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Symmetry-protected topological phases of alkaline-earth ultra-cold fermionic atoms in one dimension

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1. Introduction
2. Symmetry-protected topological order in 1D
3. Alkaline-earth cold atom & SU(N) Hubbard model
4. SU(N) symmetry-protected topological phase
5. Summary

Ref: Nonne et al. Phys.Rev.B **82**, 155134 (2010)
Euro.Phys.Lett. **102**, 37008 (2013), and work in progress

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Symmetry and Phase Transitions

- Landau picture of "phase transitions" (from L.L. vol. 5)

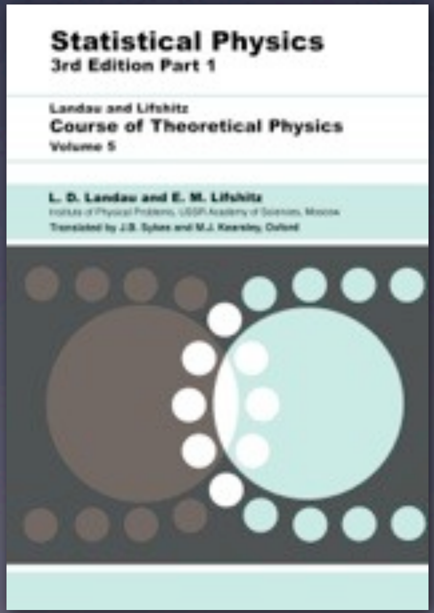
> 1. *"... the transition between phases of different symmetry .... cannot occur in a continuous manner"*
> 2. *"... in a phase transition of the second kind .... the symmetry of one phase is higher than that of the other.."*

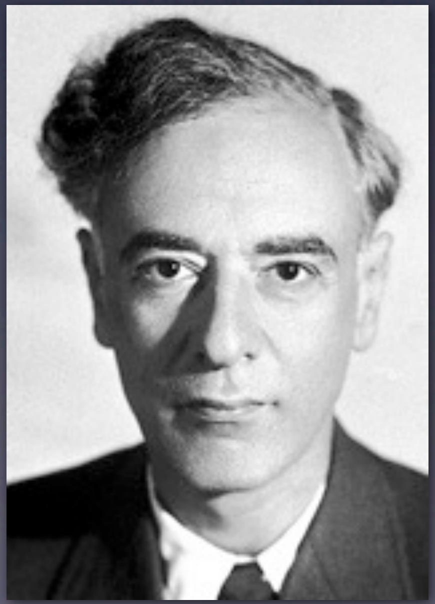
✓ phase transitions: spontaneous breaking of symmetry
G (original) ⇒ H (subgroup)

ex) G=SO(3), H=SO(2)
for (collinear) magnetic order

✓ "Order Parameters"

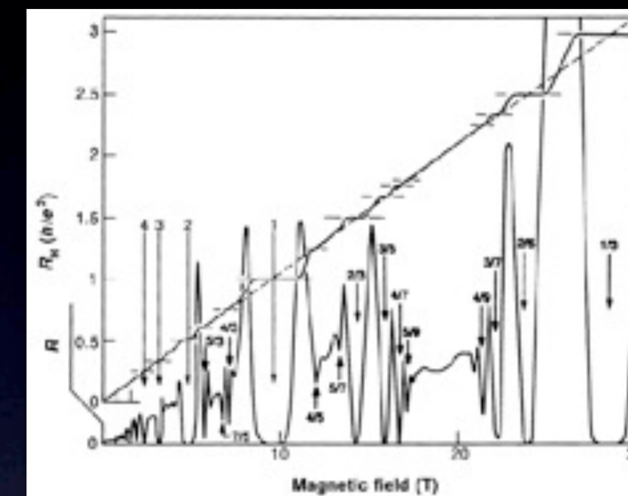
✓ classification in terms of SSB patterns of G





Challenges from quantum systems

- fractional quantum-Hall systems 1983--



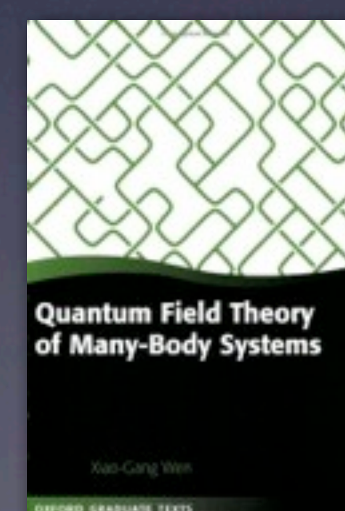
  - ✓ Laughlin wave function ...“ψ_m” ($\nu=1/m$)
  - ✓ describes featureless uniform ground states, no symmetry breaking (locally)
  - ✓ Nevertheless, different “phases” for different “m”

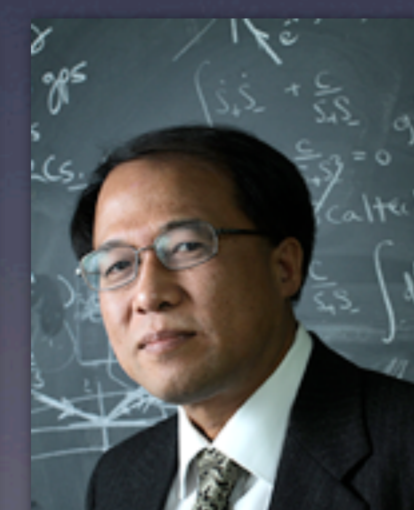
- quantum spin liquids (Z_2/chiral-SL, quantum dimer,...)

1987--

  - ✓ no magnetic LRO, no translation SSB, ...
  - ✓ global (topological) properties (e.g. γ_{top}) (≠boring paramagnet)

a new paradigm??
➡ topological order (Wen ’89)





the new “holy bible”

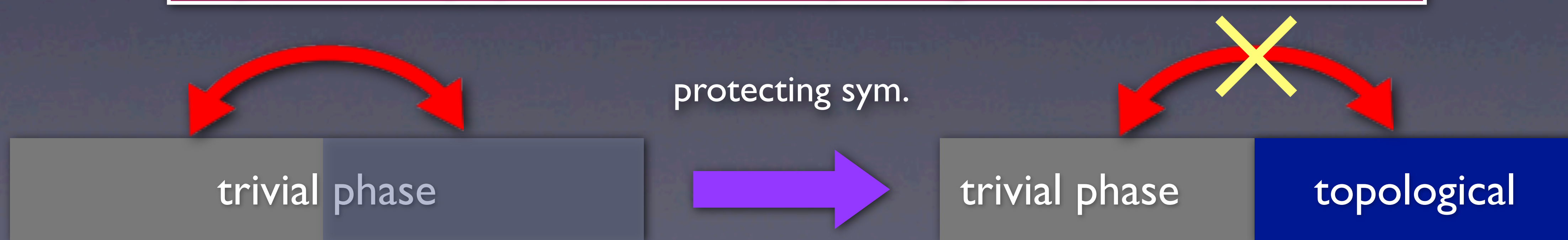
(1+1)D

"Topological Order" in 1D

- Q-information tells us:
  - ✓ any gapped states can be approximated by MPS — Hastings '07
  - ✓ only short-range entanglement in 1D (i.e. $\gamma_{top}=0$) — Verstraete et al. '05
  - ✓ can be smoothly deformed to trivial product state
    - ➡ No (genuine) topological order in 1D !!
- but... still possible to define featureless "topological phases" protected by some symmetry

