Spin liquid phases in the SU(4) and SU(3) Heisenberg model on the honeycomb lattice

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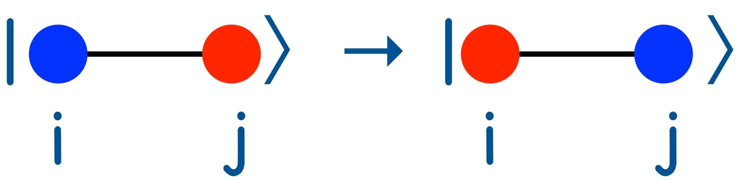
What are the SU(N) symmetric Heisenberg models that we are interested in?

$$\mathcal{H} = \sum_{i,j} \mathcal{P}_{i,j}$$

$\mathcal{P}_{i,j}$ is the transposition operator

$\mathcal{P}_{i,j}$



N species on each site that are treated equally.

$$\mathcal{P}_{ij}|\beta_i\alpha_j\rangle = |\alpha_i\beta_j\rangle$$

simplest example:
SU(2) S=1/2 (fundamental representation)
[but not the S=1 !]

Why do we care about SU(N) Heisenberg models?

(i) Spin models

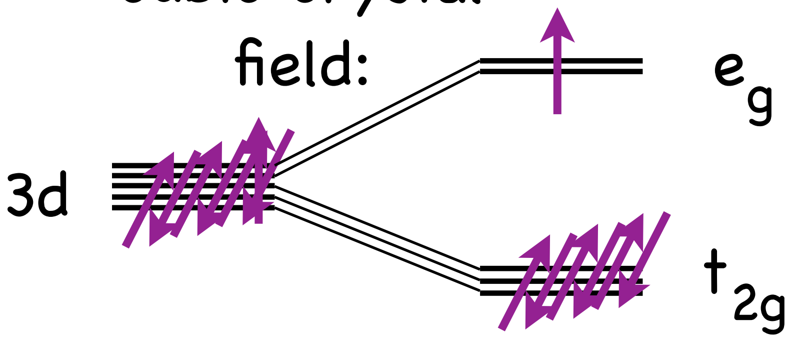
(ii) Spin-orbital models

(iii) f-electron systems

(iv) Cold alkaline-earth atoms in optical lattices

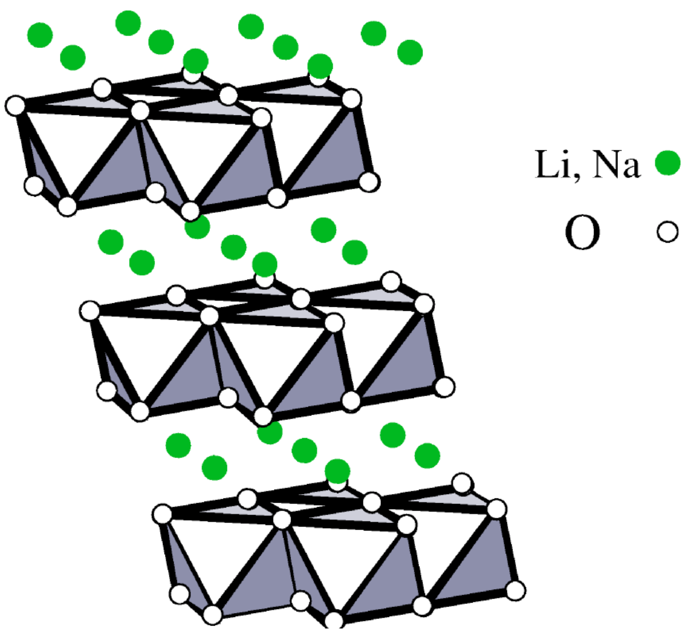
SU(4) highest symmetry of spin-orbital model (e.g. $LiNiO_2$ and $NaNiO_2$)

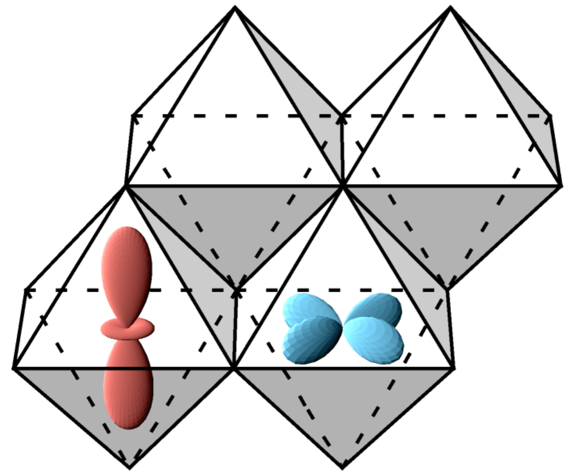
Ni^{3+} ($3d^7$) in a cubic crystal field:



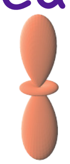
S=1/2, with 2 orbital degrees of freedom

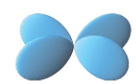
filled shell





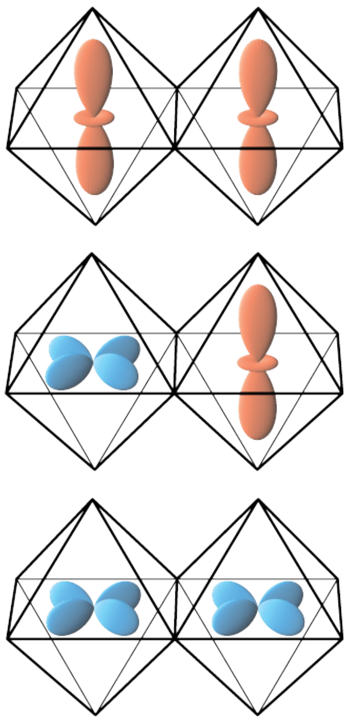
T pseudospin operators:

 : $T^z = +1/2$

: $T^z = -1/2$

Spin-orbital models : Microscopic theory

intersite hopping:



$t_{aa}=t$

$t_{ab}=0$

$t_{bb}=t'$

On-site interaction:

Coulomb-repulsion: $$\frac{\tilde{U}}{2}n^2$$

Hund's rule: $$-J_H\left(\mathbf{S}_a\mathbf{S}_b+\frac{3}{4}n_a n_b\right)$$

pair hopping: $$J_p(c^\dagger_{a,\uparrow}c^\dagger_{a,\downarrow}+c^\dagger_{b,\uparrow}c^\dagger_{b,\downarrow})\times(c_{a,\downarrow}c_{a,\uparrow}+c_{b,\downarrow}c_{b,\uparrow})$$

+ the standard perturbation theory to get the effective Hamiltonian.

Spin-orbital models: Kugel-Khomskii Hamiltonian

$$\begin{aligned}\mathcal{H}_{ij} &= -\frac{2}{\tilde{U}+2J_p}\Big[2tt'\mathbf{T}_i\mathbf{T}_j - 4tt'T_i^yT_j^y + (t-t')^2(\mathbf{n}_{ij}^z\mathbf{T}_i)(\mathbf{n}_{ij}^z\mathbf{T}_j)\\ &\qquad + \frac{1}{2}(t^2-t'^2)\left(\mathbf{n}_{ij}^z\mathbf{T}_i+\mathbf{n}_{ij}^z\mathbf{T}_j\right)+\frac{1}{4}(t^2+t'^2)\Big]\mathcal{P}_{ij}^{S=0}\\ &\quad -\frac{2}{\tilde{U}}\left[4tt'T_i^yT_j^y+\frac{1}{2}(t^2+t'^2)+\frac{1}{2}(t^2-t'^2)\left(\mathbf{n}_{ij}^z\mathbf{T}_i+\mathbf{n}_{ij}^z\mathbf{T}_j\right)\right]\mathcal{P}_{ij}^{S=0}\\ &\quad -\frac{2}{\tilde{U}-J_H}\left[-2tt'\mathbf{T}_i\mathbf{T}_j-(t-t')^2(\mathbf{n}_{ij}^z\mathbf{T}_i)(\mathbf{n}_{ij}^z\mathbf{T}_j)+\frac{1}{4}(t^2+t'^2)\right]\mathcal{P}_{ij}^{S=1}\end{aligned}$$

$$\mathcal{P}_{ij}^{S=0} = \frac{1}{4} - \mathbf{S}_i\mathbf{S}_j \qquad \mathcal{P}_{ij}^{S=1} = \mathbf{S}_i\mathbf{S}_j + \frac{3}{4}$$

For t=t′ and J_p=0

SU(2)xSU(2) symmetric

$$\mathcal{H}_{ij} = \frac{4t^2}{\tilde{U}}\left(\mathbf{T}_i\mathbf{T}_j+\frac{3}{4}\right)\left(\mathbf{S}_i\mathbf{S}_j-\frac{1}{4}\right)+\frac{4t^2}{\tilde{U}-J_H}\left(\mathbf{T}_i\mathbf{T}_j-\frac{1}{4}\right)\left(\mathbf{S}_i\mathbf{S}_j+\frac{3}{4}\right)$$

t=t′ and J_p=J_H=0

SU(4) symmetric

$$\mathcal{H}_{ij} = \frac{8t^2}{\tilde{U}}\left(\mathbf{T}_i\mathbf{T}_j+\frac{1}{4}\right)\left(\mathbf{S}_i\mathbf{S}_j+\frac{1}{4}\right) \sim \mathcal{P}_{ij}$$

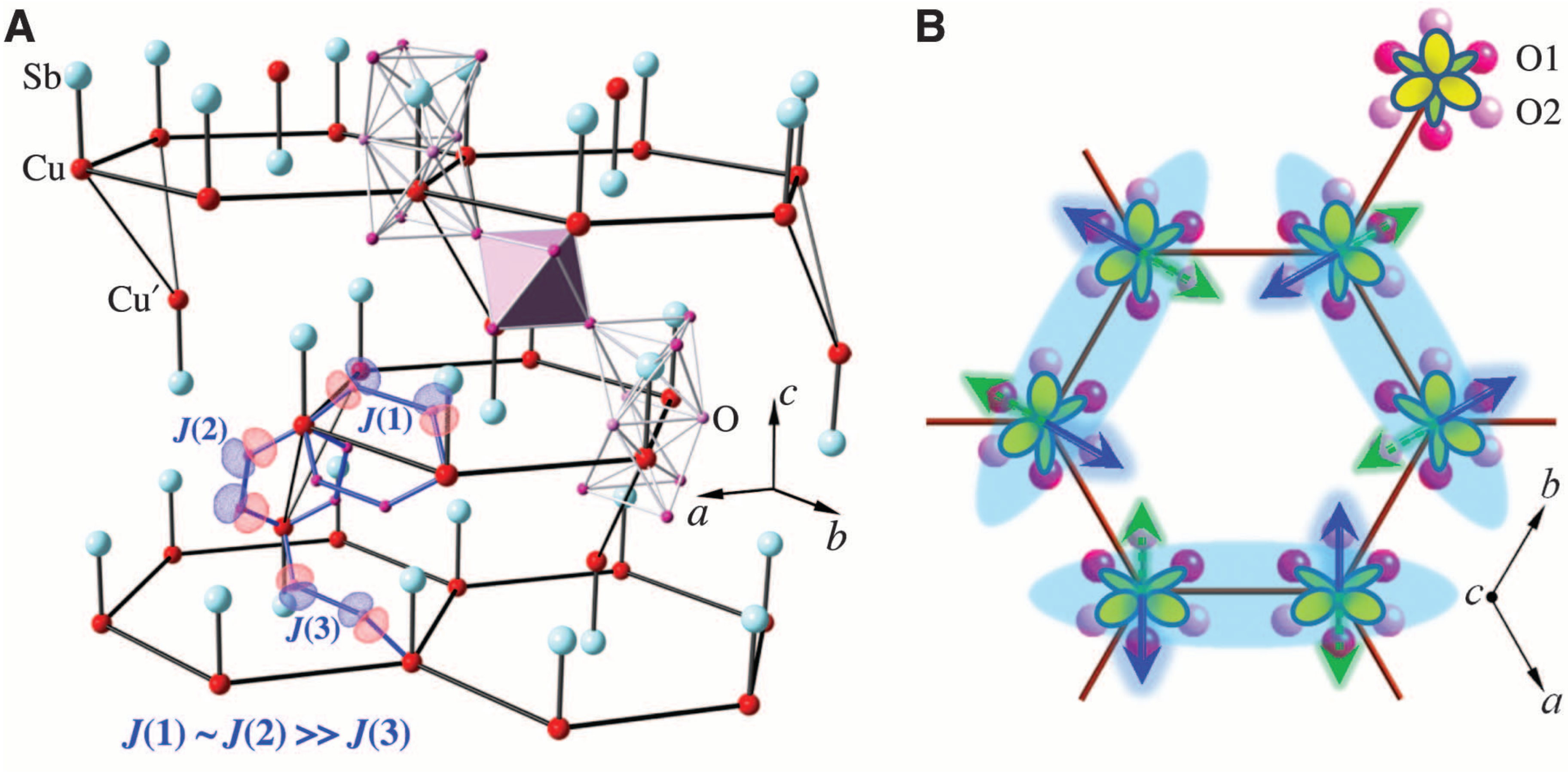
permutation operator

SU(4) on honeycomb lattice

Motivation: Spin-Orbital Short-Range Order on a Honeycomb-Based Lattice

S. Nakatsuji[1,*], K. Kuga[1], K. Kimura[1], R. Satake[2], N. Katayama[2], E. Nishibori[2], H. Sawa[2], R. Ishii[3], M. Hagiwara[3], F. Bridges[4], T. U. Ito[5], W. Higemoto[5], Y. Karaki[6], M. Halim[7], A. A. Nugroho[7], J. A. Rodriguez-Rivera[8,9], M. A. Green[8,9], C. Broholm[8,10]

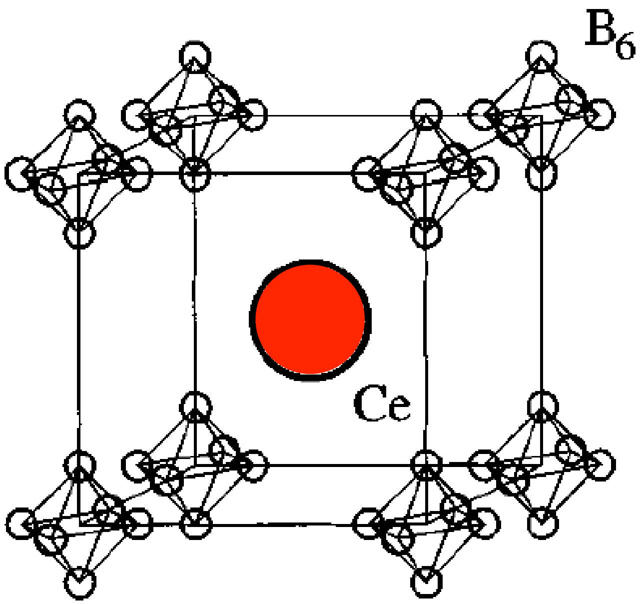
Science **336**, 559 (2012)



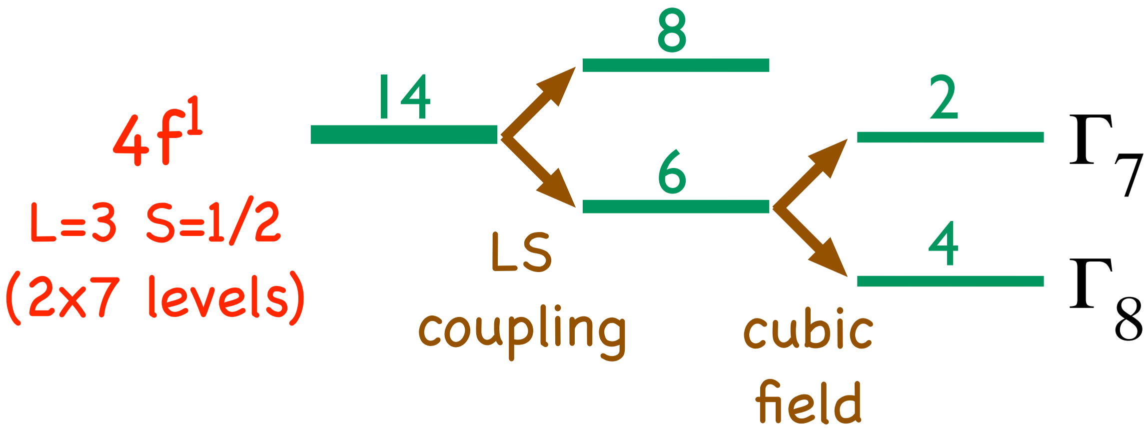
$Ba_3CuSb_2O_9$

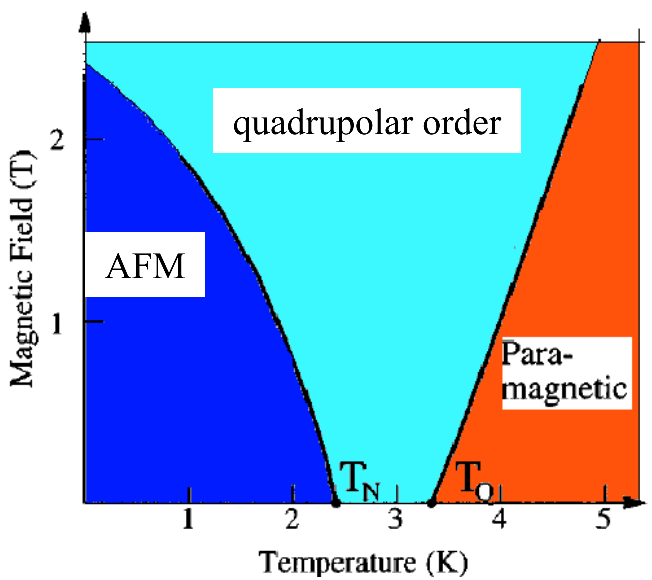
We consider the Kugel-Khomskii model for the S=1/2 and two Cu orbitals at the symmetric SU(4) point.

