Hydrodynamic theory for Higgs-confining and Coulomb phases in quantum spin ice

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Thanks to: Y. Tanaka, Y. Kato (RIKEN), Y.-J. Kao (Natl. Taiwan Univ.),
L. Balents, S. Lee (KITP, UCSB)

SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).
SO, J. Phys.: Conf. Series **320**, 012065 (2011).
L.-J. Chang, SO, Y. Su et al. Nature Comm. 3:992 (2012).
S. Lee, SO, L. Balents, PRB 86, 104412 (2012).
SO, J. Phys.: Cond. Matter, Topical Review (invited).

Diamond lattice

Diamond

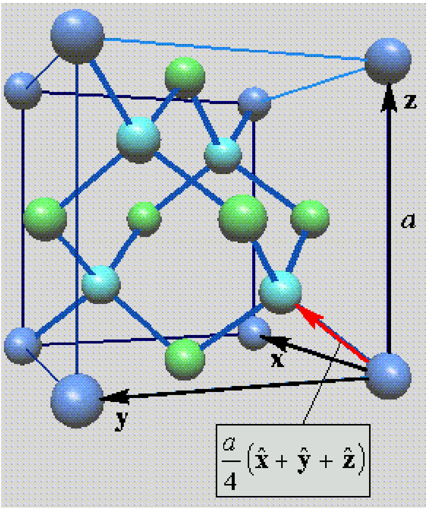
$(Dy,Ho)_2Ti_2O_7$
Classical spin ice

$Yb_2Ti_2O_7$, $Pr_2Zr_2O_7$
Quantum spin ice

Cut, Carat, Clarity

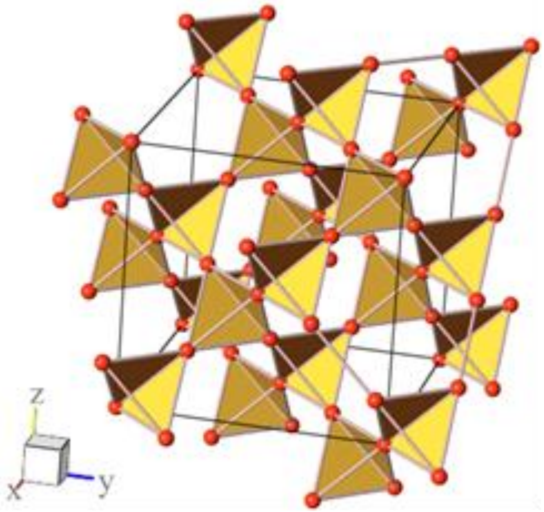
courtesy of Y. Yasui

Diamond lattice



Dual lattice

Pyrochlore lattice

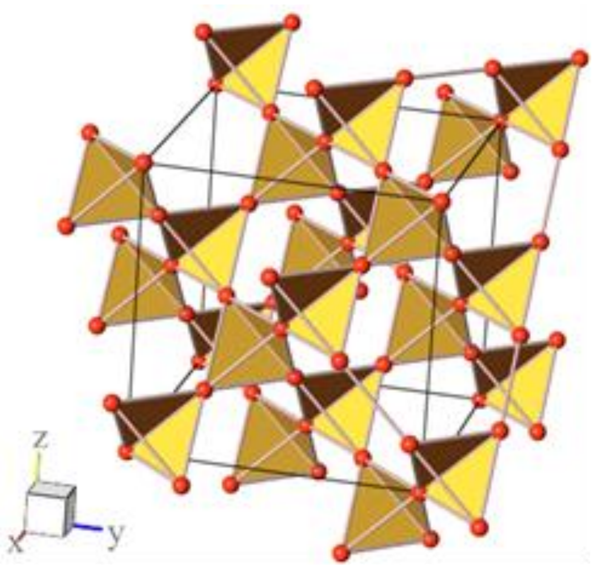


Spin ice & emergent monopoles

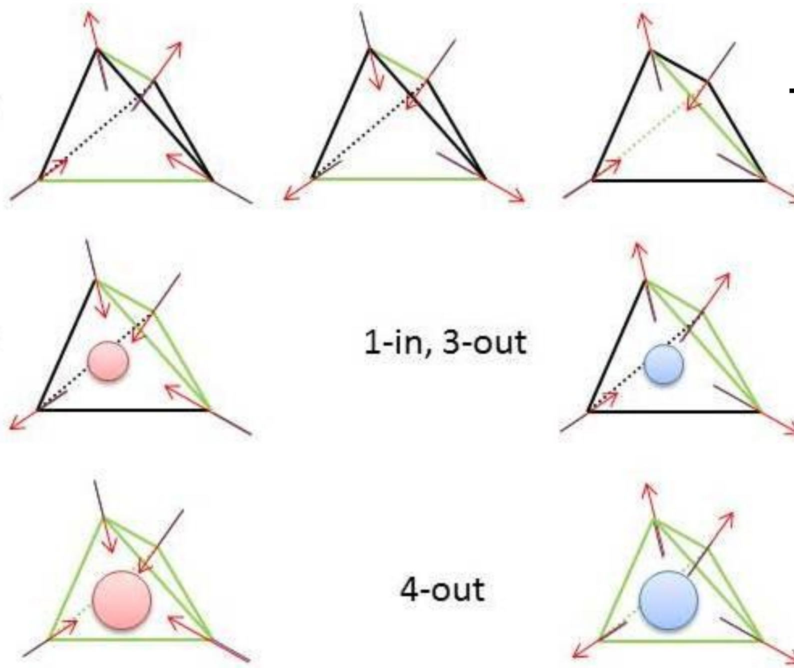
AF Ising model on a pyrochlore lattice

Moessner-Sondhi

$$H = 4J \sum_{<r,r'>}^{n.n.} S_r^z S_{r'}^z \qquad S_r^z \text{ : Ising (S=1/2) spin}$$



Energy/Monopole charge



$-2J / 0$

$0 / \pm 1$

$6J / \pm 2$

Classical Coulomb-phase physics: divergence-free $\nabla \cdot S^z = 0$

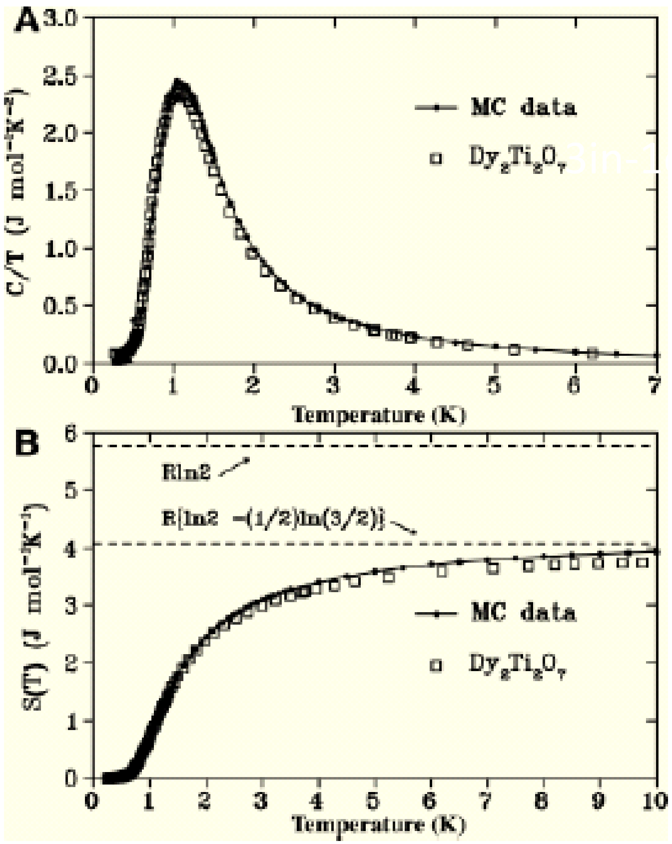
Experiments and numerics on dipolar spin ice