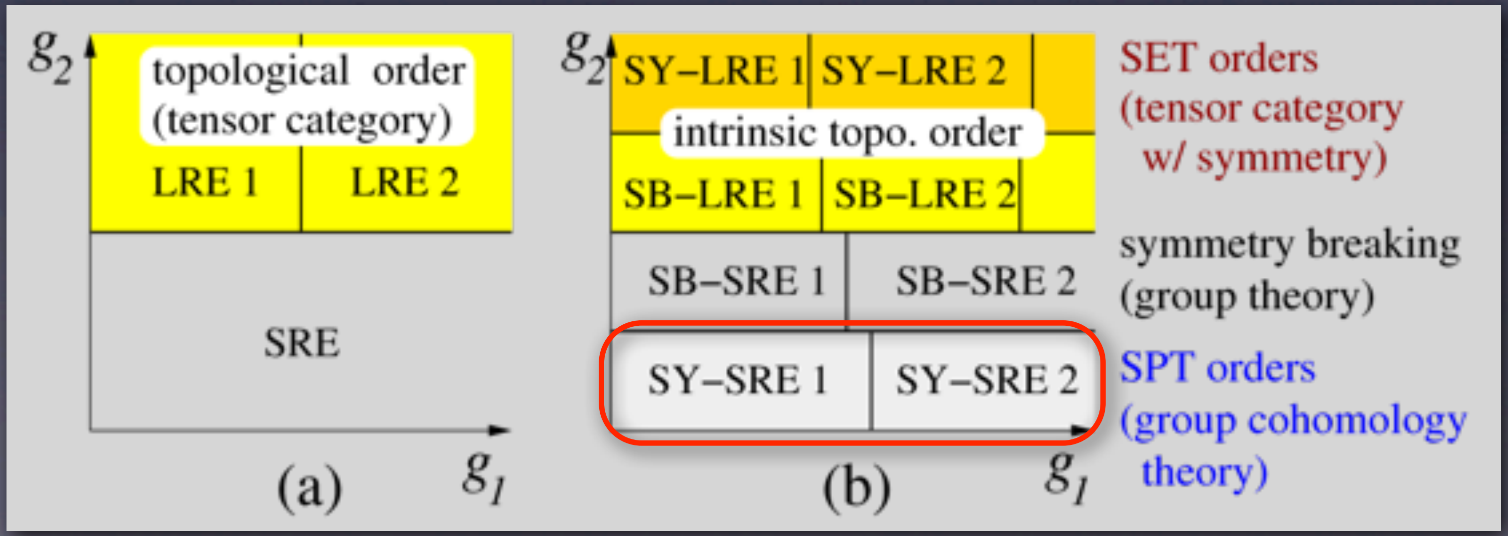
Gu-Wen '09, Chen et al. '11

symmetry-protected topological (SPT) order



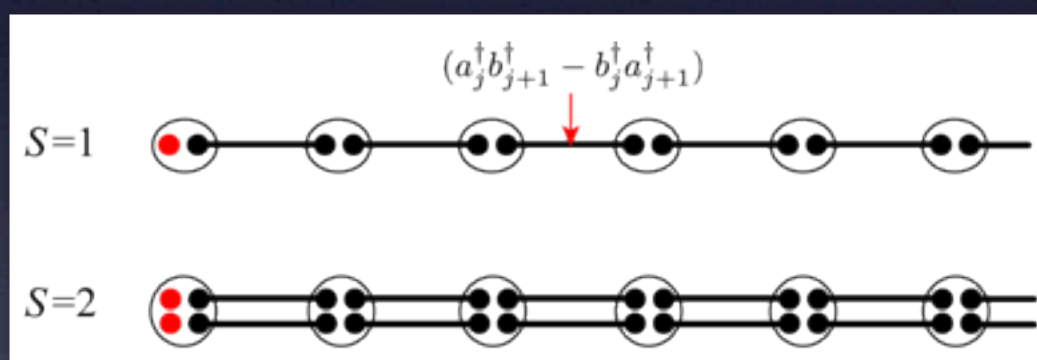
"Topological Order" in 1D

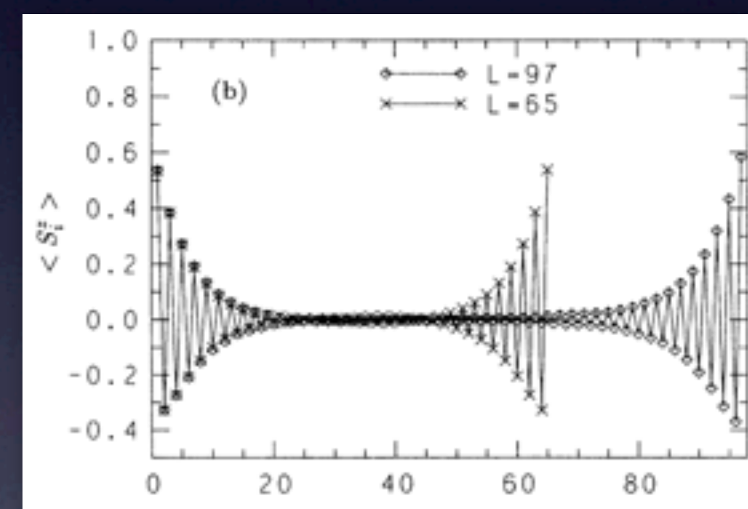
- Phases-I ... long-range entanglement (LRE):
  - ✓ genuine topological phases in (2+1)D, (3+1)D
  - ✓ w/ symmetries, possible to have topological order + SSB ("symmetry-enriched" etc.)
- Phases-II ... short-range entanglement (SRE)
  - ✓ wo/ symmetries, only a single trivial product state
  - ✓ w/ "protecting symmetries", various topological phases



"Topological Order" in 1D

- Paradigmatic example: "Haldane phase"
  - ✓ 1D integer-S spin chain — Haldane '83, Affleck et al. '88
  - ✓ featureless non-magnetic state w/ exponentially-decaying cor.
  - ✓ existence of edge states (cf. ESR)





Miyashita -Yamamoto '93

- Non-local (string) order parameters:
  - ✓ $Z_2 \times Z_2$ symmetry — Kennedy-Tasaki '92
  - ✓ relation to SPT order

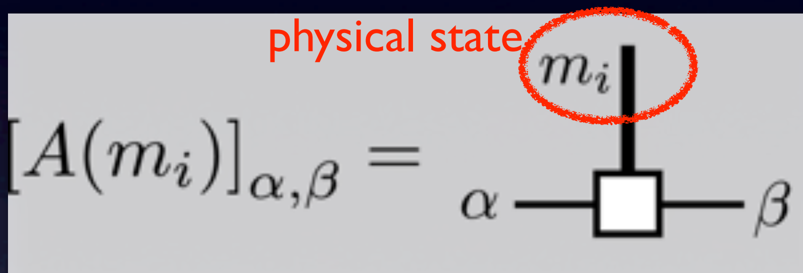
Pollmann-Turner '12, Hasebe-KT '13

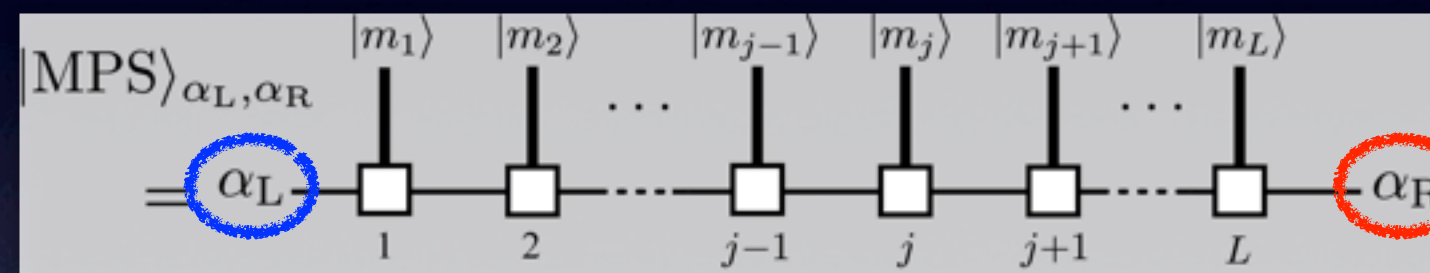
$$\mathcal{O}^z_{\text{string}} \equiv \lim_{|i-j|\nearrow\infty} \left\langle S_i^z \exp\left[i\pi \sum_{k=i}^{j-1} S_k^z \right] S_j^z \right\rangle$$

$$\mathcal{O}^x_{\text{string}} \equiv \lim_{|i-j|\nearrow\infty} \left\langle S_i^x \exp\left[i\pi \sum_{k=i+1}^{j} S_k^x \right] S_j^x \right\rangle .$$

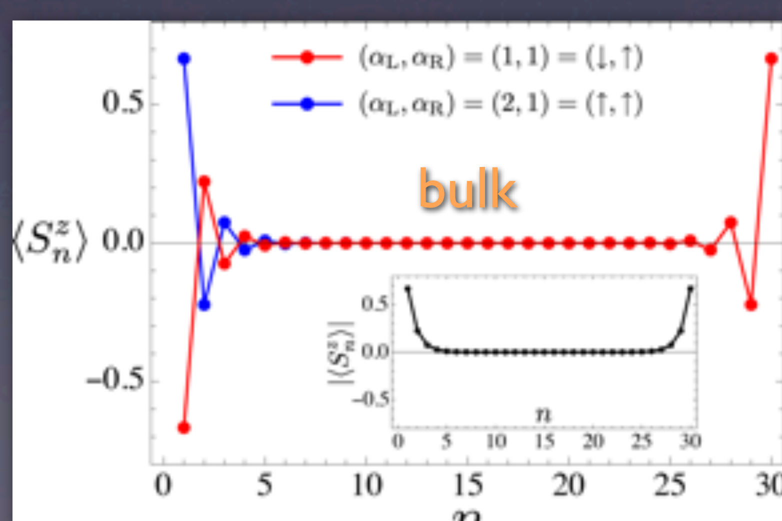
“Topological Order” in 1D

- Matrix-Product States (MPS): $\begin{pmatrix} |0\rangle_i & \sqrt{2}|-1\rangle_i \\ -\sqrt{2}|1\rangle_i & -|0\rangle_i \end{pmatrix}$: S=1 VBS
  - ✓ convenient rep. for *generic* gapped states in 1D (w/ SRE)





- existence of “edge states” (α,β) cf. FQHE, topological insulators
  - ✓ featureless in the bulk
  - ✓ special structure (“edge states”) localized at boundaries



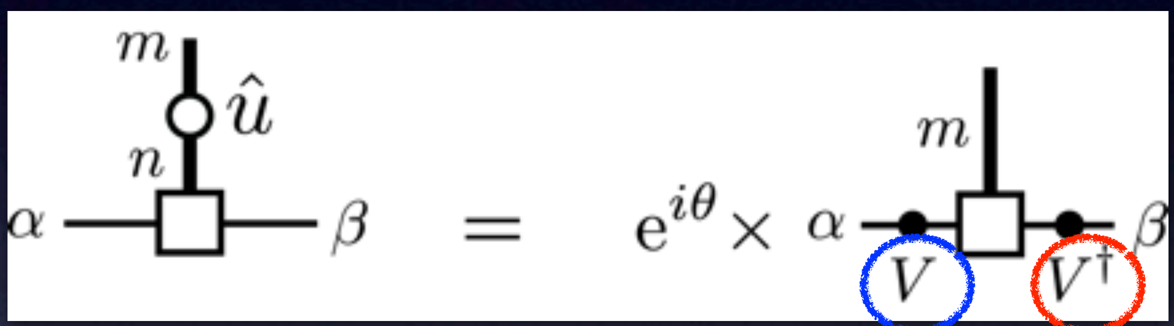
Kintaro Ame

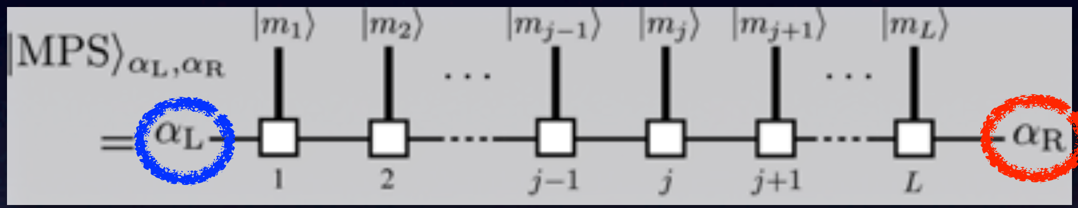
"Topological Order" in 1D

- edge states & "symmetry fractionalization":

  ✓ symmetry operation on MPS :

Perez-Garcia et al. '08





symmetry fractionalizes

Pollmann et al. '10, '12

- edge states = obey "projective representation"

  ✓ classifying "topological phases" = classifying possible "edge states"

Chen-Gu-Wen '11, Schuch et al. '11

  ✓ "V" does not necessarily follow group-multiplication rule:

$$V(g_1)V(g_2) = \omega(g_1, g_2)V(g_1 g_2)$$

  ✓ catalogue of gapped symmetry-protected topological phases in 1D (in terms of proj. rep.)

