THE UNIVERSITY OF TOKYO

ISSP Int Symposium
"Emergent Quantum Phases in Condensed Matter",
Kashiwa, 14 June 2013

How we can manipulate graphene ---
chiral symmetry, topology and charged vacuum

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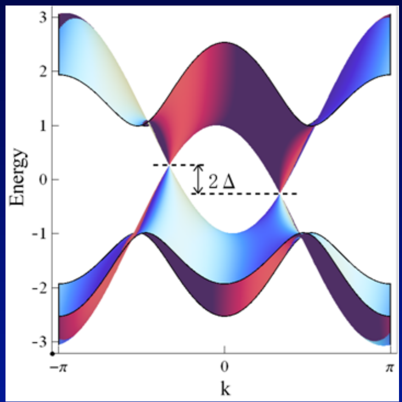
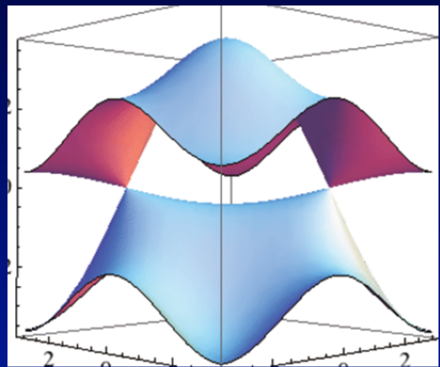
Ryo Shimano, Univ Tokyo

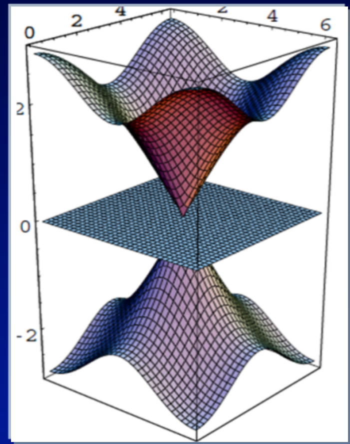
Peter Maksym, Univ Leicester

Plan of the talk

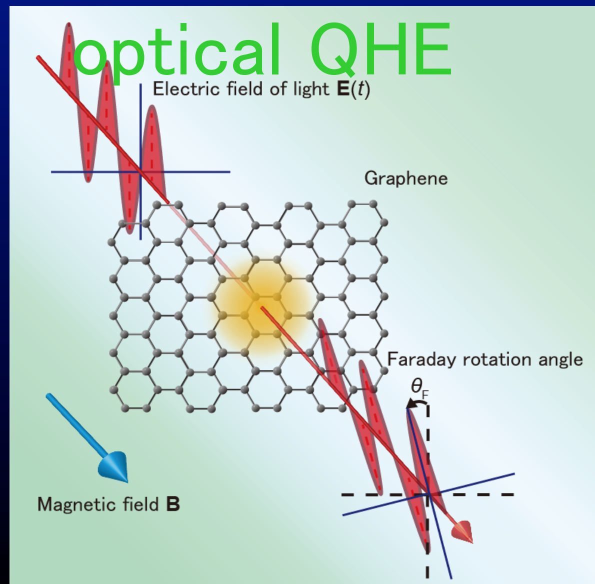
(a) Topological and chiral aspects in graphene
--- how general? unexpectedly robust





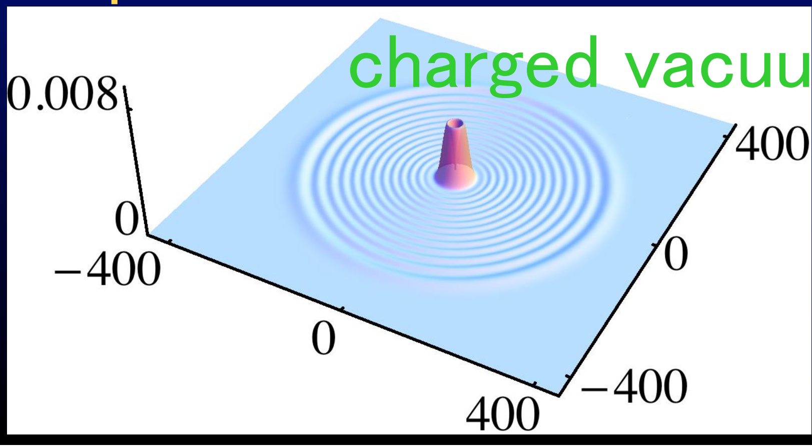


(b) Does graphene QHE appear in optics?

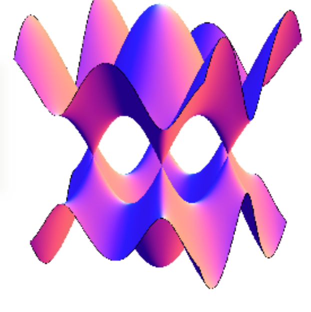


(c) Graphene quantum dot?

charged vacuum



Chirality in graphene



$$H_K = v_F(\sigma_x p_x + \sigma_y p_y)$$ → K

$$H_{K'} = v_F(- \sigma_x p_x + \sigma_y p_y)$$ → K'

Chiral symmetry

$$\{H, \Gamma\} = 0 \qquad \Gamma = \sigma_z$$

$$\Rightarrow \quad E : |\psi\rangle \qquad -E : \Gamma|\psi\rangle$$

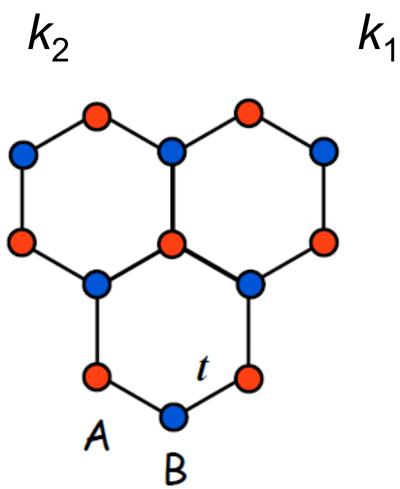
$$\Gamma c_i \Gamma^{-1} = c_i \quad i \in A \text{ sublattice}$$
$$\Gamma c_i \Gamma^{-1} = -c_i \quad i \in B \text{ sublattice}$$

(3+1)	(2+1)
γ_0	σ_x
γ_1	σ_y
γ_2	
γ_3	
γ_4	
γ_5	σ_z

E=0 is special

made eigenstates of Γ

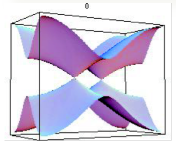
$$\psi_\pm = (1 \pm \Gamma)\psi_{E=0} \rightarrow \Gamma\,\psi_\pm = \pm\,\psi_\pm$$



$$H = -\sum_{<i,j>} t_{ij} c_i^\dagger c_j$$

ψ_+(ψ_-) has its amplitude only on B (A) sub-lattice

Why doubled Dirac cones ? --- topological stability



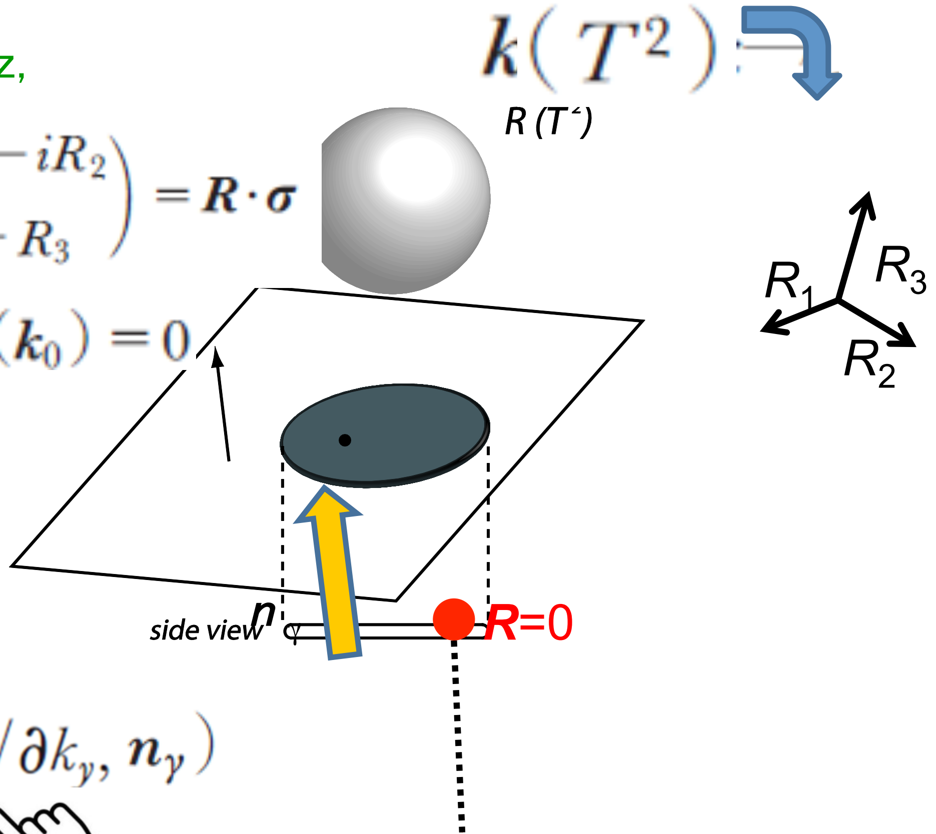
cf. Hatsugai, this workshop

(Hatsugai & Aoki in *Physics of Graphene*, ed. by H. Aoki & M. Dresselhaus, to appear)

Generically, for the entire Bz,

$$H(\boldsymbol{k}) = \begin{pmatrix} R_3(\boldsymbol{k}) & D(\boldsymbol{k}) \\ D^*(\boldsymbol{k}) & -R_3(\boldsymbol{k}) \end{pmatrix} \equiv \begin{pmatrix} R_3 & R_1 - iR_2 \\ R_1 + iR_2 & -R_3 \end{pmatrix} = \boldsymbol{R}\cdot\boldsymbol{\sigma}$$

$$E^2 = R^2 \qquad E_\pm = \pm R\ (\boldsymbol{k}) \qquad \boldsymbol{R}(\boldsymbol{k}_0) = 0$$



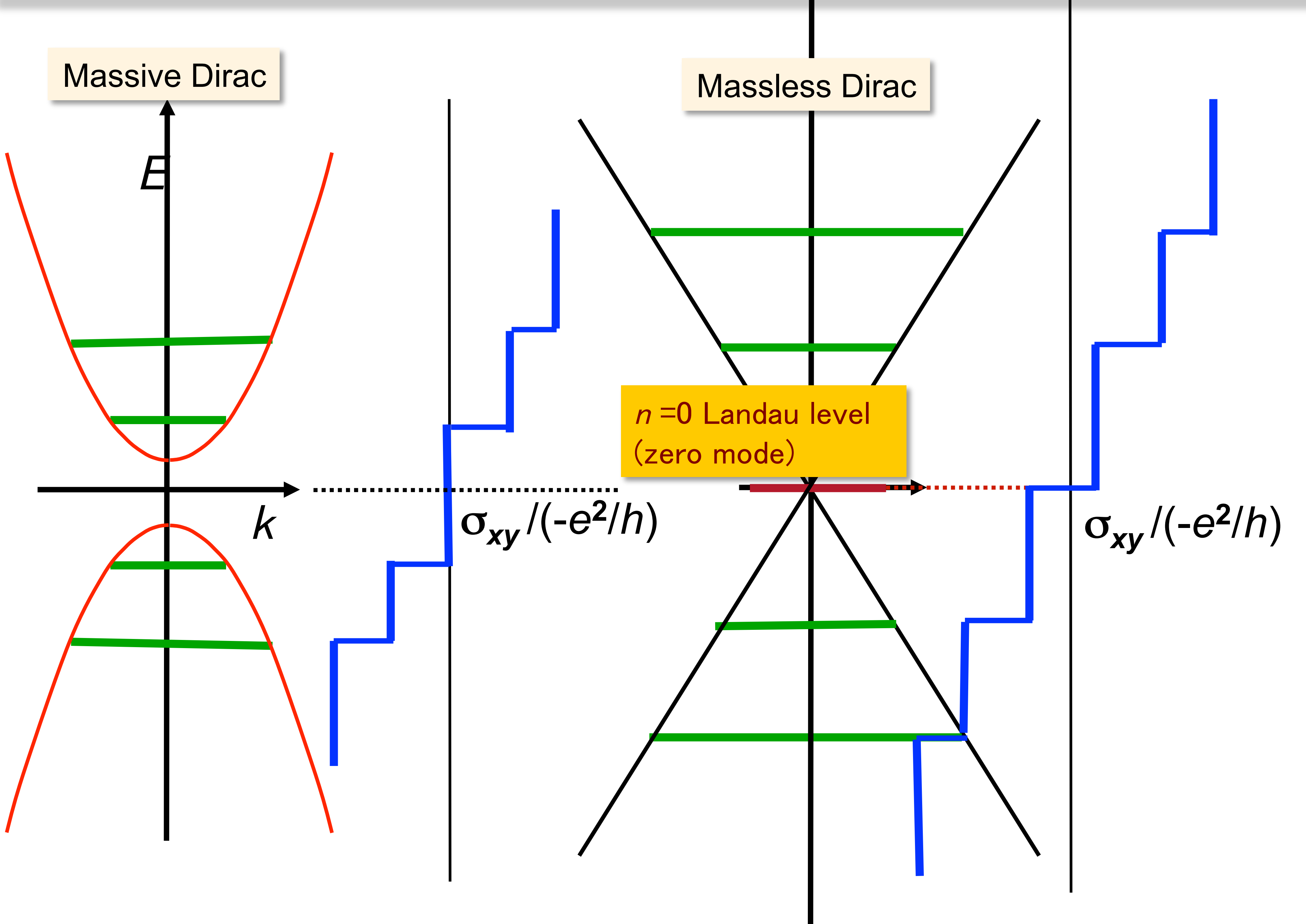
If $H = \boldsymbol{R}\cdot\boldsymbol{\sigma}$ is chiral-symmetric
with a chiral operator $\gamma = \boldsymbol{n}_\gamma \cdot \boldsymbol{\sigma}$
→ $\boldsymbol{R} \perp \boldsymbol{n}_\gamma$