Harris, Ramirez, Bramwell, Sakakibara , Hiroi, Maeno, Gingras

$Dy_2Ti_2O_7$, $Ho_2Ti_2O_7$

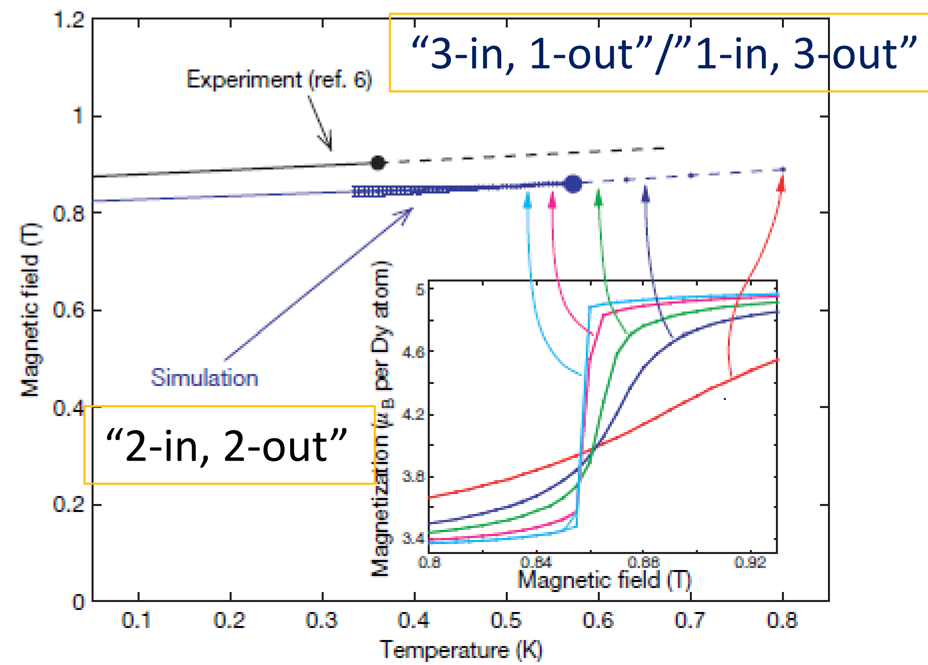
N.N. Ising coupling J∼ 2.4 K

Castelnovo-Moessner-Songhi, Nature 451, 42-45 (2008)



Metamagnetic transition under H // (111)

→ **liquid-gas phase transition of monopoles**



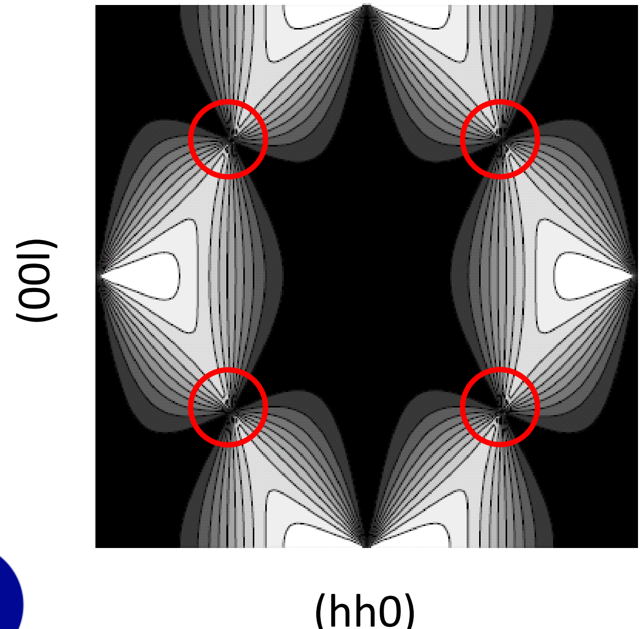
Exp., Sakakibara et al., *Phys. Rev. Lett.* 90, 207205 (2003).

Dipolar spin correlations: Coulomb physics

- O(N) Heisenberg antiferromagnet

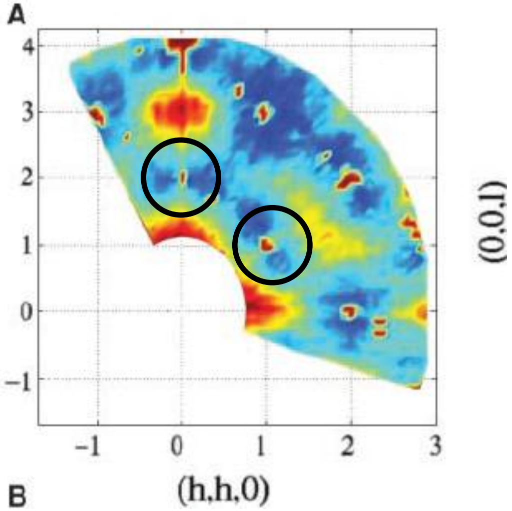
S.V. Isakov, K. Gregor, R. Moessner, S. L. Sondhi, *Phys. Rev. Lett.* 93, 167204 (2004).

Works well for N=1 (Ising) and infinity.



N=∞, 1

S.T. Bramwell and M.J.P. Gingras
Science **294**, 1495 (2001)



cf. pinch-point singularity

C. L. Henley, *Phys. Rev. B* 71 014424 (2005)

divergence-free condition, i.e., spin-ice rule

div M → 0

More recent experiments on dipolar spin ice: Morris et al., Fennell et al.