“Topological Order” in 1D

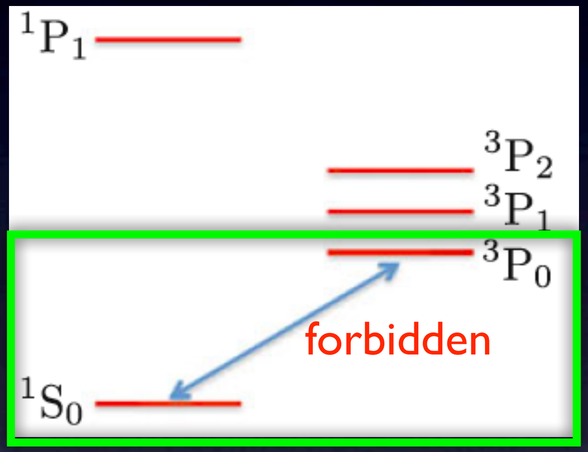
- Motivation....
- Given a “protecting symmetry”
  - ✓ zoo of symmetry-protected topological (SPT) phases
- Realization of topological phases
  - ✓ existence of (many) symmetry-breaking perturbations in real systems (fine-tuning necessary to have high sym.)
  - ✓ SPT phases protected by high symmetry unlikely ??

> How to realize SPT phases in realistic systems ??

Alkaline-earth cold atoms

... a realistic system bearing SPT phases

- alkaline-earth atoms: two electrons in the outer shell



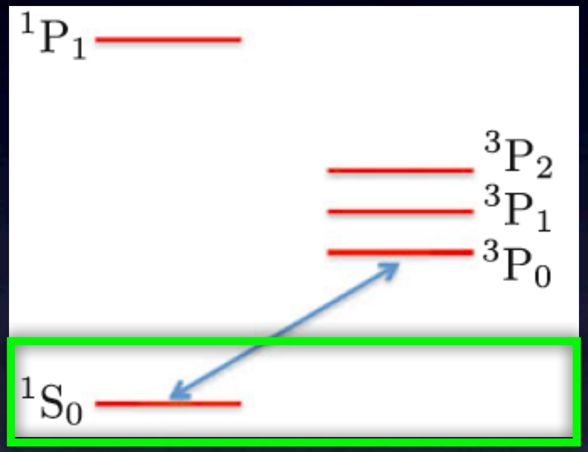
- J=0 for both G.S. ($^1S_0$) and (metastable) excited state ($^3P_0$)
- decoupling of "nuclear spin I" from electronic states
  ➡ I-independent scattering length
- SU(N) (N=2I+1) symmetry (to a very good approx.) Gorshkov et al '10
- ^{171}Yb (I=1/2, SU(2)), ^{173}Yb (I=5/2, SU(6)), ^{87}Sr (I=9/2, SU(10)) ,....

Takahashi group '10-'12 DeSalvo et al. '10

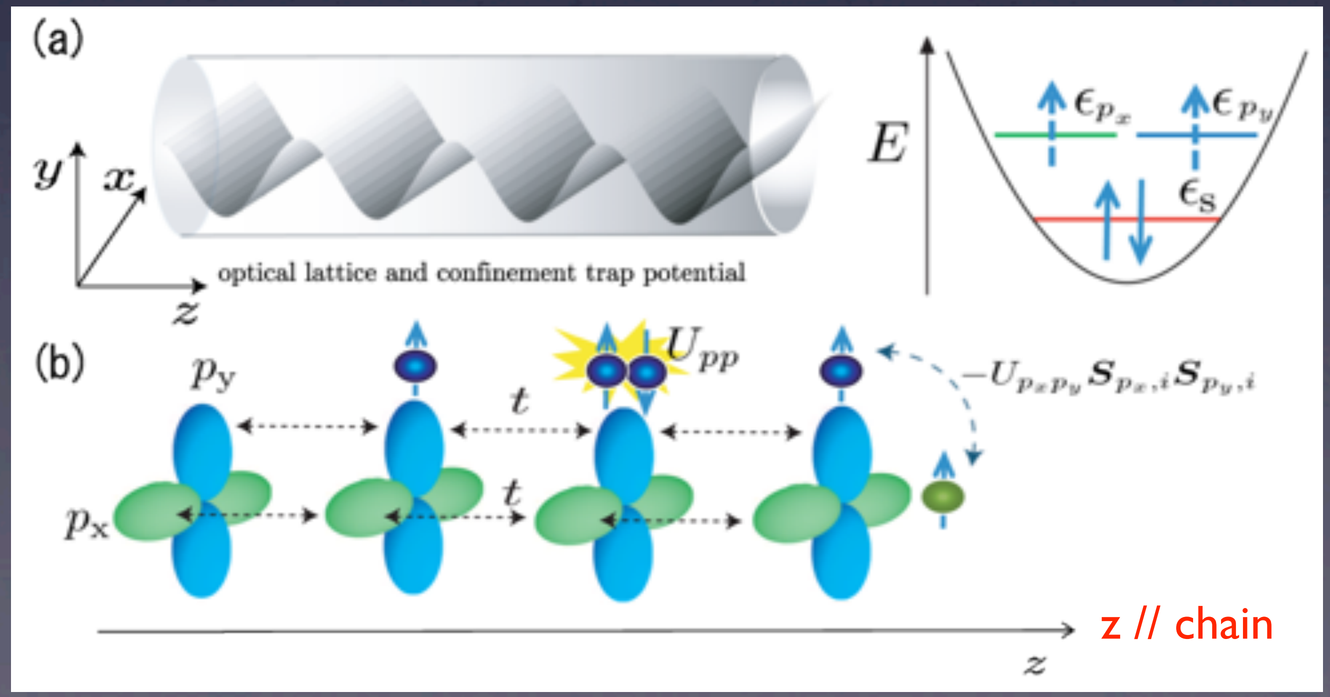
Alkaline-earth cold atoms

... a realistic system bearing SPT phases

- alkaline-earth atoms: two electrons in the outer shell



- p-band model ... another way of introducing 2-orbitals



✓1D optical lattice
✓confinement in (xy) not too strong
✓fill s-band only

Kobayashi et al. '12

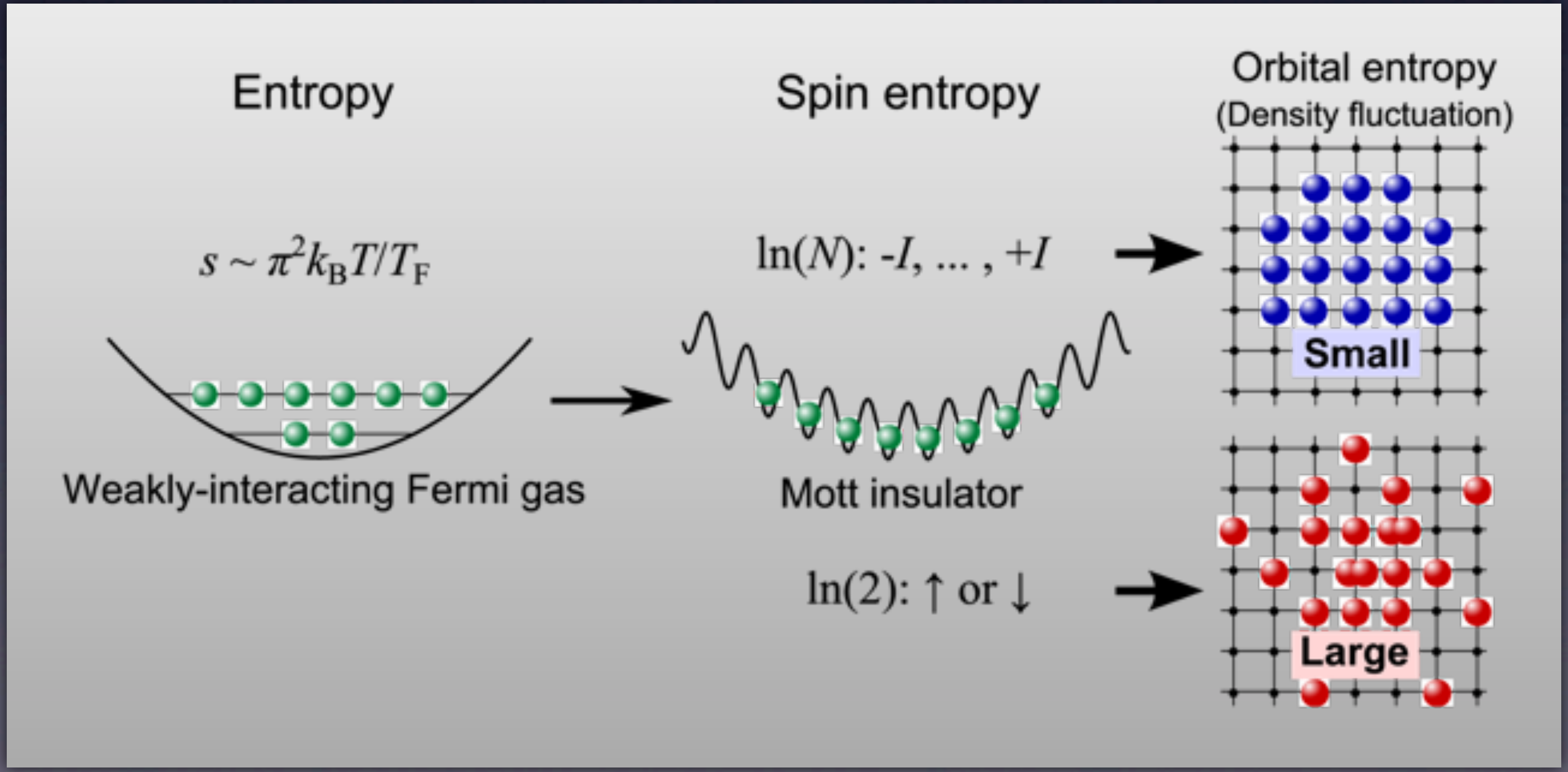
Alkaline-earth cold atoms

... Mott insulating phase (expt.)

