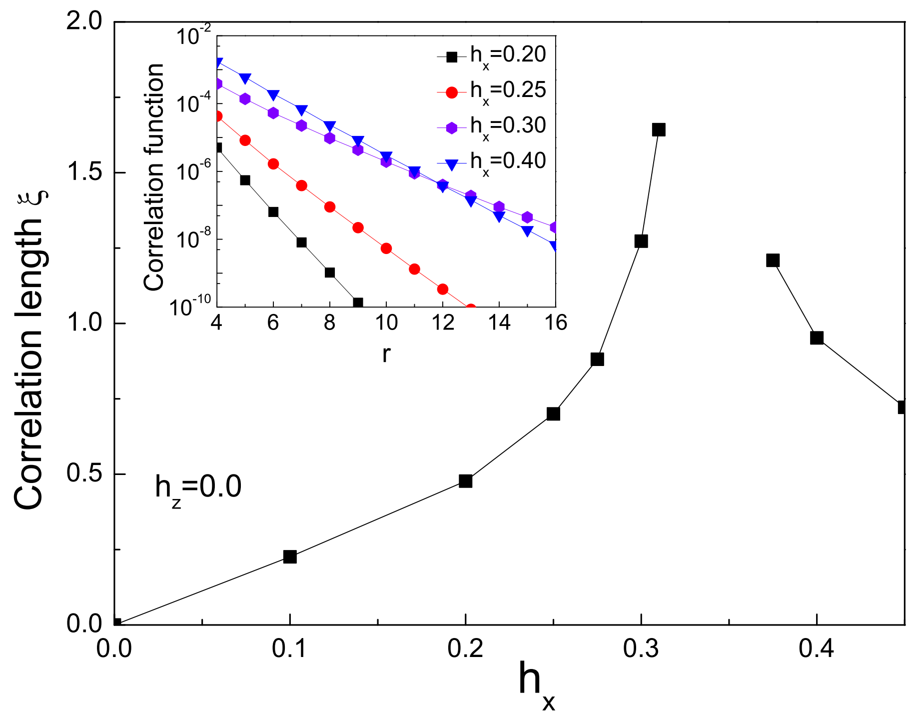


non-monotonic behavior: still far from converging for Renyi entropies

TEE

- Correlation length criteria?