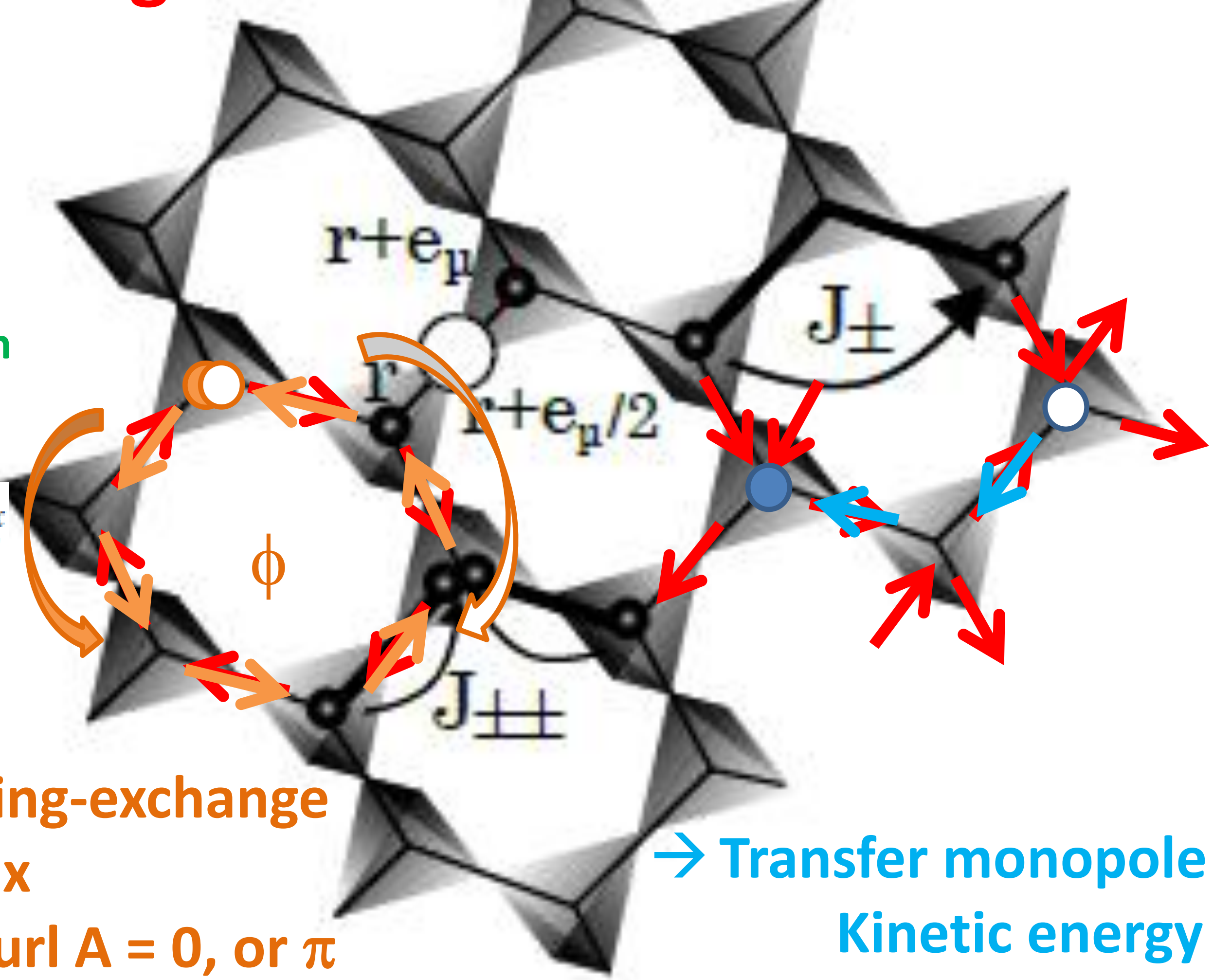
Classical to quantum spin ice

Spin-flip exchange interaction

$\delta J\, S_r^{+} S_{r'}^{-}$

c.f. Quite different from a quantum tunneling of protons in water ice

$$\mathcal{H} = J\sum_{\langle i,j\rangle} S_i^z S_j^z - \Gamma \sum_i S_i^x$$



→Effective ring-exchange

→ A fixed flux

ϕ = curl A = 0, or π

→ Transfer monopole charge

Kinetic energy

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Classical-to-quantum Coulomb-phase physics

- Classical case: particles obeying a Coulombic law

$$H_{cl} \approx \frac{1}{8\pi}\mathbf{E}^2 - \mu\psi^+\psi + u\psi^+\psi\psi^+\psi$$

→ Coulomb propagator

$$\mathbf{E} = \hat{S}^z\mathbf{z} \rightarrow \nabla\cdot\mathbf{E} = g(\psi^+\psi)$$

ψ^+, ψ : **Spinon** operators creating and annihilating the gauge charge

- Quantum case: kinetic energy with gauge field (QED)

U(1) quantum spin liquid, unless condensed

$$H_{qm} \approx \frac{1}{8\pi}(\mathbf{E}^2 + \mathbf{B}^2) + \frac{1}{2m}\psi^+(-i\hbar\nabla + g\mathrm{A})^2\psi - \mu\psi^+\psi + u\psi^+\psi\psi^+\psi$$

$$\nabla\cdot\mathbf{E} = g(\psi^+\psi) \leftarrow \mathbf{E} = \hat{S}^z\mathbf{z}$$

$$\mathbf{B} = \nabla\times\mathbf{A} \leftarrow \hat{S}_r^{\pm} = \psi_{r\pm d}^+ e^{\pm iA_{r+d,r-d}}\psi_{r\mp d}$$

$$[A_{r+d,r-d}, E_{r+d,r-d}] = i$$

Abelian Higgs models:
Savary-Balents
(non-interacting spinons)
S.Lee-S.O.-Balents
(interacting spinons)

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Bose condensation of spinons ψ → Higgs transition, monopole supercurrent

Gapless "Photon" excitations in a quantum Coulomb phase (T=0)

Lattice U(1) gauge theory Hamiltonian

$$\mathcal{H}'_{U(1)} = \frac{\mathcal{U}}{2}\sum_{\mathbf{r}\in A,n}\left[(\nabla_{\circ}\times\mathcal{A})_{(\mathbf{r},n)}\right]^2 + \frac{1}{2\mathcal{K}}\sum_{\mathbf{s}\in A',m}\left[\frac{\partial\mathcal{A}_{(\mathbf{s},m)}}{\partial t}\right]^2 + \frac{\mathcal{W}}{2}\sum_{\mathbf{s}\in A',m}\left[(\nabla_{\circ}\times\nabla_{\circ}\times\mathcal{A})_{(\mathbf{s},m)}\right]^2$$

$U{\sim}\delta^3 J$

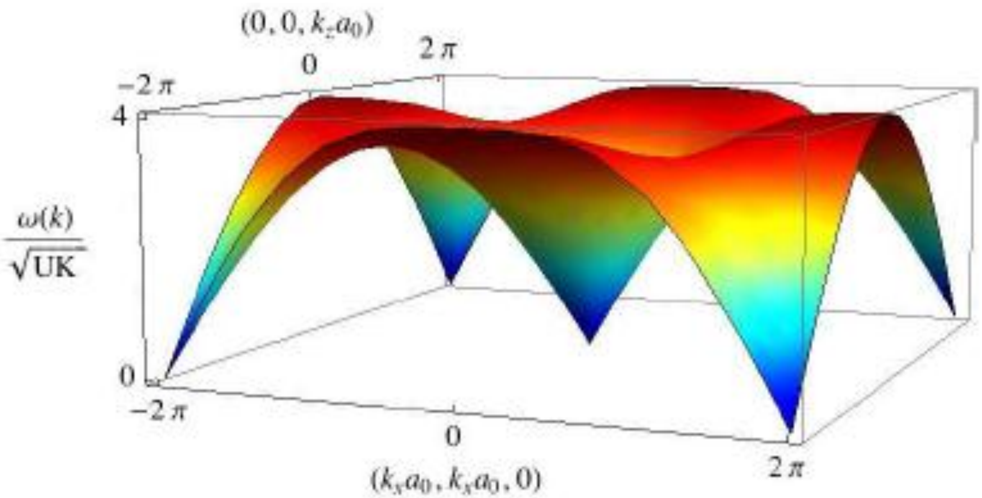
$K{\sim}1/J$

Velocity c$\sim\delta^{3/2}J$

$\longrightarrow$

A measure of quantum Coulomb regime

T < $\delta^{3/2}J \sim 1\ K$ (Yb2Ti2O7)



Hermele-Fischer-Balents 2004

Benton-Sikora-Shannon 2012

In the Higgs phase, however, absorbed into Higgs bosons.

At a T > 0 Coulomb phase no well defined photons!