- alkaline-earth atoms (^{173}Yb, I=5/2): SU(6) Mott phase

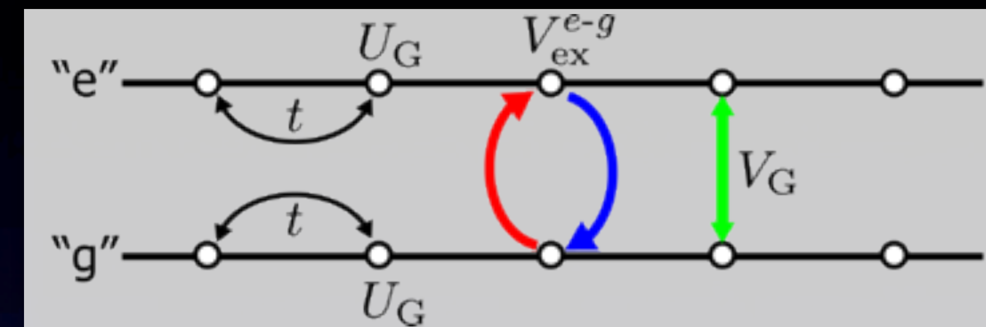
Taie et al. '12

- ✓ optical lattice, large-U: Mott phase (w/ n=1)
- ✓ charge gap, doublon, compressibility
- ✓ But.... still in high-T region (i.e. SU(N) paramagnet)



2-orbital SU(N) Hubbard model

... a minimal model



- "ingredients"
  - ✓ charge
  - ✓ nuclear-spin multiplet ($I^z=-I,\ldots,+I$, $N=2I+1$, $\alpha=1,\ldots,N$)
  - ✓ orbitals: ground state "g", excited state "e" (m=1,2)
    cf. 2-orbitals p_x, p_y in p-band model   Machida et al. '12

- Hubbard-like Hamiltonian:

Gorshkov et al '10
Nonne-Moliner-Capponi-Lecheminant-KT '13

$$H_G = -t\sum_{i,m\alpha}\left(c^\dagger_{m\alpha,\,i}c_{m\alpha,\,i+1} + \text{h.c.}\right) - \mu_G\sum_i n_i + \sum_i\sum_{m=e,g}\frac{U_G}{2}n_{m,\,i}(n_{m,\,i}-1)$$

Hubbard-like int. within "e" & "g"

$$+V_G\sum_i n_{g,\,i}n_{e,\,i} + V_{ex}^{e\text{-}g}\sum_{i,\alpha\beta}c^\dagger_{g\alpha,\,i}c^\dagger_{e\beta,\,i}c_{g\beta,\,i}c_{e\alpha,\,i}$$

Coulomb between "g"-"e"   "e"-"g" exchange

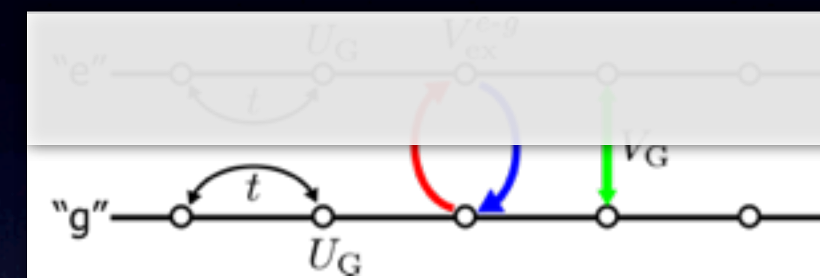
$c^\dagger_{m\alpha,i}$: creation op. for "orbital"=m, nuclear-spin=α

generic symmetry: $\mathrm{U}(1)_c\times\mathrm{SU}(N)_s\times\mathrm{U}(1)_o$

SU(N) Hubbard model

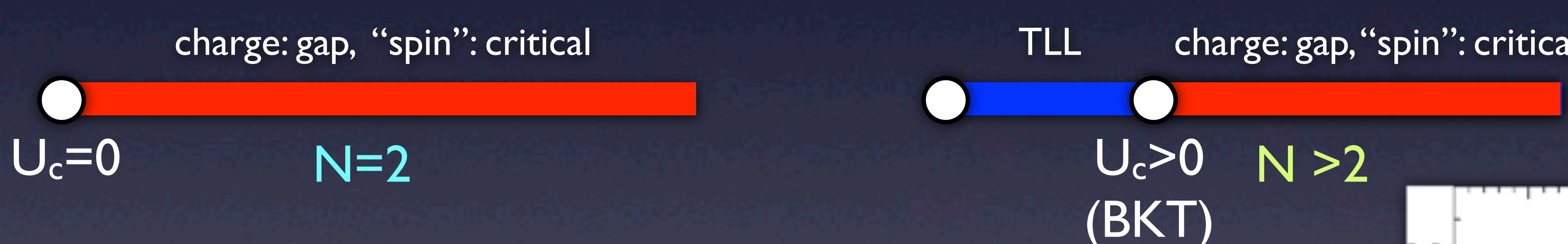
... simpler case

- quench "e"-orbital ➡ (single-band) SU(N) Hubbard



- insulating phases:
  - ✓ 1/N-filling: MIT@U=U_c
    ➡ charge: gapped, "SU(N)-spin": gapless

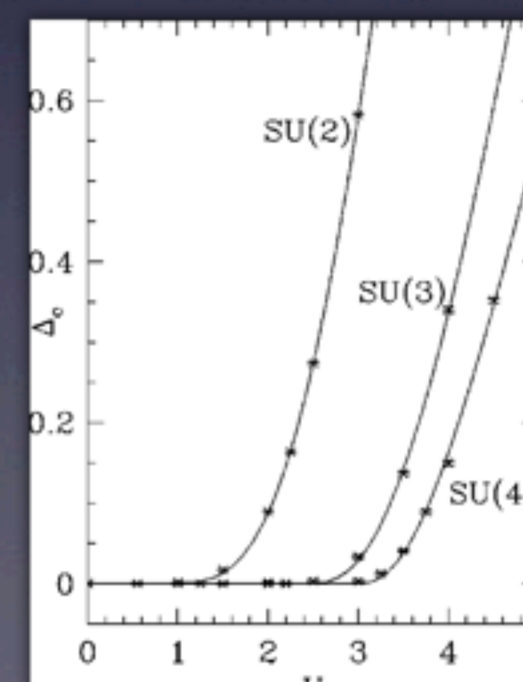
Lieb-Wu '68, Assaraf et al. '99
Manmana et al. '11



  - ✓ deep inside Mott phase: SU(N)-Heisenberg
    (typically, U/t >11)

Sutherland '75, Affleck '88
Manmana et al. '11

  - ✓ for other commensurate fillings... f=p/q ??



Assaraf et al. '99

SU(N) Hubbard model

... simpler case

- single-band SU(N) Hubbard
- For other commensurate fillings ... f=p/q

Szirmai et al. '05, '08

✓ q > N: gapless charge/"spin", C1S(N-1) (c=n)
➡ critical phase

✓ q < N: full gap (C0S0), spatially non-uniform insulating phase

SU(5) Hubbard@comm.fillings

Szirmai et al. '05, '08

➡ No featureless fully-gapped phase

possible to have
"fully-gapped uniform states" ??

in realistic interacting models....

Phases of 2-orbital SU(N) Hubbard

- plan of attack:
  - ✓ 2-orbital (“g” & “e”), SU(N)-fermion, @half-filling
  - ✓ Maximal filling: 2N fermions/site (2N boxes)
    ex) half-filling = N fermions/site (N boxes out of 2N)
  - ✓ Mathods:
    1. weak-coupling (RG + duality)
       ➡ low-energy field theory: 4N Majorana fermions (+ int.)
    2. strong-coupling (approach from Mott sides)
       ➡ effective spin/orbital models
    3. numerical ➡ DMRG

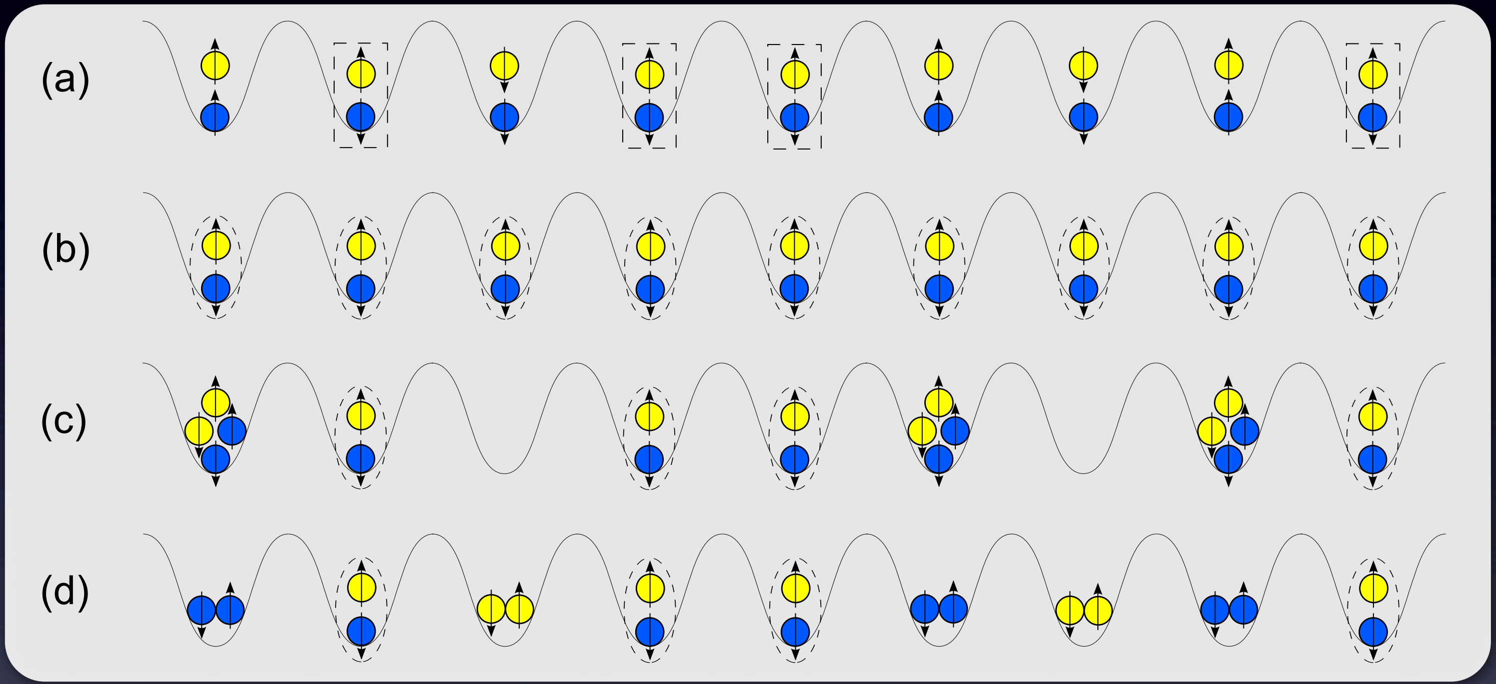
Phases of 2-orbital SU(N) Hubbard

... N=2, half-filling

- 4 Mott phases wo/ G.S. degeneracy:

orbital-"g"

orbital-"e"



spin-Haldane $S_{spin}=1$

rung-singlet orbital large-D

charge-Haldane

orbital-Haldane $S_{orb}=1$

	"charge"	"orbital"	"spin"
(a) spin-Haldane	gapped	local singlet	spin-1 Haldane
(b) rung-singlet	gapped	Tz=0 (large-D)	local singlet
(c) charge-Haldane	gapped	singlet/triplet	local singlet
(d) orbital-Haldane	gapped	spin-1 Haldane	local singlet