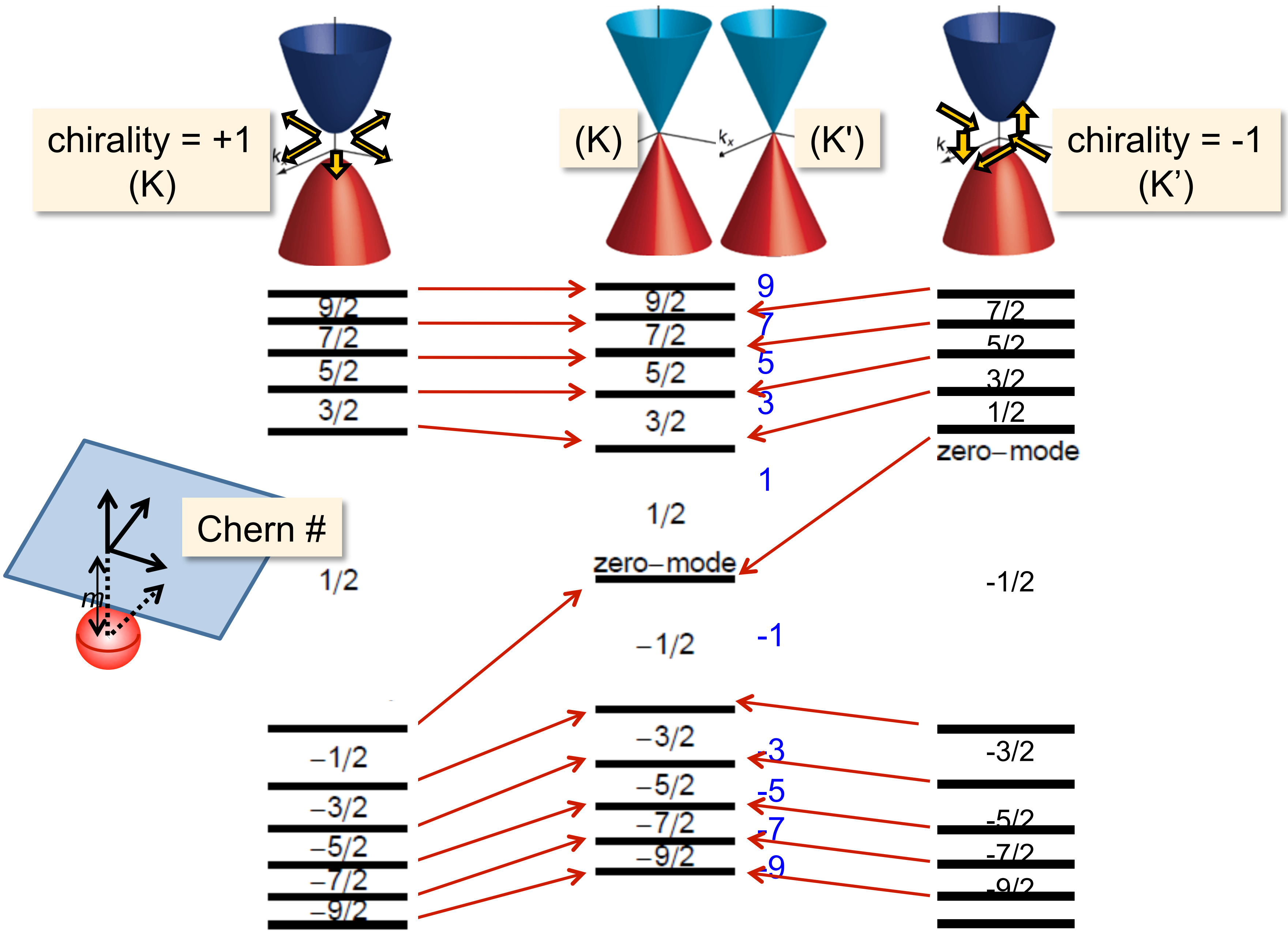
$$(\partial\boldsymbol{R}/\partial k_x,\ \partial\boldsymbol{R}/\partial k_y,\ \boldsymbol{n}_\gamma)$$

→ chirality +1
→ chirality -1

Fermion doubling
((2+1)D analogue of Nielsen-Ninomiya)

QHE


Massive Dirac
E
k
σxy /(-e²/h)
Massless Dirac
n =0 Landau level
(zero mode)
σxy /(-e²/h)



chirality = +1
(K)
(K)
(K')
chirality = -1
(K')
9/2
7/2
5/2
3/2
1/2
−1/2
−3/2
−5/2
−7/2
−9/2
9/2
7/2
5/2
3/2
1/2
zero−mode
−1/2
−3/2
−5/2
−7/2
−9/2
9
7
5
3
1
-1
-3
-5
-7
-9
7/2
5/2
3/2
1/2
zero−mode
-1/2
-3/2
-5/2
-7/2
-9/2
Chern #

QHE in graphene

(Novoselov *et al*, Nature 2005; Nature Phys 2006; Zhang *et al*, Nature 2005)

$B = 14\,\mathrm{T}$ and $T = 4\,\mathrm{K}$

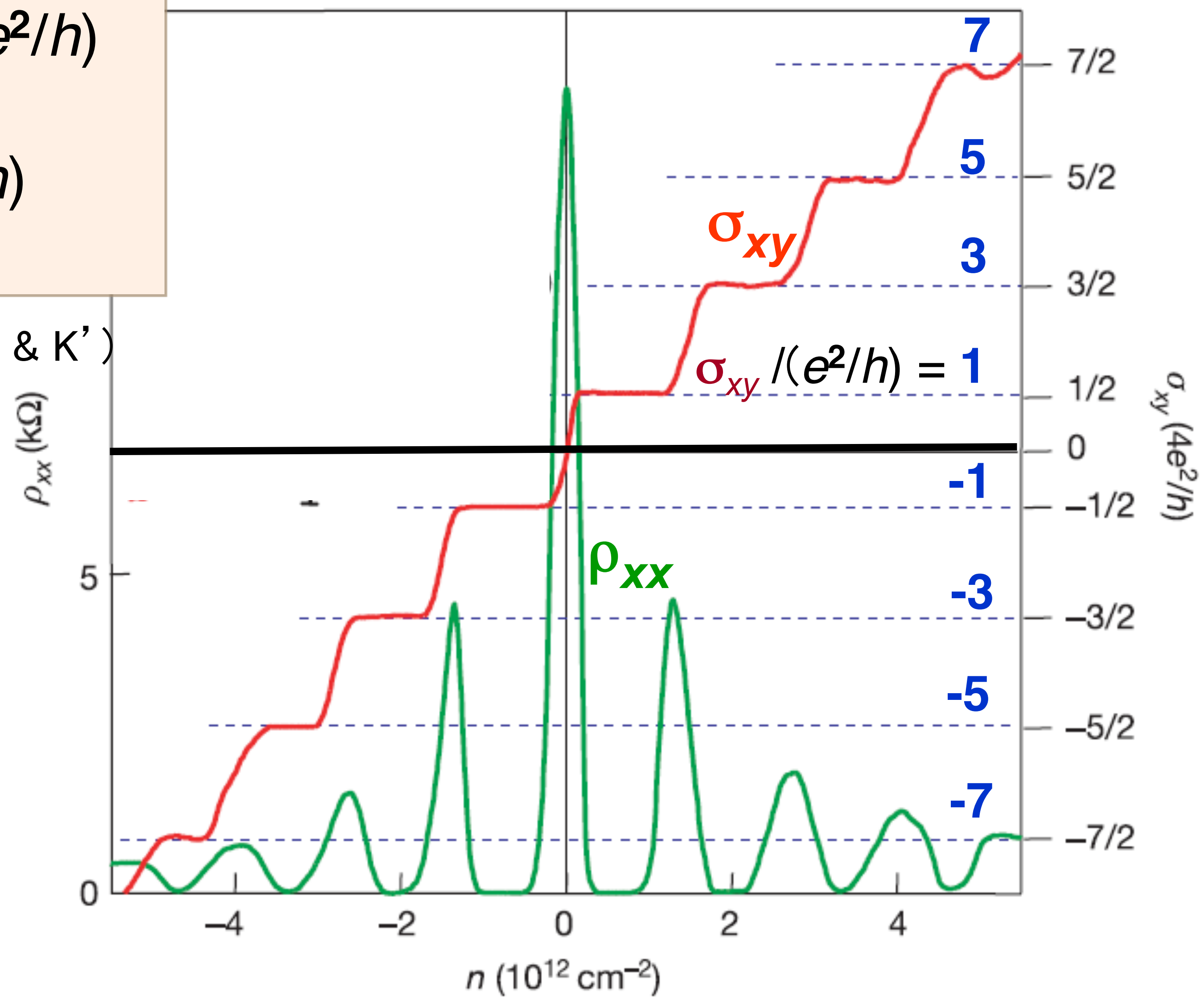
(SCBA; Zheng & Ando, PRB 2002;
Gusynin & Sharapov, PRB, PRL 2005)

$-\sigma_{xy} = (2n+1)(e^2/h)$

$= 2\,(n+1/2)(e^2/h)$

Valley degeneracy (K & K')

Can we lift this?

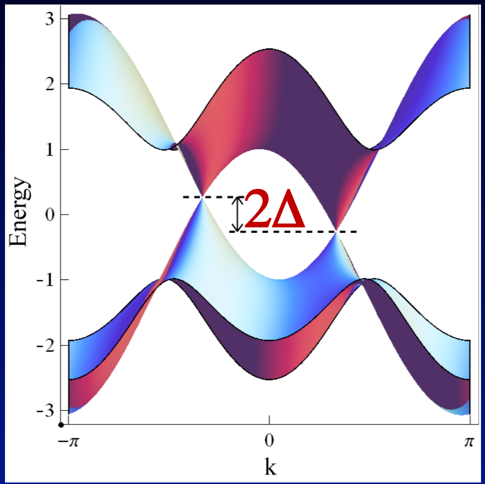


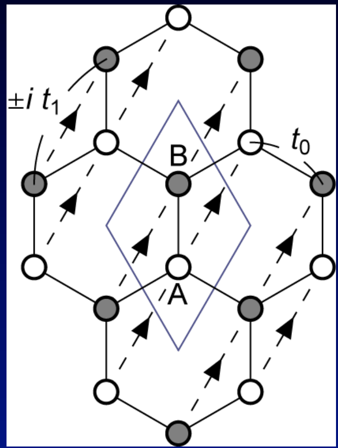
- Lattice models should always
  have integer Chern #
  (Thouless et al (TKNN), PRL 1982)

→ We cannot go around this, but at least
decompose into contributions from each cone

Shifted Dirac cones

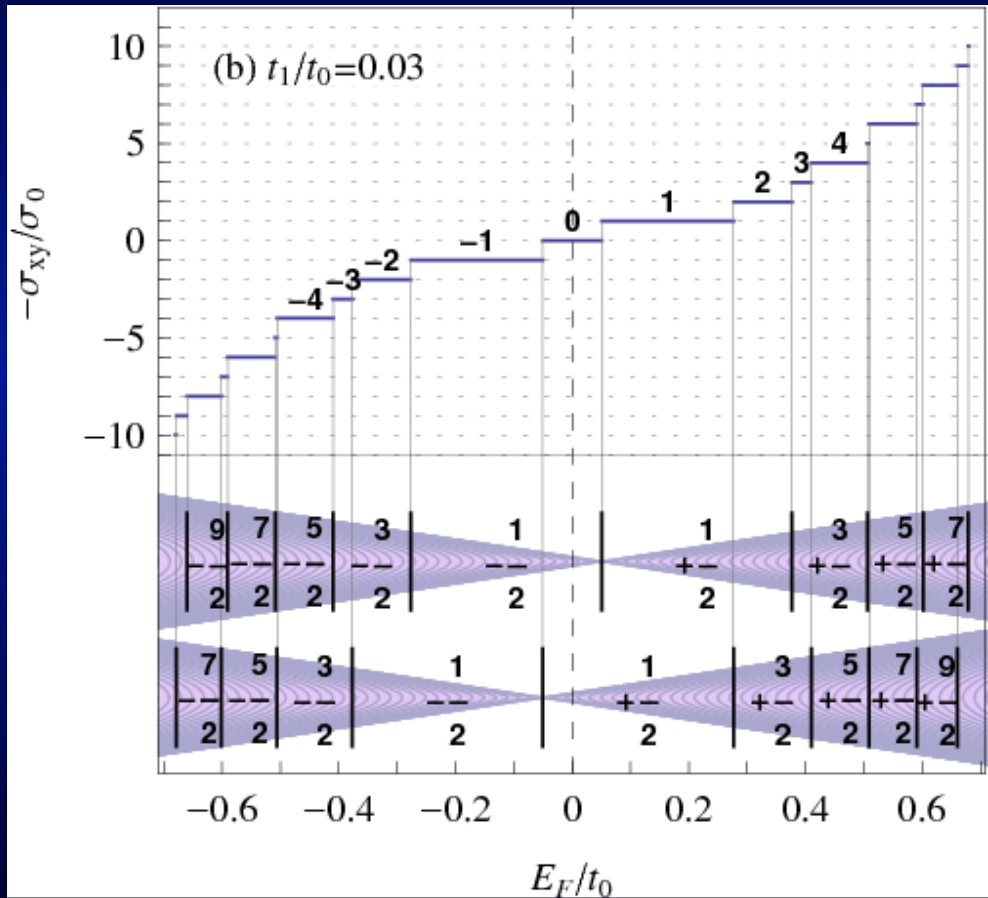
(Watanabe et al, PRB(R) 2010)





$$\mathcal{H} = \chi\Delta\sigma_0 - hv_F[\chi\delta k_x\sigma_1 + \delta k_y\sigma_2]$$

χ = 1 (-1) for K(K')