CeB_6: almost SU(4) on cubic lattice



level scheme:

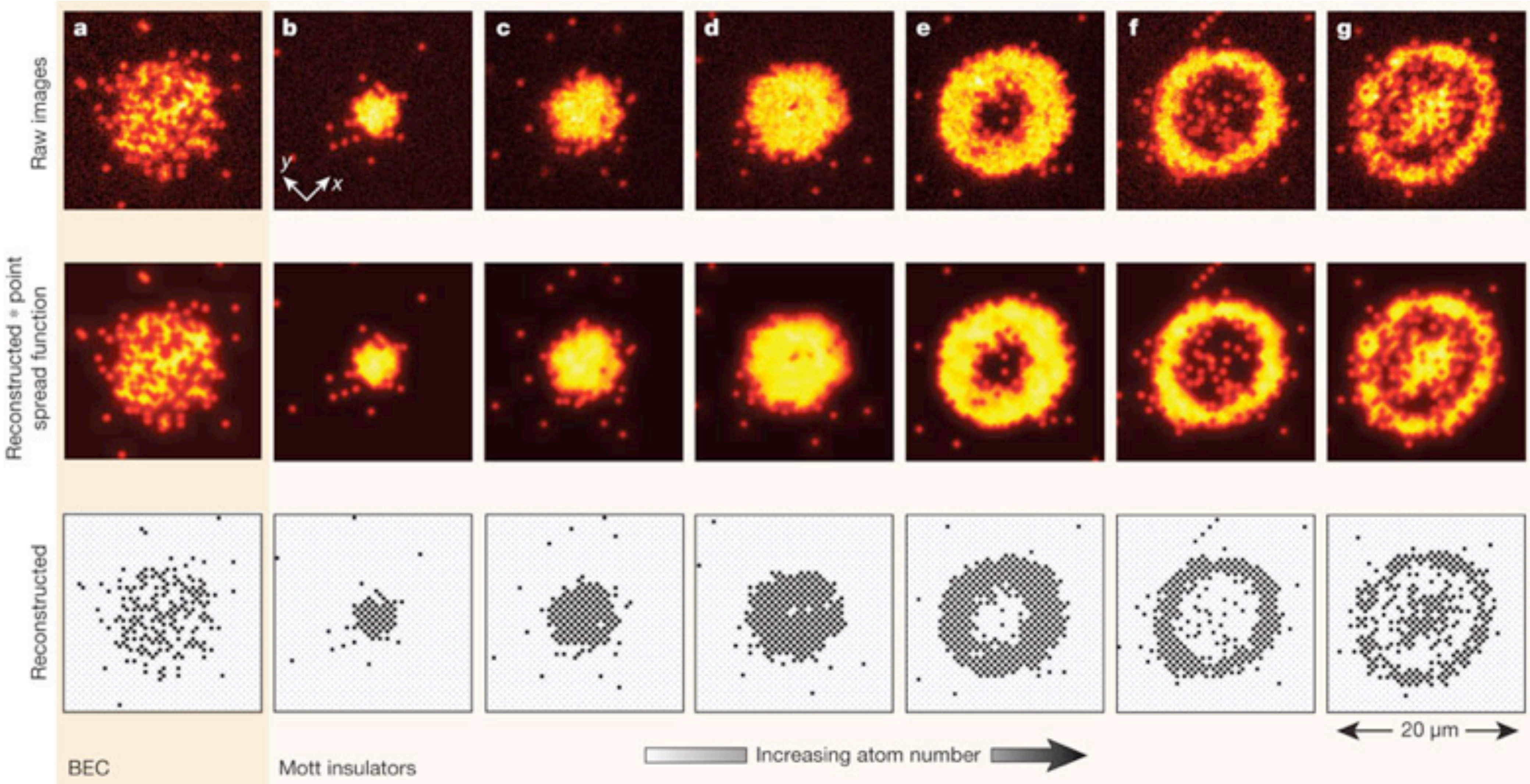




Anisotropic SU(4) spin wave treatment sufficient.
(R. Shiina et al,
J. Phys. Soc. Jpn. **66**, 1741 (1997)

Single-atom-resolved fluorescence imaging of an atomic Mott insulator

Jacob F. Sherson *et al.*, Nature **467**, 68 (2010) [bosonic]



also

Probing the Superfluid–to–Mott Insulator Transition at the Single-Atom Level

W. S. Bakr *et al.*, Science **329**, 547 (2010) [bosonic]

Single-atom-resolved fluorescence imaging of an atomic Mott insulator

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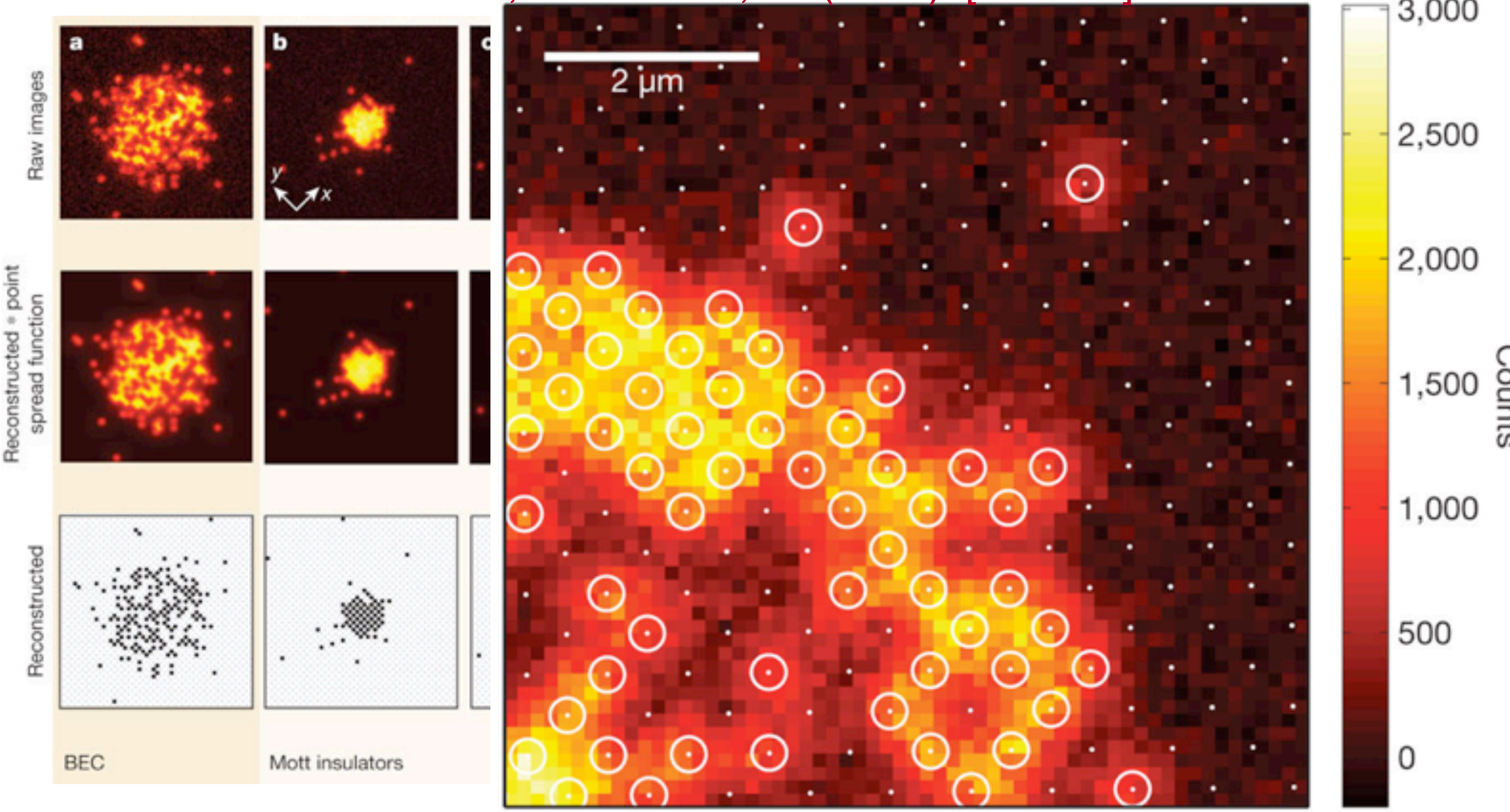


also

Probing the Superfluid–to–Mott Insulator Transition at the Single-Atom Level

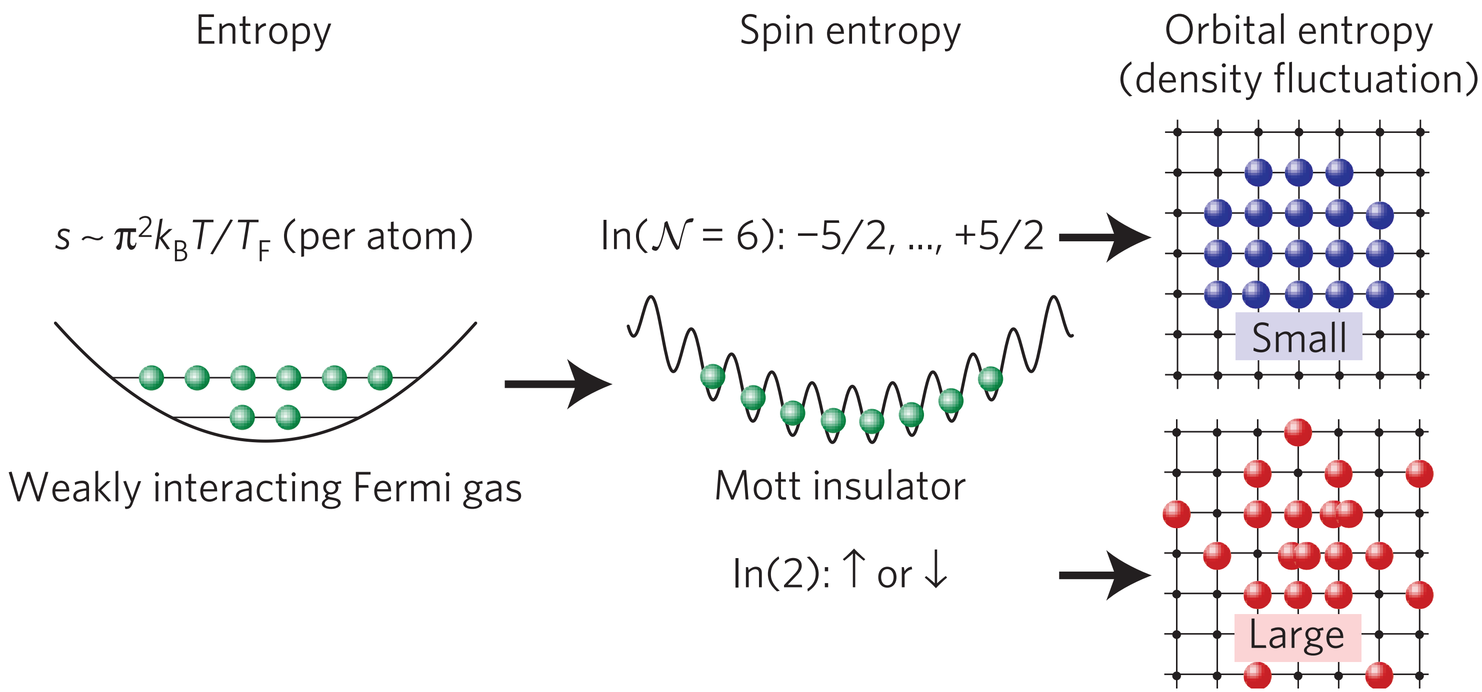
W. S. Bakr *et al.*, Science **329**, 547 (2010) [bosonic]

SU(6) Mott physics in cold atoms

An SU(6) Mott insulator of an atomic **Fermi** gas realized by large-spin Pomeranchuk cooling

Shintaro Taie, Rekishu Yamazaki, Seiji Sugawa & Yoshiro Takahashi

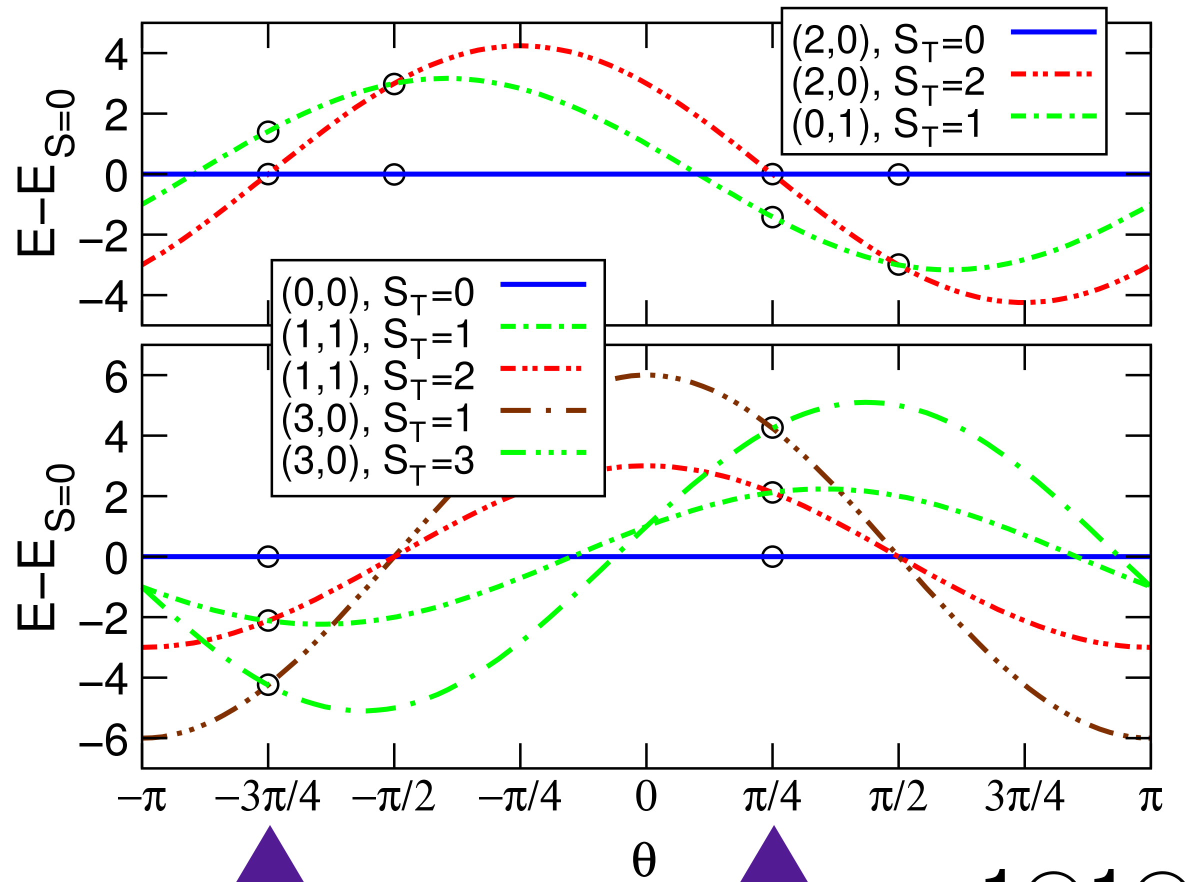
Nature Physics **8**, 825–830 (2012) doi:10.1038/nphys2430



SU(3) in S=1 spin model
(how to find higher symmetry points)

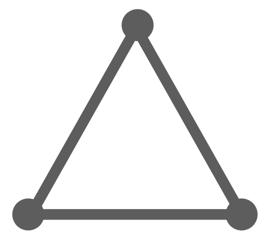
$$\mathcal{H} = J \sum_{i,j} \left[\cos\vartheta\, \mathbf{S}_i \mathbf{S}_j + \sin\vartheta \left(\mathbf{S}_i \mathbf{S}_j \right)^2 \right]$$

degeneracy as a signature of increased symmetry



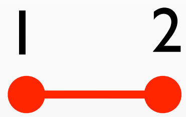
S=1

$1 \otimes 1 = 0 \oplus 1 \oplus 2$



$1 \otimes 1 \otimes 1 = 0 \oplus 1 \oplus 1 \oplus 1 \oplus 2 \oplus 2 \oplus 3$

SU(2) vs. SU(3) - two sites

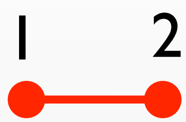


$$\mathcal{H} = \mathcal{P}_{12}$$

$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$ E=−1 , odd wave function

$\mathcal{P}_{12}(|\alpha\beta\rangle + |\beta\alpha\rangle) = +(|\alpha\beta\rangle + |\beta\alpha\rangle)$ E=+1, even wave function

SU(2) vs. SU(3) - two sites



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Addition of two S=½ SU(2) spins:

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

using Young diagrams:

$2 \times 2 = 1 + 3$

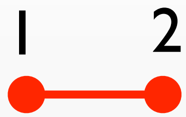
$\square \otimes \square = \begin{matrix}\square\\\square\end{matrix} \oplus \square\square$

$\square$ ↑ or ↓ spin

$\begin{matrix}\square\\\square\end{matrix}$ |↑↓⟩−|↓↑⟩ singlet, odd (anti-symmetrical)

$\square\square$ |↑↑⟩, |↑↓⟩+|↓↑⟩, |↓↓⟩ triplet even (symmetrical)

SU(2) vs. SU(3) - two sites



$$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$$ E=−1 , odd wave function

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Addition of two S=½ SU(2) spins:

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2 × 2 = 1 + 3

$\square \otimes \square = \begin{array}{c}\square\\\square\end{array} \oplus \square\square$

$\square$ ↑ or ↓ spin

$\begin{array}{c}\square\\\square\end{array}$ |↑↓⟩−|↓↑⟩ singlet, odd (anti-symmetrical)

$\square\square$ |↑↑⟩, |↑↓⟩+|↓↑⟩, |↓↓⟩ triplet even (symmetrical)

Addition of two SU(3) spins:

3 × 3 = 3 + 6

$\square \otimes \square = \begin{array}{c}\square\\\square\end{array} \oplus \square\square$

$\square$ |a⟩, |b⟩, and |c⟩.

$\begin{array}{c}\square\\\square\end{array}$ |ab⟩−|ba⟩, |ac⟩−|ca⟩, |bc⟩−|cb⟩: odd (anti-symmetrical).

$\square\square$ |aa⟩, |bb⟩, |cc⟩, |ab⟩+|ba⟩, |ac⟩+|ca⟩, and |bc⟩+|cb⟩. even (symmetrical)

SU(3) irreps on 3 sites

Addition of three SU(3) spins (27 states):

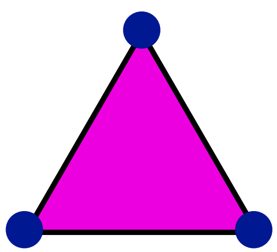
$$3 \times 3 \times 3 \quad = 1 + 2 \times 8 + 10$$

$$\begin{array}{|c|}\hline \; \\ \hline\end{array} \otimes \begin{array}{|c|}\hline \; \\ \hline\end{array} \otimes \begin{array}{|c|}\hline \; \\ \hline\end{array} \quad = \begin{array}{|c|}\hline \; \\ \hline \; \\ \hline \; \\ \hline\end{array} \oplus 2\times \begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline\end{array}$$

SU(3) singlet

$$\begin{array}{|c|}\hline \; \\ \hline \; \\ \hline \; \\ \hline\end{array} = |ABC\rangle + |CAB\rangle + |BCA\rangle - |BAC\rangle - |ACB\rangle - |BCA\rangle$$

spins fully antisymmetrized



in the SU(3) singlet the spins are fully entangled:
we cannot write it in a product form

What methods do we use?

(i) Variational - site factorized wave function

(ii) Flavor wave calculations

(iii) Exact diagonalization of small clusters

(iv) iPEPS: infinite project entangled pair states(variational approach based on tensor ansatz)

(v) Variational - Gutzwiller projected fermionic wave functions

Variational (classical) approach

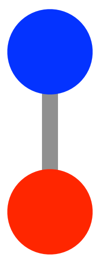
a site-product wave function for e.g. SU(3):

$$|\Psi\rangle = \prod_i |\psi_i\rangle$$

$$|\psi_i\rangle = d_{A,i}|A\rangle_i + d_{B,i}|B\rangle_i + d_{C,i}|C\rangle_i$$

$$E_{\text{var}} = \frac{\langle\Psi|\mathcal{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = J\sum_{\langle i,j\rangle}\left|\mathbf{d}_i \cdot \bar{\mathbf{d}}_j\right|^2$$

minimal, when the $\mathbf{d}_i$ and $\mathbf{d}_j$ on the bond are orthogonal



two different colors on a bond

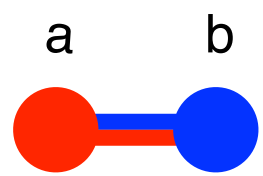
SU(3) flavour-wave theory

$$\mathcal{P}_{ij} = \sum_{\mu,\nu \in \{A,B,C\}} a^\dagger_{\mu,i} a^\dagger_{\nu,j} a_{\nu,i} a_{\mu,j}$$

1/M expansion:

$$\tilde{a}^\dagger_A, \tilde{a}_A \to \sqrt{M - \tilde{a}^\dagger_B \tilde{a}_B - \tilde{a}^\dagger_C \tilde{a}_C}$$
$$\to \sqrt{M} - \frac{1}{2\sqrt{M}} \left(\tilde{a}^\dagger_B \tilde{a}_B + \tilde{a}^\dagger_C \tilde{a}_C \right) + \dots$$

Holstein-Primakoff bosons



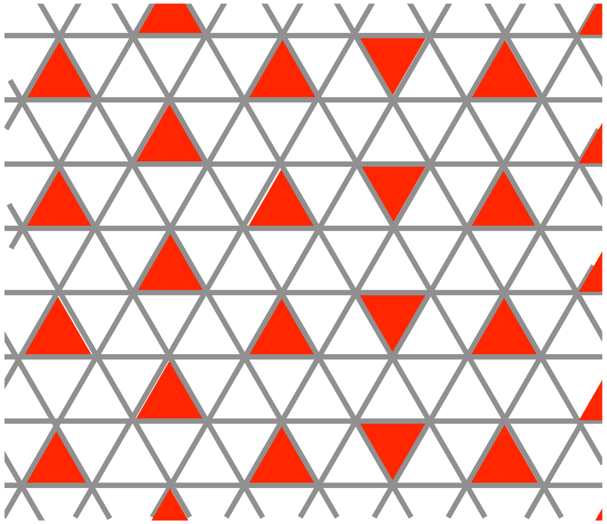
$$\mathcal{H} = (a^\dagger + b)(a + b^\dagger)$$

quadratic in operators: we know how to diagonalize it (spin wave)

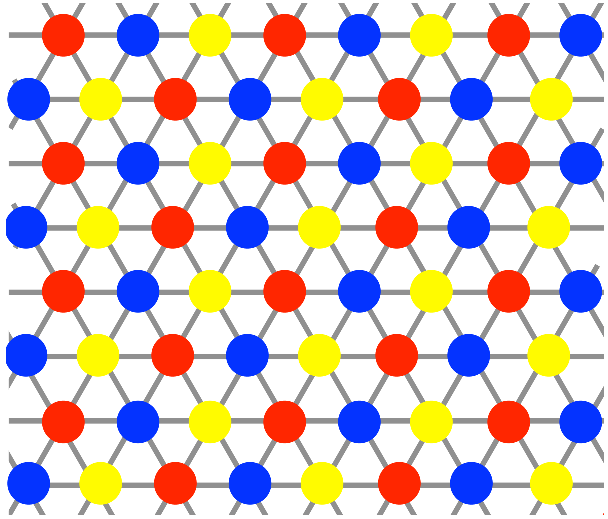
$$\mathcal{H} = -MJL + M \sum_\nu \sum_{\mathbf{k}} \omega_\nu(\mathbf{k}) \left(\alpha^\dagger_\nu(\mathbf{k}) \alpha_\nu(\mathbf{k}) + \frac{1}{2} \right)$$

The fate of SU(3) on triangular lattice

SU(2) frustrated!



crystal of singlets?