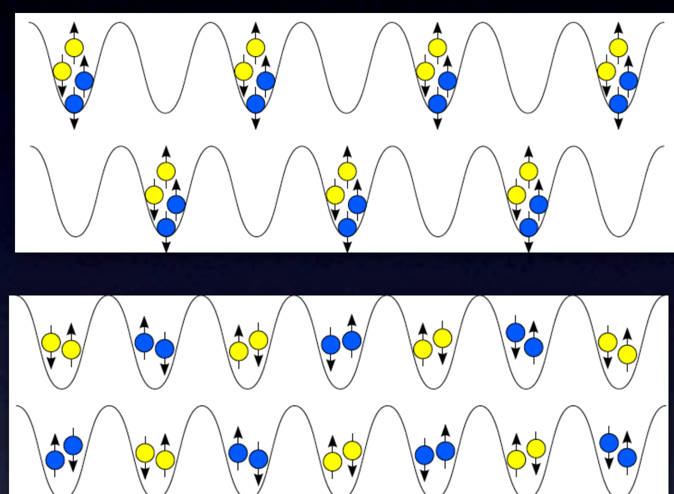
Kobayashi et al. '12

Della Torre et al. '06
Nonne et al. '10

Phases of 2-orbital SU(N) Hubbard

... N=2, half-filling (weak-coupling)

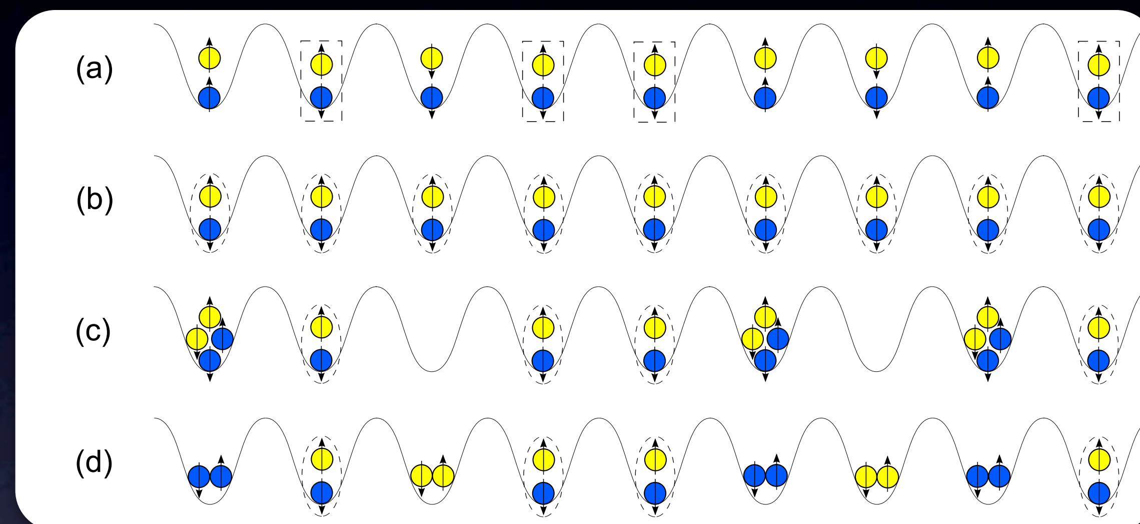
- 4 Mott phases + 3 phases w/ degenerate G.S.:



CDW ($2k_F$)

ODW ($2k_F$)

degenerate



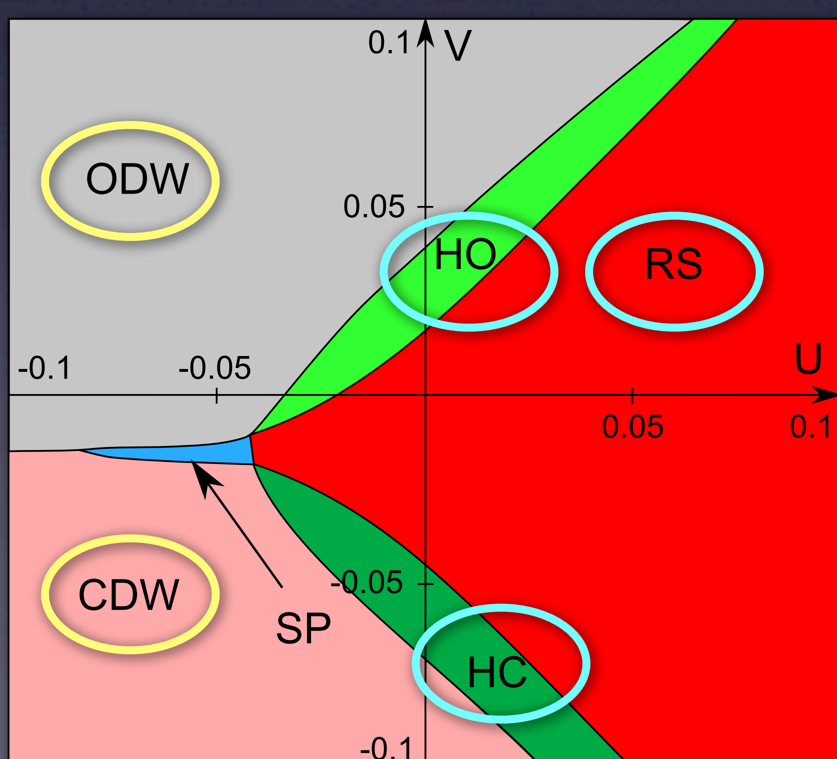
spin-Haldane S_{spin}=1 (“RT”)

rung-singlet orbital large-D (“RS”)

charge-Haldane (“HC”)

orbital-Haldane S_{orb}=1 (“HO”)

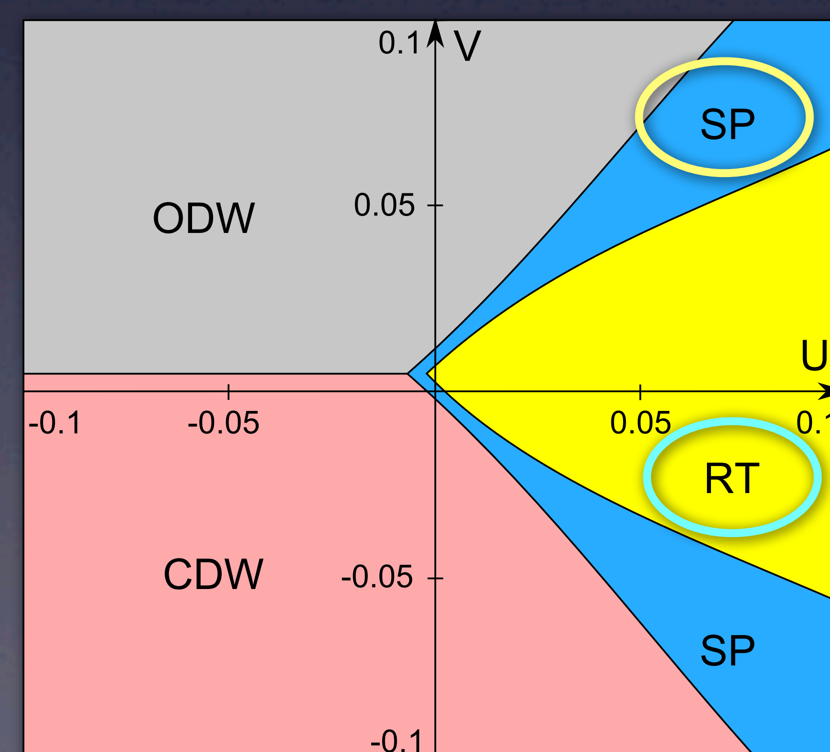
non-degenerate, Mott-like



$V_{\rm ex} = -0.03t$

weak-coupling phase diag.

Nonne et al. ’10

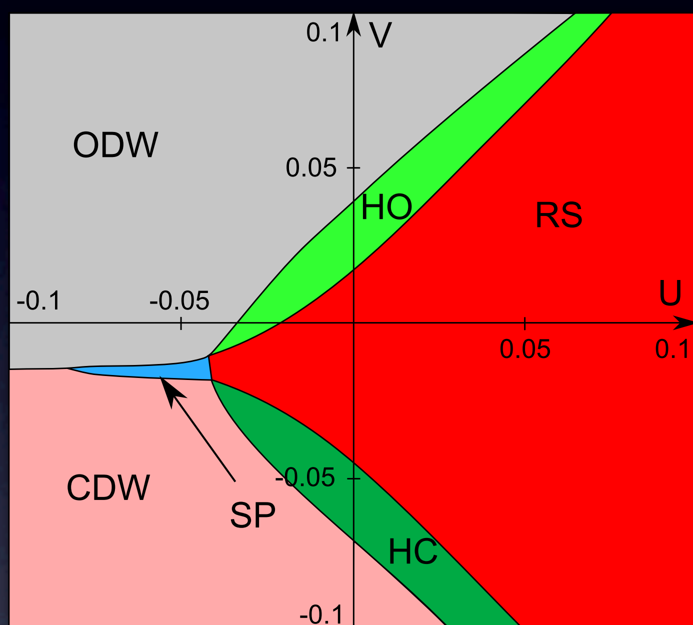


$V_{\rm ex} = 0.01t$

Phases of 2-orbital SU(N) Hubbard

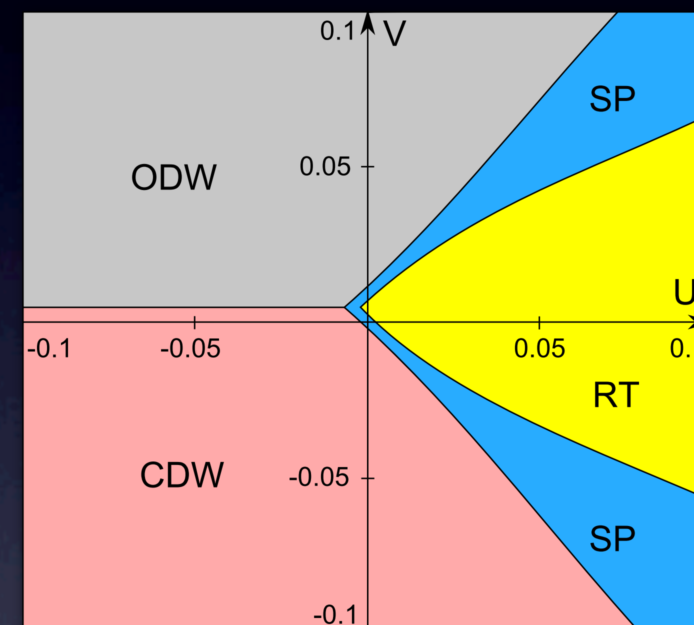
... N=2, half-filling (numerical)