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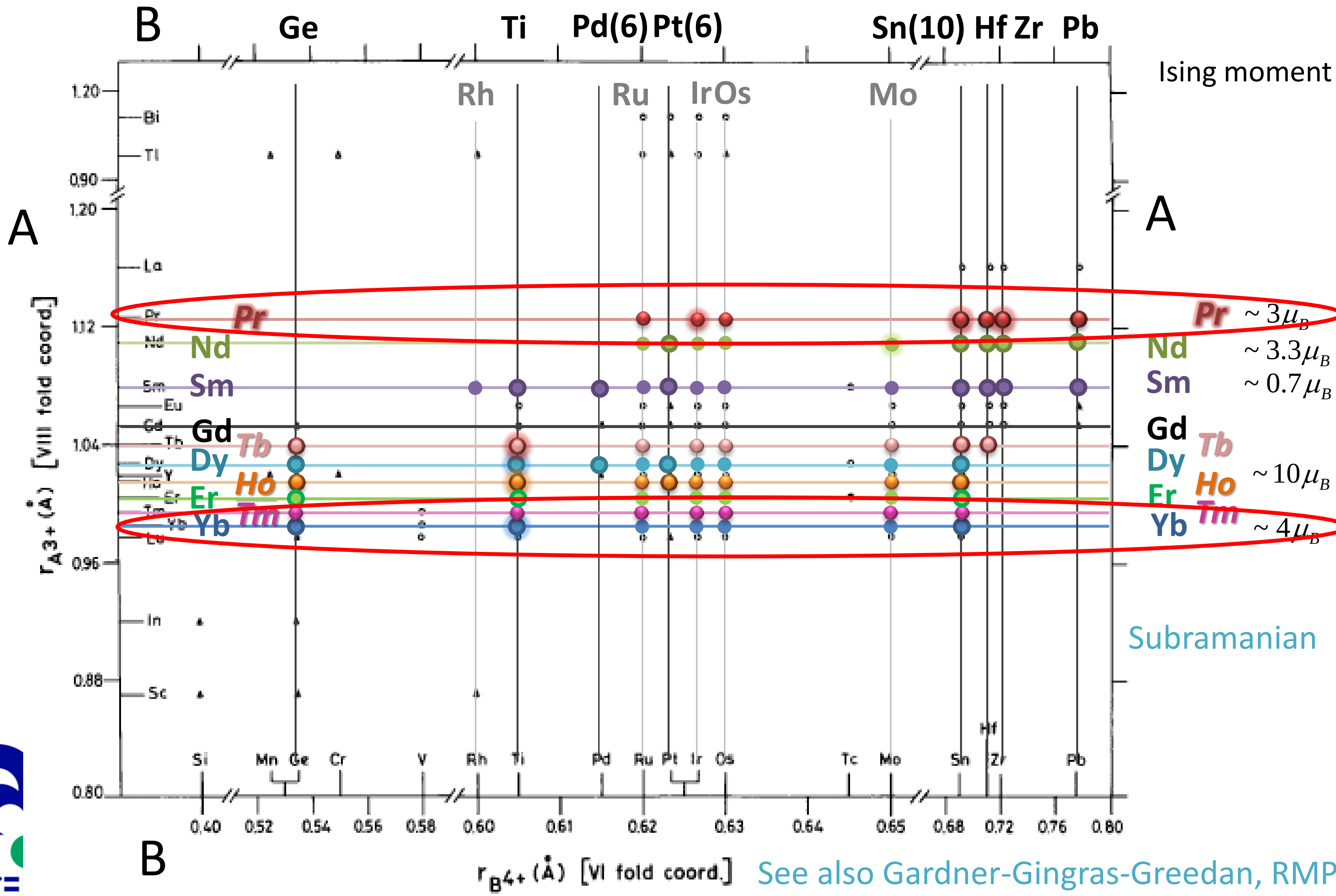
Candidate pyrochlore magnets $A_2B_2O_7$



Subramanian

See also Gardner-Gingras-Greedan, RMP

Specific examples:
Derivation of realistic superexchange int.

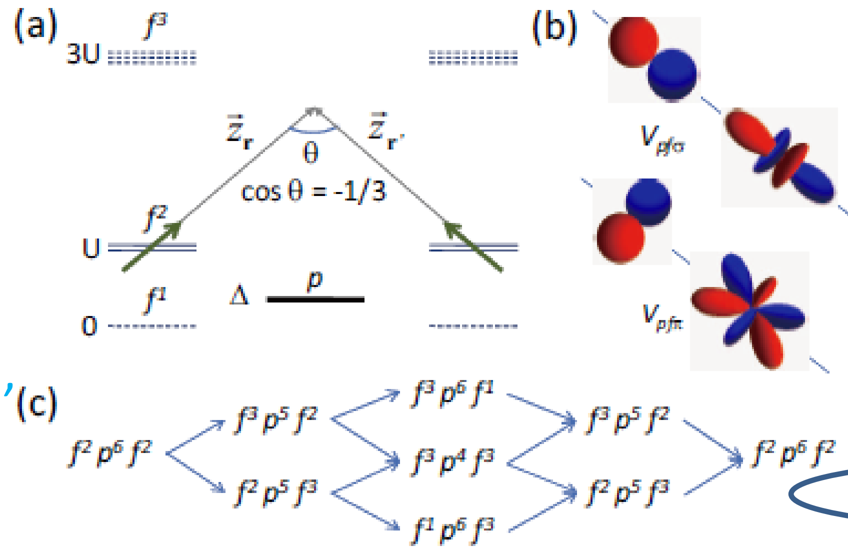
Anderson's superexchange int. →Project onto the gr. doublets

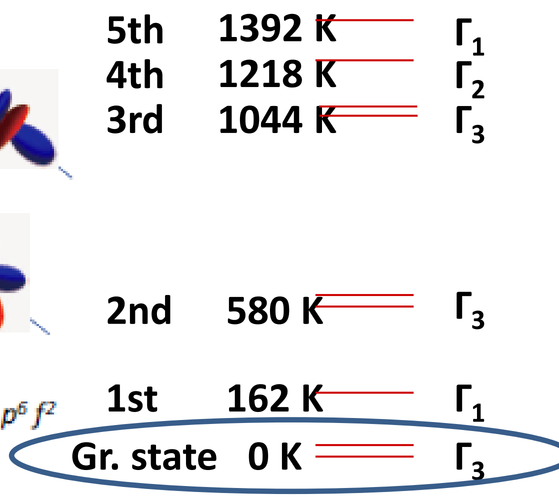
- $Pr_2TM_2O_7$

non-Kramers doublet

(integer-spins)

SO-Tanaka,
PRL **105**, 047201 (2010),
PRB **83**, 094411 (2011).





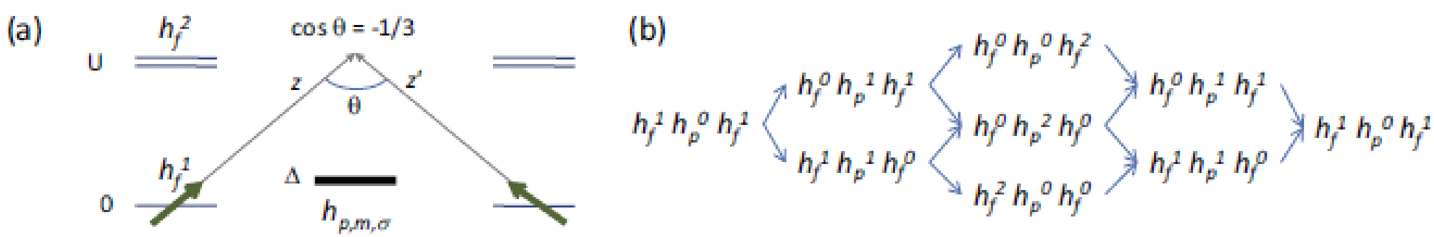
$$|\sigma\rangle = \alpha|M = 4\sigma\rangle + \sigma\beta|M = \sigma\rangle - \gamma|M = -2\sigma\rangle$$

- $Yb_2TM_2O_7$

Kramers doublet (half-integer spins)

SO, J. Phys.: Conf. Series 320, 012065 (2011)

$$|\sigma\rangle_D = -\alpha\sigma|J_z = \frac{7}{2}\sigma\rangle + \beta|J_z = \frac{1}{2}\sigma\rangle + \gamma\sigma|J_z = -\frac{5}{2}\sigma\rangle$$



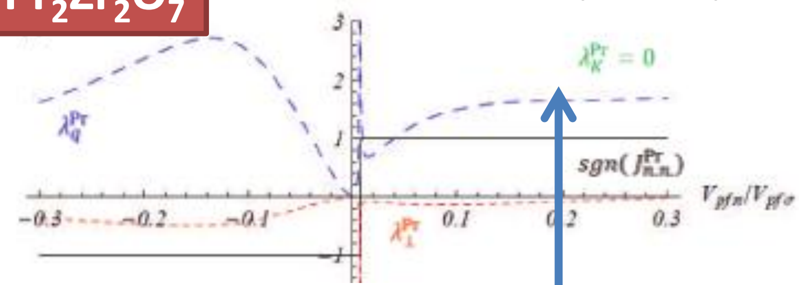
Effective pseudospin-1/2 model

Anisotropic superexchange interaction
[SO-Tanaka (2009, 2010), SO (2011)]