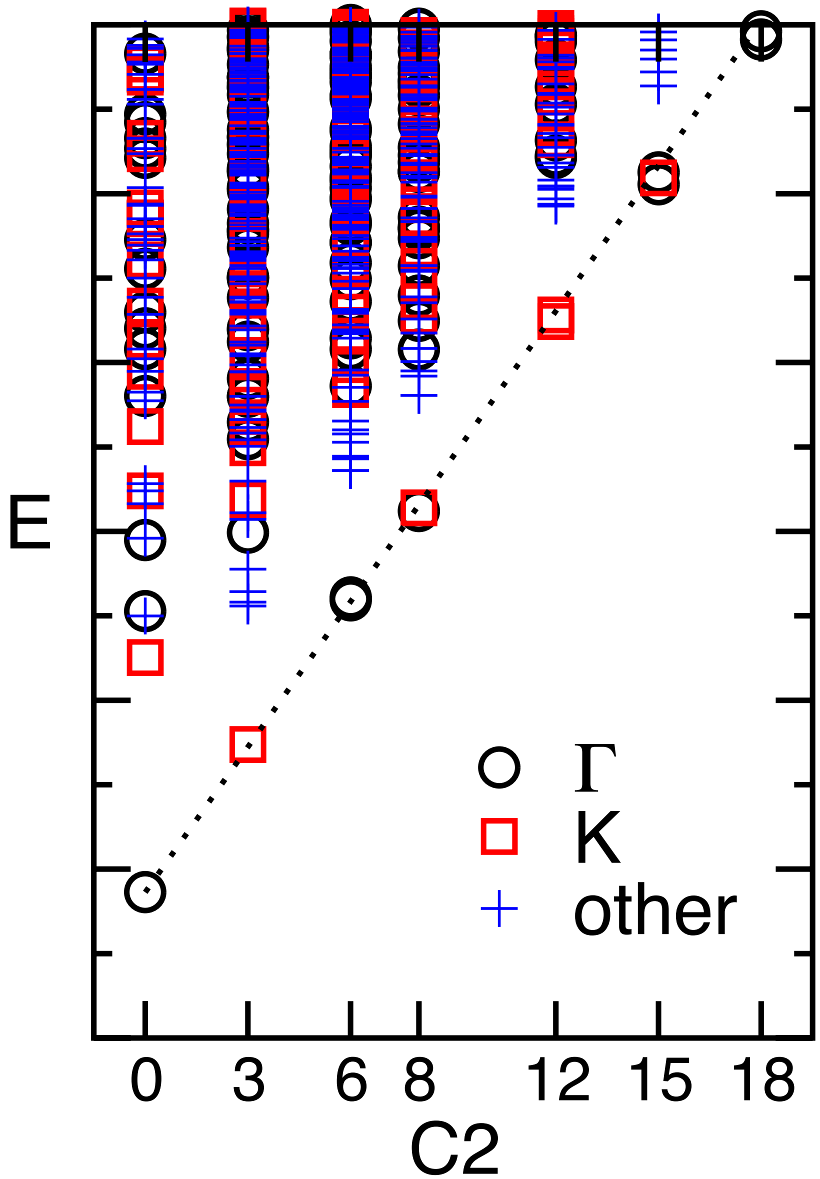
“classical” solution?

SU(3) classical state is perfectly happy on the triangular lattice - the 3 mutually perpendicular **d**’s form a 3 sublattice structure.

H. Tsunetsugu and M. Arikawa, J. Phys. Soc. Jpn. **75**, 083701 (2006)
[NiGa2S4, Nakatsuji]

A. M. Läuchli, F. Mila, and K. Penc, Phys. Rev. Lett. **97**, 087205/1-4 (2006)

SU(3) on triangular lattice - exact diagonalization



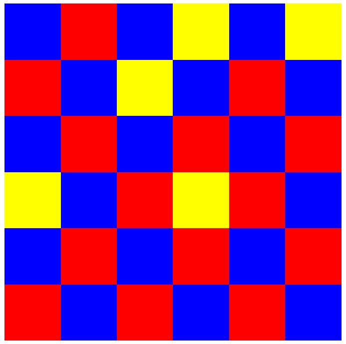
Signature of SU(3) breaking in the excitation spectrum:
Anderson towers compatible with 3 sublattice order

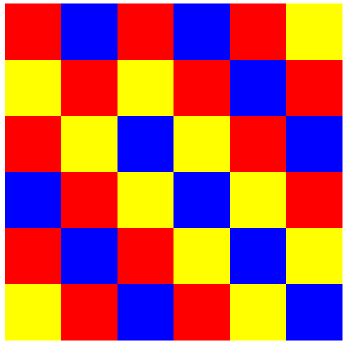
C2 - Casimir operator, analog of the total spin S^2

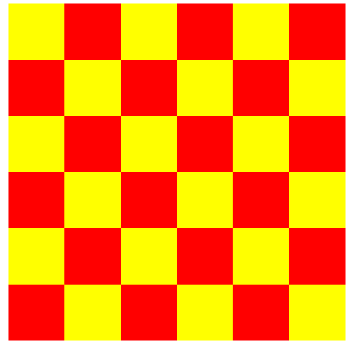
K. Penc, A. M. Läuchli, in `Introduction to Frustrated Magnetism', p. 331-362, Springer Series in Solid-State Sciences, Vol. **164**, eds. C. Lacroix, F. Mila, and P. Mendels (Springer, 2011)

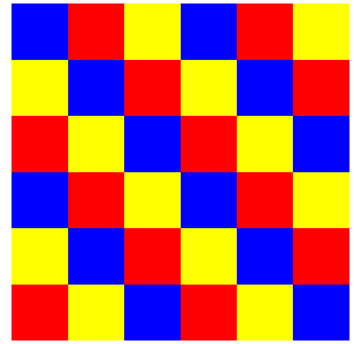
SU(3) square lattice, classical solutions: macroscopically degenerate

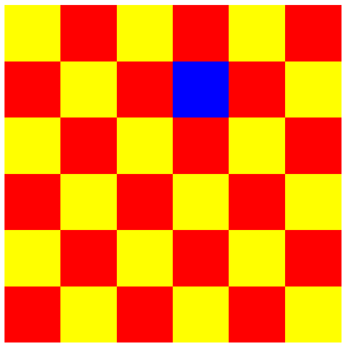
SU(3) square lattice, classical solutions: macroscopically degenerate



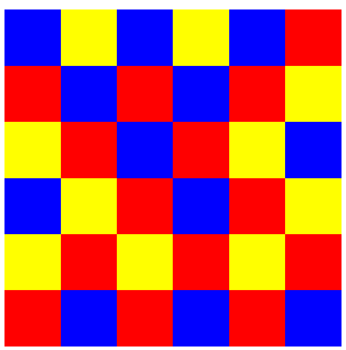




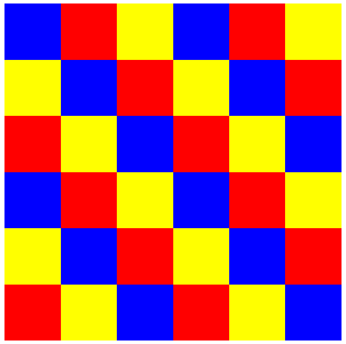




All bonds happy at the mean field level,
frustration due to abundance of choices



SU(3) square lattice, classical solutions: macroscopically degenerate



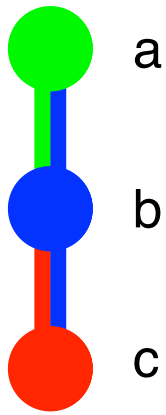
All bonds happy at the mean field level,
frustration due to abundance of choices

Order by disorder:
the zero point energy of the quantum fluctuations over a mean field solution selects the ground state

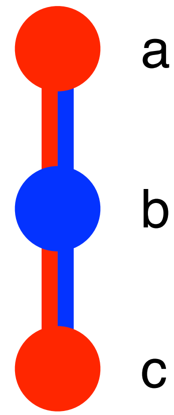
$$E_{ZP} = \frac{M}{2} \sum_{\nu} \sum_{\mathbf{k}} \omega_{\nu}(\mathbf{k})$$

structure of the flavor wave Hamiltonian

$$\mathcal{H} = \\ + (a^\dagger + b)(b^\dagger + a) \\ + (b^\dagger + c)(c^\dagger + b)$$



$$\mathcal{H} = \\ + (a^\dagger + b)(b^\dagger + a) \\ + (b^\dagger + c)(c^\dagger + b)$$

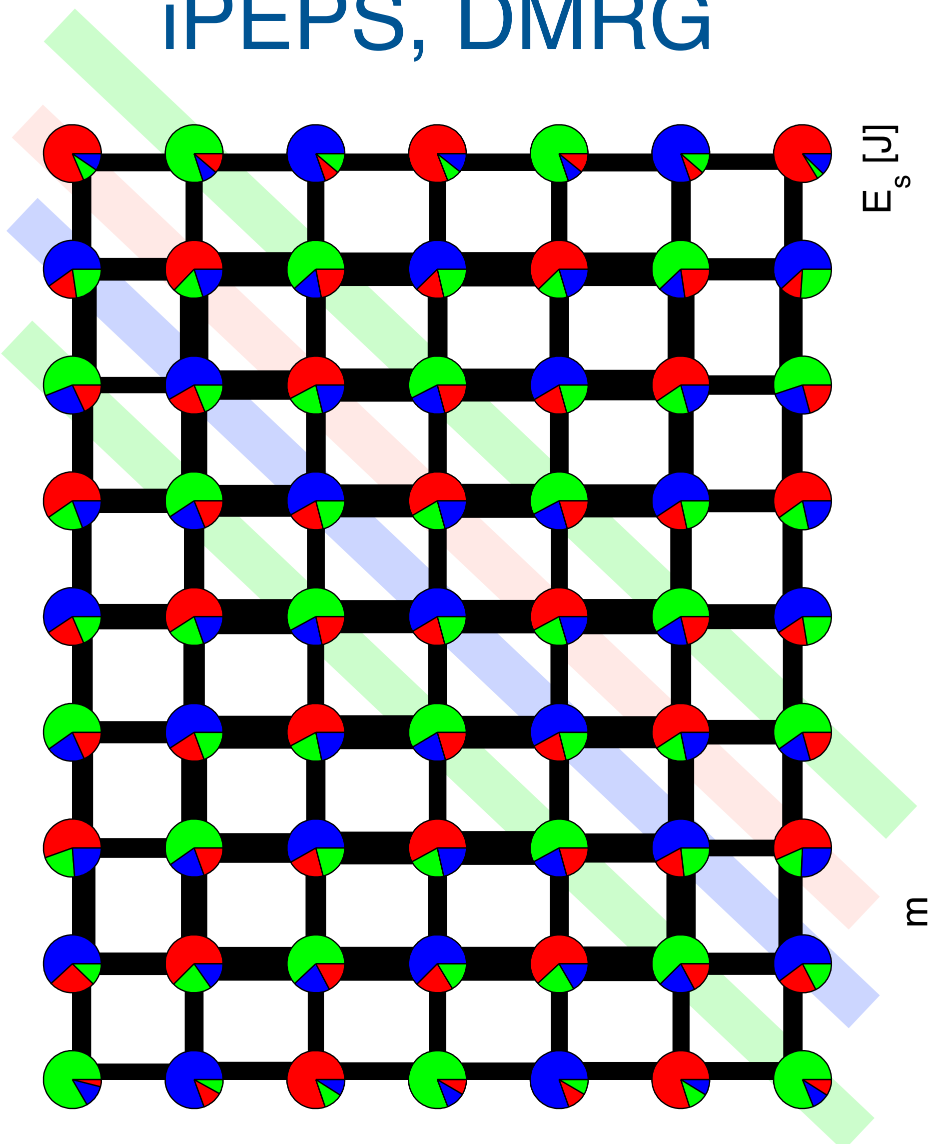


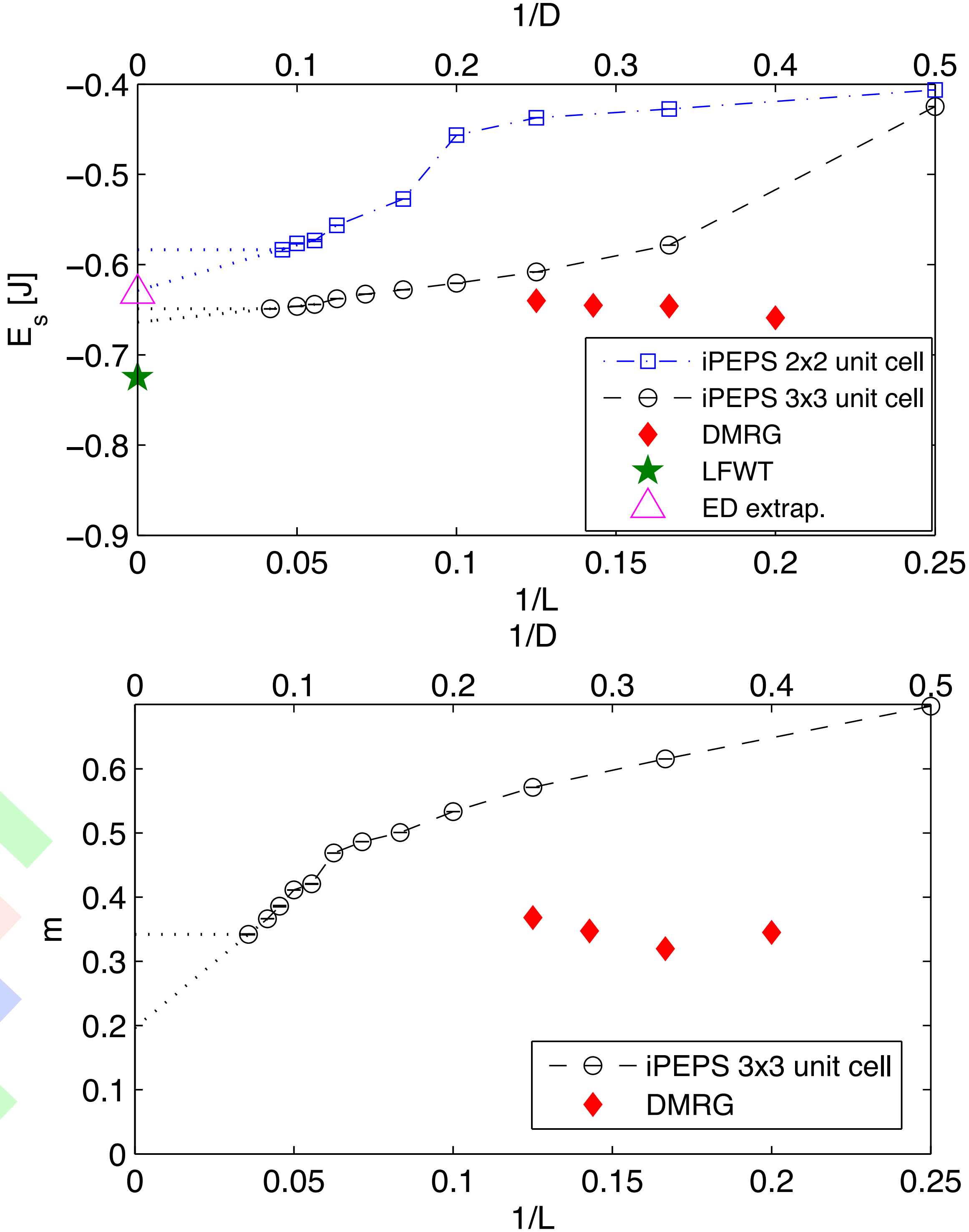
each term separately $E_{ZP} = 0$

the 3-site term gives $E_{ZP} > 0$

energy minimal
if next nearest neighbor spins
are also of different color

Unbiased calculation: iPEPS, DMRG



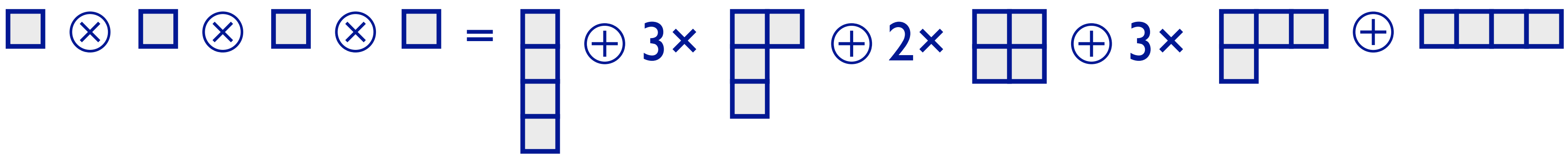


B. Bauer, P. Corboz, A. M. Läuchli, L. Messio,
K. Penc, M. Troyer, F. Mila:
Phys. Rev. B **85**, 125116/1-11 (2012)

SU(4) irreps on 4 sites

Addition of four SU(4) spins (256 states):

$$4 \times 4 \times 4 \times 4 = 1 + 3\times 15 + 2\times 20 + 3\times 45 + 35$$



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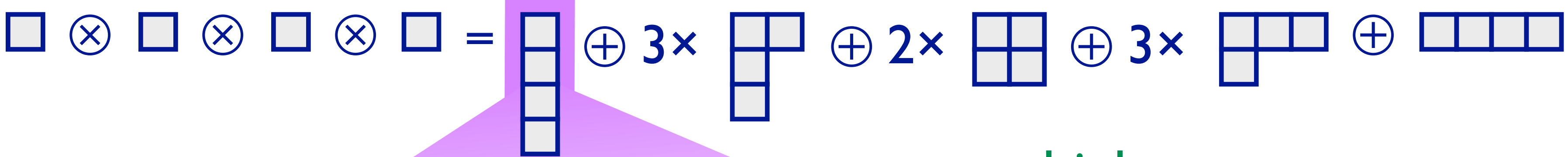
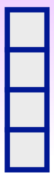


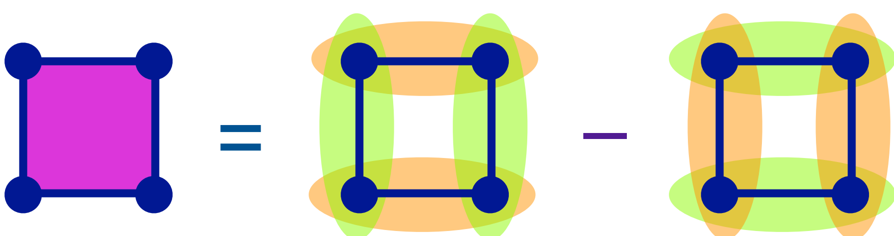
multiplets

SU(4) singlet



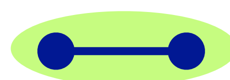
$= |abcd\rangle - |bacd\rangle + |badc\rangle - |bdac\rangle - \ldots$

spins fully antisymmetrized



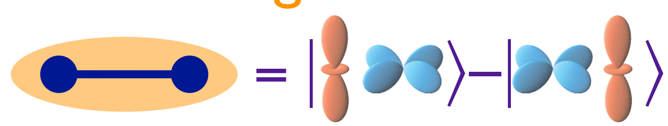
SU(4) singlet plaquette
entangled spins and orbitals

spin singlet bond:



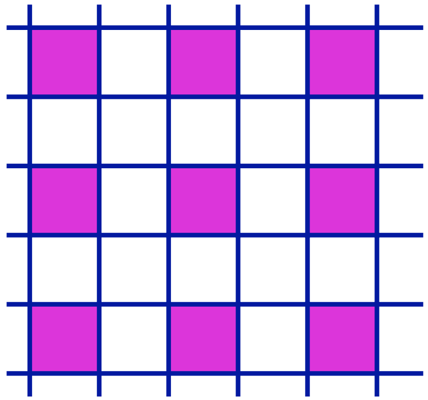
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

orbital singlet bond:



SU(4) on 2D-square lattice

Ground state 4-fold degenerate?