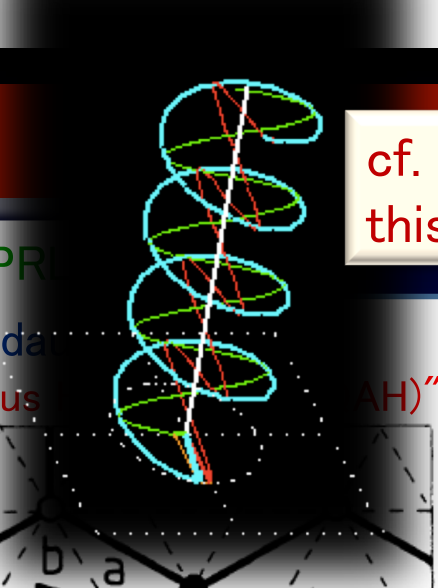
ac cones

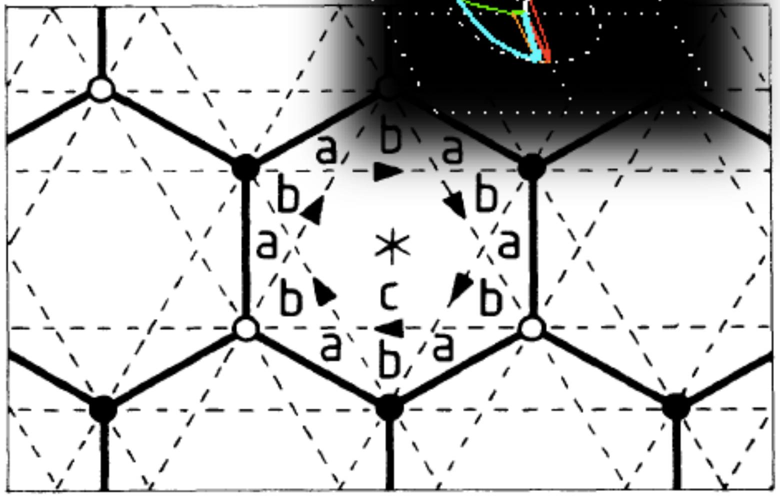
cf. Oka,
this symposium

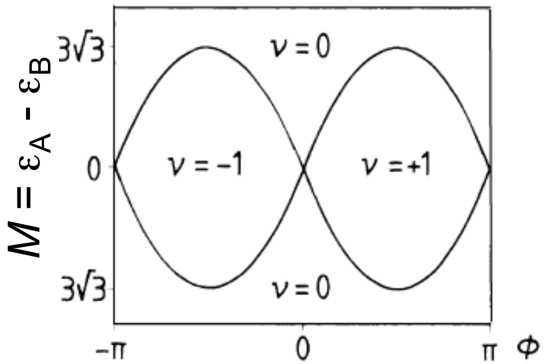
(Haldane, PR

QHE without Landa

"Quantum anomalous AH)"

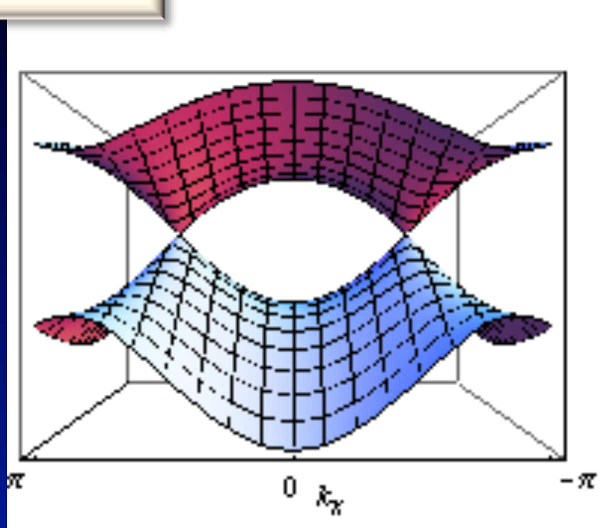




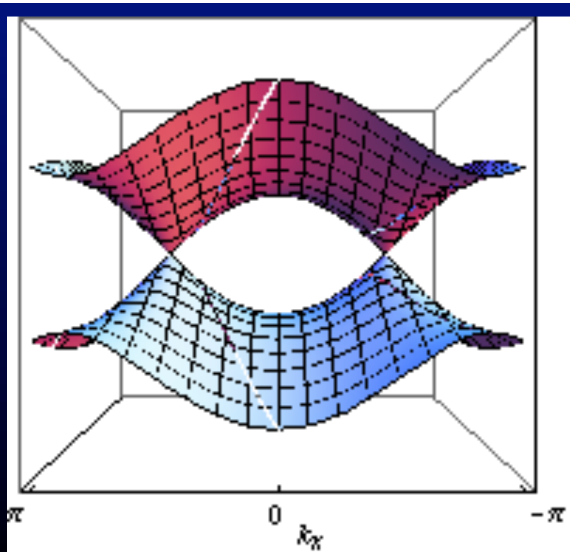


ν: Chern #

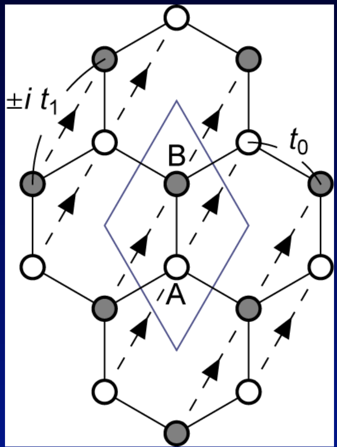
On the boundaries one of the cones becomes massless



(Watanabe et al, PRB(R) 2010)



Complex hopping unrealistic?



--- realisable in cold atoms in optical lattices

(Osterloh et al, PRL 2005;
Ruseckas et al, PRL 2005)

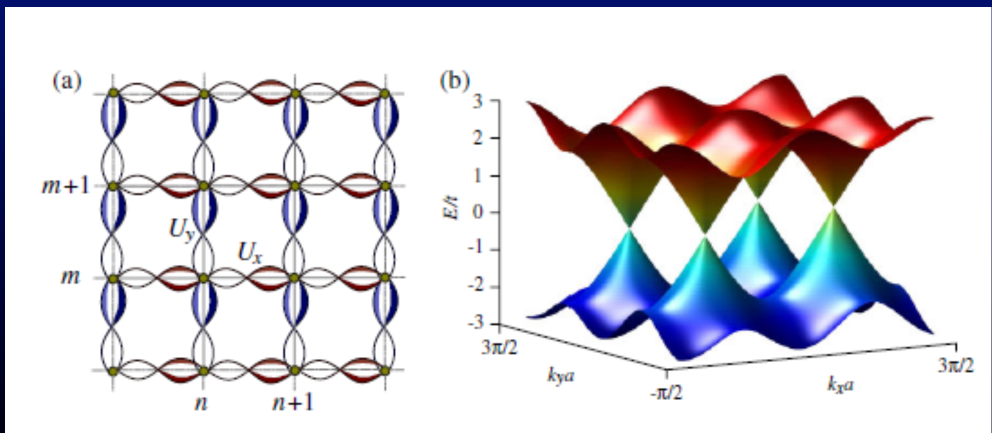
$$H = -t \sum_{\langle \mathbf{r},\mathbf{r}'\rangle} \sum_{\tau\tau'} c^{\dagger}_{\tau'}(\mathbf{r}') e^{-i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A}\cdot d\mathbf{l}} c_{\tau}(\mathbf{r})$$

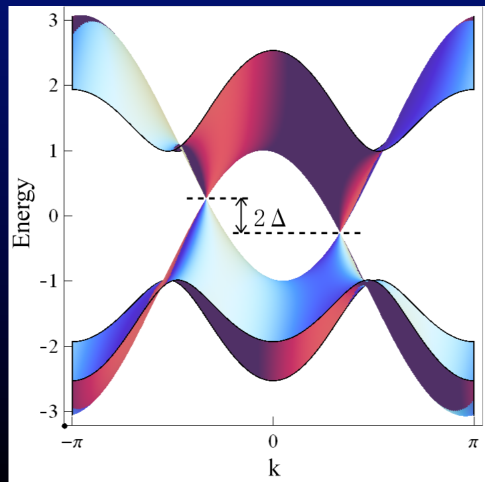
Two-component(hyperfine) fermion systems

→ $\boldsymbol{A} = \frac{B_0}{2}(-y, x) + a(B_\alpha \sigma_y, B_\beta \sigma_x)$

Abelian Nonabelian (Wilczek-Zee 84)

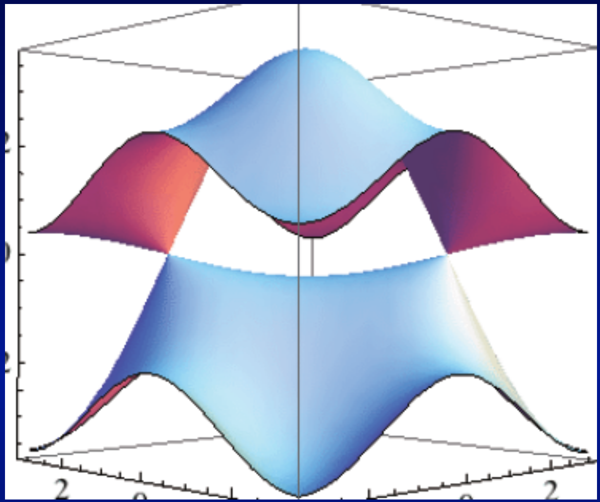
(Goldman et al, PRL 2009)

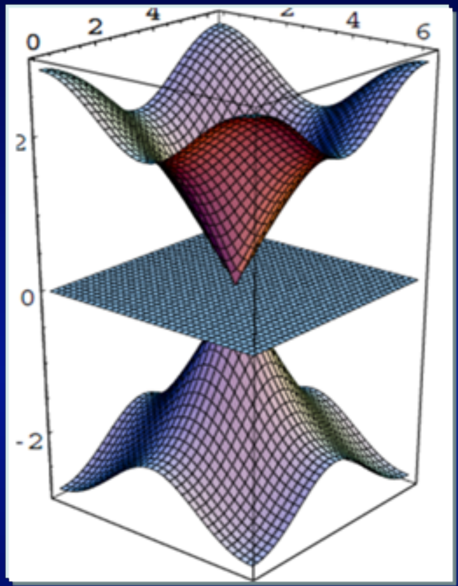




(Mei et al,
PRA 2011)

Various extensions of Dirac cones
← "Generalised chiral symmetry"





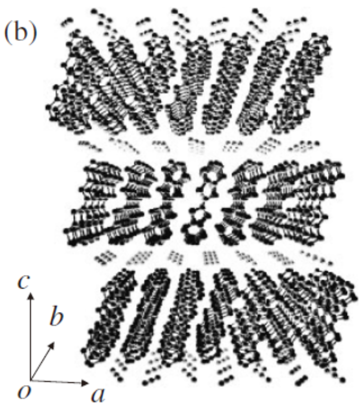
$$H_{\mathrm{K}} = v_{\mathrm{F}}(\sigma_x p_x + \sigma_y p_y)$$

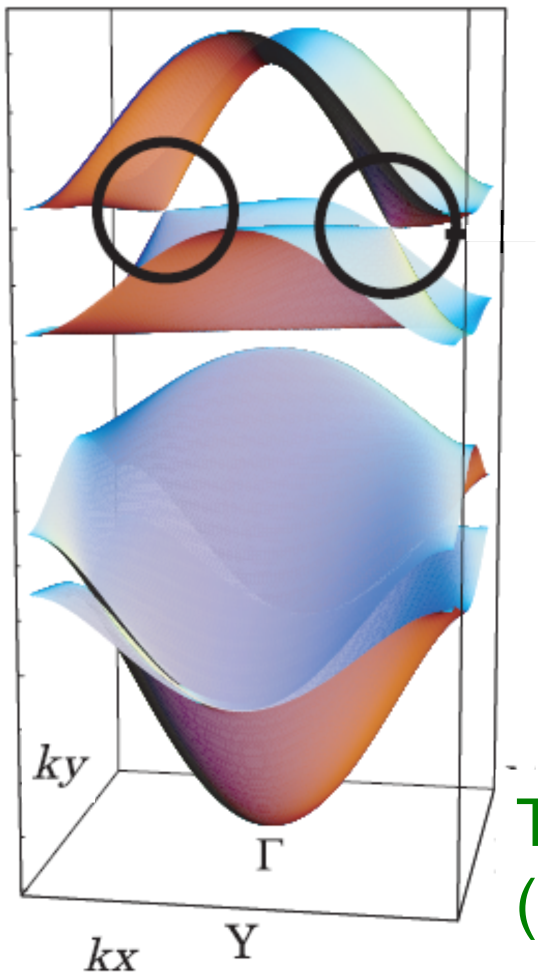
$$= v_{\mathrm{F}}$$

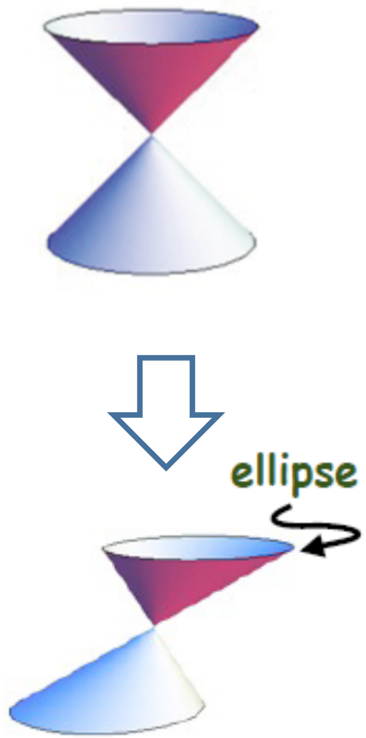
"Tilted" Dirac cones wash out the anomaly?