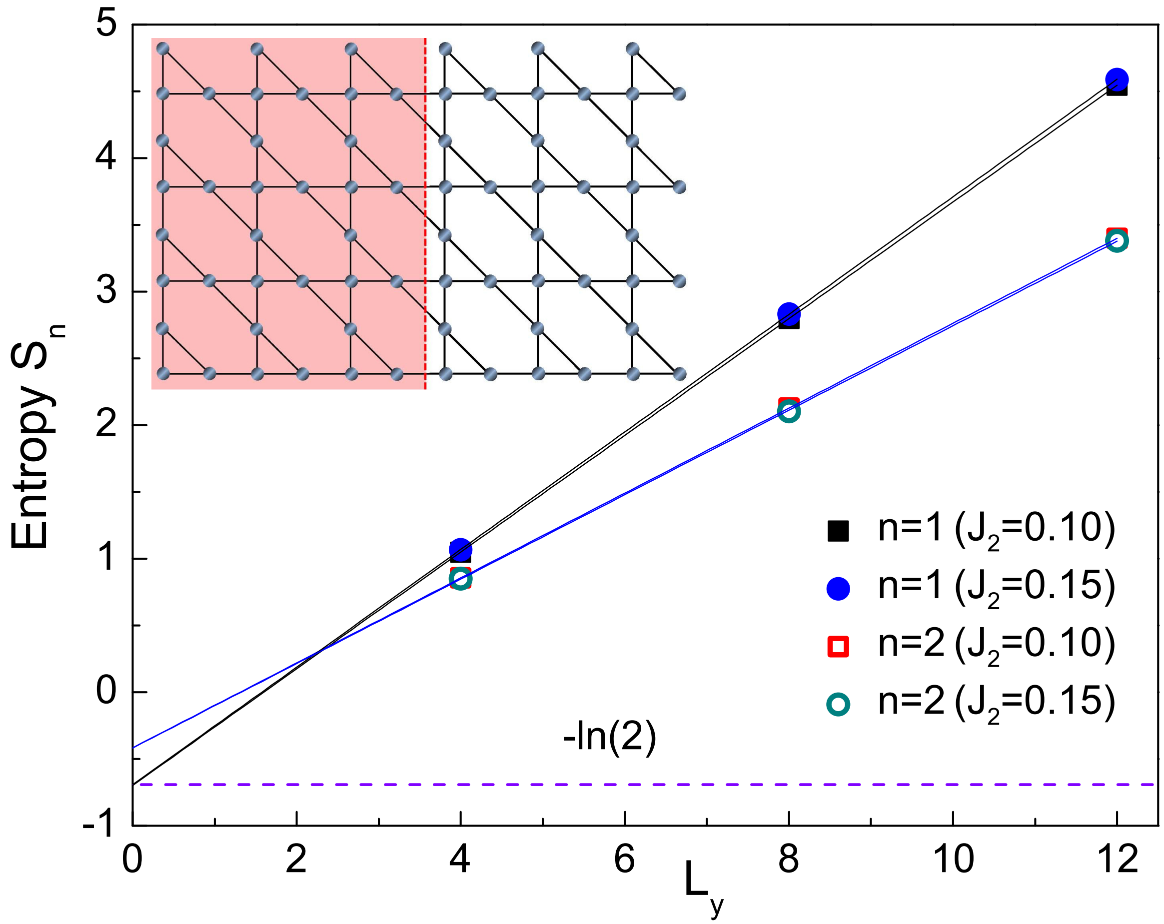




S_{vN} determines TEE to < 1% for $L_y/\xi \approx 10$

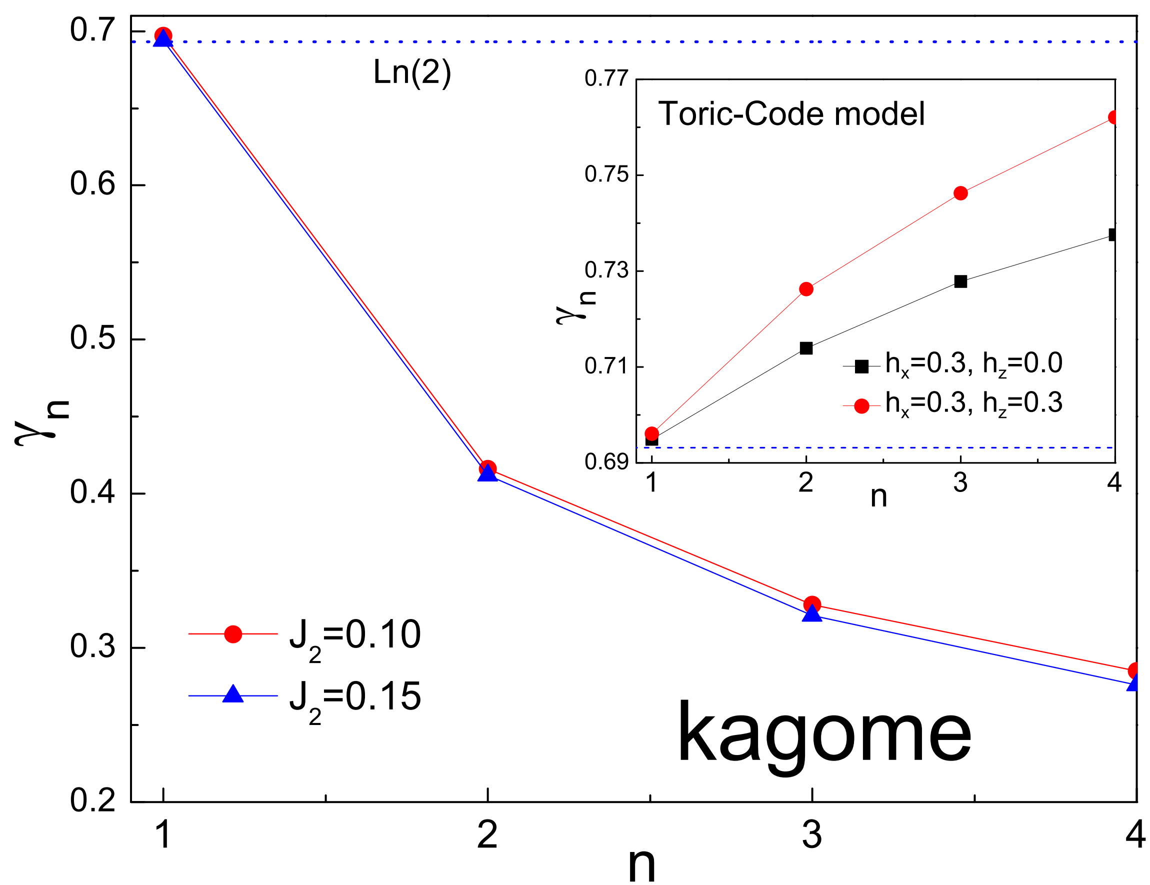
Kagome

- Problems even more severe



Kagome

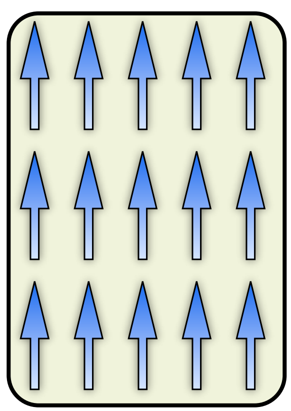
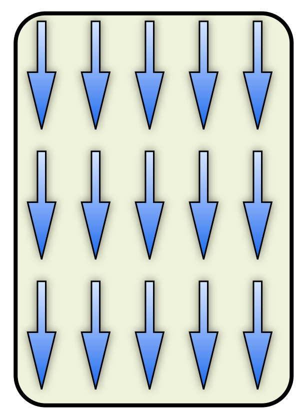
- Problems even more severe