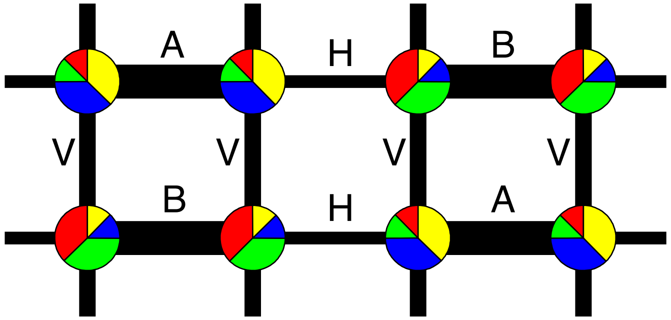


minimal energy for a plaquette bond, but
not so good energy between plaquettes

Z2 liquid ?
Wang & Viswanath (PRB 2009)
using Majorana fermions

SU(4) isomorphic to SO(6)
6 Majorana fermions

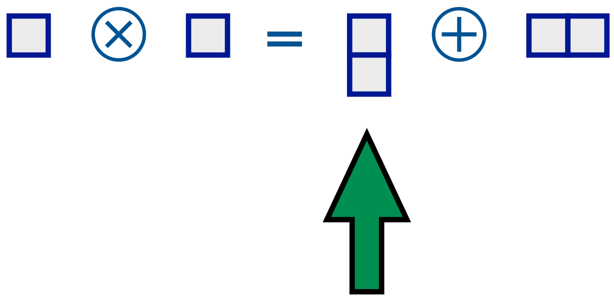
SU(4) on 2D-square lattice: iPEPS



D = 12 and a unit cell 4 × 2

dimerization and Neel-like state: both spatial and the SU(4) symmetry is broken

$$4 \times 4 = 6 + 10$$



the 6 dimensional irreducible representation is realized on the dimers, can Neel order

2-step scenario:

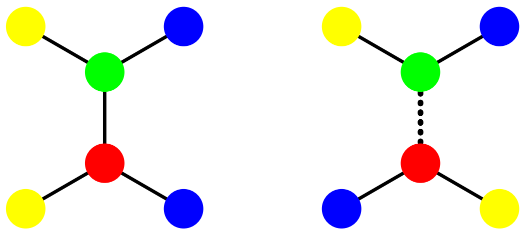
(i) Dimerization: 6-dimensional irreps are formed

(ii) the 6-imensional irreps can possibly Néel order

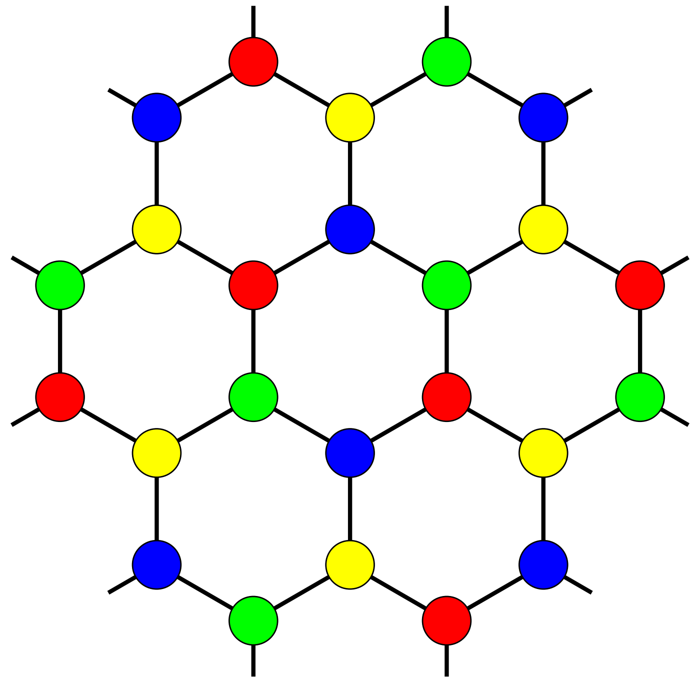
P. Corboz, A. M. Läuchli, K. Penc, M. Troyer, F. Mila, PRL **107**, 215301 (2011).

SU(4) honeycomb: Lifting of the degeneracy in flavor wave theory

Basic building blocks:
nearest (mean field) and next nearest
(fluctuations) neighbor colors different.



Order by disorder does not work!



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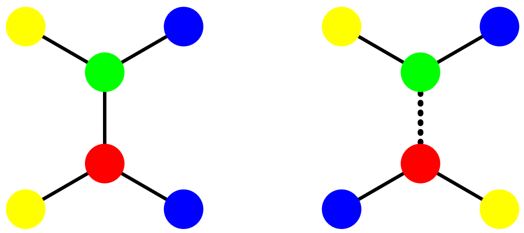

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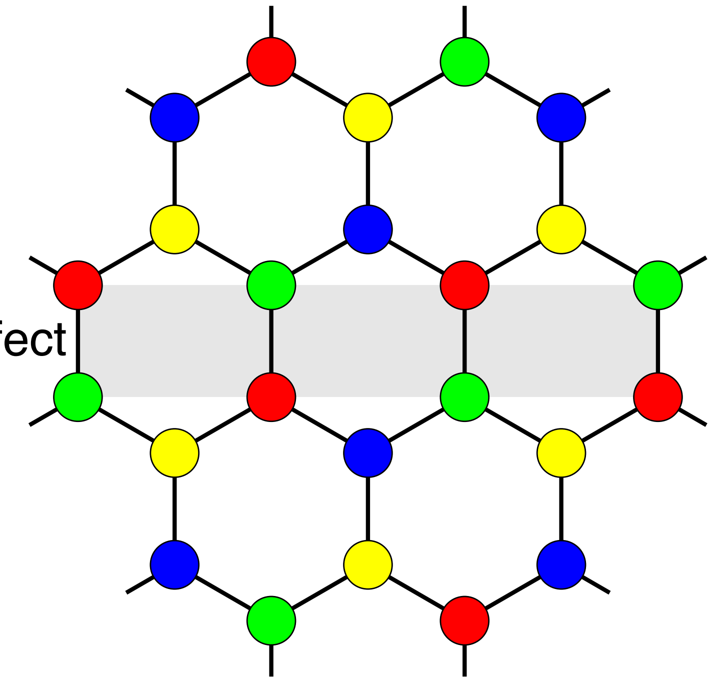

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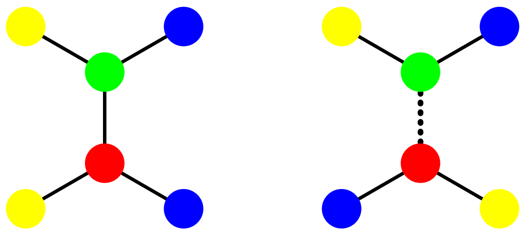

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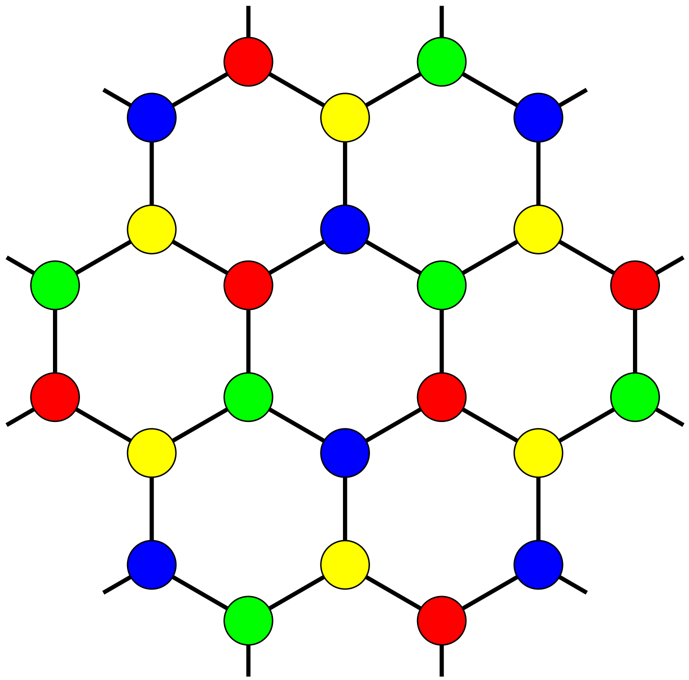
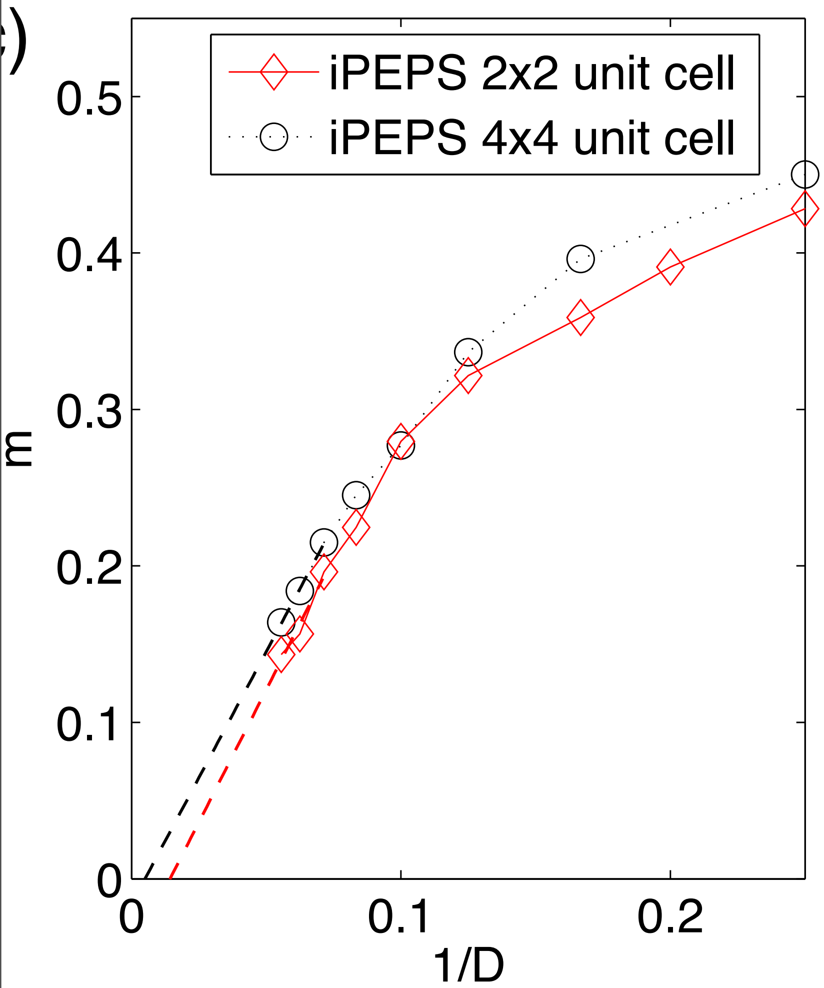
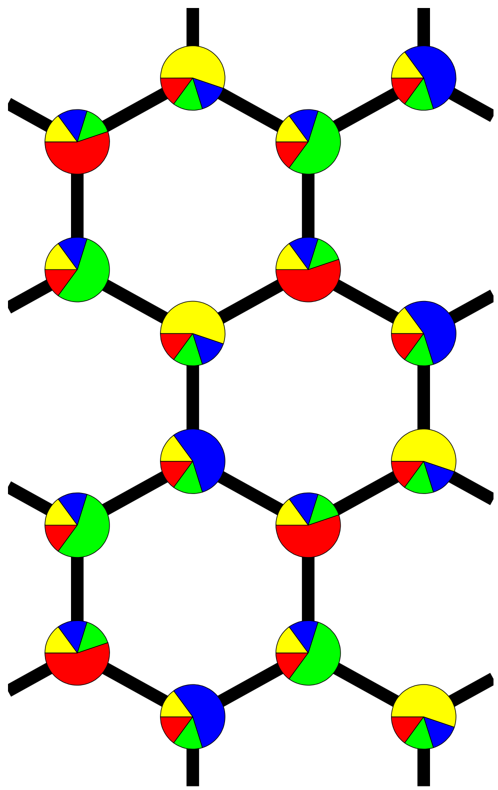
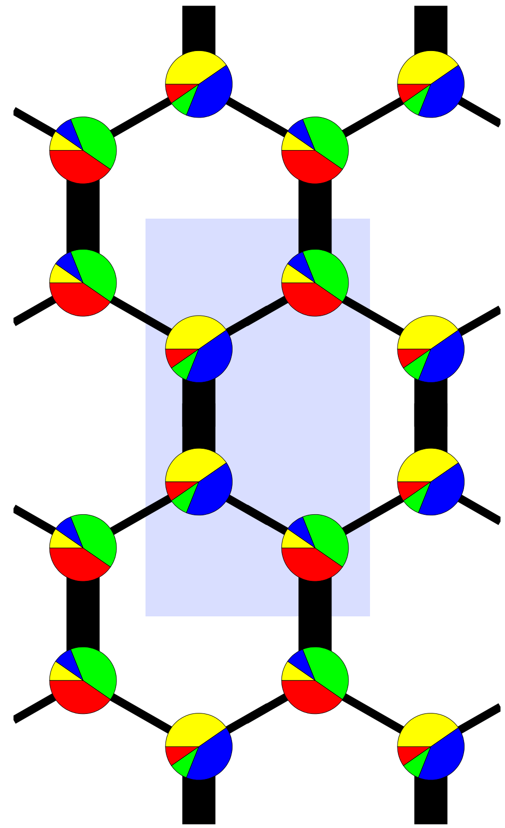


iPEPS - local magnetization vanishes





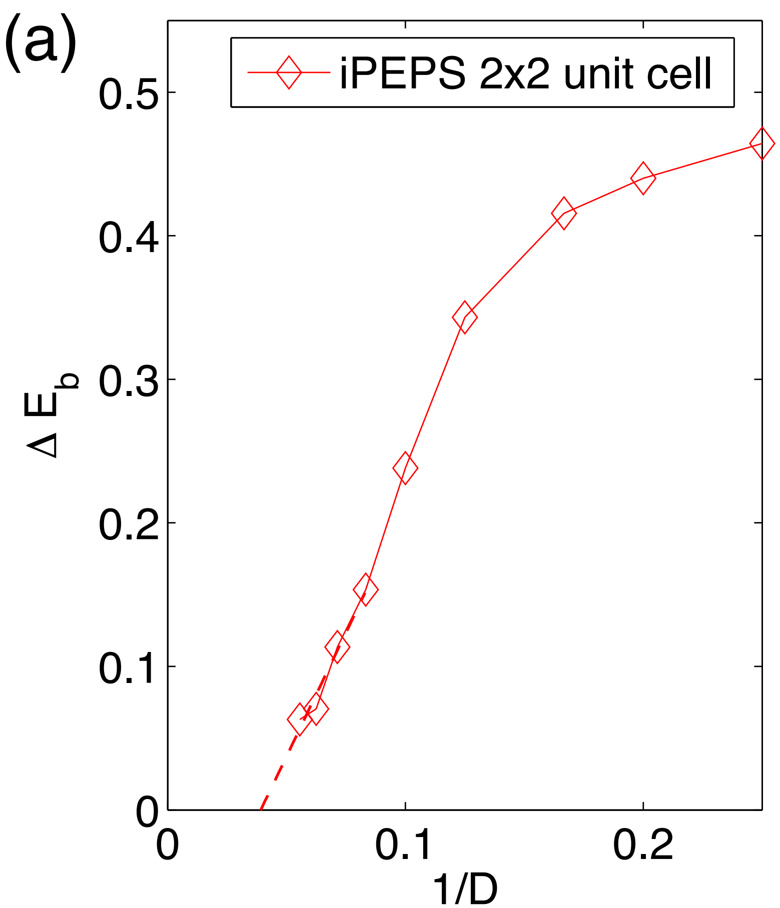
4x4 unit cell,
D=6

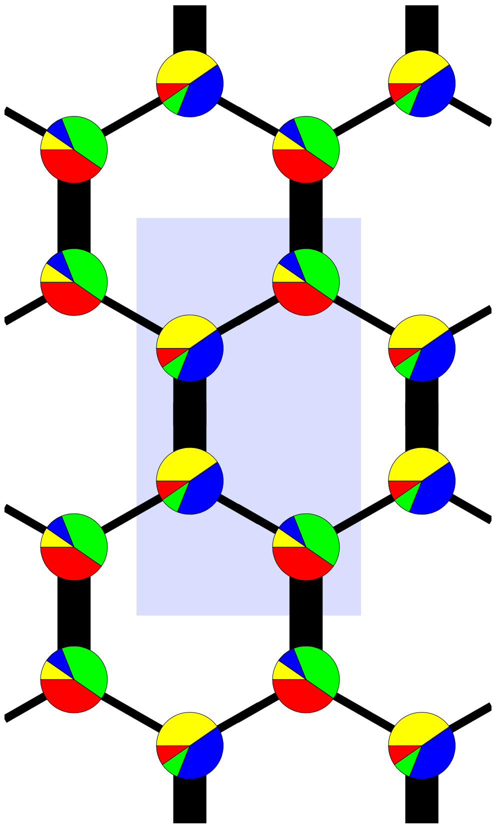


2x2 unit cell,
D=6

P. Corboz, M. Lajkó, A. M. Läuchli, K. Penc, F. Mila:
Phys. Rev. X **2**, 041013/1-11 (2012).

iPEPS - dimerization vanishes





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P. Corboz, M. Lajkó, A. M. Läuchli, K. Penc, F. Mila:
Phys. Rev. X **2**, 041013/1-11 (2012).