α-(BEDT-TTF)$_2$I$_3$

(Tajima et al, JPSJ 2000; 2002; 2006; Kobayashi et al, JPSJ 2004)





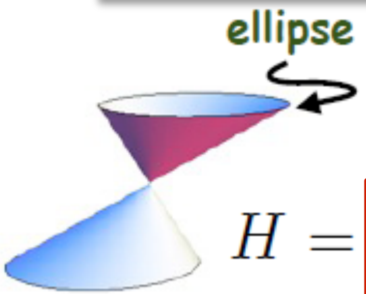


Tilted Dirac cones

(usual chiral symm does not apply)

"Generalised chirality" for tilted Dirac cones

(Kawarabayashi et al, RPB 2011)



$$H = \boxed{\sigma_0(\boldsymbol{W}\cdot\boldsymbol{\pi}/\hbar)} + (\boldsymbol{\sigma}\cdot\boldsymbol{X})\pi_x/\hbar + (\boldsymbol{\sigma}\cdot\boldsymbol{Y})\pi_y/\hbar$$

$$\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$$

$\Delta^2 > 0$

We can still define $\gamma = \boldsymbol{\sigma}\cdot[(\boldsymbol{X}\times\boldsymbol{Y}) - \mathrm{i}(W_y\boldsymbol{X} - W_x\boldsymbol{Y})]/\Delta$

$$\gamma^\dagger H \gamma = -H,$$

γ: nonhermitian, but has eigenvalues ±1, since $\gamma^2 = (\gamma^\dagger)^2 = \sigma_0$

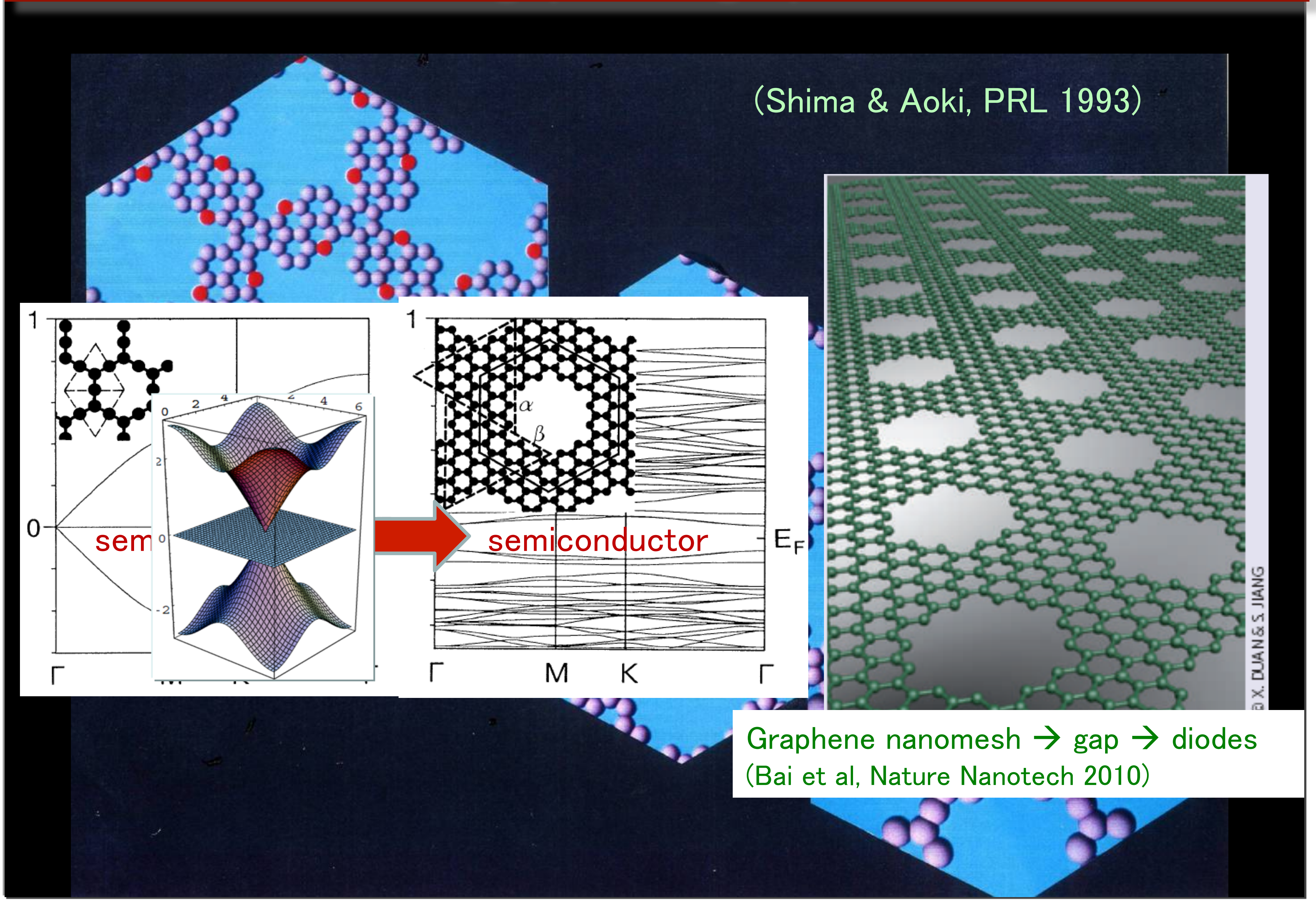
Generalised chiral symm definable when dispersion = cone

↔ H: elliptic as a differential operator ($\Delta^2 > 0$)

↔ index theorem applicable

--- rigorous link!

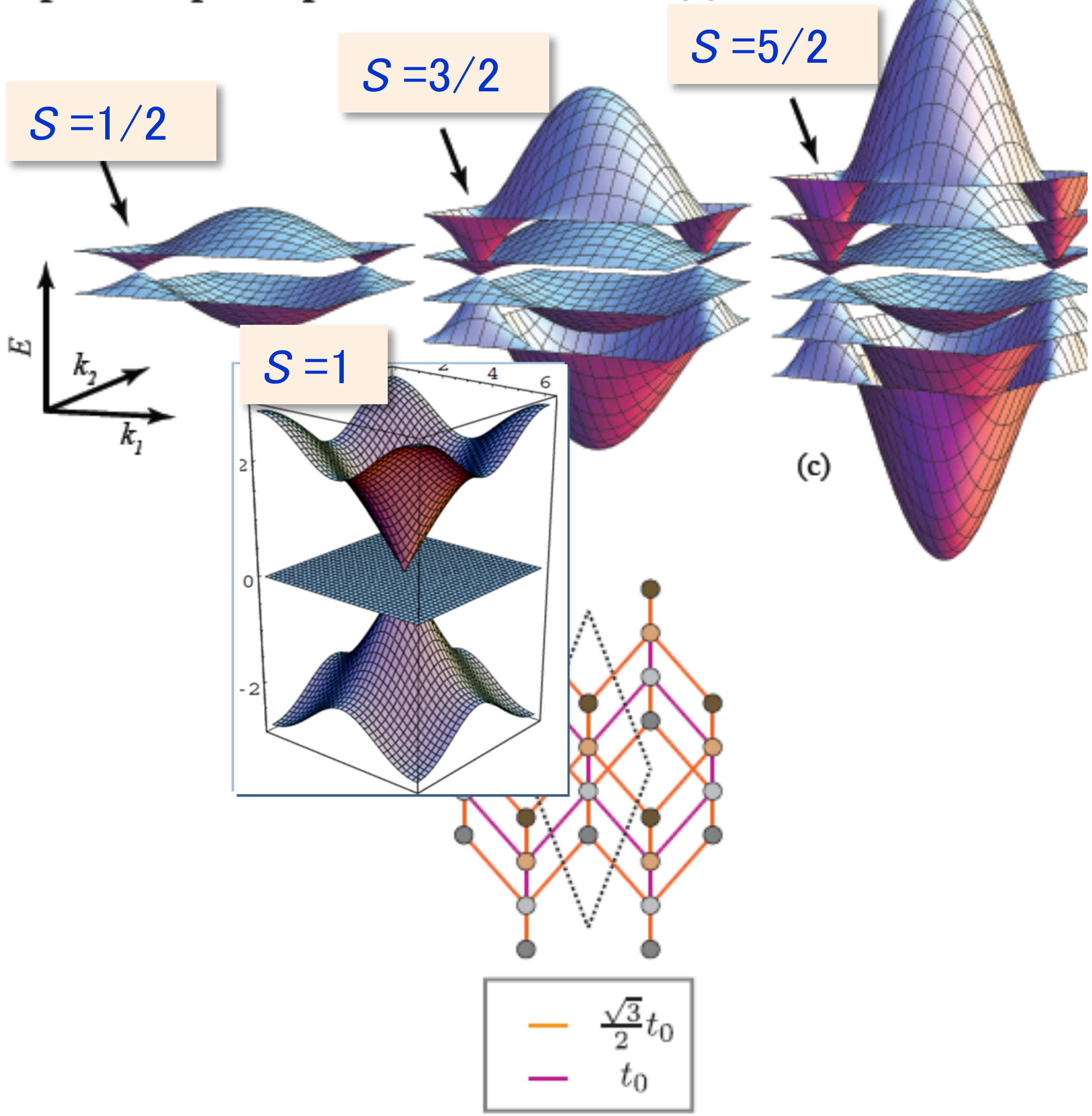
Long-period graphene

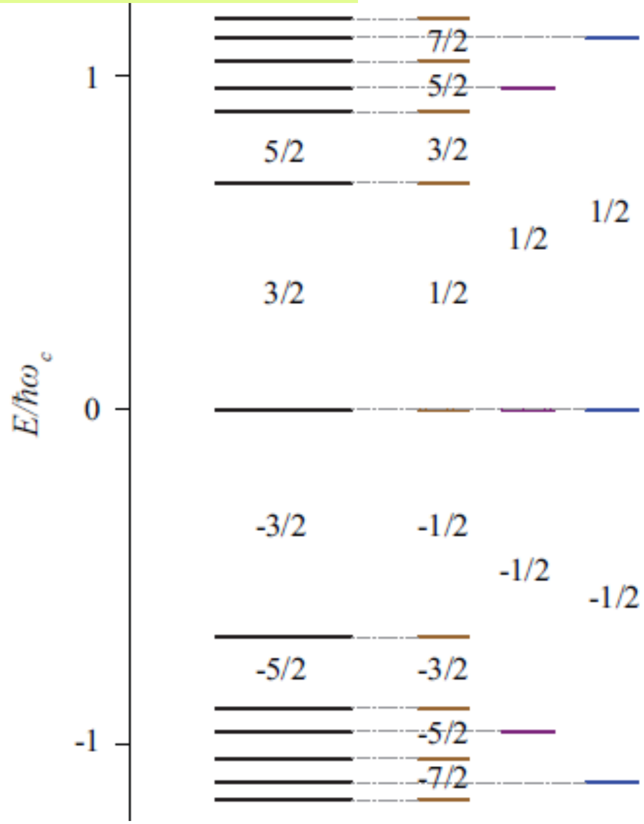


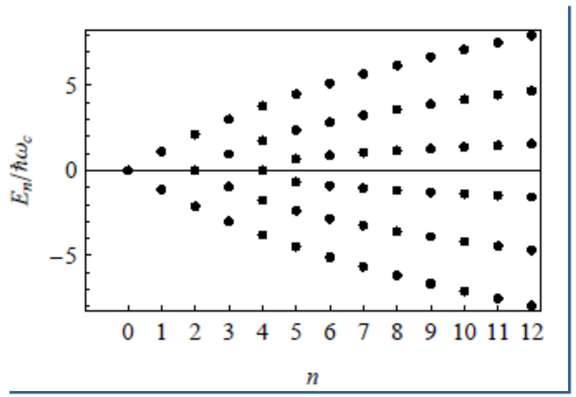
(Watanabe et al, Proc HMF 2010)

Spin S rep of SU(2) → each Dirac pt comprises (S+1/2) cones

higher-pseudospin representations of SU(2)

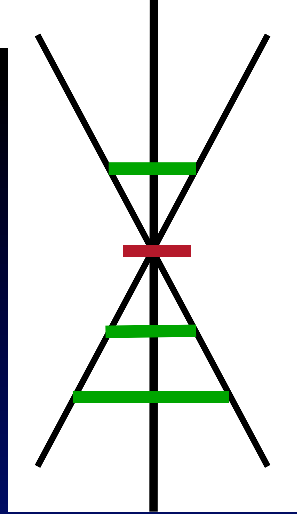


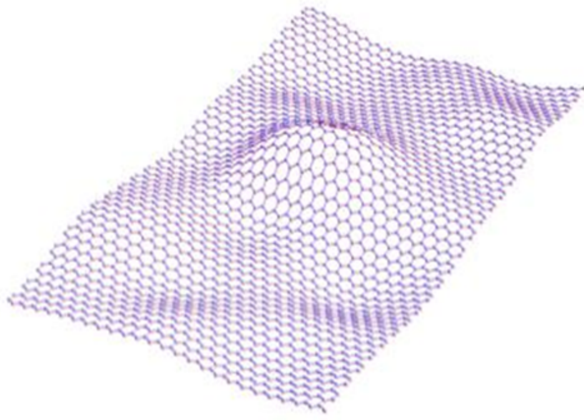




Disorder →

n =0 Landau level robust?