$Pr_2Zr_2O_7$

$$\hat{H}^R_{\rm SE} = \frac{|J^R_{\rm n.n.}|}{2} \sum_{(\boldsymbol{r},\boldsymbol{r}')}^{\rm n.n.} \left[sgn(J^R_{\rm n.n.}) \hat{S}^z_{\boldsymbol{r}} \hat{S}^z_{\boldsymbol{r}'} + \lambda^R_\perp \hat{S}^+_{\boldsymbol{r}} \hat{S}^-_{\boldsymbol{r}'} + \lambda^R_q e^{2i\phi_{\boldsymbol{r},\boldsymbol{r}'}} \hat{S}^+_{\boldsymbol{r}} \hat{S}^+_{\boldsymbol{r}'} + \lambda^R_K e^{i\phi_{\boldsymbol{r},\boldsymbol{r}'}} \left(\hat{S}^z_{\boldsymbol{r}} \hat{S}^+_{\boldsymbol{r}'} + \hat{S}^+_{\boldsymbol{r}} \hat{S}^z_{\boldsymbol{r}'} \right) \right] + h.c.$$

Vpfπ / Vpfσ

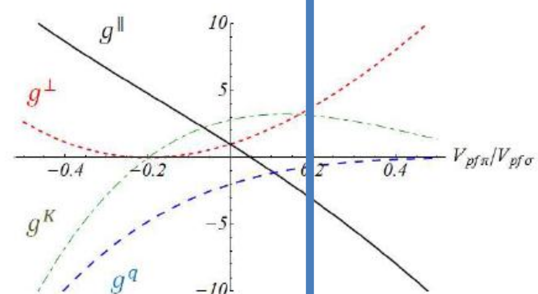


$S^{\pm}$: magnetic dipole for Kramers doublets (Yb, Nd, Sm, Dy)
quadrupole for non-Kramers doublets (Pr, Tb, …)

Magnetic moment

Yb2Ti2O7

$$\hat{\boldsymbol{m}}^R_{\boldsymbol{r}} = g_J \mu_B \hat{\boldsymbol{J}}^R_{\boldsymbol{r}} = \mu_B \left[g^R_\perp \left(\hat{S}^x_{\boldsymbol{r}} \boldsymbol{x}_i + \hat{S}^y_{\boldsymbol{r}} \boldsymbol{y}_i \right) + g^R_\parallel \hat{S}^z_{\boldsymbol{r}} \boldsymbol{z}_i \right]$$



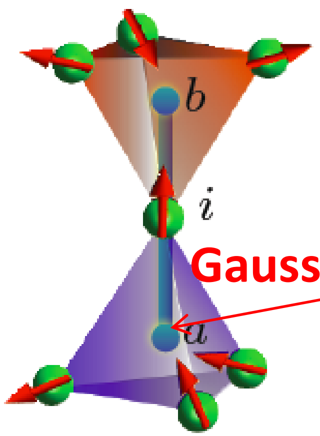
Magnetic dipole interaction

$$\hat{H}^R_{\rm D} = \frac{\mu_0}{4\pi} \sum_{(\boldsymbol{r},\boldsymbol{r}')} \left[\frac{\hat{\boldsymbol{m}}^R_{\boldsymbol{r}} \cdot \hat{\boldsymbol{m}}^R_{\boldsymbol{r}'}}{(\Delta r)^3} - 3 \frac{(\hat{\boldsymbol{m}}^R_{\boldsymbol{r}} \cdot \Delta \boldsymbol{r})(\Delta \boldsymbol{r} \cdot \hat{\boldsymbol{m}}^R_{\boldsymbol{r}'})}{(\Delta r)^5} \right]$$

Best fit to
neutron-scattering exp.

Interacting U(1) Higgs model: QED with charged bosonic spinons

S. Lee, S.O., L. Balents
PRB (2012)



Gauss' law

$$S_i^z = \eta_a E_{ab} \qquad \eta_a = \pm 1 [a \in A(B)]$$

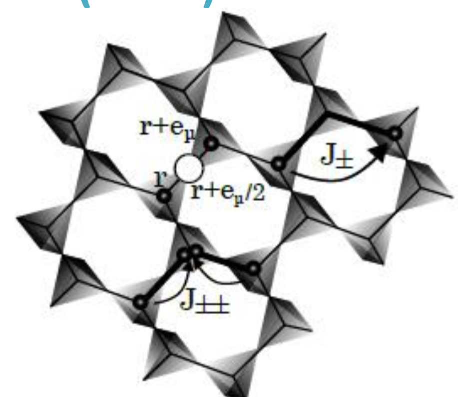
$$S_i^+ = \Phi_a^\dagger e^{iA_{ab}} \Phi_b \qquad \Phi_a = e^{-i\varphi_a}$$

$$Q_a = (divE)_a \qquad \Phi_a^\dagger \Phi_a = 1$$

$$[A_{ab}, E_{ab}] = i$$

$$[\Phi_a, Q_a] = \Phi_a$$

Monopolar spinons
(Higgs bosons)
Increasing/decreasing the charge



$$H_{QED} = \frac{J_{zz}}{2} \sum_{\mathbf{r}} Q_{\mathbf{r}}^2 - J_\pm \sum_{\mathbf{r}} \sum_{\mu \neq \nu} \Phi^\dagger_{\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\mu} \Phi_{\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu} \mathrm{s}^{-\eta_\mathbf{r}}_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\mu} \mathrm{s}^{+\eta_\mathbf{r}}_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu}$$

$$+ \frac{J_{\pm\pm}}{2} \sum_{\mathbf{r}} \sum_{\mu \neq \nu} \left(\gamma_{\mu\nu}^{-2\eta_\mathbf{r}} \Phi^\dagger_\mathbf{r} \Phi^\dagger_\mathbf{r} \Phi_{\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\mu} \Phi_{\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu} \mathrm{s}^{\eta_\mathbf{r}}_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\mu} \mathrm{s}^{\eta_\mathbf{r}}_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu} + h.c \right)$$

$$+ J_{z\pm} \sum_{\mathbf{r}} \sum_{\mu \neq \nu} \mathrm{s}^z_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\mu} \left(\gamma_{\mu\nu}^{-\eta_\mathbf{r}} \Phi^\dagger_\mathbf{r} \Phi_{\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu} \mathrm{s}^{\eta_\mathbf{r}}_{\mathbf{r},\mathbf{r}+\eta_\mathbf{r} \mathbf{e}_\nu} + h.c. \right) + \text{const.}.$$

+δ

+q

+K

Starting from spin ice
with deconfined spinons

Non-Kramers doublets (integer spins) (Pr)

Classification of mean-field phases

Let's study the case of integer spins: non-Kramers doublets (Pr)

	$\langle s^z_{\mathbf{r},\mathbf{r}\pm\mathbf{e}_\mu}\rangle$	$\langle s^\pm_{\mathbf{r},\mathbf{r}\pm\mathbf{e}_\mu}\rangle$	$\langle \Phi_\mathbf{r}\rangle$	$\langle \Phi_\mathbf{r}\Phi_\mathbf{r}\rangle$	$\langle \Phi^\dagger_\mathbf{r}\Phi_{\mathbf{r}\pm\mathbf{e}_\mu}\rangle$
Ising order (confined)	$\neq 0$	0	0	0	0
QSL					
U(1)	0	$\neq 0$	0	0	0
Z_2 (charge-2 Higgs)	0	$\neq 0$	0	$\neq 0$	0
XY order					
U(1)	0	$\neq 0$	0	0	$\neq 0$
Classical (confined Higgs)	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$