Summary of iPEPS results [SU(4) honeycomb]

- dimerization vanishes
  (actually no point group symmetry breaking)

- local magnetization vanishes
  (no SU(4) symmetry breaking)

spin-orbital liquid

How to characterize it?

fermionic representation:

$$\mathcal{P}_{ij} = \sum_{\mu,\nu \in \text{colors}} f^\dagger_{\alpha,i} f_{\beta,i} f^\dagger_{\beta,j} f_{\alpha,j}$$

$$\mathcal{P}^{\text{MF}}_{ij} = \sum_{\alpha,\beta \in \text{colors}} \langle f_{\beta,i} f^\dagger_{\beta,j} \rangle f^\dagger_{\alpha,i} f_{\alpha,j}$$

$$= - \sum_{\alpha \in \text{colors}} t^\alpha_{ij} f^\dagger_{\alpha,i} f_{\alpha,j}$$

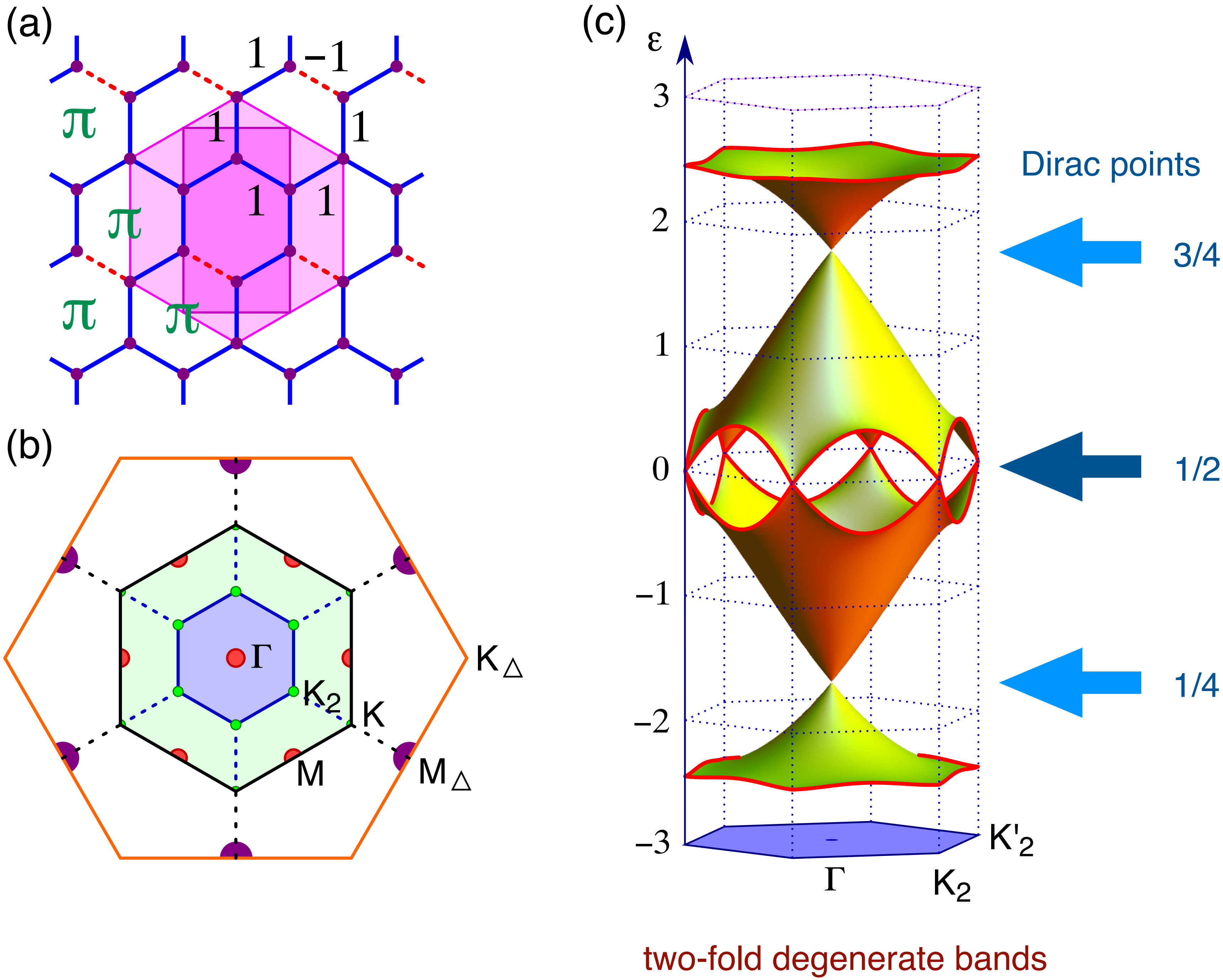
Mean-field decoupling of the fermionic Hamiltonian gives a hopping Hamiltonian and a variational wave function

$$|\Psi_{\text{vari}}\rangle = P_{\text{Gutzwiller}} |\Psi_{\text{FS}}\rangle$$

Using different Ansätze for the hoppings, we evaluate the expectation value of the Hamiltonian

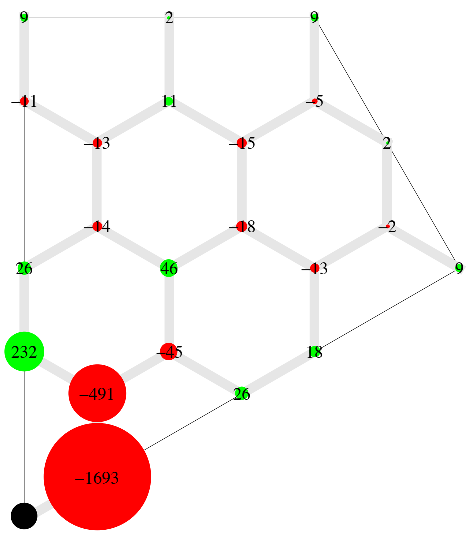
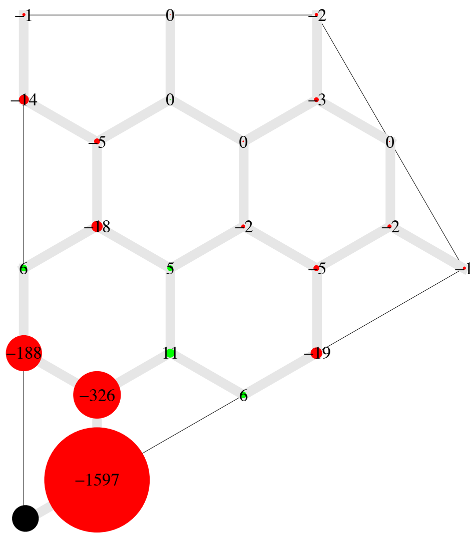
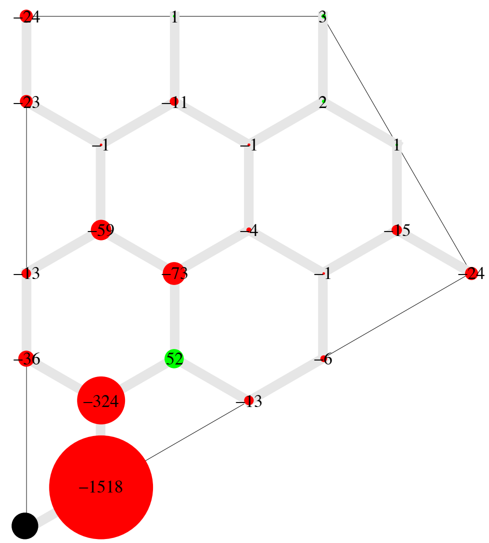
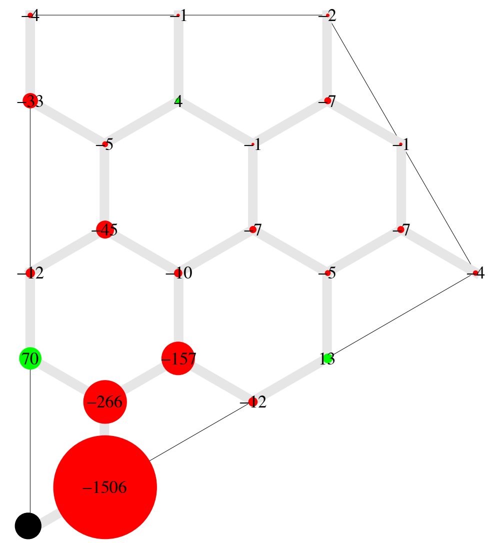
$$E_{\text{vari}} = \frac{\langle \Psi_{\text{vari}} | \mathcal{H} | \Psi_{\text{vari}} \rangle}{\langle \Psi_{\text{vari}} | \Psi_{\text{vari}} \rangle}$$

The fermionic wave function of the pi-flux state



96-site cluster - real space correlations from Gutzwiller projected wavefunction

$$\langle S_\bullet S_\delta \rangle \propto \langle P_{\bullet,\delta} \rangle - \frac{1}{4}$$

π-flux	Majorana 0-flux	0-flux	Majorana π-flux
E/N = –0.89466	E/N = –0.822479	E/N = –0.763304	E/N = –0.754662

Majoranna fermions for square lattice: F. Wang and A. Vishwanath, Phys. Rev. B **80**, 064413 (2009).

96-site cluster - real space correlations from Gutzwiller projected wavefunction

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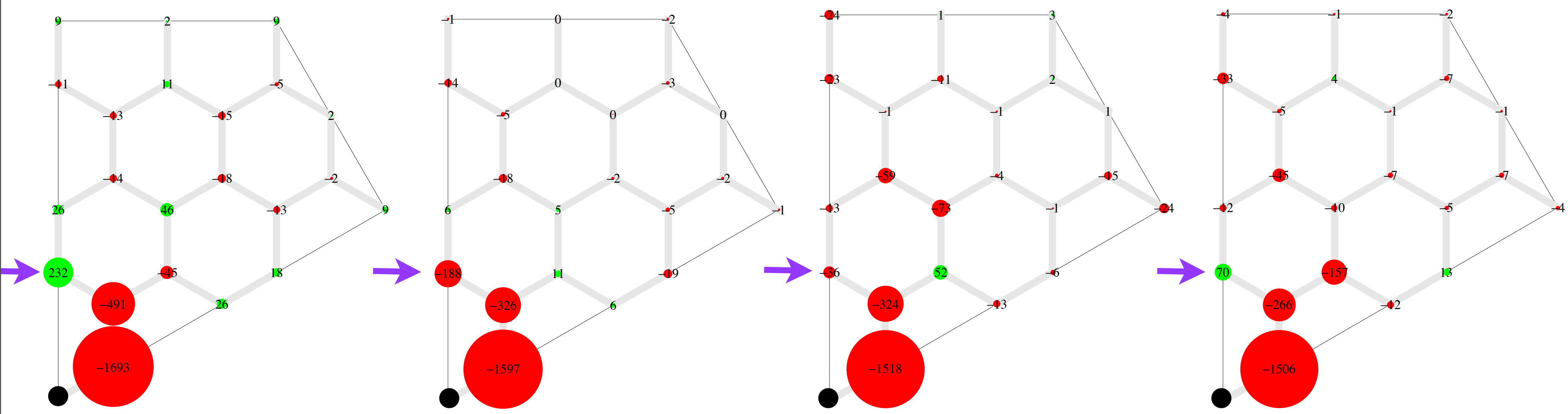


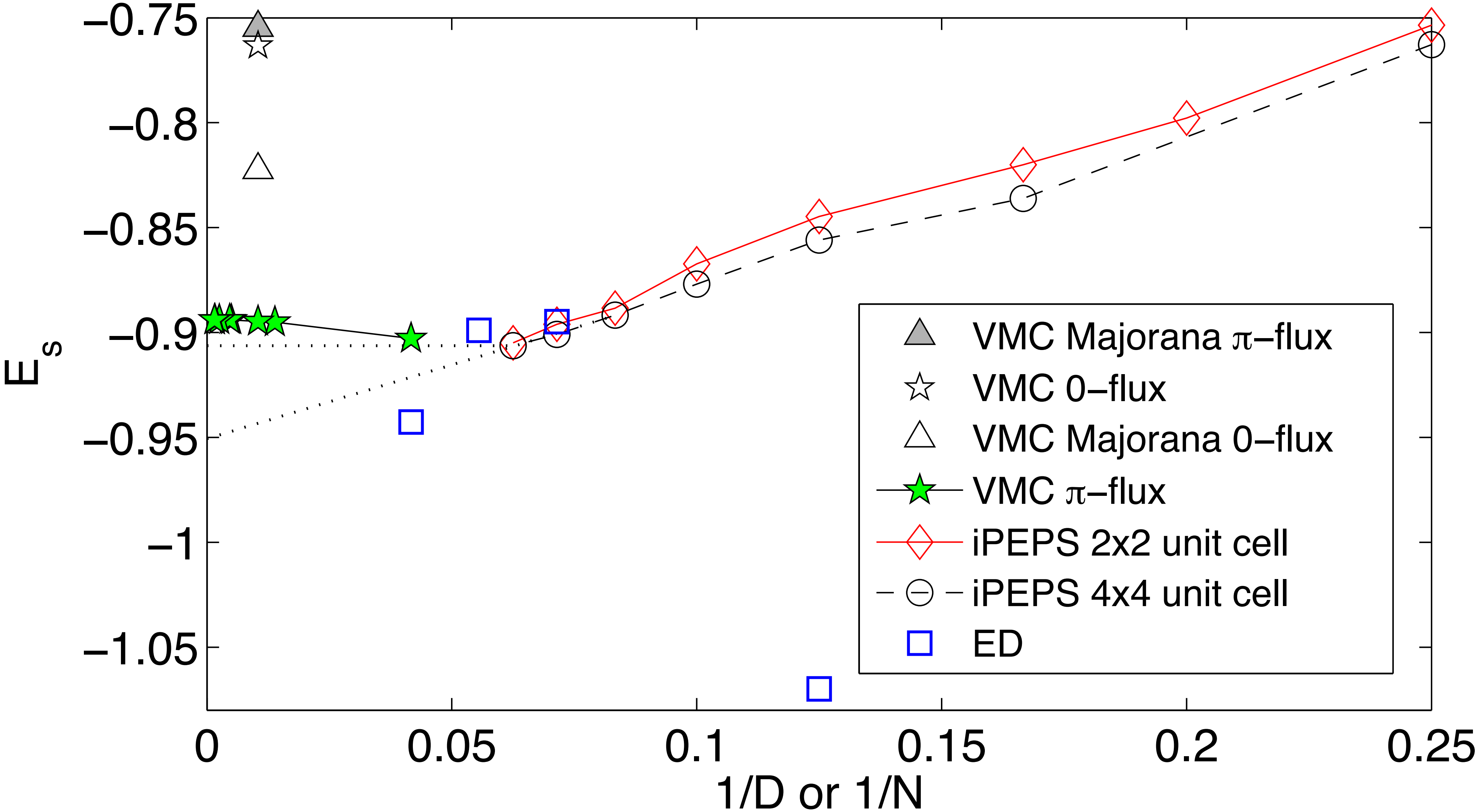
π-flux | Majorana 0-flux | 0-flux | Majorana π-flux

E/N = −0.89466 | E/N = −0.822479 | E/N = −0.763304 | E/N = −0.754662

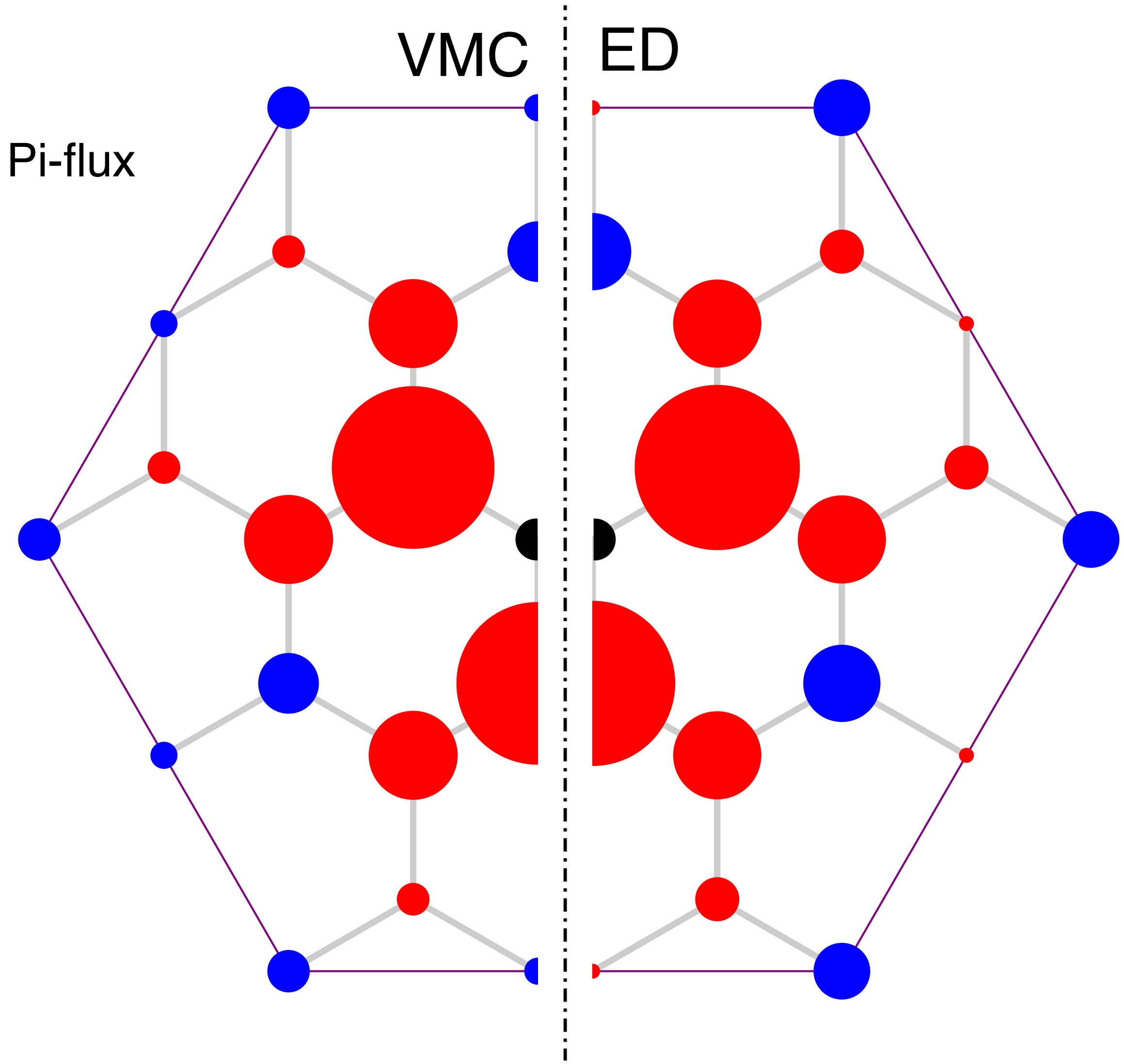
marked differences in
3rd neighbor correlations

Majoranna fermions for square lattice: F. Wang and A. Vishwanath, Phys. Rev. B **80**, 064413 (2009).