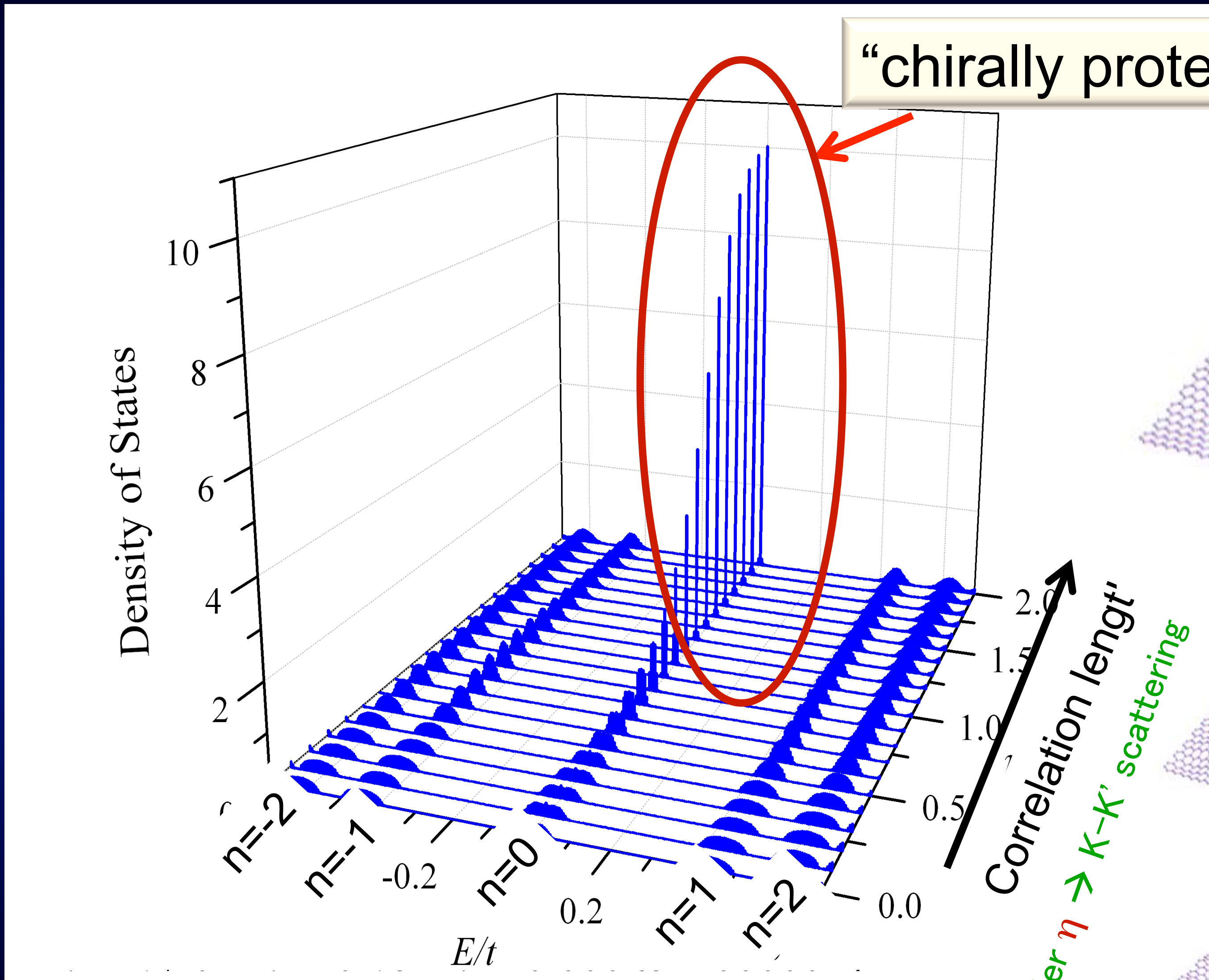


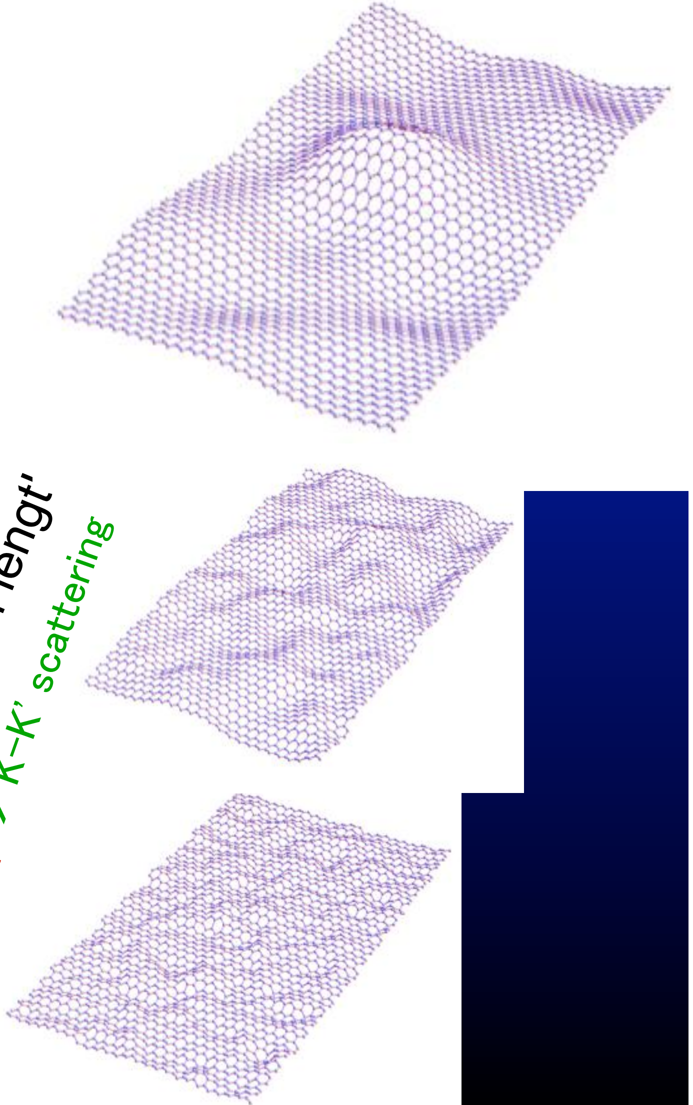
--- yes, for chiral-symmetric disorders

Disorder in graphene	*Chiral symmetry*
impurities	no
random bonds	yes
random mag fields	yes
ripple	yes
Kekulean bond orders	yes

Ripple (bond disorder) ← respects chiral symm

(Kawarabayashi et al, PRL 2009)

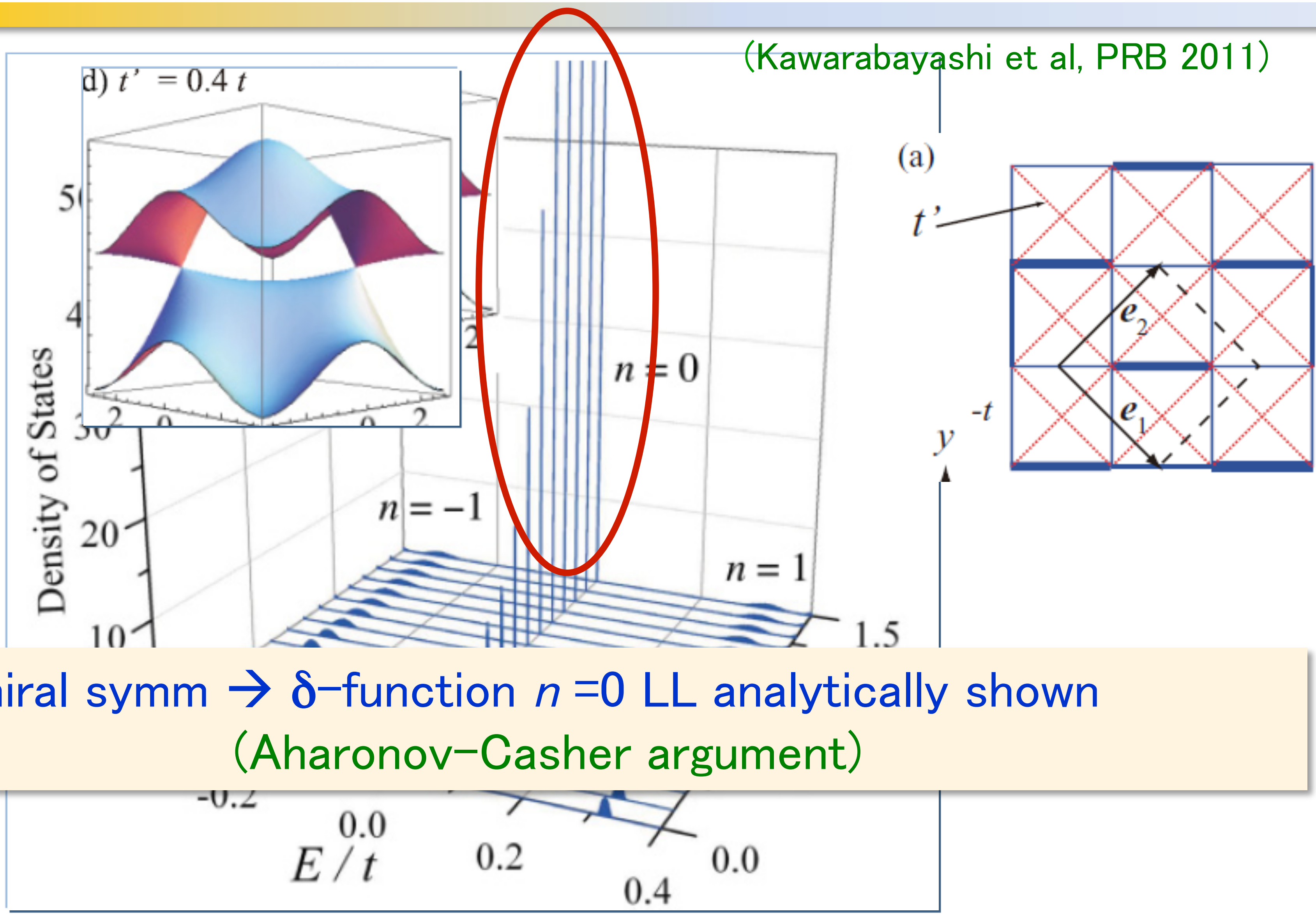




Cf. Random Dirac fermion with chiral symm
(Ludwig et al, RB 1994)

A tilted-cone lattice model + bond disorder

(Kawarabayashi et al, PRB 2011)



Gen. chiral symm → δ–function n =0 LL analytically shown

(Aharonov–Casher argument)

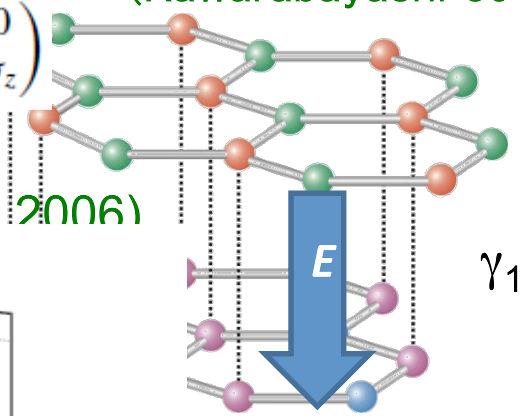
QHE in bilayer graphene

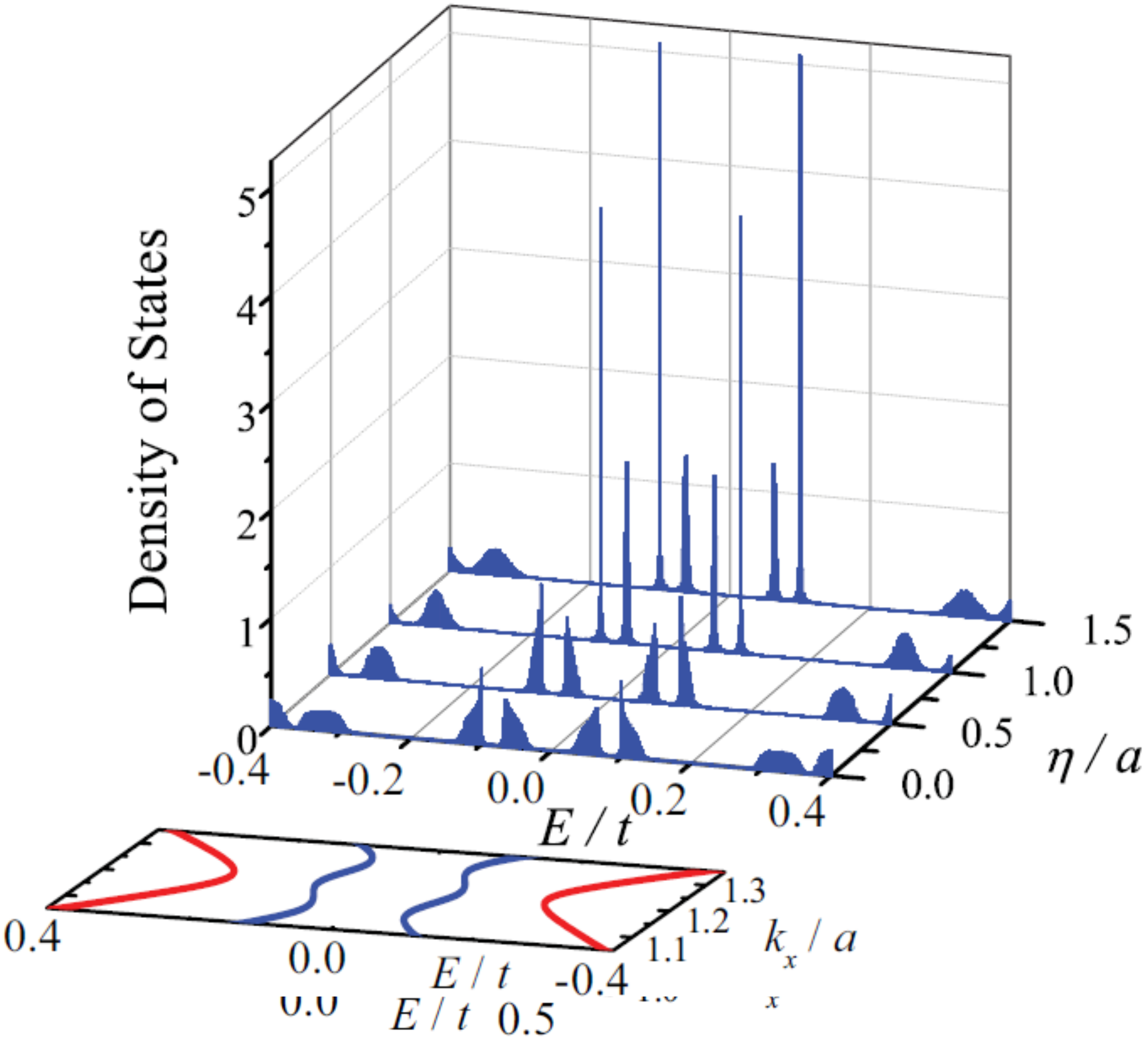
(Kawarabayashi et al, PRB 2012)

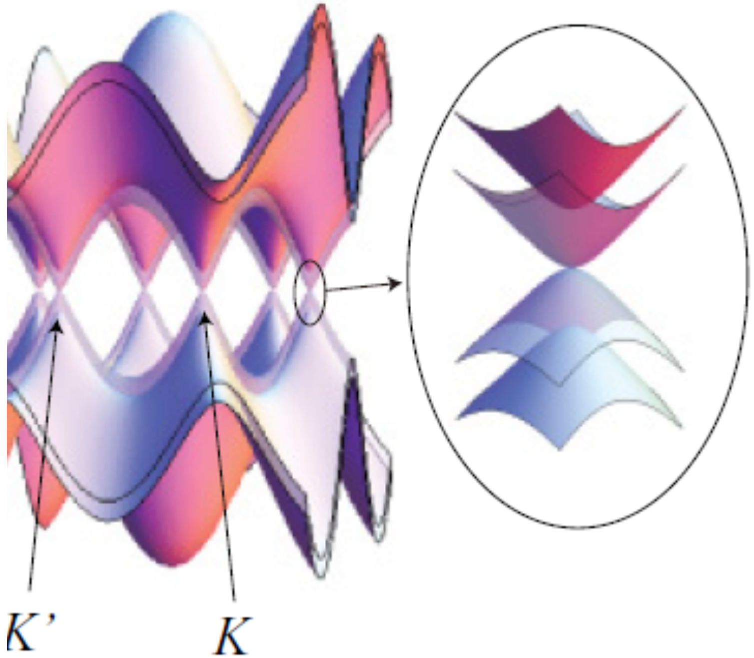
Chiral symmetric model $\Gamma = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$