more on MES

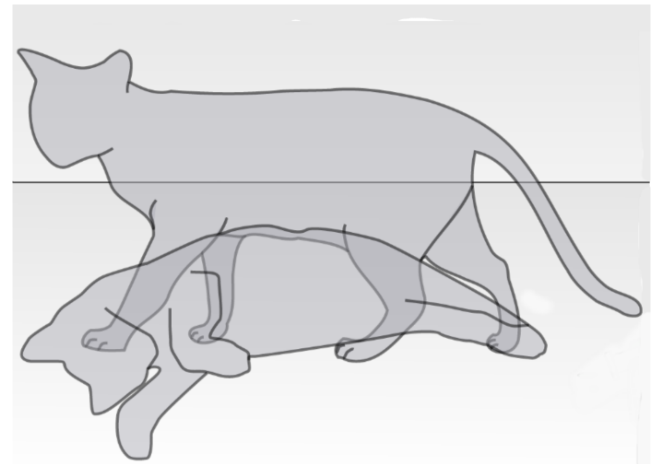
- Long-range entanglement can arise even in non-exotic systems
- Symmetry breaking

$$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z - h \sum_i S_i^x$$

$h \lesssim J$: $|0\rangle =$  or $|1\rangle =$ 

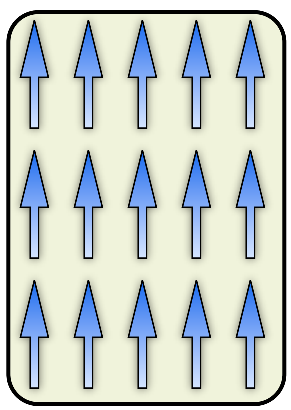
more on MES

- Schrödinger cat

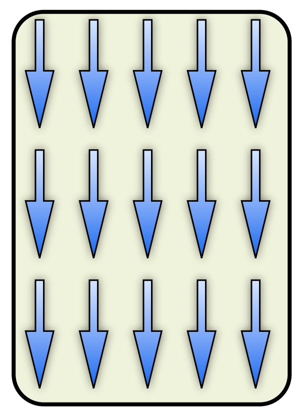


$$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z - h \sum_i S_i^x$$

h≲J: $|\pm\rangle = \frac{1}{\sqrt{2}} \Bigg($

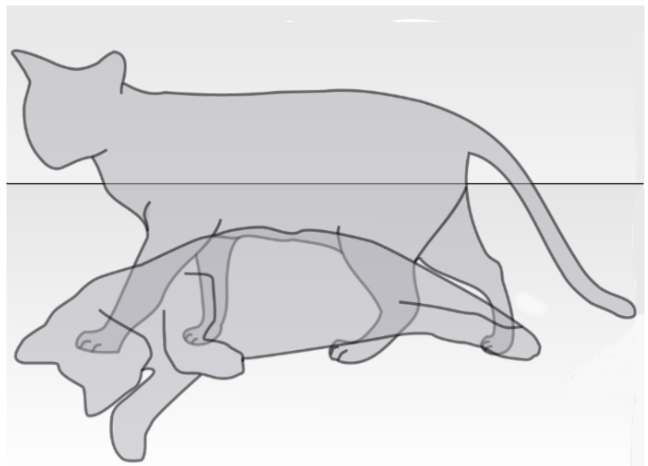


$\pm$



$\Bigg)$

Schrödinger cat



- Cat state is the global ground state
- BUT...undesirable
  - Not physical: decoherence collapses the cat into a symmetry broken state
  - Cannot directly measure local observables

Long-range entanglement

- The cat state has higher entanglement

$$S(|\pm\rangle) = S(|0\rangle, |1\rangle) + \ln 2$$

- To get it requires capturing entanglement between farthest pairs of spins in the system
- DMRG will avoid this state for *very long systems*, finding MES (physical!)