S. Lee, SO, L. Balents, PRB 86, 104412 (2012).

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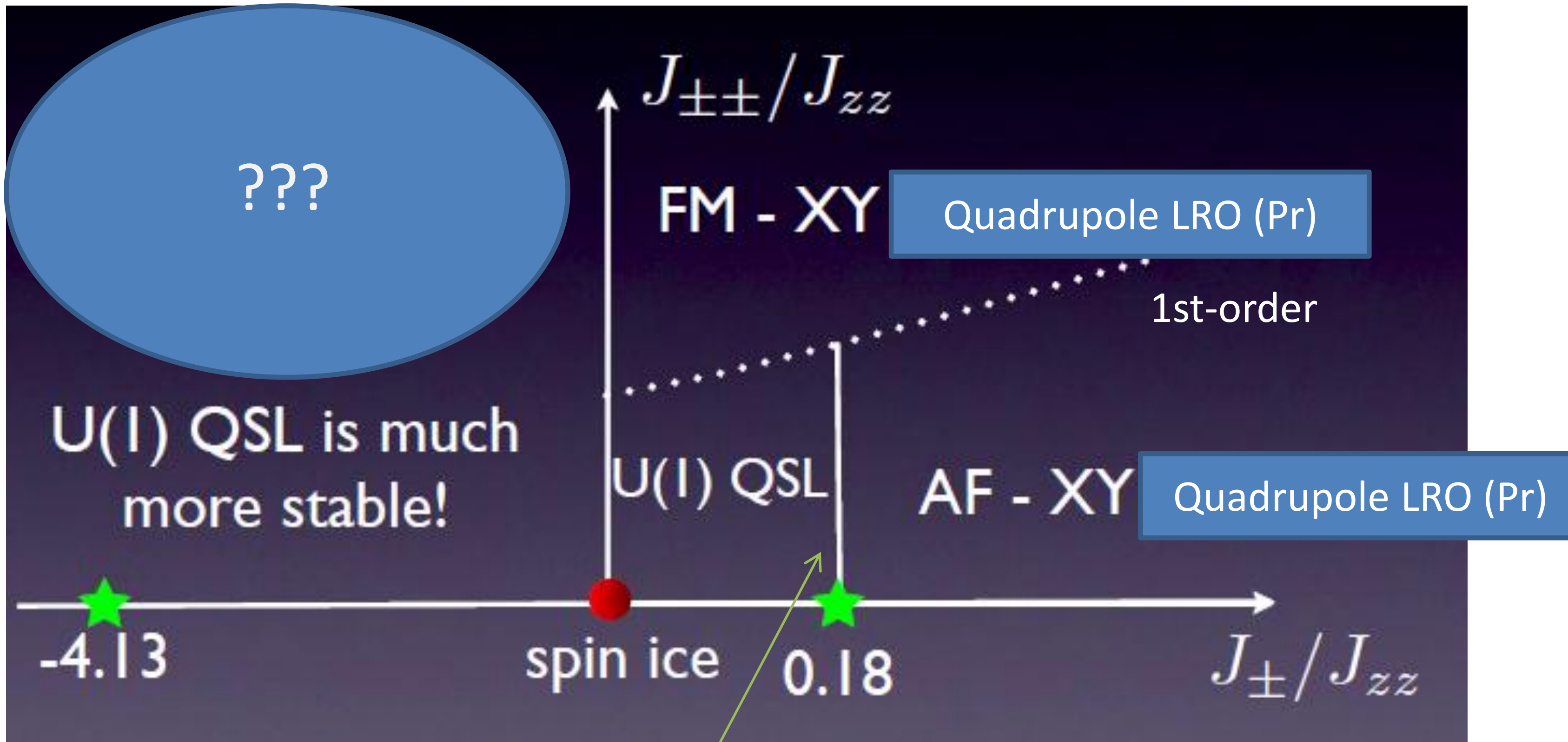
Mean-field phase diagram in the case of non-Kramers doublets (Pr)



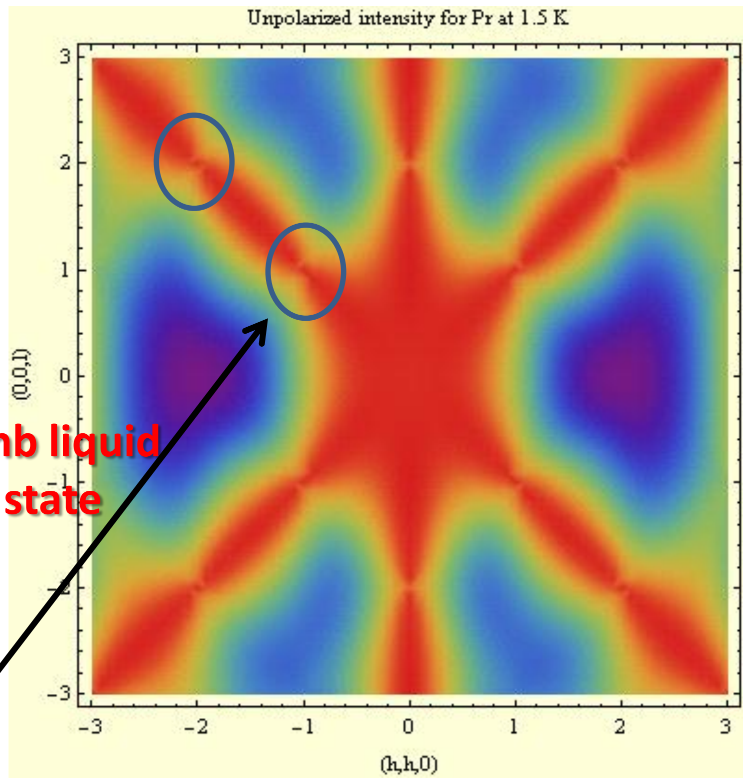
Weakly first-order Higgs transition
c.f. Fradkin

S. Lee, SO, L. Balents

Is $Pr_2Zr_2O_7$ a U(1) QSL?

For exchange parameters
for $Pr_2Zr_2O_7$

(1/N) expansion



Indication of Coulomb liquid down to the ground state

SO, unpublished

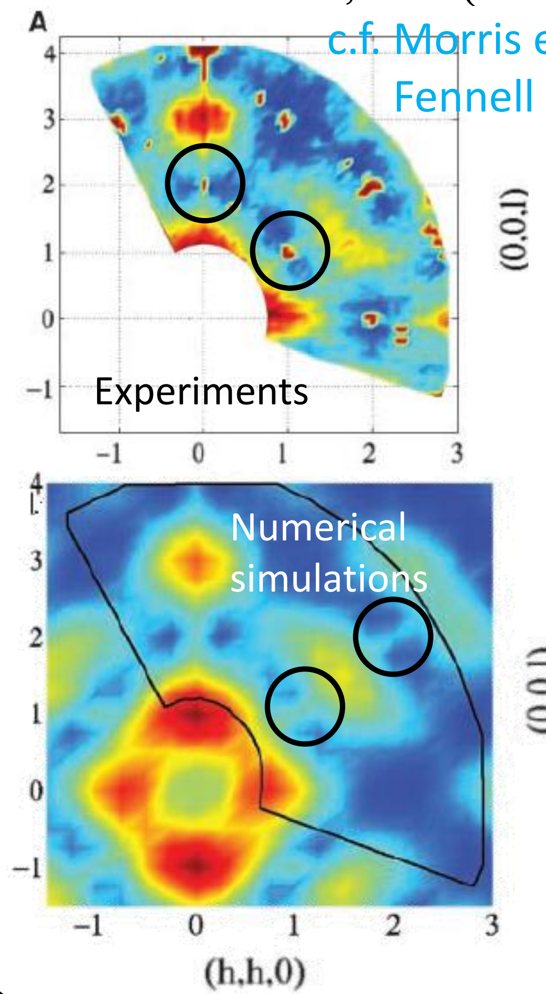
Pinch point singularity is broadened by a dynamical violation of the ice rule

Dipolar spin ice

S.T. Bramwell and M.J.P. Gingras

Science **294**, 1495 (2001)

c.f. Morris et al.
Fennell et al.