Ground state energy from different methods



24-site cluster - real space correlations

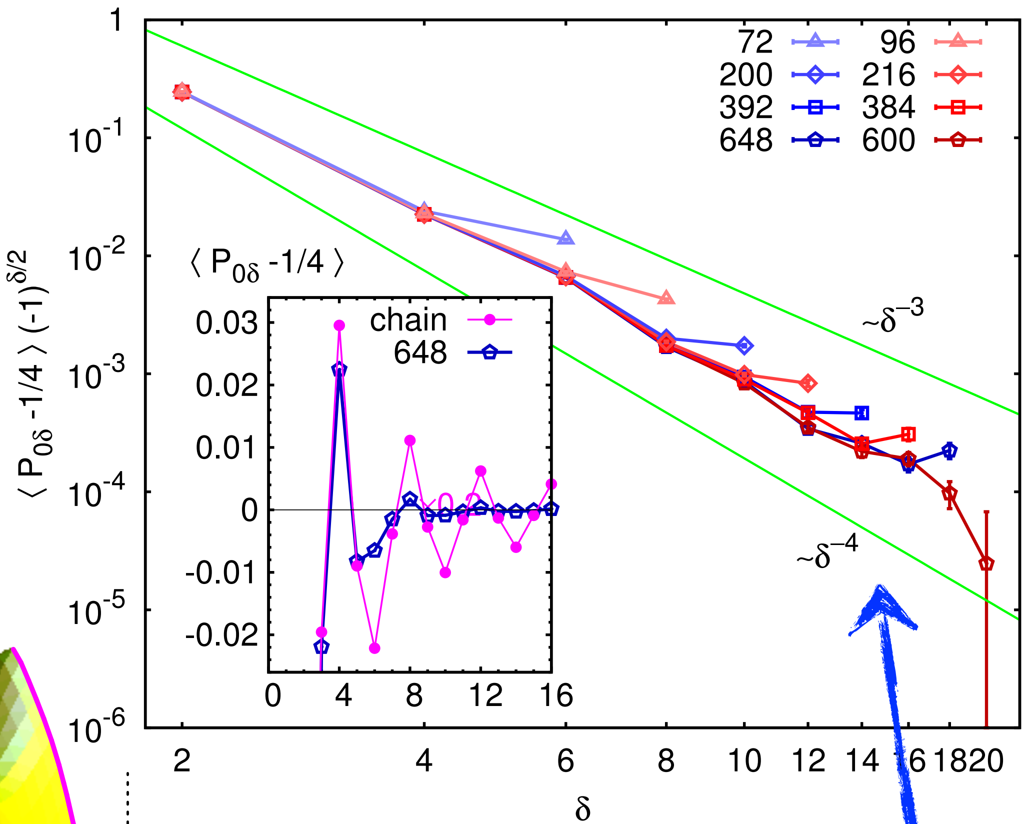


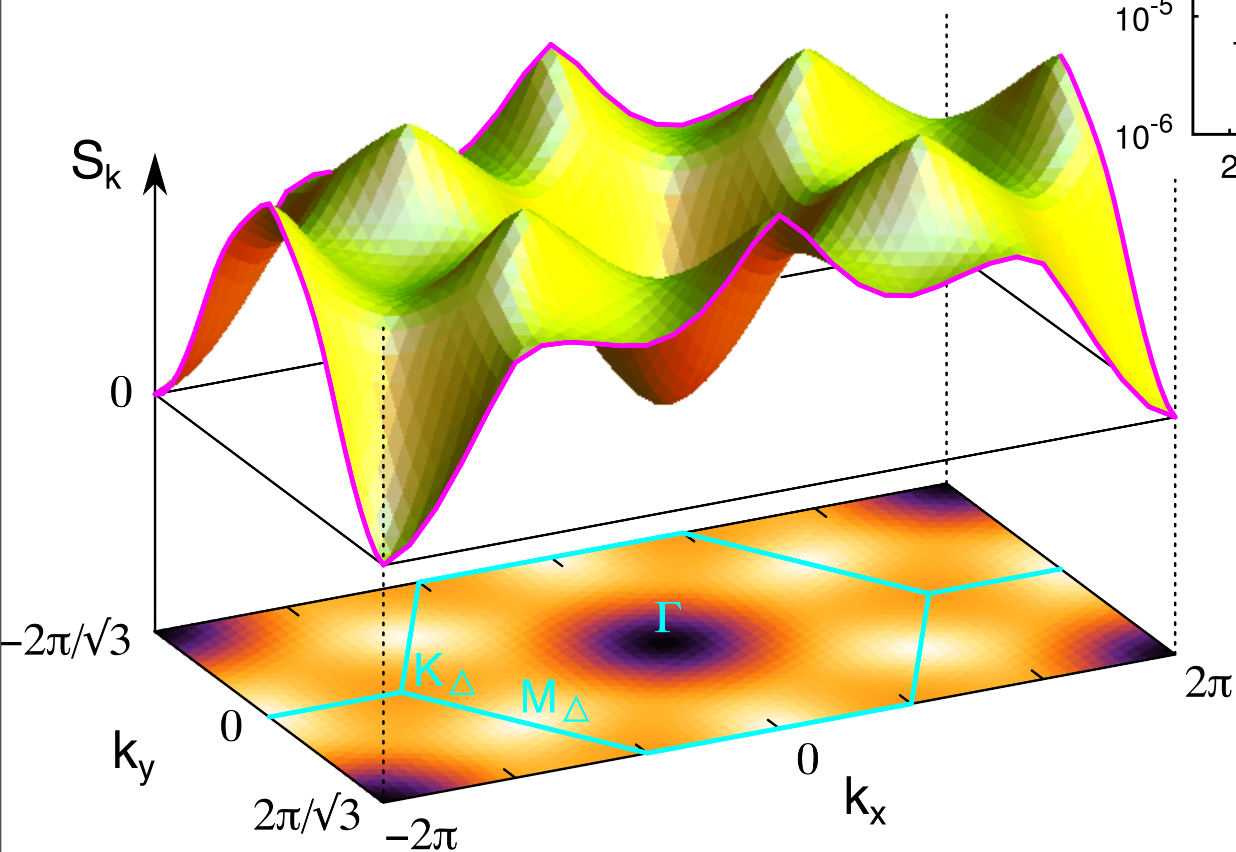
Dimension of the Hilbert space is $24!/(6!)^4$ = 2 308 743 493 056
using symmetries makes it tractable

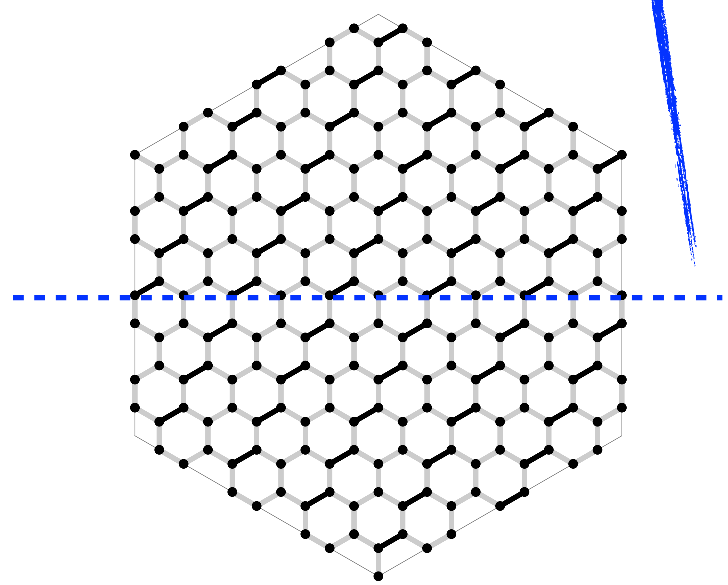
algebraic correlations and structure factor

$$|\langle S_0 S_\delta \rangle| \propto \left| \langle P_{0,\delta} \rangle - \frac{1}{4} \right| \propto \delta^\eta$$

$$\eta \approx -3.5$$

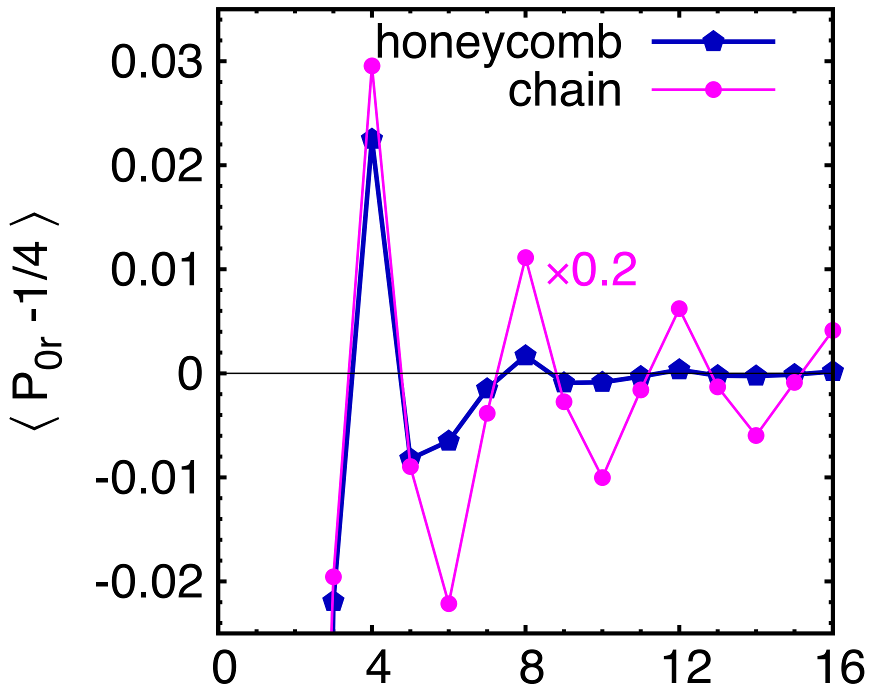


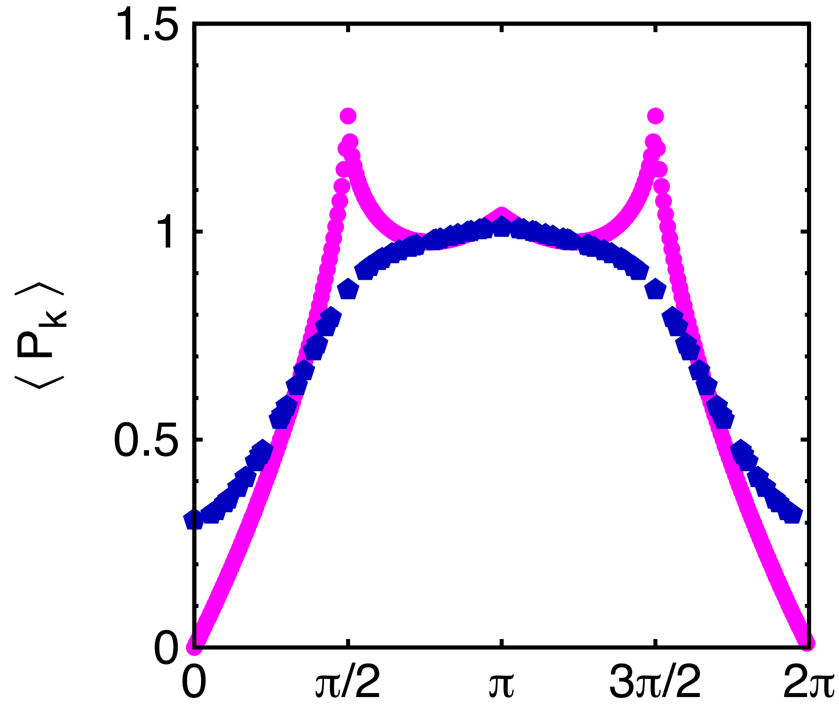


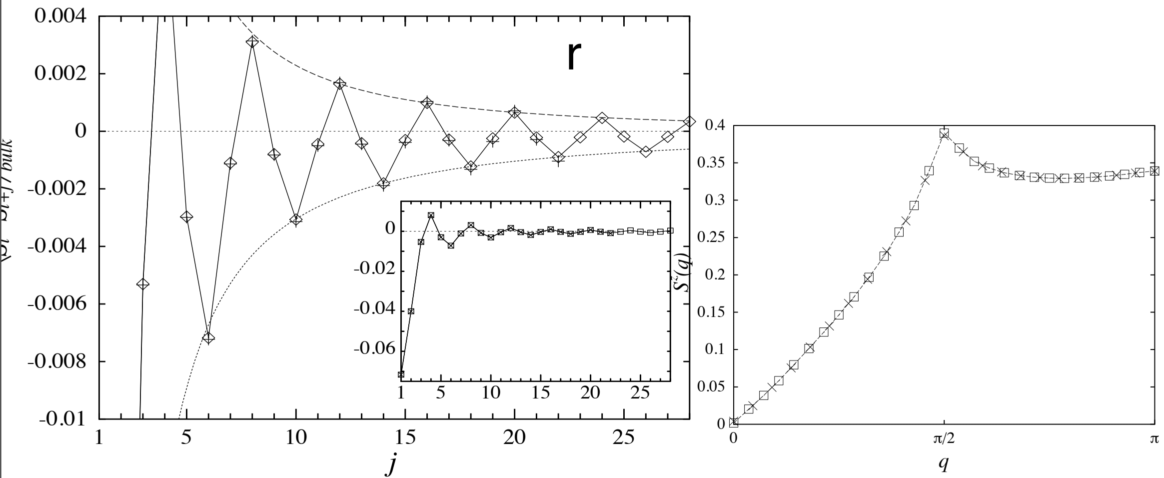


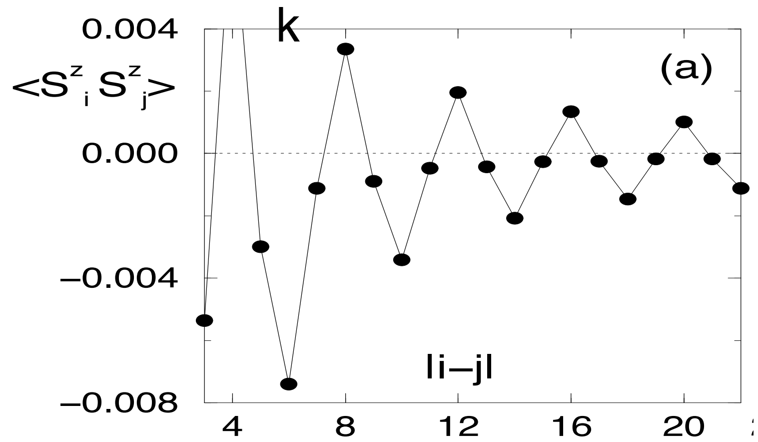
comparison to 1D chains

variational MC (Gutzwiller
projected Fermi sea)









Y. Yamashita, N. Shibata, K. Ueda
Phys. Rev. B **58**, 9114-9118 (1998)

Beat Frischmuth, Frederic Mila, Matthias Troyer
Phys. Rev. Lett. **82**, 000835 (1999)

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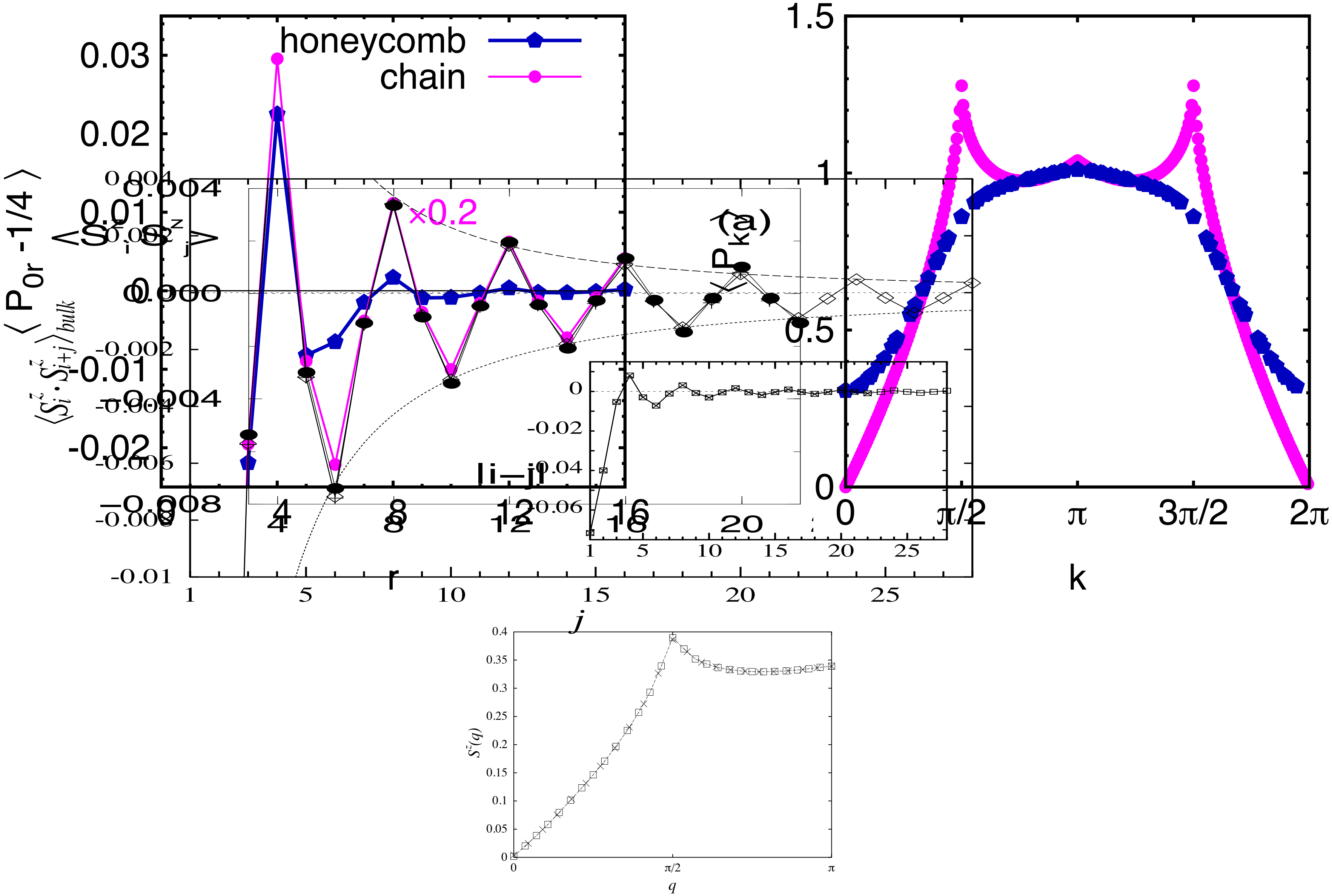

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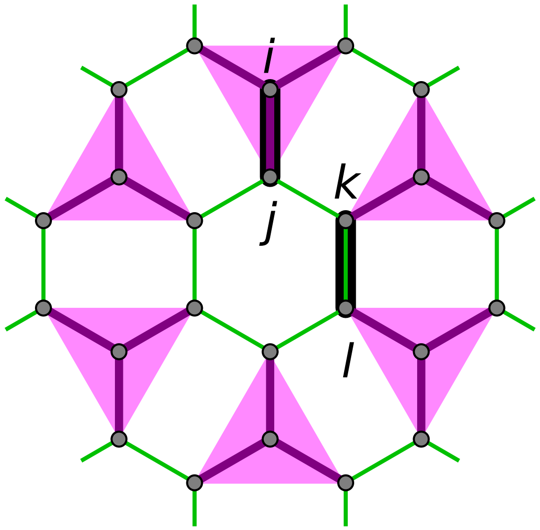
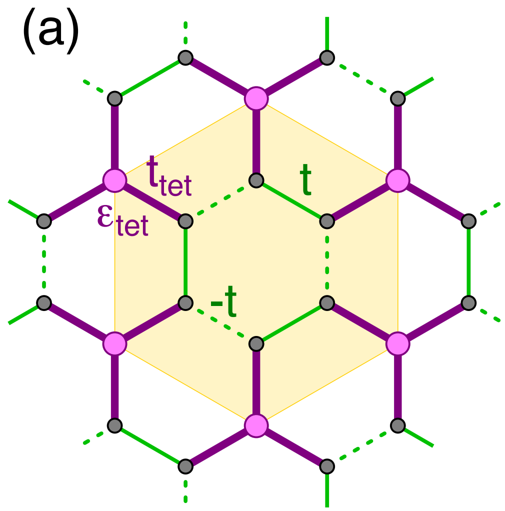
SU(4) honeycomb: Tetramerization