No Dirac cones

$E_n = \pm\hbar\omega_c\sqrt{n(n-1)}$ (McCann & Fal'ko 2006)



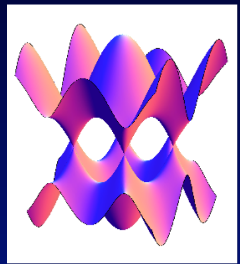


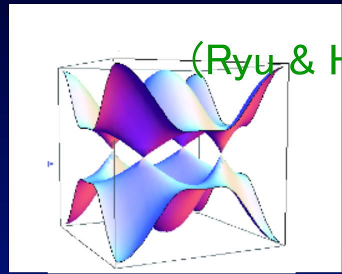


Graphene ⟷ d-wave SC

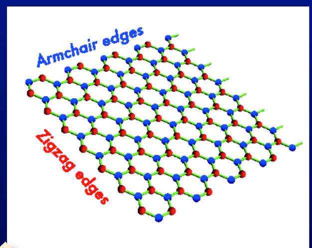
Dirac Hamiltonian ~ Bogoliubov-de Gennes Hamiltonian (2x2)

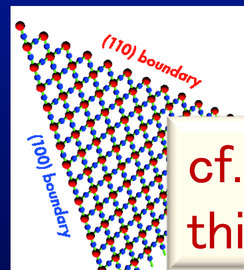
(chiral symmetry ⟷ time-reversal symm)





(Ryu & Hatsugai, PRL 2002)





cf. Tanaka,
this symposium

zero edge modes

Z_2 Berry phase

localised Andreev modes

Chiral symmetry

can also dominates many-body physics

as in a "chiral condensate" in graphene QHE

(Hamamoto et al, arXiv:1305.7314)

that can explain Philip Kim's experiment

(PRL 2012; nat phys 2012) for a ν=0 gap $\propto$ B

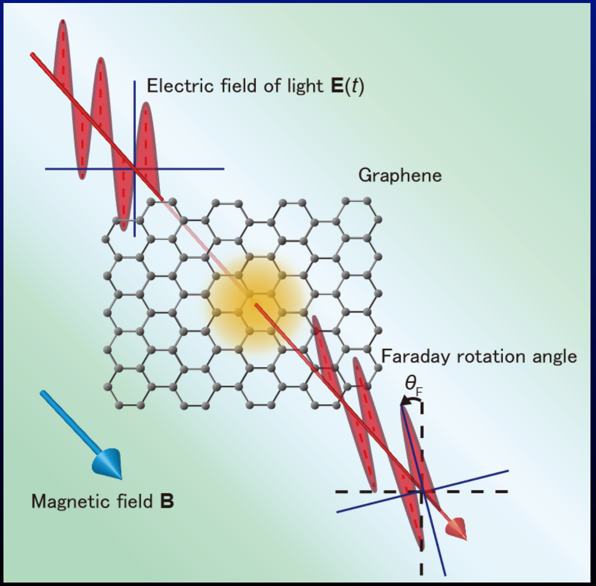
Plan of the talk

(a)Topological and chiral aspects in graphene

--- how general?

(b)Optical Hall effect in graphene in QHE regime

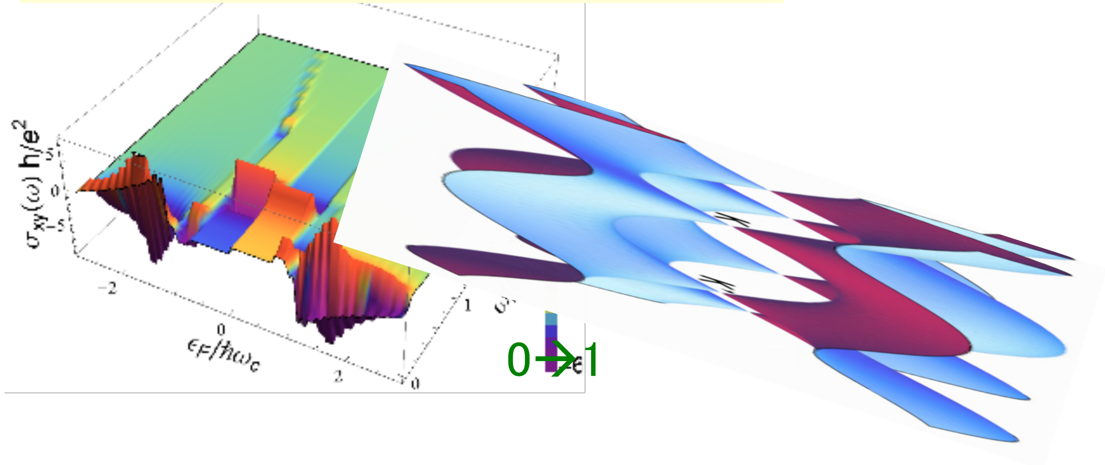
(c)Charged vacuum in graphene dot



Optical Hall σ_{xy} (ω) for honeycomb lattice

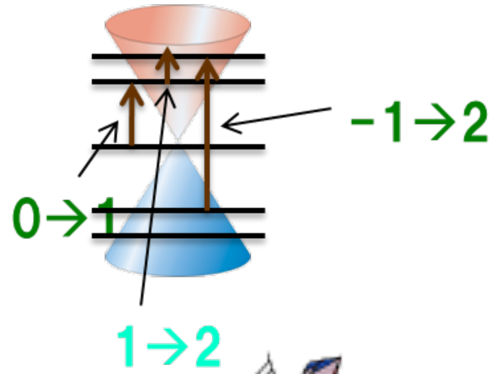
(Morimoto, Hatsugai & Aoki, PRL 2009)

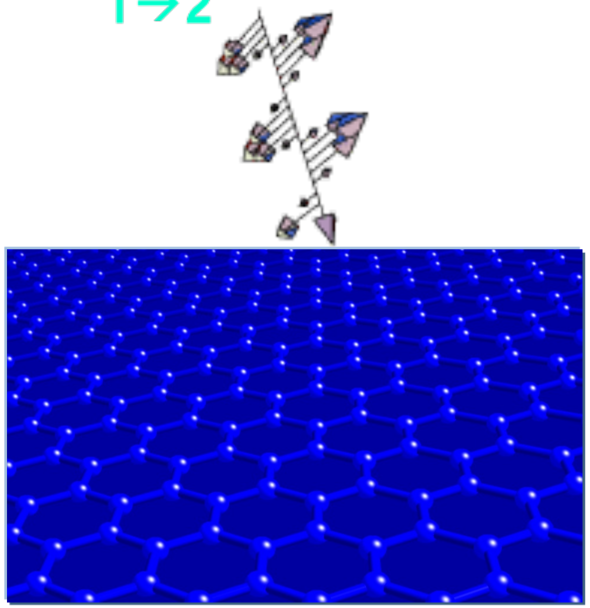
Chiral symmetric bond disorder $\langle|\delta t|\rangle = 0.1t$



1→2

Resonance at ω_c
(clean lim → $\frac{n}{h}\frac{\omega_c^2}{\omega_c^2-\omega^2}$





Faraday rotation (in THz)~ fine structure const:

"α seen as a rotation"

Faraday rotation

Faraday rotation ∝ optical Hall cond.

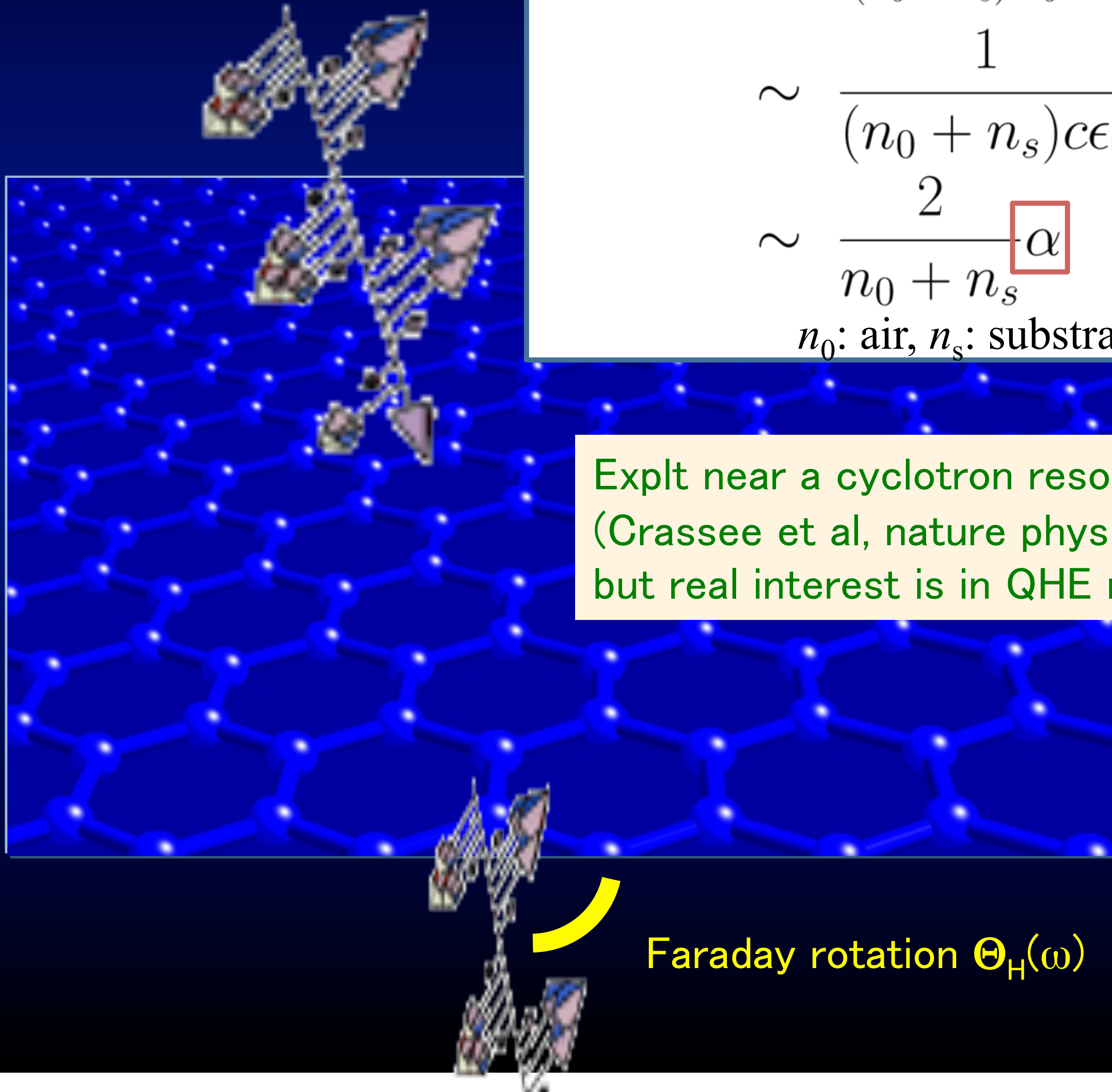
$$\Theta_H(\omega) \sim \frac{1}{(n_0+n_s)c\epsilon_0}\sigma_{xy}(\omega)$$