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Higgs transition in $Yb_2Ti_2O_7$

nature
COMMUNICATIONS

ARTICLE

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DOI: 10.1038/ncomms1989

Higgs transition from a magnetic Coulomb liquid to a ferromagnet in $Yb_2Ti_2O_7$

Lieh-Jeng Chang[1,2], Shigeki Onoda[3], Yixi Su[4], Ying-Jer Kao[5], Ku-Ding Tsuei[6], Yukio Yasui[7,8], Kazuhisa Kakurai[2] & Martin Richard Lees[9]

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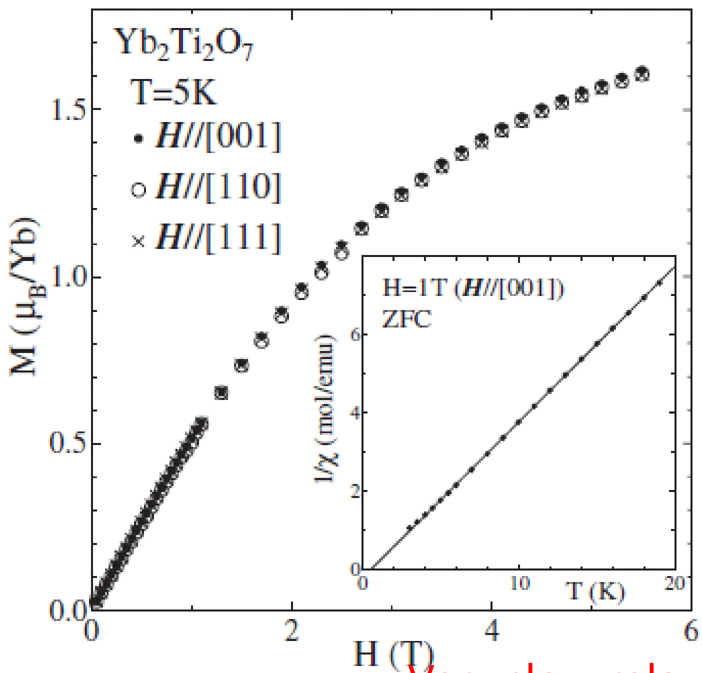
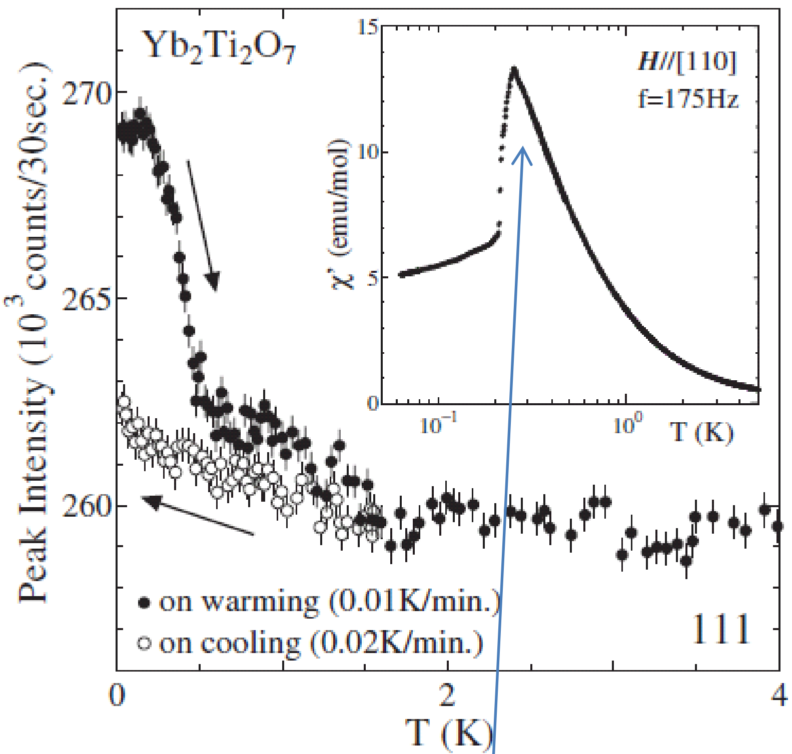
Next talk by L.-J. Chang!

Evidence of the 1st–order phase ferromagnetic transition at ~0.21 K

Yasui et al. JPSJ (2003)





Very slow relaxation of magnetization
Magnetic Bragg peak evolves in 2 hours!

Spin excitations are gapped.

Related anomaly in the specific heat [Blote et al. 1969]

c.f. Sample dependence: the best available sample shows FM, while others does not.
Hodges et al, Thompson et al, Gardner et al, Ross et al

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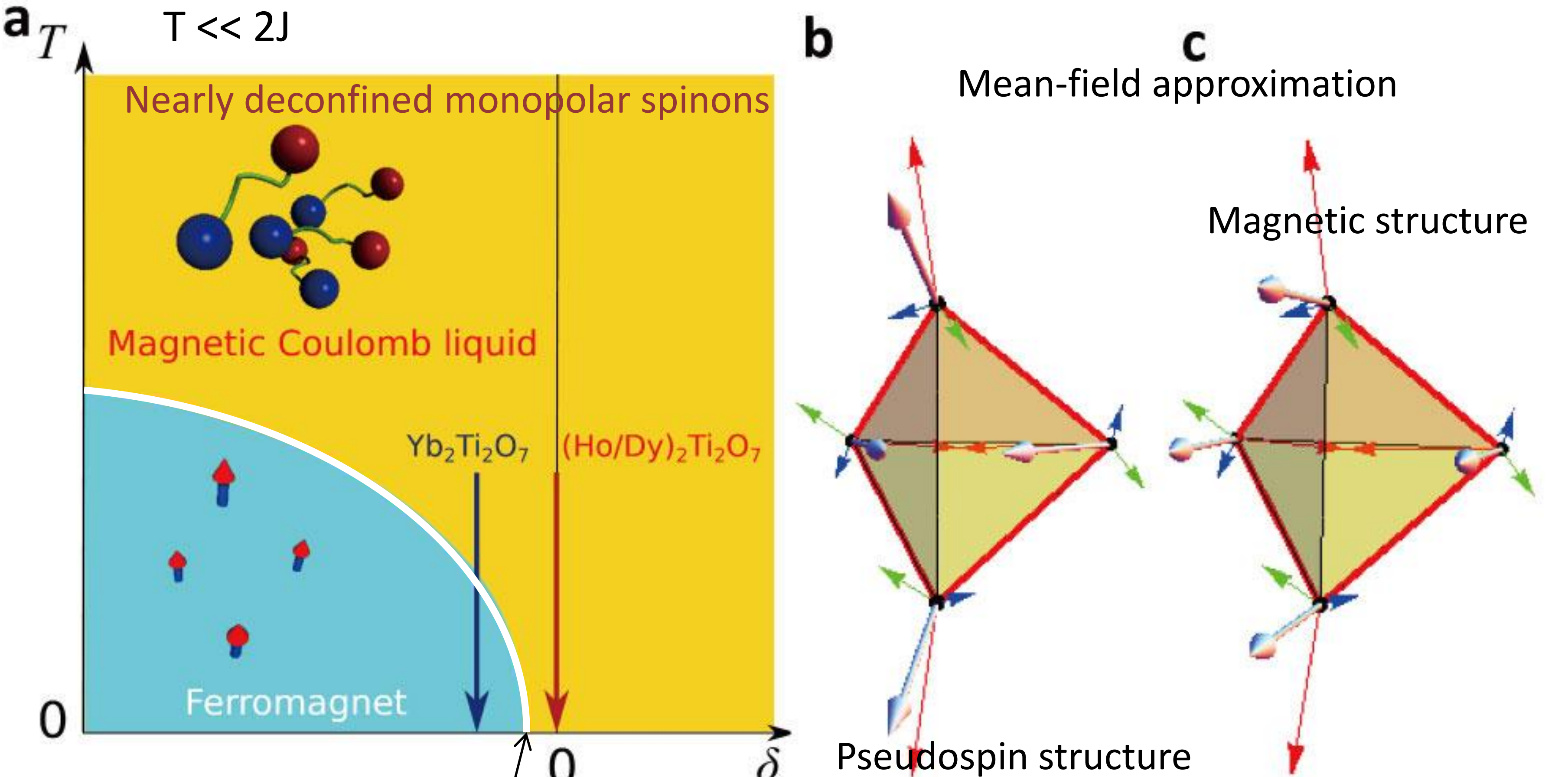
Phase diagram and the hypothetical magnetic structure