Long-range entanglement

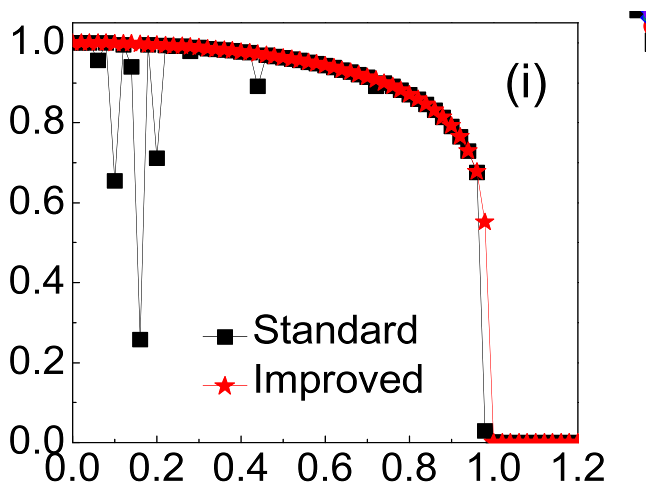
- The cat state has higher entanglement

$$S(|\pm\rangle) = S(|0\rangle, |1\rangle) + \ln 2$$

- To get it requires capturing entanglement between farthest pairs of spins in the system
- BUT, for shorter systems, or long simulations, the cat appears

Cat states

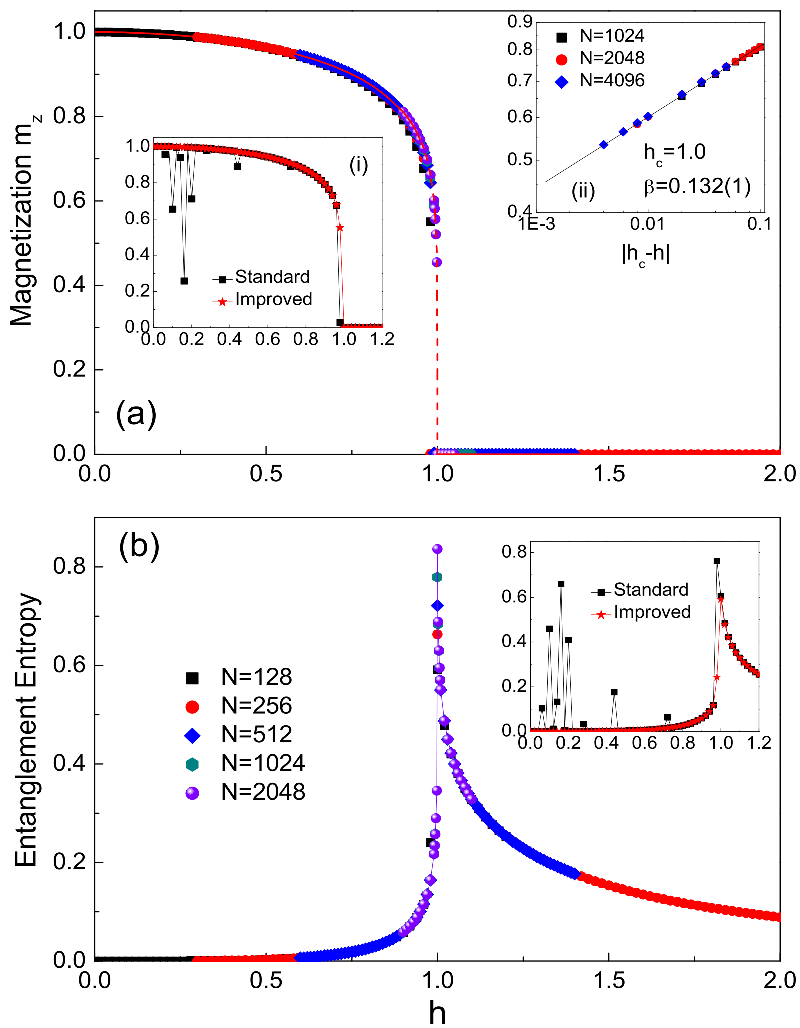
- The “best” simulations find a superposition (partial cat)
  - local quantities, e.g. magnetization of Ising model, can be any value between ±m



Collapse the cat

- It can be useful to select a MES by making small "observations" of the state
- This can be done in DMRG by introducing a "quench" - a short period of increased truncation error - followed by recovery

TFIM



- This procedure allows a *direct* measurement of order parameters, critical properties, etc

- One obtains a MES *without a priori knowledge* of broken symmetry, topological order, etc., even for "short" systems

Fin

- DMRG is remarkably efficient at obtaining TEE for realistic quantum spin models with short correlation lengths from the *von Neumann* entropy
- Empirically, the scaling limit occurs only in prohibitively large systems for the Renyi entropies
- Definitive identification of the kagome lattice QSL as a Z_2 topological phase. In the future we should use the methods to push connections to experiments on e.g. herbertsmithite
- MES are also useful for study of more conventional phases