plateau

$$\sim \frac{1}{(n_0+n_s)c\epsilon_0}\frac{e^2}{h}$$

$$\sim \frac{2}{n_0+n_s}\boxed{\alpha} \quad \sim \boxed{10\ \mathrm{mrad}}$$

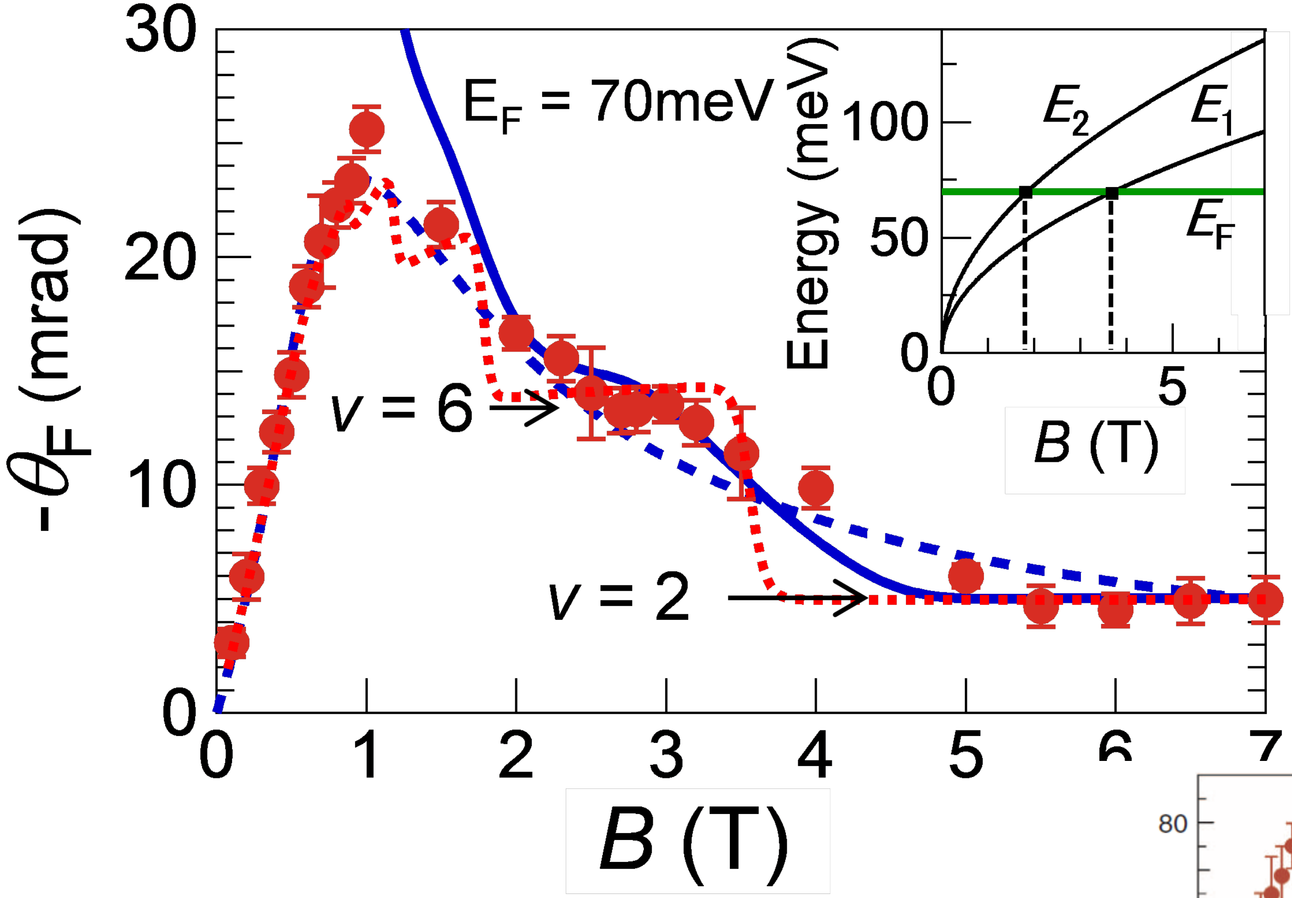
n_0: air, n_s: substrate



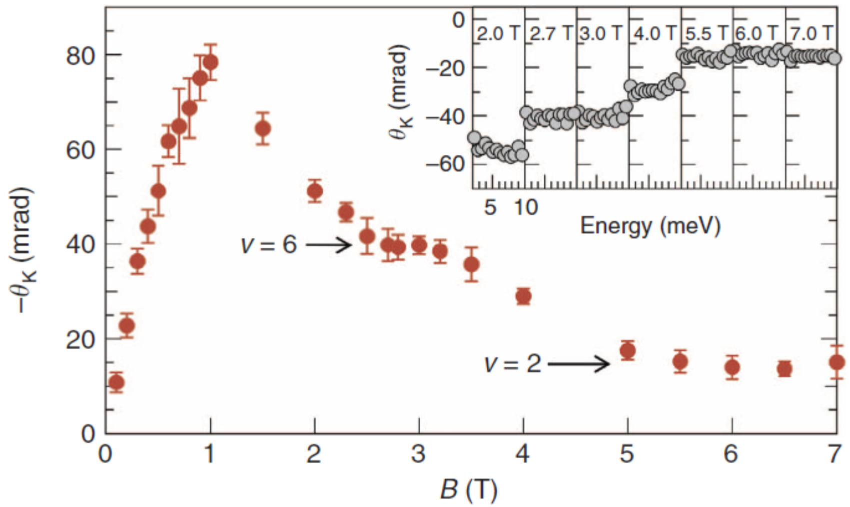
Explt near a cyclotron resonance
(Crassee et al, nature phys 2010),
but real interest is in QHE regime

(Shimano, Yumoto, Yoo, Matsunaga, Tanabe, Hibino, Morimoto & Aoki, Nature Commun 2013)

Faraday rotation



Kerr rotation

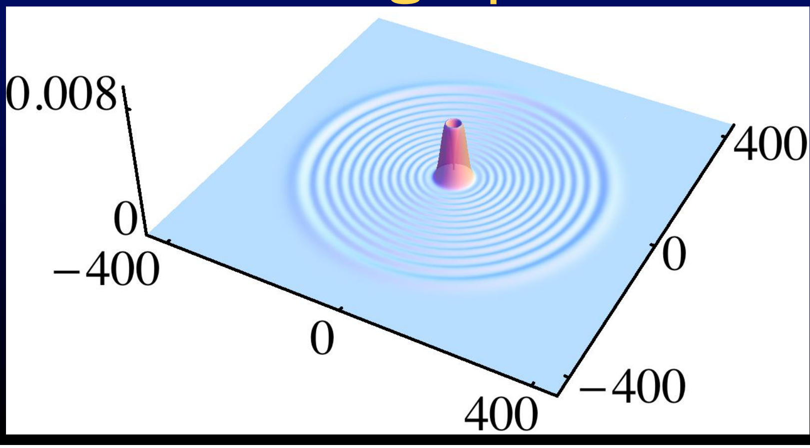


Plan of the talk

(a)Topological and chiral aspects in graphene
--- how general?

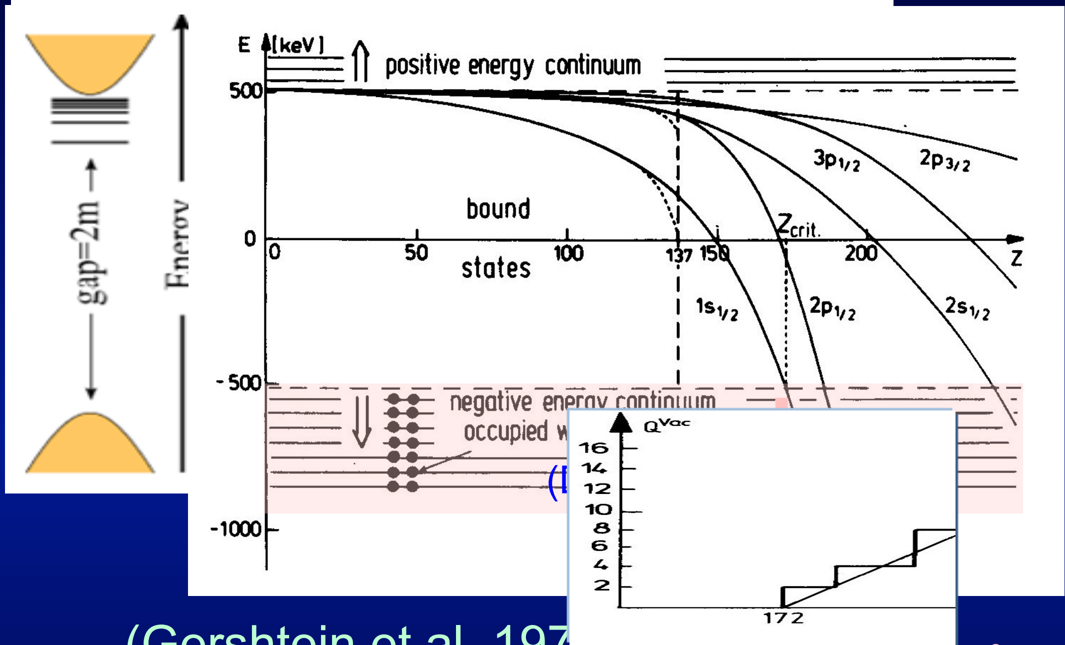
(b)Optical quantum Hall effect in graphene

(c)Charged vacuum in graphene dot

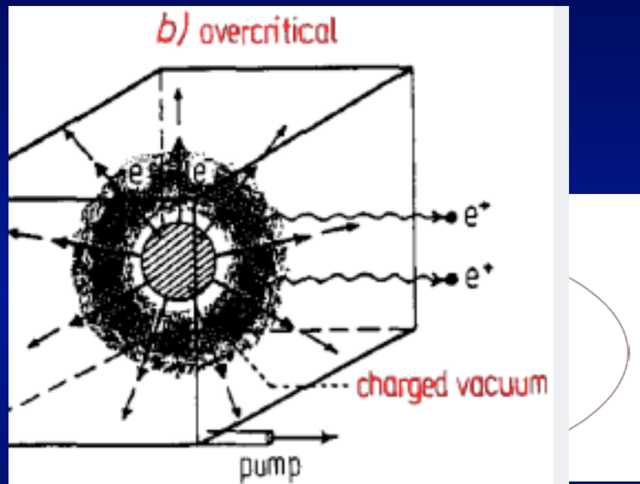


Atomic physics: "Supercritical nuclei" → charged vacuum

Diving states (Hydrogen atom in QED)



(Gershtein et al, 1972,
Rafelski et al, Phys Rep 1978)



→ a bound state enters negative continuum: supercritical

$Z > 1/\alpha = 137$, $\alpha = e^2/hc$,
more accurately, $Z > 172$)

- Despite much experimental effort this remains unobserved

"Supercritical nuclei" in graphene dots

Relativistic hydrogen-like atom in *D*-spatial dimensions

$$E/m = \left[1 + \frac{(Z\alpha)^2}{\left(n - |\kappa| + \frac{D-3}{2} + \sqrt{\kappa^2 - (Z\alpha)^2} \right)^2} \right]^{-1/2}$$

(Katsura & Aoki, J Math Phys, 2006)

$Z\alpha > (D - 1) / 2 \quad (\alpha = e^2/hc)$

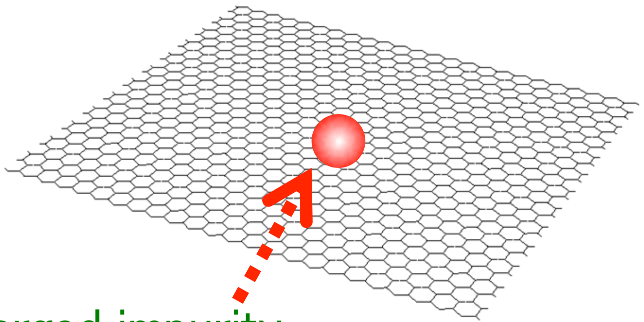
$Z > 137$ for $D = 3$

$Z >\sim 1$ for $D = 2$, $\alpha = e^2/hv_F$
for a (massive) graphene

First pointed out for graphene + Coulomb impurity by
Pereira et al, PRL 2007; PRB 2008

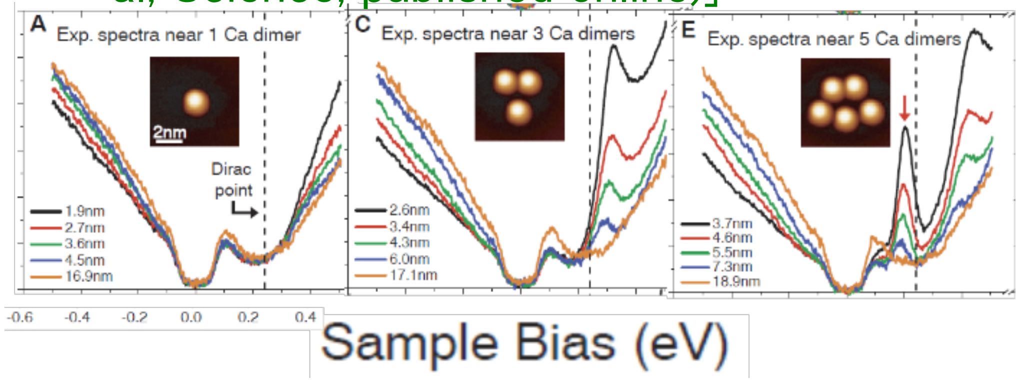
Point (1): general potential other than Coulombic produces supercritical situations

Pereira et al, PRL 2007; PRB 2008

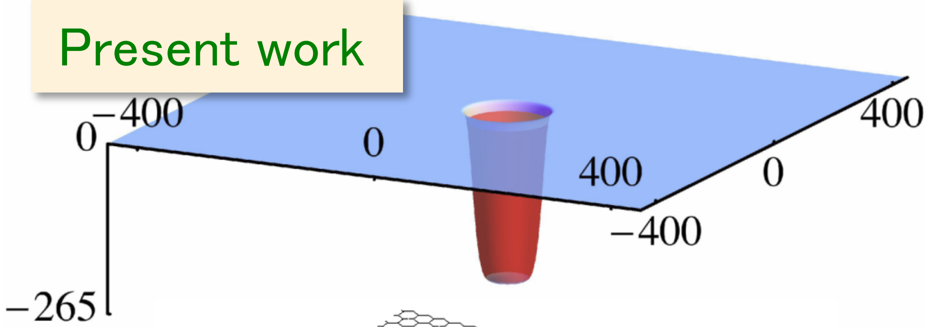


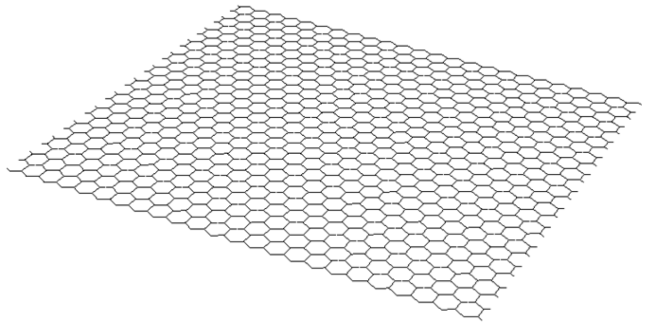
Charged impurity
→change the charge (difficult)

[cf. Crommie's group (Wang et al, Science, published online)]

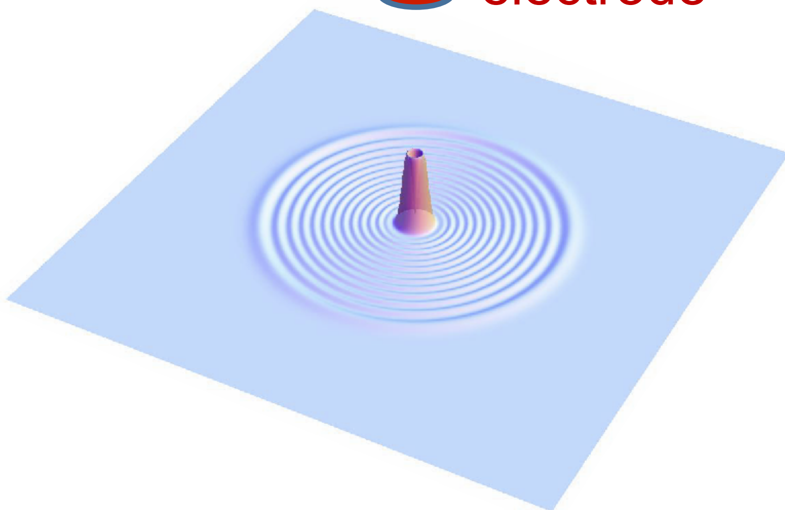


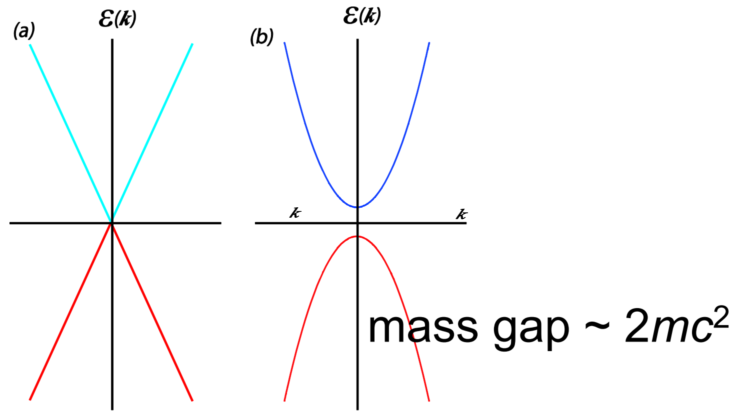
Present work





electrode





A mass gap required

--- Recently a mass gap ~ 0.1 eV has been found in

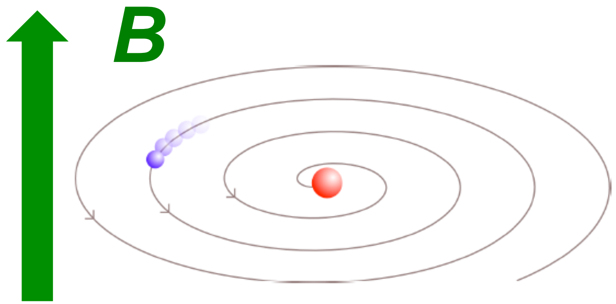
graphene

on BN (Giovannetti et al 2007; Ci et al 2012; Yelge et al 2012),

on Ru (Enderlein et al 2010);

silicene on ZrB2 (Fleurence et al PRL 2012)

Point (2): Magnetic–field control of charged vacuum



Magnetic field modifies supercritical situation when

rest mass energy ~ cyclotron energy

	Real electron	Graphene
m_0c^2	0.5 MeV	100 meV
c	3×10^8 ms^{-1}	10^6 ms^{-1}
$m_0c^2 = \hbar\omega_c$ when	$B \sim 10^{10}$ T	$B \sim 10$ T

Orders of magnitude (~ 10 $^{-9}$) smaller *B* suffices in graphene!