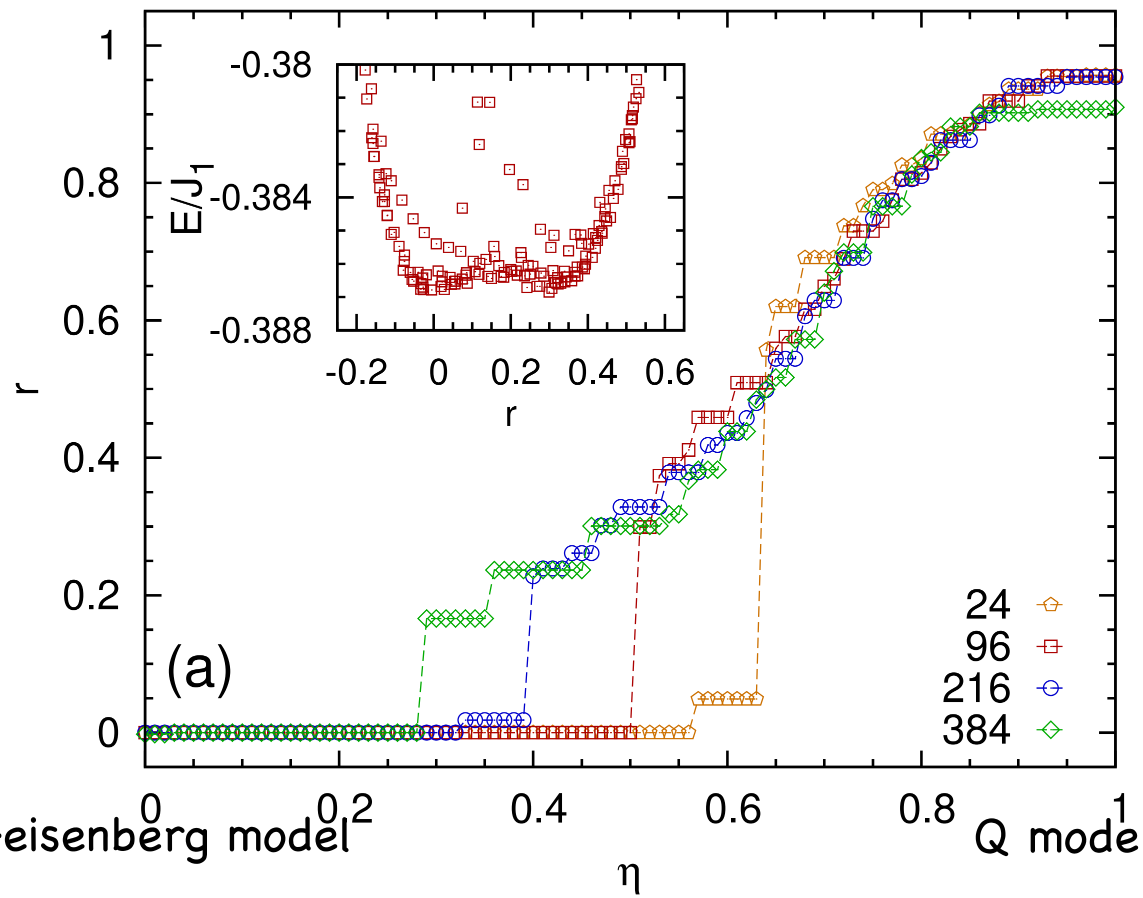
$$Q_{(ij),(kl)} = \frac{1}{4}(1+\mathcal{P}_{ij})(1+\mathcal{P}_{kl})$$



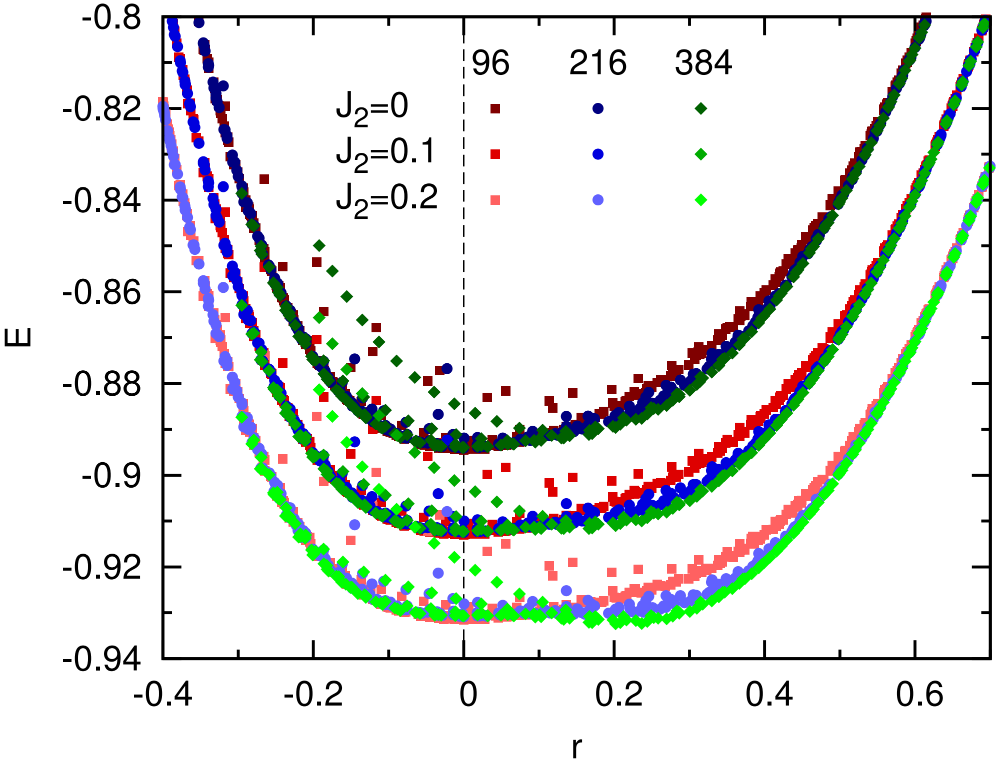
$$\mathcal{H}_\eta = \sum_{(ij)} \mathcal{P}_{ij} + \frac{\eta}{4} \sum_{(ij),(kl)} (1 + \mathcal{P}_{ij}\mathcal{P}_{kl})$$

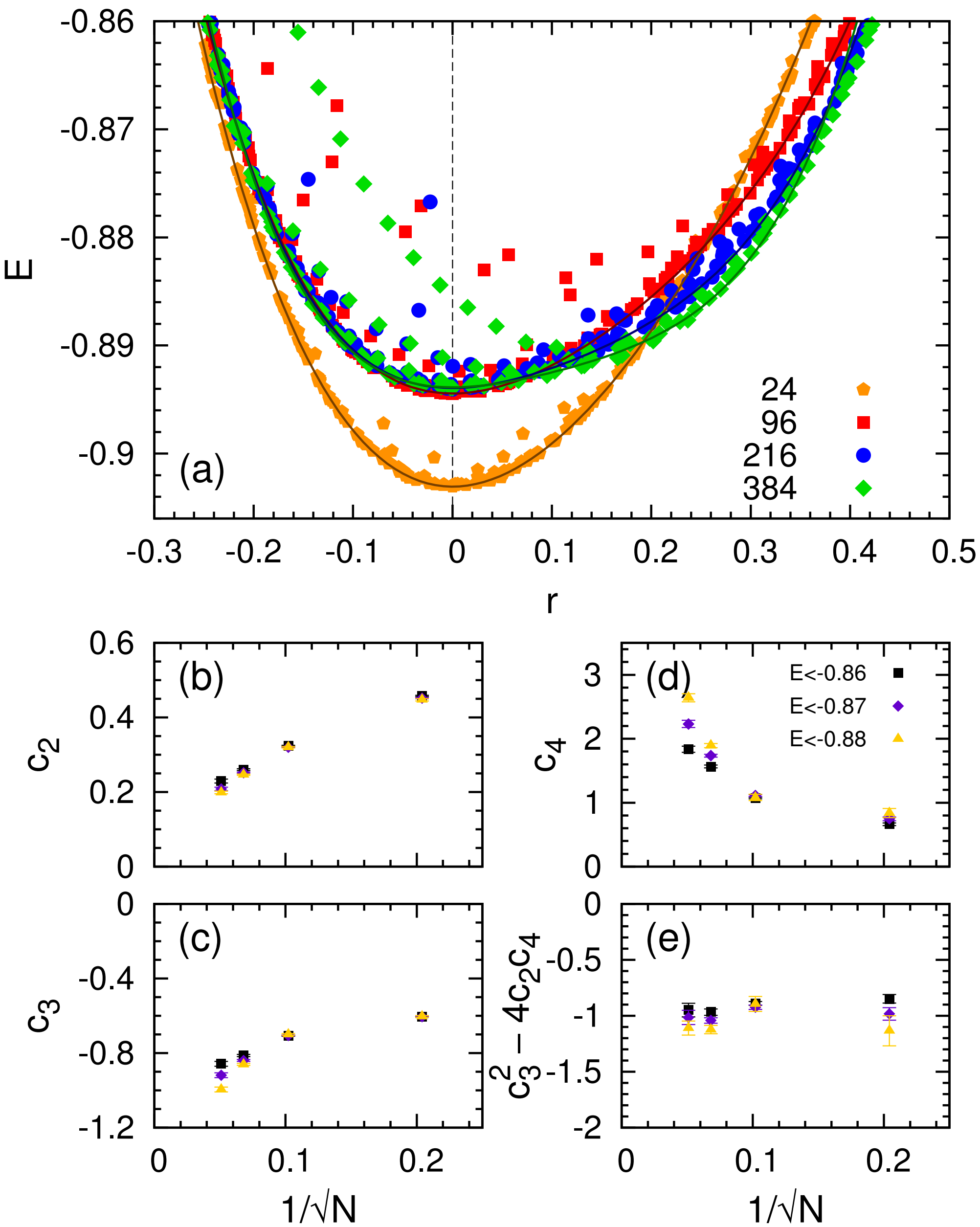
exact ground state for a Hamiltonian that is a sum of Q operators





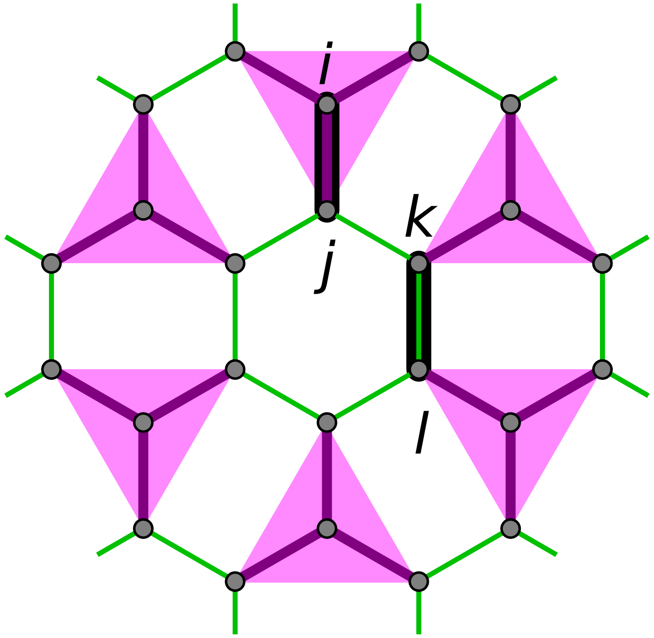
SU(4) honeycomb: Tetramerization

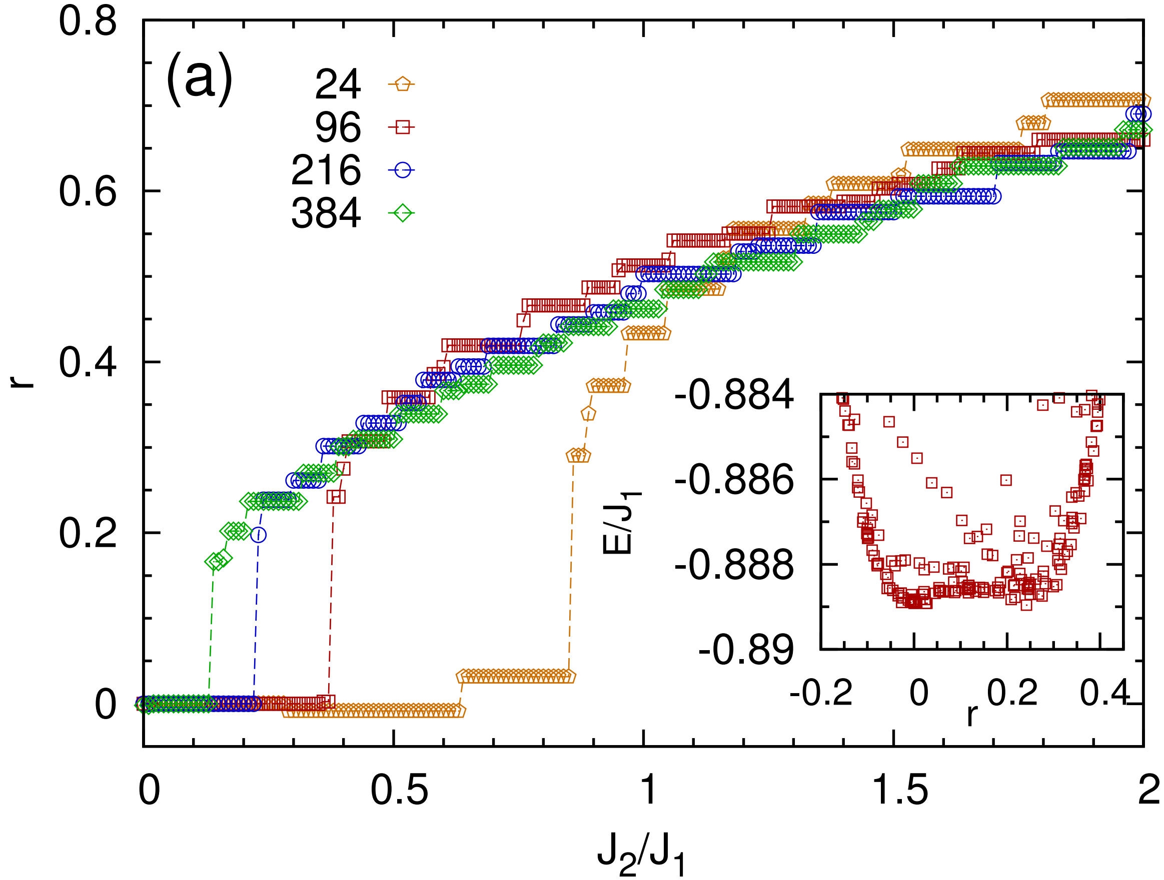




$$E_{\text{fit}} = E_0 + c_2 r^2 + c_3 r^3 + c^4 r^4$$

J₁-J₂ SU(4) honeycomb: Tetramerization





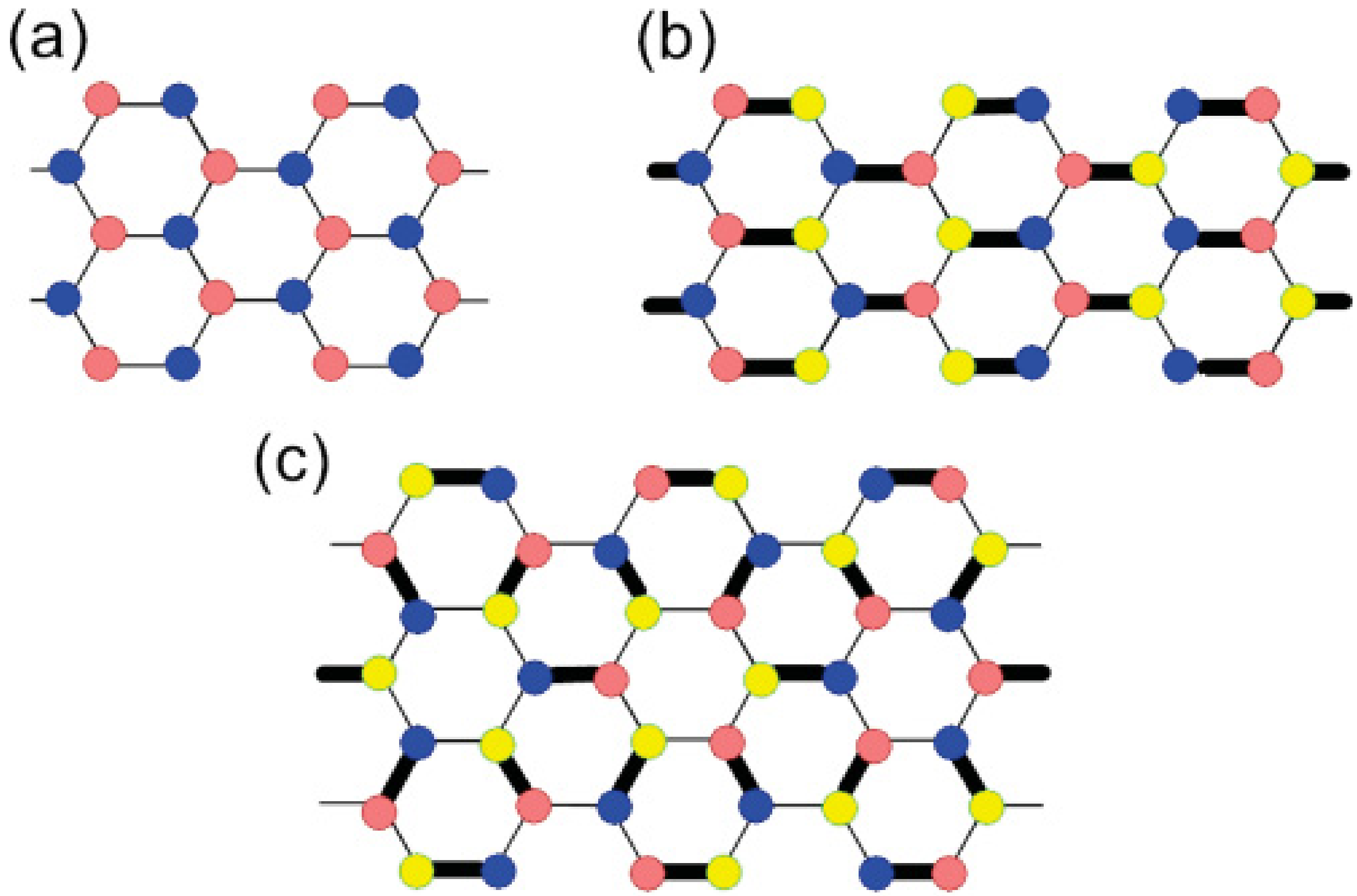
Heisenberg model

Q model

SU(3) honeycomb lattice: flavor wave

Lee and Yang,

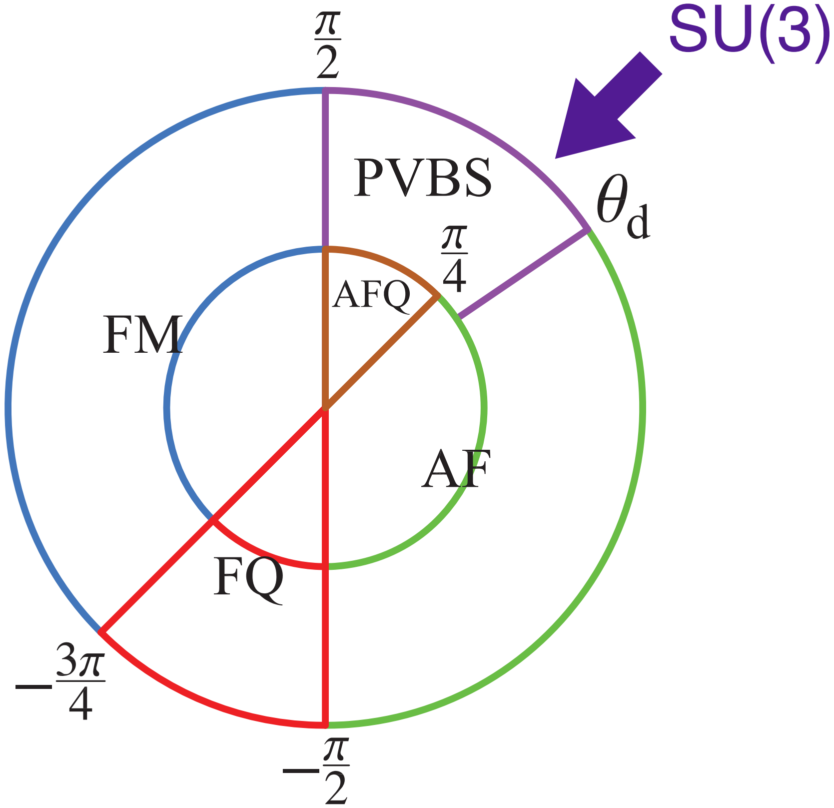
Phys. Rev. B **85**, 100402 (2012)