Spinons are confined
Gapped spin-wave

Higgs transition

Nearly collinear ferromagnet (~1 degree canting)
M//[100]
Consistent with the magnetic structure analysis

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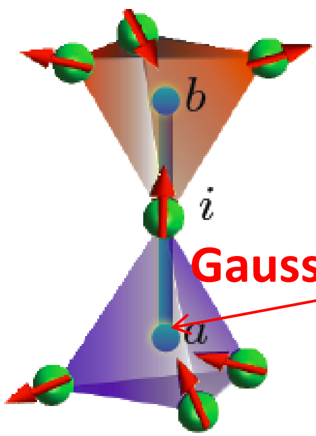
Interacting U(1) Higgs model: QED with charged bosonic spinons revisited…



$$S_i^z = \eta_a E_{ab} \qquad \eta_a = \pm 1[a \in A(B)]$$

$$S_i^+ = \Phi_a^\dagger e^{iA_{ab}} \Phi_b \qquad \Phi_a = e^{-i\varphi_a}$$

Gauss' law $Q_a = (divE)_a$ $\qquad \Phi_a^\dagger \Phi_a = 1$

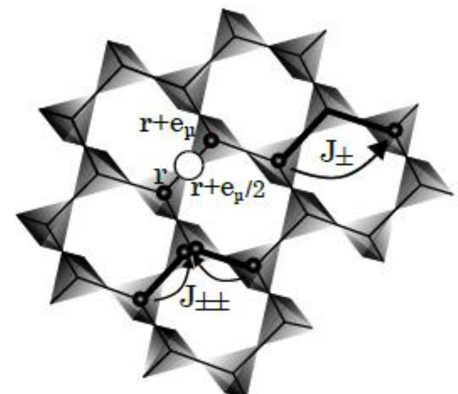
$$[A_{ab}, E_{ab}] = i$$

$$[\Phi_a, Q_a] = \Phi_a$$

Monopolar spinons (Higgs bosons)

Increasing/decreasing the charge

S.O.



$$H_{QED} = \frac{J_{zz}}{2}\sum_{\mathbf{r}} Q_{\mathbf{r}}^2 - J_{\pm}\sum_{\mathbf{r}}\sum_{\mu\neq\nu} \Phi^\dagger_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{-\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu} \mathrm{s}^{+\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu}$$

+δ

$$+\frac{J_{\pm\pm}}{2}\sum_{\mathbf{r}}\sum_{\mu\neq\nu}\left(\gamma_{\mu\nu}^{-2\eta_{\mathbf{r}}}\Phi^\dagger_{\mathbf{r}}\Phi^\dagger_{\mathbf{r}}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} + h.c\right)$$

+q

$$+J_{z\pm}\sum_{\mathbf{r}}\sum_{\mu\neq\nu} \mathrm{s}^z_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\left(\gamma_{\mu\nu}^{-\eta_{\mathbf{r}}}\Phi^\dagger_{\mathbf{r}}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} + h.c.\right) + \mathrm{const..}$$

+K

Starting from spin ice with deconfined spinons

Non-Kramers doublets (integer spins) (Pr)

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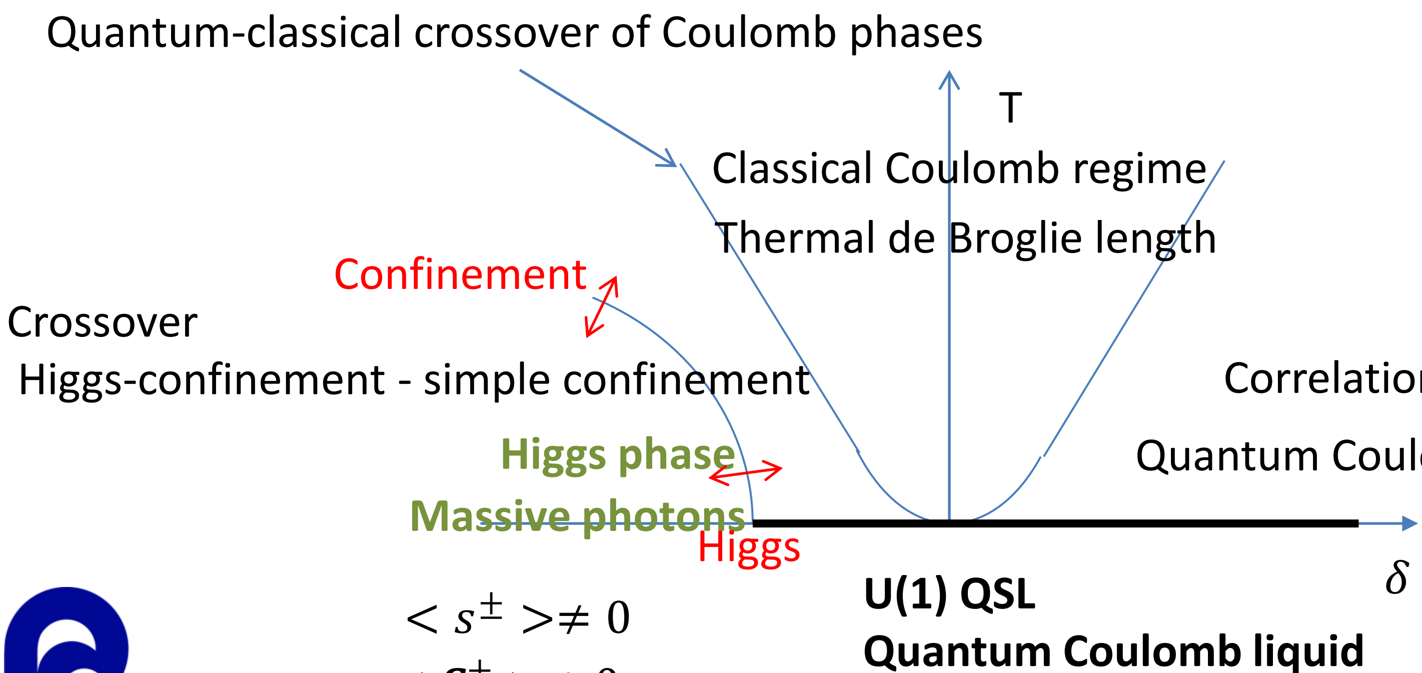
Effects of thermal fluctuations on gauge fields

- Immediately kill the gauge fields and confine spinons

c.f. Fradkin, Shenker 1986

Castro-Neto, Pujol, Fradkin 2006

1-loop calculations beyond gauge mean-field theory



$< s^{\pm} > \neq 0$

$< \boldsymbol{S}^{\pm} > \neq 0$

$< s^{\pm} > \neq 0$

c.f. Banerjee-...-YBKim

c.f. Lattice Maxwell calc.
Sikora-Benton-Shannon

Summary

- Quantum spin ice for (Pr,Yb)$_2$*TM*$_2$O$_7$
  - Magnetic monopole charges ($\nabla \cdot M \neq 0$) carried by spinons!

→ Emergent gapless U(1) spin liquid (Fictitious dual QED)

→ Higgs transitions to classical spin-gapped ferromagnets
"Superconductivity" of magnetic monopoles
(gauge group U1 -> Z2)

→ Neutron-scattering on high-quality single crystal $Yb_2Ti_2O_7$

1. deconfined bosonic spinons carrying monopole charge in the high-T phase
2. Confined spinons to form classical ferromag. in the low-T phase

→ $Pr_2Zr_2O_7$: U(1) quantum spin liquid?
Remnants of pinch-point singularity

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