- 4 Mott phases + 3 insulating phases w/ degenerate G.S.:

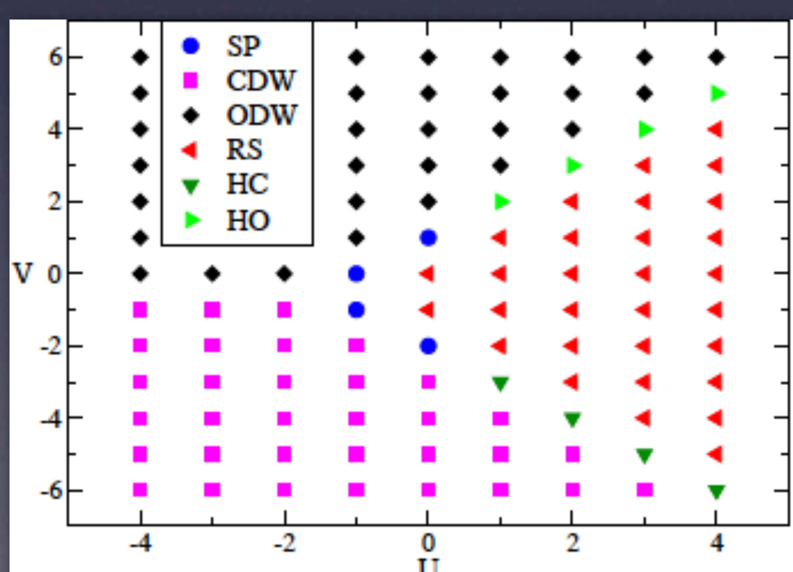


$V_{\rm ex} = -0.03t$

weak-coupling phase diag.



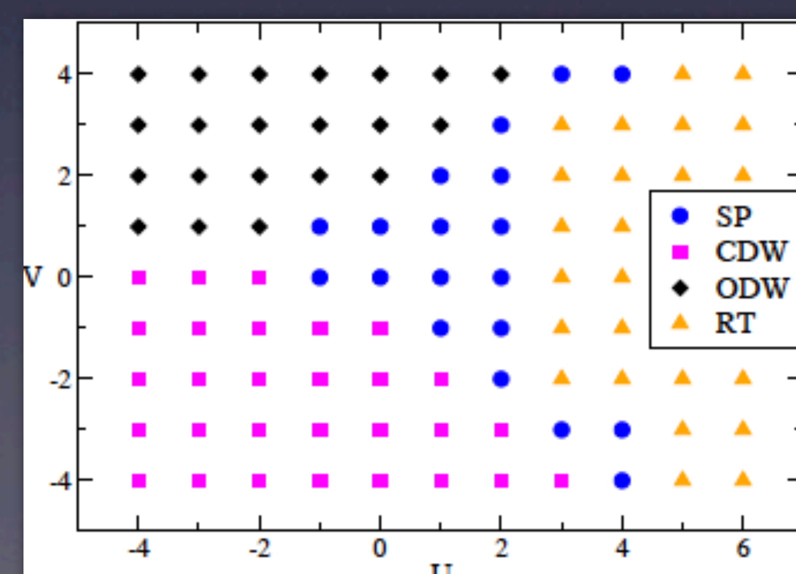
$V_{\rm ex} = 0.01t$



$V_{\rm ex} = -t$

DMRG

Nonne et al. '10



$V_{\rm ex} = t$

2-orbital SU(N) (N>2) Hubbard
... half-filling

- weak-coupling: (very) complicated, but still doable...

$$\dot{g}_1 = \frac{N}{4\pi} g_1^2 + \frac{N}{8\pi} g_2^2 + \frac{N}{16\pi} g_3^2 + \frac{N+2}{4\pi} g_7^2 + \frac{N-2}{2\pi} g_8^2 + \frac{N-2}{4\pi} g_9^2$$

$$\dot{g}_2 = \frac{N}{2\pi} g_1 g_2 + \frac{N^2-4}{4N\pi} g_2 g_3 + \frac{1}{2\pi} g_3 g_4 + \frac{1}{2\pi} g_2 g_5 + \frac{N}{\pi} g_7 g_8 + \frac{N-2}{\pi} g_8 g_9$$

$$\dot{g}_3 = \frac{N}{2\pi} g_1 g_3 + \frac{N^2-4}{4\pi N} g_2^2 + \frac{1}{\pi} g_2 g_4 + \frac{N}{\pi} g_7 g_9 + \frac{N-2}{\pi} g_8^2$$

$$\dot{g}_4 = \frac{1}{2\pi} g_4 g_5 + \frac{N^2-1}{2\pi N^2} g_2 g_3 + \frac{2(N-1)}{N\pi} g_8 g_9$$

$$\dot{g}_5 = \frac{N^2-1}{2\pi N^2} g_2^2 + \frac{1}{2\pi} g_4^2 + \frac{2(N-1)}{N\pi} g_8^2$$

$$\dot{g}_6 = \frac{N+1}{4\pi N} g_7^2 + \frac{N-1}{2\pi N} g_8^2 + \frac{N-1}{4N\pi} g_9^2$$

$$\dot{g}_7 = \frac{N^2+N-2}{2N\pi} g_1 g_7 + \frac{2}{\pi} g_6 g_7 + \frac{N-1}{4\pi} g_3 g_9 + \frac{N-1}{2\pi} g_2 g_8$$

$$\dot{g}_8 = \frac{N+1}{4\pi} g_2 g_7 + \frac{N^2-N-2}{4N\pi} g_2 g_9 + \frac{2}{\pi} g_6 g_8 + \frac{N^2-N-2}{2N\pi} g_1 g_8 + \frac{1}{2\pi} g_5 g_8 + \frac{1}{2\pi} g_4 g_9 + \frac{N^2-N-2}{4N\pi} g_3 g_8$$

$$\dot{g}_9 = \frac{N+1}{4\pi} g_3 g_7 + \frac{2}{\pi} g_6 g_9 + \frac{N^2-N-2}{2N\pi} g_1 g_9 + \frac{1}{\pi} g_4 g_8 + \frac{N^2-N-2}{2N\pi} g_2 g_8$$

RG-flow pattern different from N=2

✓ SP
✓ $2k_F$-CDW
✓ orbital-Heisenberg (S=N/2)
⬅ regardless of "N"

Nonne et al. '13

2-orbital SU(N) (N>2) Hubbard
... half-filling

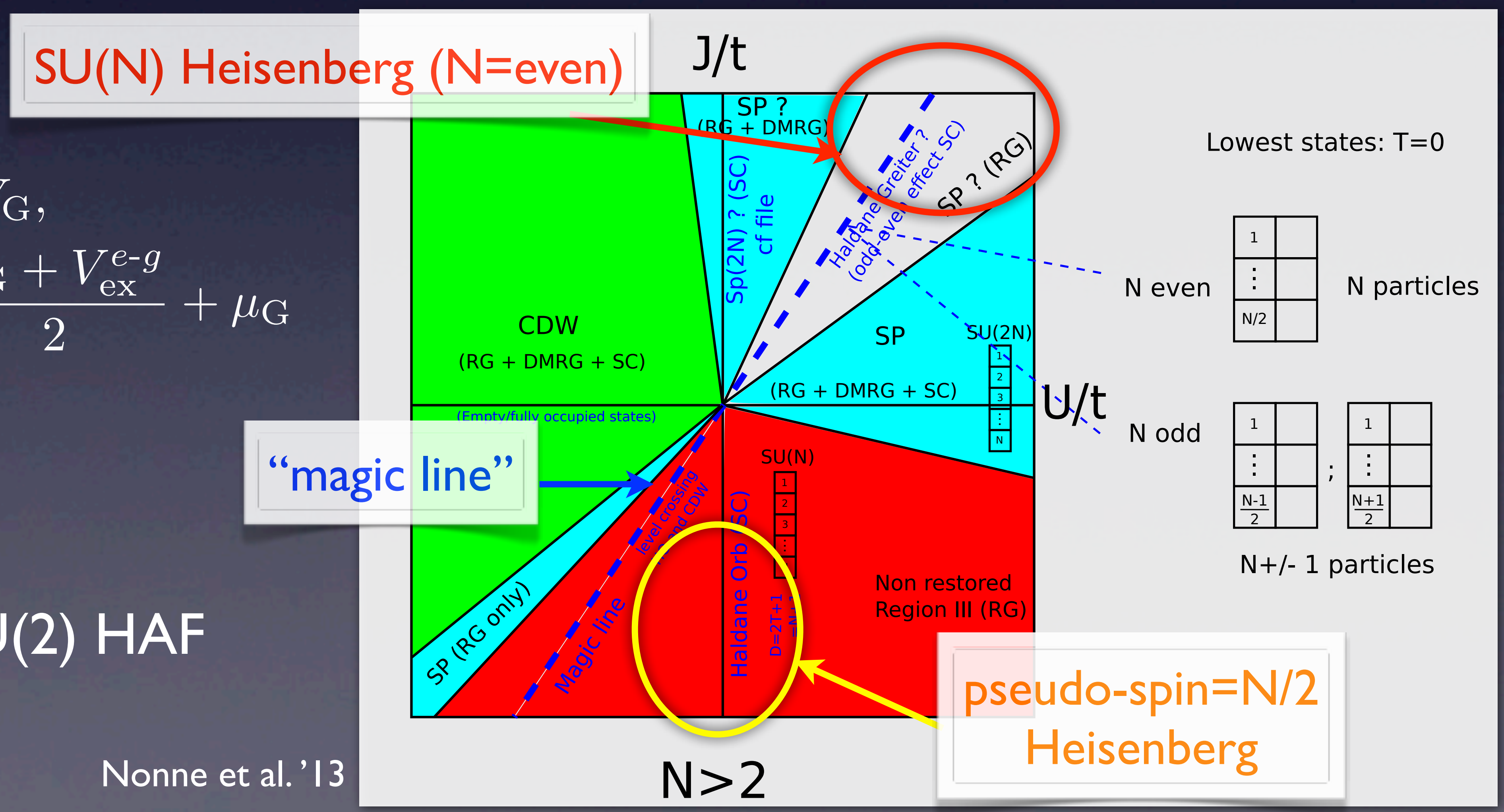
- (very) schematic phase diagram for N=even:
- $$\mathcal{H}_{\rm H} = -t\sum_i\sum_{m=1}^{2}\sum_{\alpha=1}^{N}\left(c^\dagger_{m\alpha,\,i}c_{m\alpha,\,i+1}+{\rm h.c.}\right)-\mu\sum_i n_i+\frac{U}{2}\sum_i n_i^2 + J\sum_i\{(T_i^x)^2+(T_i^y)^2\}+J_z\sum_i(T_i^z)^2,$$