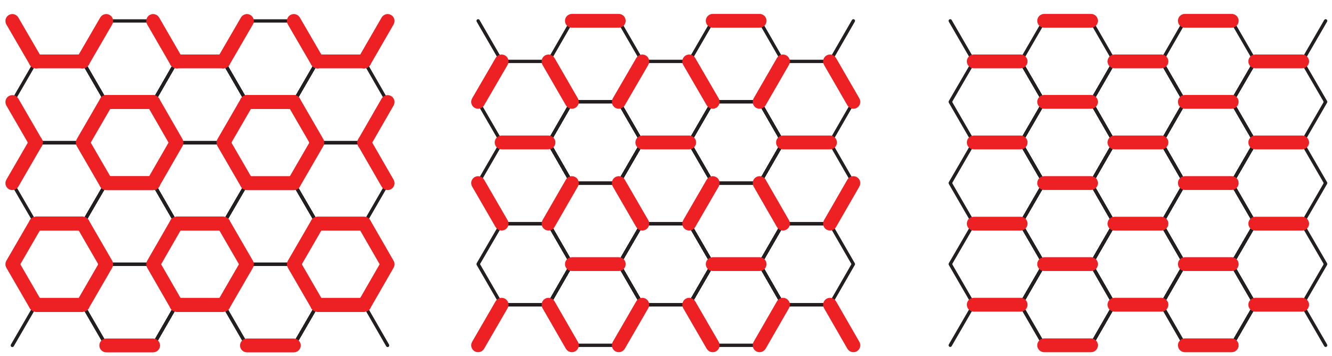


SU(3) honeycomb lattice: tensor network

S=1 bilinear-biquadratic model

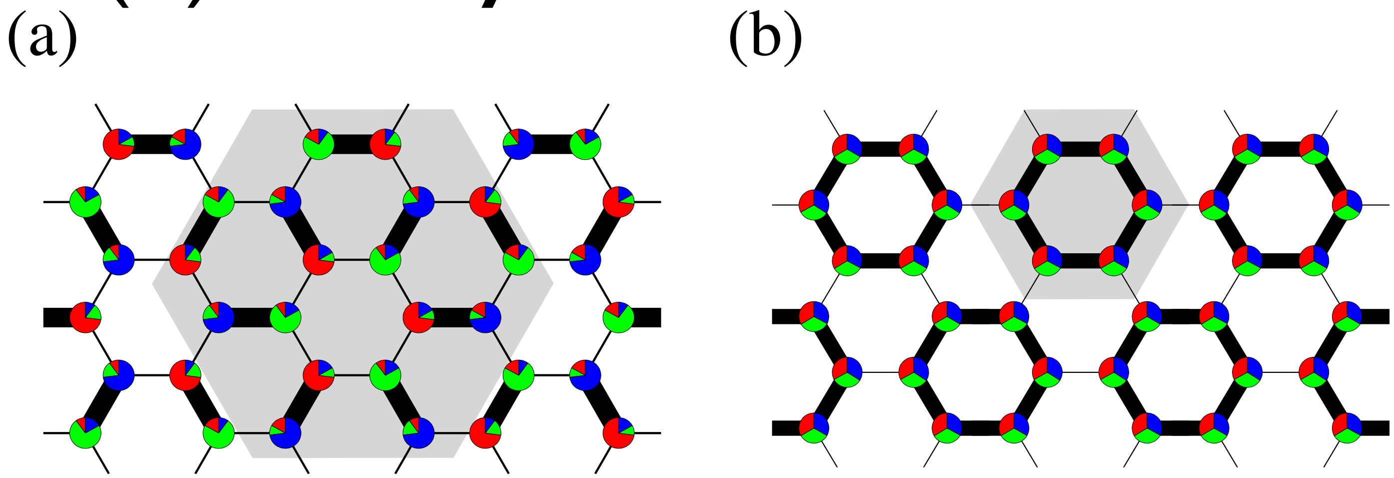
H.H.Zhao,C.Xu,Q.N.Chen,Z.C.Wei,
M.P.Qin,G.M. Zhang, and T. Xiang,
Phys. Rev. B **85**, 134416 (2012).

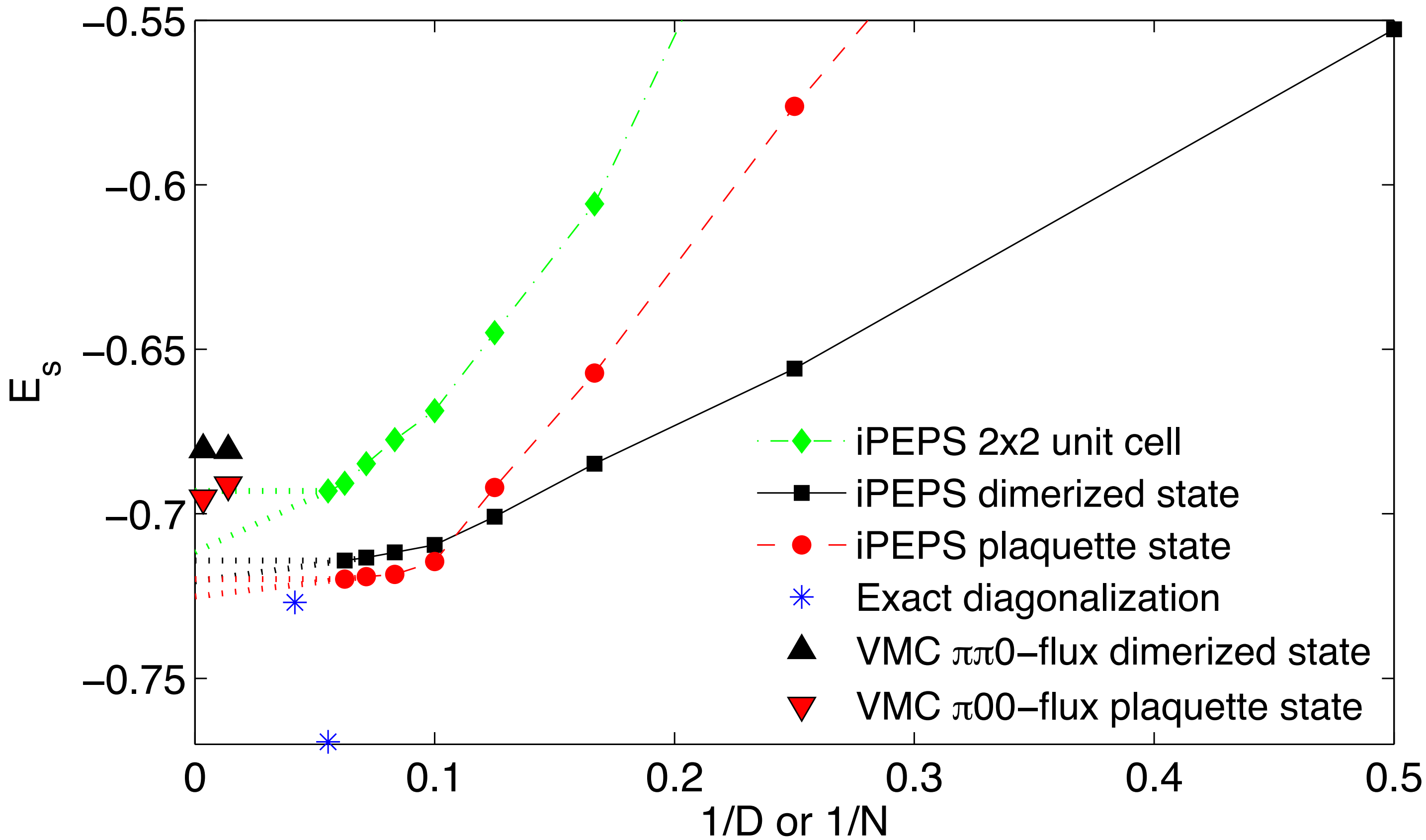




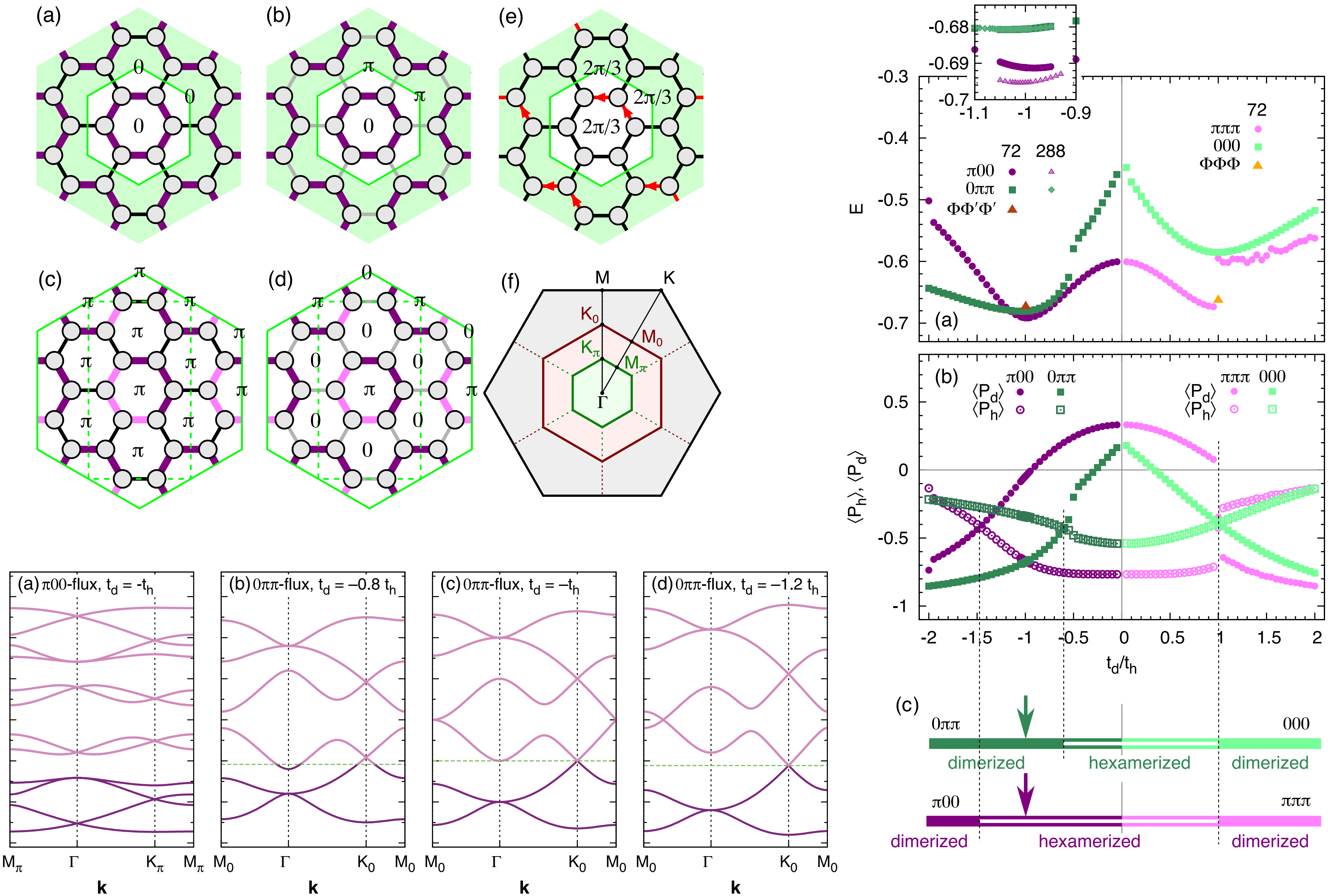
(a) (b) (c)

SU(3) honeycomb lattice: iPEPS



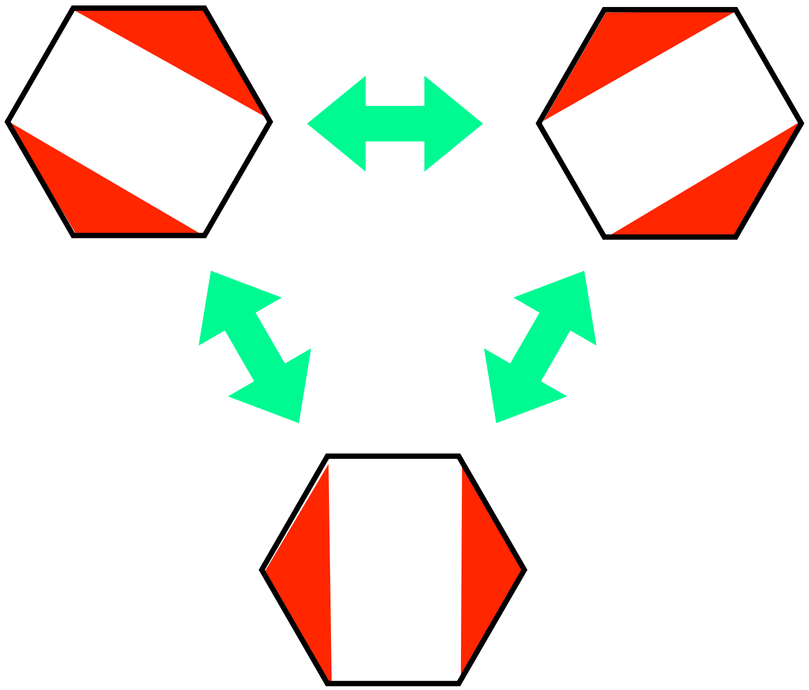


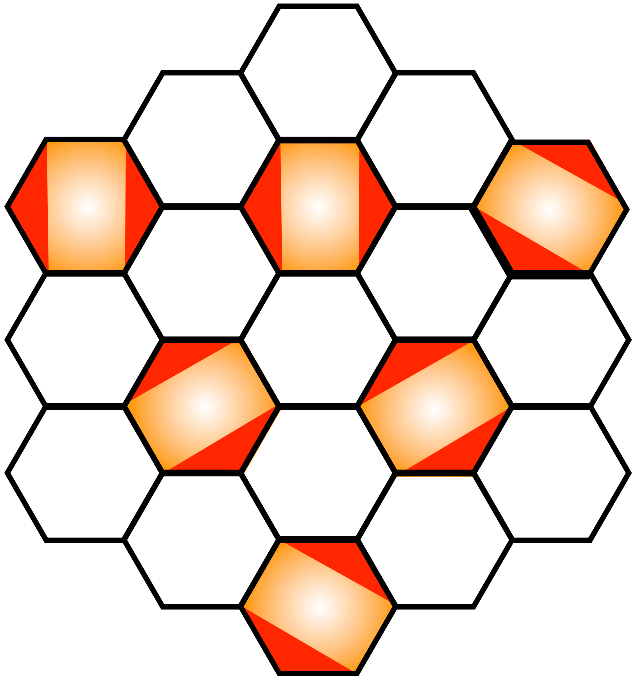
SU(3) honeycomb lattice: projected fermions



SU(3) honeycomb lattice: cartoon picture

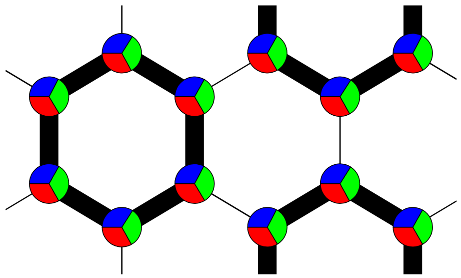
Resonance of SU(3) singlets





SU(N) on honeycomb

SU(2) is a Néel state



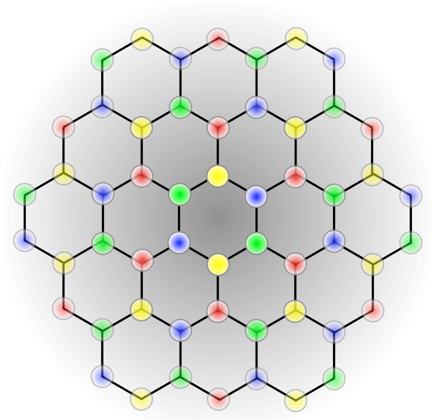
P. Corboz, unpublished

SU(3) is a plaquette state

[Y.-W. Lee and M.-F. Yang, Phys. Rev. B **85**, 100402 (2012).
H.H.Zhao,C.Xu,Q.N.Chen,Z.C.Wei,M.P.Qin,G.M. Zhang, and T. Xiang,
Phys. Rev. B **85**, 134416 (2012).]

SU(4) is most probably an algebraic flavor liquid

[P. Corboz, M. Lajkó, A. M. Läuchli, K. Penc, F. Mila, arXiv:1207.6029]



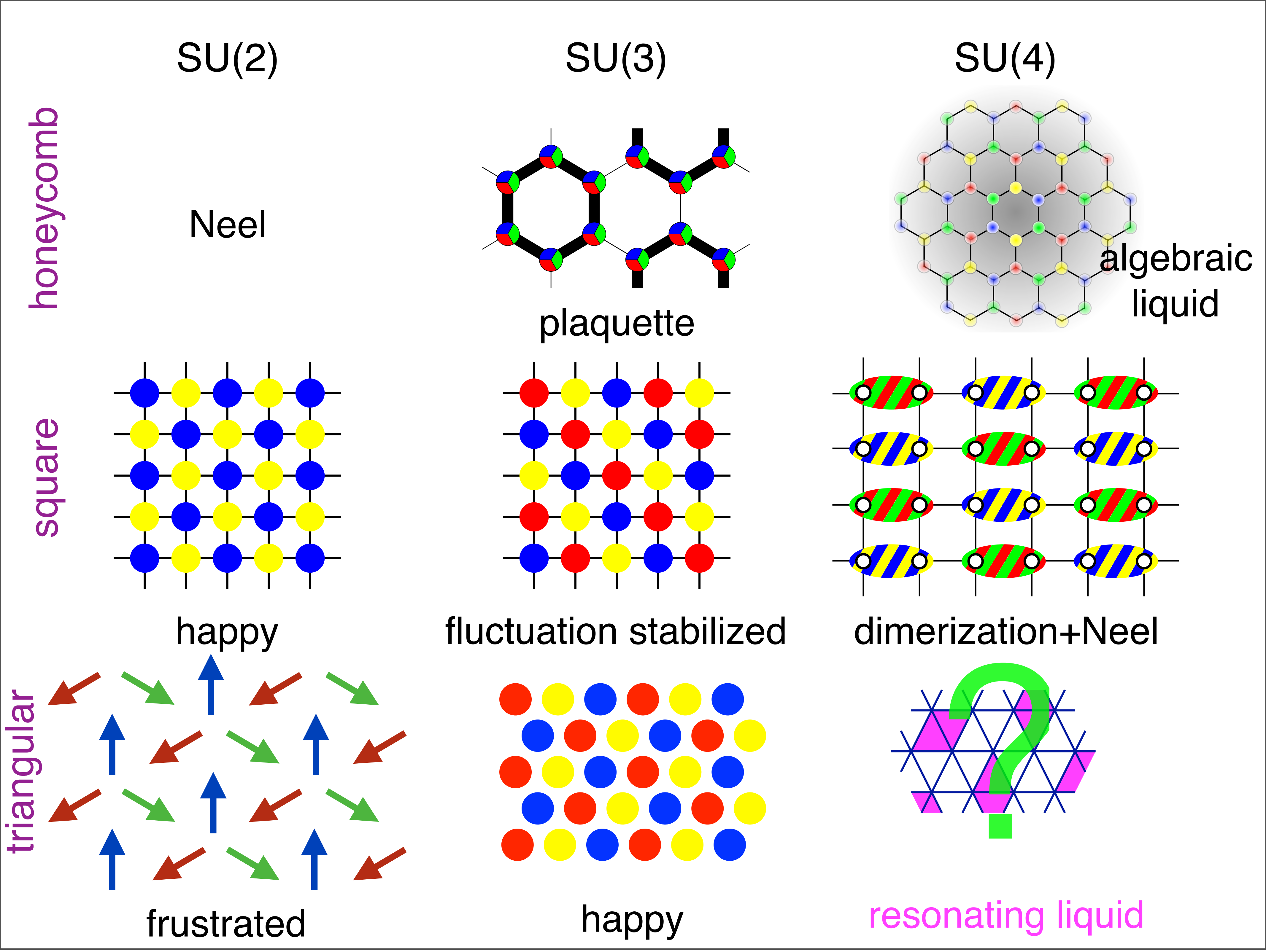
SU(6) is possibly a chiral flavor liquid

[G. Szirmai E. Szirmai, A. Zamora, and M. Lewenstein, Phys. Rev. A **84**, 011611 (2011)],

similarly, SU(N) is also a chiral liquid

[extending the results of M. Hermele, V. Gurarie, & A. M. Rey, Phys. Lett. **103**, 135301 (2009)]

honeycomb optical lattices can be realized


SU(2)
SU(3)
SU(4)
honeycomb
Neel
plaquette
algebraic
liquid
square
happy
fluctuation stabilized
dimerization+Neel
triangular
frustrated
happy
?
resonating liquid

the end

thank you for your attention