$$J = V_{\rm ex}^{e\text{-}g},\quad J_z = U_{\rm G}-V_{\rm G},$$

$$U = \frac{U_{\rm G}+V_{\rm G}}{2},\quad \mu = \frac{U_{\rm G}+V_{\rm ex}^{e\text{-}g}}{2}+\mu_{\rm G}$$

✓ spin-Peierls (SP)
✓ CDW
✓ orbital T=N/2 SU(2) HAF
✓ SU(N) HAF

Nonne et al. '13

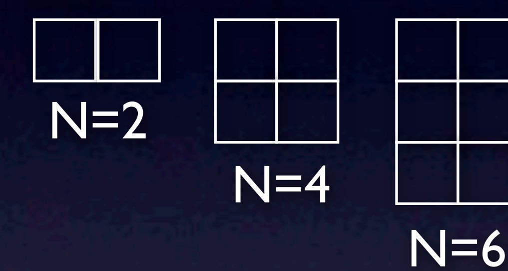


2-orbital SU(N) (N>2) Hubbard
... SU(4) SPT phase (new!)

- "Magic line": N=even $\quad J_z = J \, , \quad J = \dfrac{2NU}{N+2}$
- strong-coupling effective Hamiltonian:

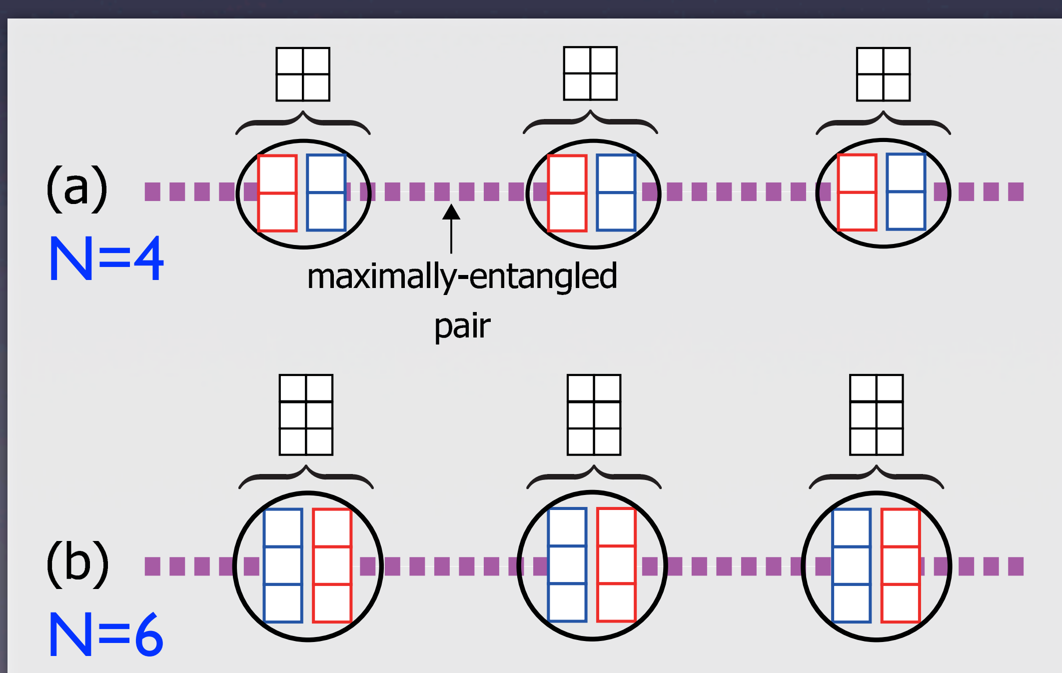
$$\mathcal{H}_{\text{eff}} = \sum_{A=1}^{N^2-1} S_i^A S_{i+1}^A$$

N/2 rows, 2 columns



- generalize AKLT-idea: ex) SU(4) VBS Nonne et al. '13

parent Hamiltonian: $\mathcal{H}_{\text{VBS}} = J_{\text{s}} \sum_i \left\{ S_i^A S_{i+1}^A + \frac{13}{108} (S_i^A S_{i+1}^A)^2 + \frac{1}{216} (S_i^A S_{i+1}^A)^3 \right\}$



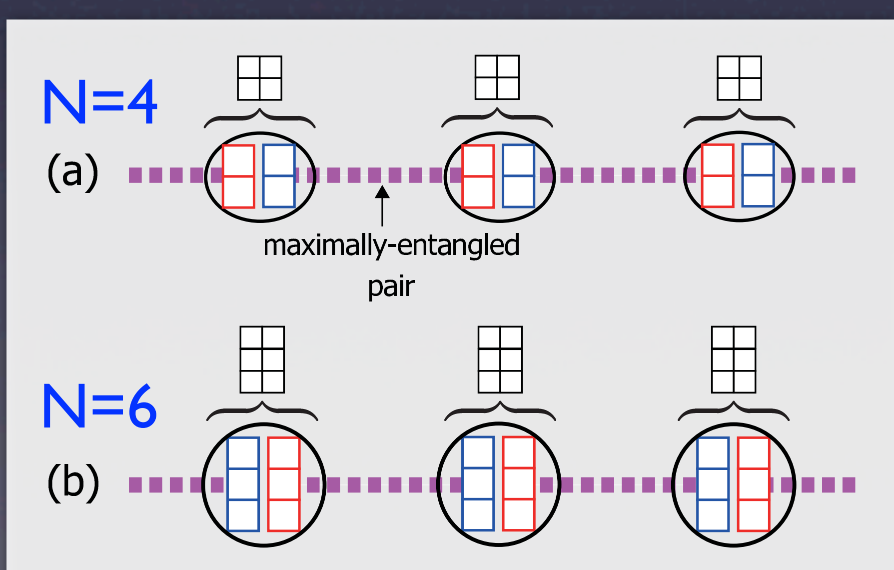
$$\langle S^A(j) S^A(j+n) \rangle = \begin{cases} \frac{12}{5} \left(-\frac{1}{5}\right)^n & n \neq 0 \\ \frac{4}{5} & n = 0 \end{cases}$$

NB) consistent w/ DMRG sim.

Nonne et al. '13

2-orbital SU(N) (N>2) Hubbard
... SU(4) SPT phase (new!)

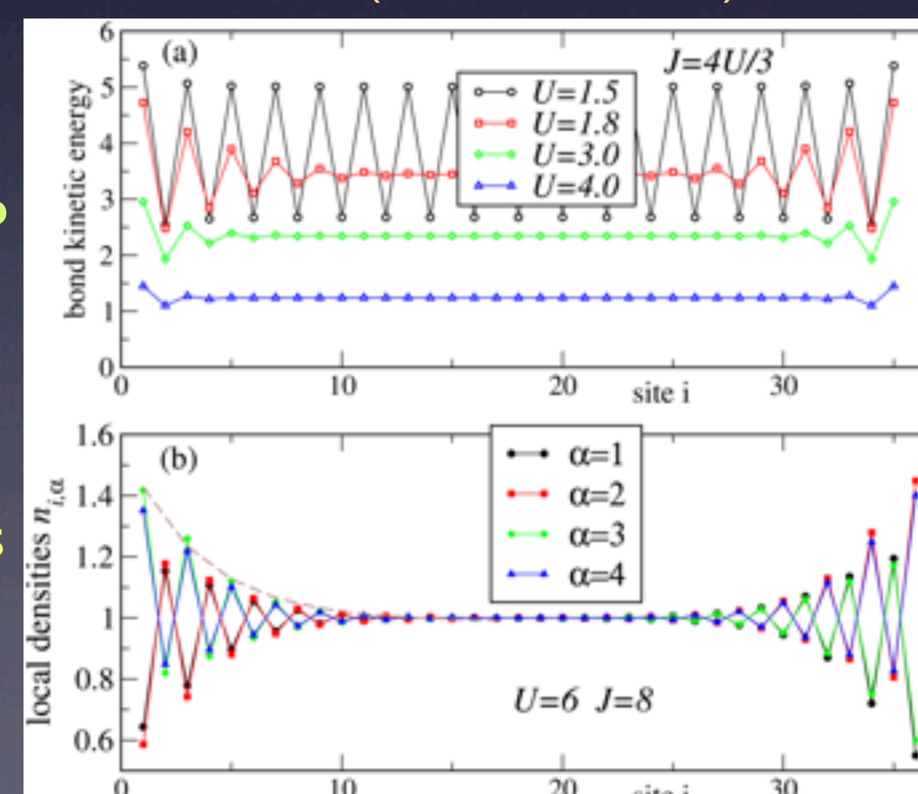
- SU(4) VBS $\mathcal{H}_{\rm VBS} = J_{\rm s} \sum_i \left\{ S_i^A S_{i+1}^A + \frac{13}{108}(S_i^A S_{i+1}^A)^2 + \frac{1}{216}(S_i^A S_{i+1}^A)^3 \right\}$
  - ✓ 6-fold degenerate edge states
  - ✓ one of N(=4) SPT phases (protected by SU(N)) Duivenvoorden-Quella '12, '13
  - ✓ When U↘, QPT out to ("spin"-orbital entangled) "SP" phase (univ. class $SU(N)_2$)



N=4 (DMRG, L=36)

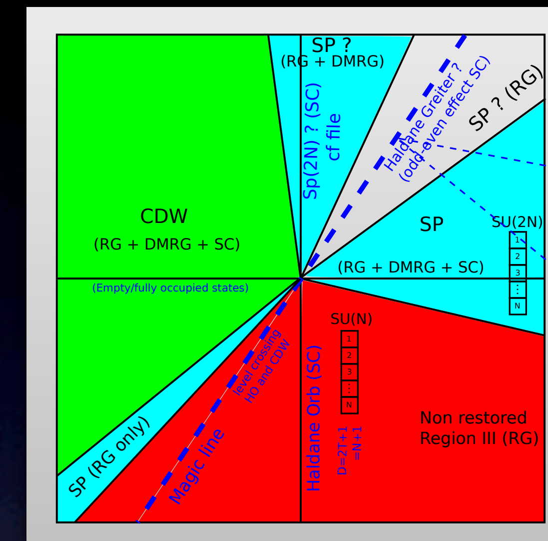
QPT: top ⇒ SP

edge states (n_α)



Nonne et al. '13

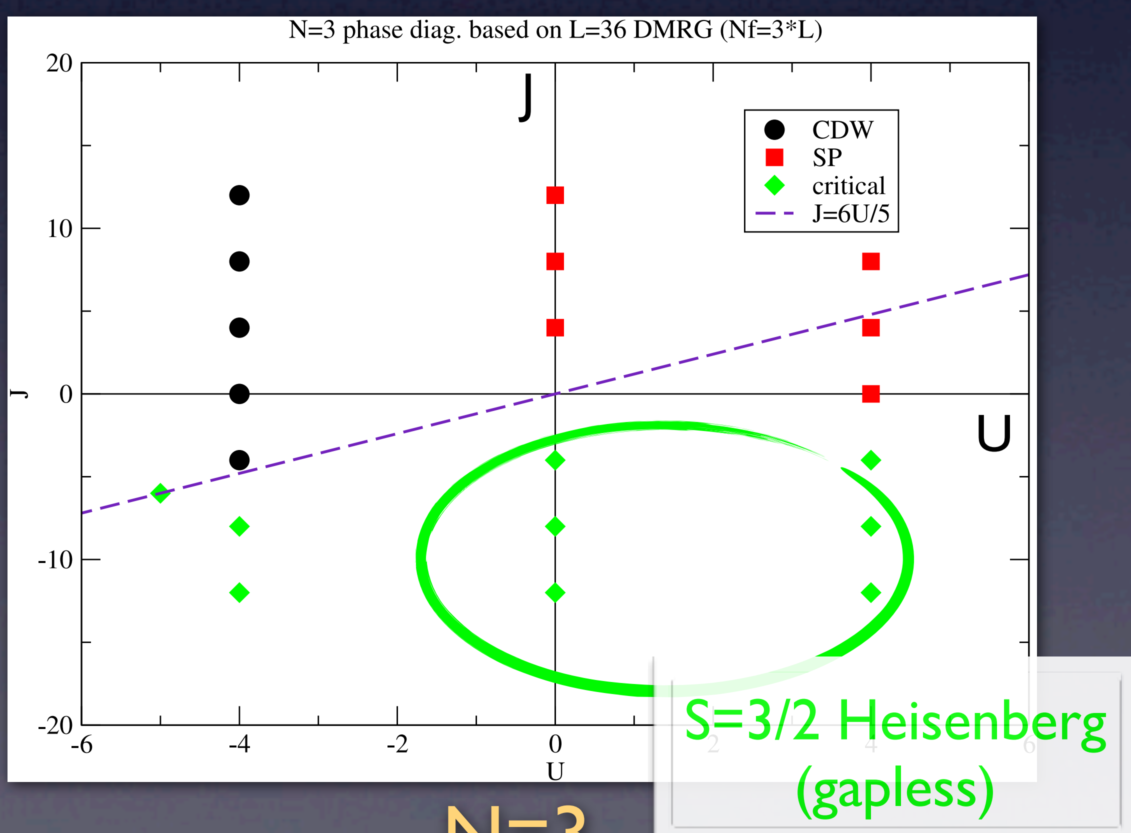
Comparison: N=3 and N=4 cases
... numerics (DMRG)



- drastic difference between N=odd / even
- (preliminary) DMRG results:

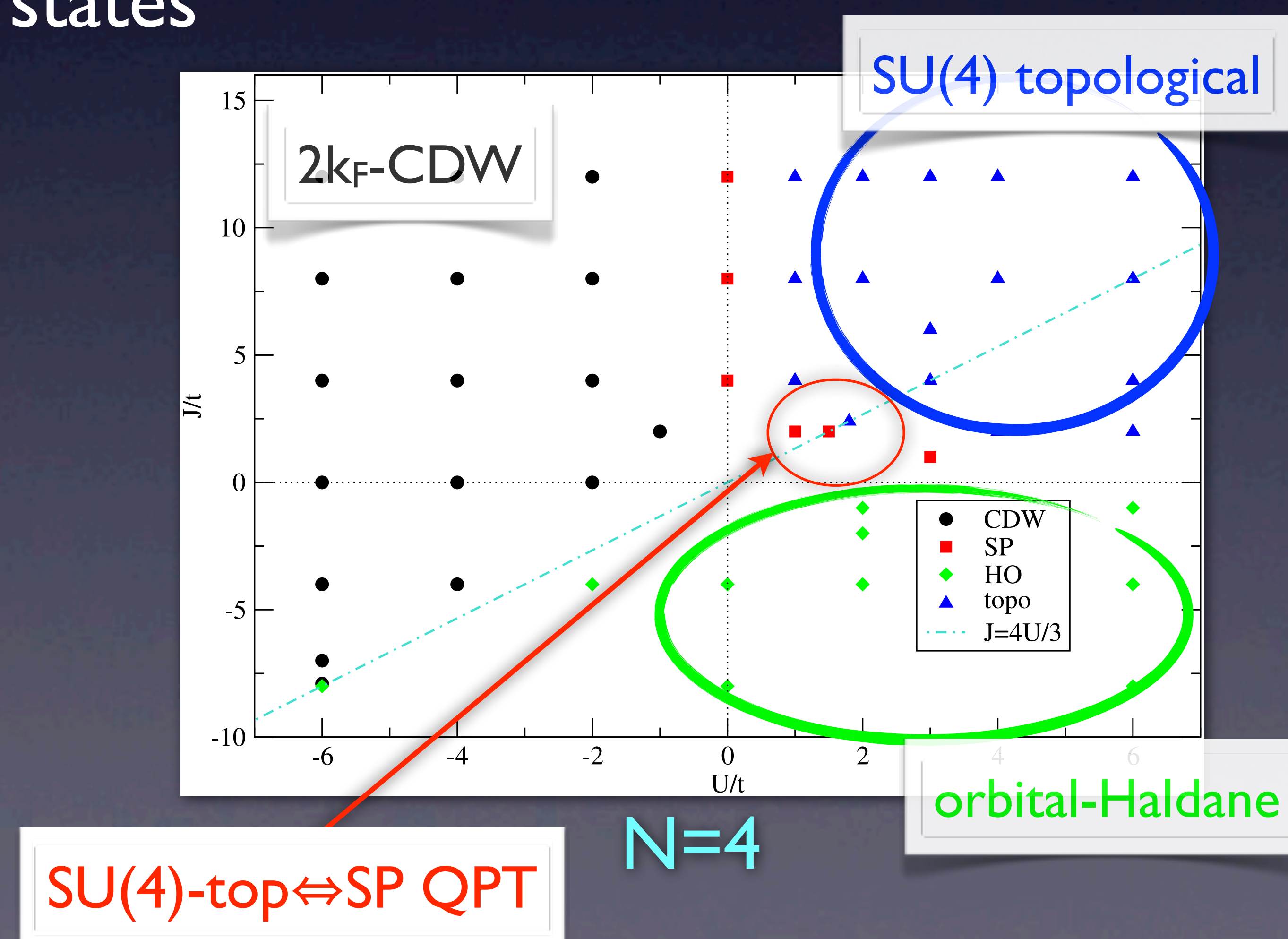
cf. Nonne et al. '13

- ✓ L=36 sites
- ✓ order parameter & edge states



N=3

3 phases (2 gapped, 1 gapless)

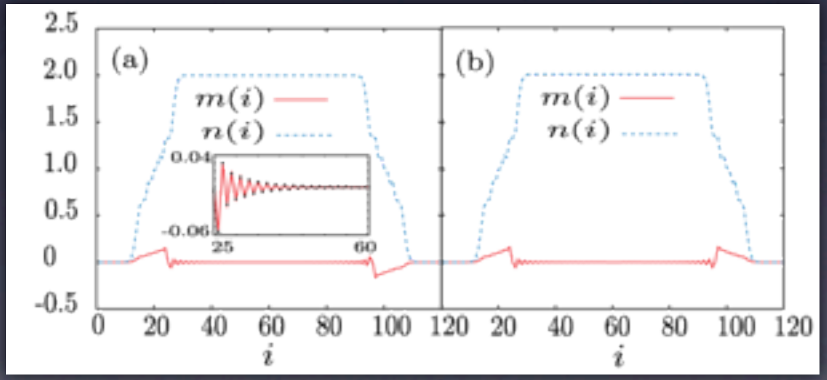


N=4

Summary

- Symmetry-protected topological order in 1D
- SU(N) Hubbard model with 2-orbitals (N=2I+1)
  - ✓ alkalline-earth Fermi gas in optical lattice (@half-filling)
  - ✓ phase diagram depends on N=even / odd
  - ✓ symmetry-protected topological phases for N=even (I=1/2,3/2,..)
  - ✓ stable topological phases for N/2=odd, i.e. I=1/2 (^{171}Yb), 5/2 (^{173}Yb), 9/2 (^{87}Sr), ...

"Mott core"



Kobayashi et al. '12

- Outlook:
  - ✓ Effects of trap (SPT phase & Mott core) ?
  - ✓ quantum-optical detection of topological order ??
  - ✓ higher-D ???