Formalism

Dirac Hamiltonian:

$$H = (\gamma/\hbar)\boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A}) + V_{\text{dot}} + m_0 c^2 \sigma_z$$

$$c = \gamma/\hbar$$

γ = 646 meV, m_0c^2 = 100 meV

Wavefunction of a circularly-symmetric dot:

$$\phi(\mathbf{r}) = (\chi_1(r)\exp(i(m-1)\theta), \chi_2(r)\exp(im\theta))$$

in spherical coordinates (r, θ),
m: angular momentum quantum #

Radial functions satisfy, with $f_1 = \sqrt{r}\chi_1, if_2 = \sqrt{r}\chi_2$

$$\frac{V + m_0c^2}{\gamma} f_1 + \left(\frac{d}{dr} + \frac{m - \frac{1}{2}}{r} + \frac{e}{\hbar}A_\theta\right) f_2 = \frac{E}{\gamma} f_1,$$

$$\left(-\frac{d}{dr} + \frac{m - \frac{1}{2}}{r} + \frac{e}{\hbar}A_\theta\right) f_1 + \frac{V - m_0c^2}{\gamma} f_2 = \frac{E}{\gamma} f_2.$$

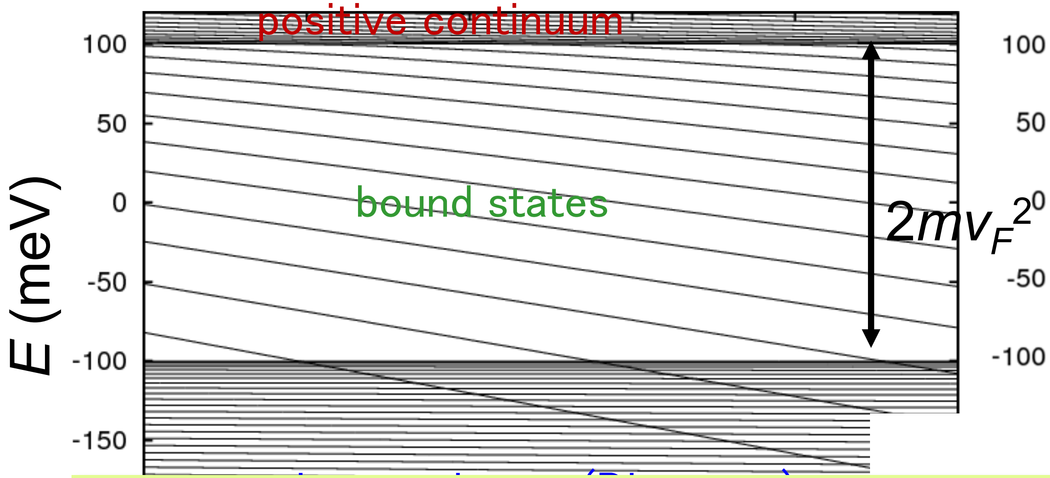
Result

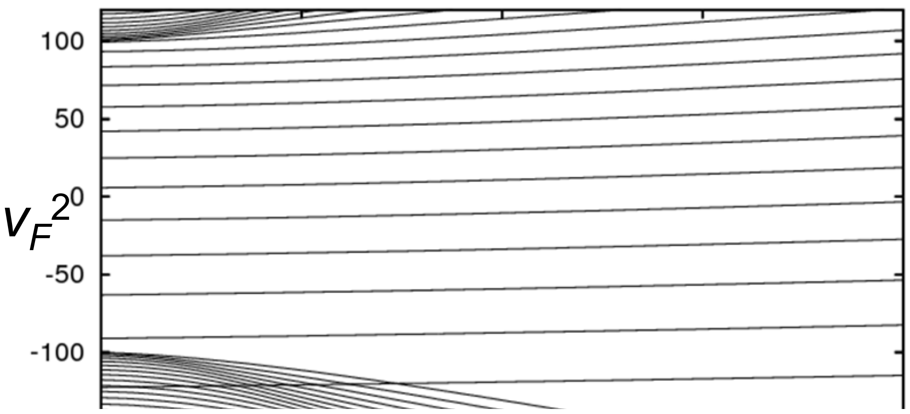
B = 0, *p*=2

K point, *m*=1 (*l* =0)

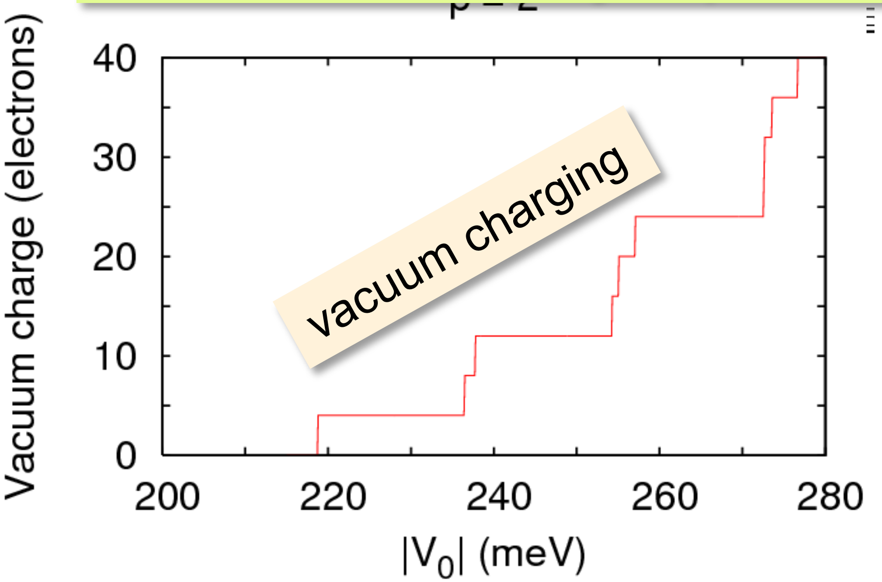
B > 0

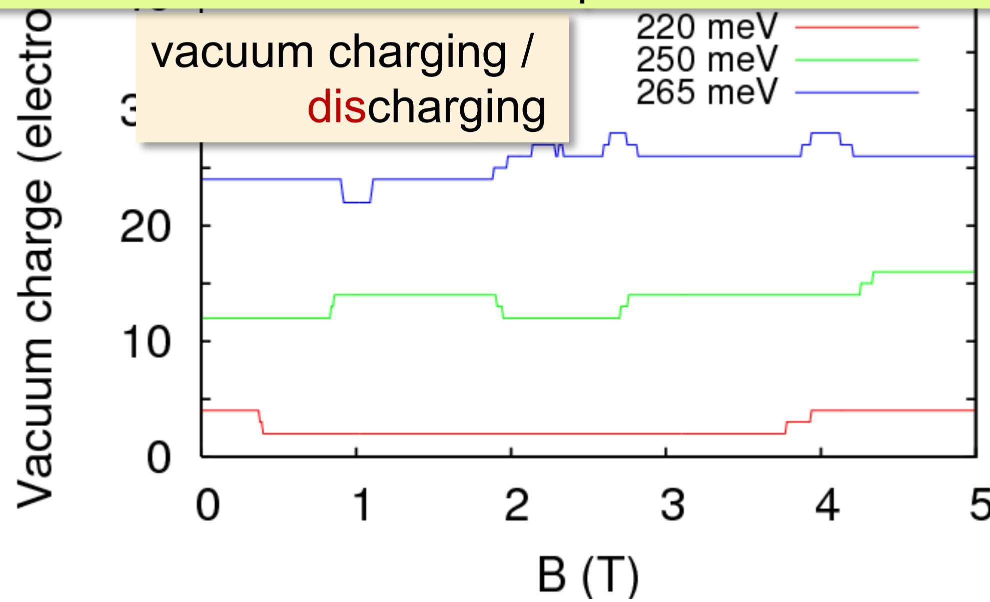
Fixed V_0 = 265 meV, K point, *m*=1

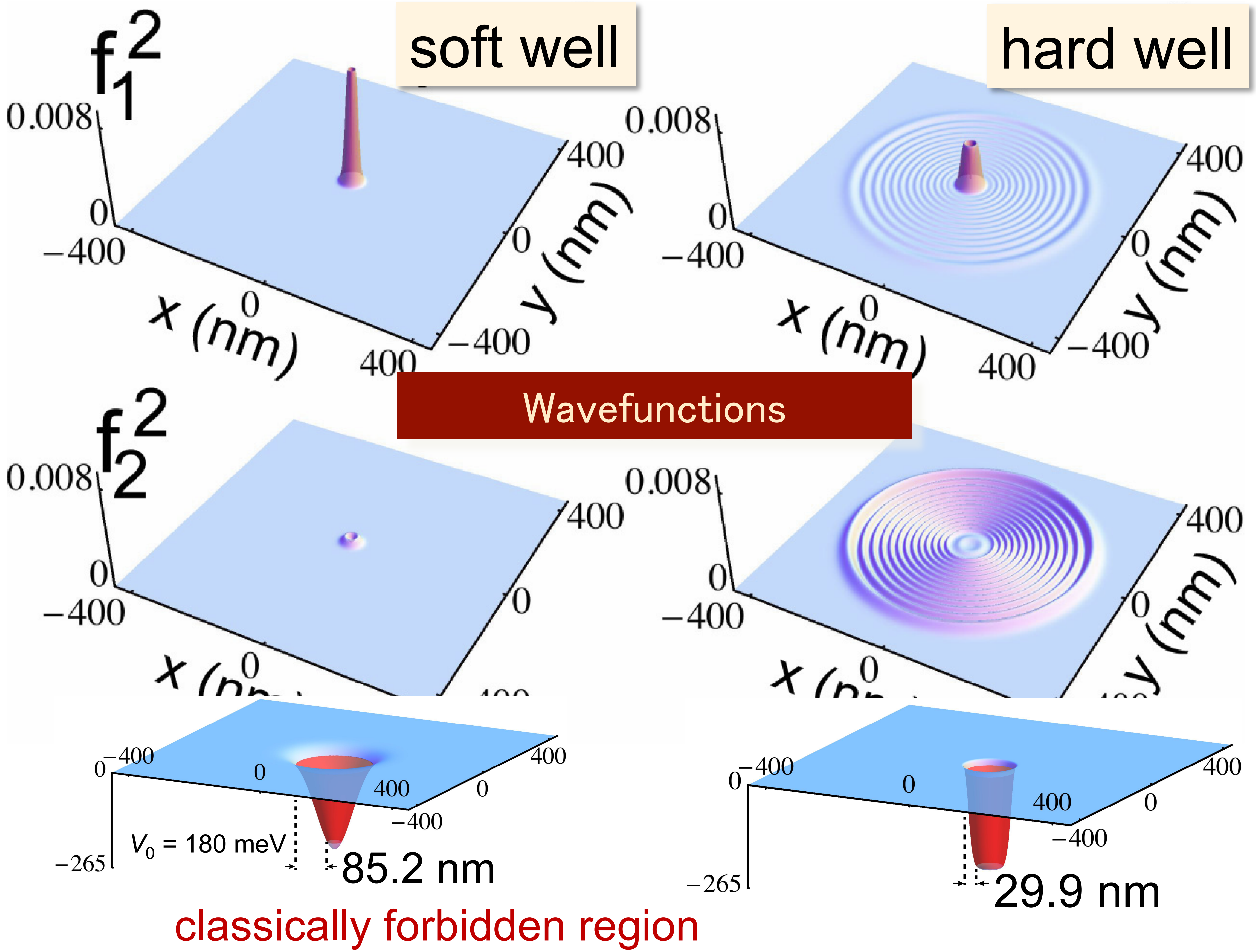




•Vacuum charge jumps when a level crosses E_F at *K* and *K'*



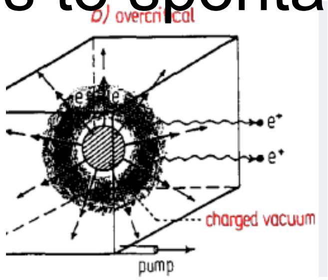




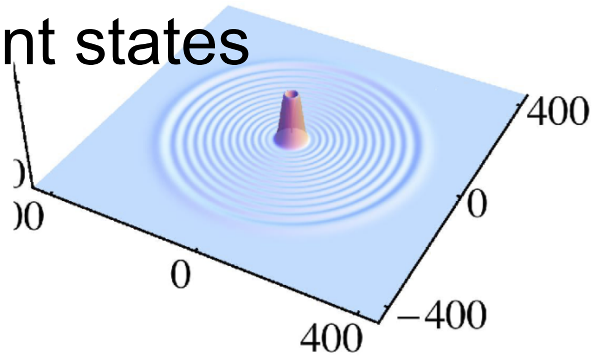
soft well
hard well
Wavefunctions
f_1^2
f_2^2
0.008
0
−400
x (nm)
y (nm)
400
V_0 = 180 meV
−265
85.2 nm
29.9 nm
classically forbidden region

Experimental detection

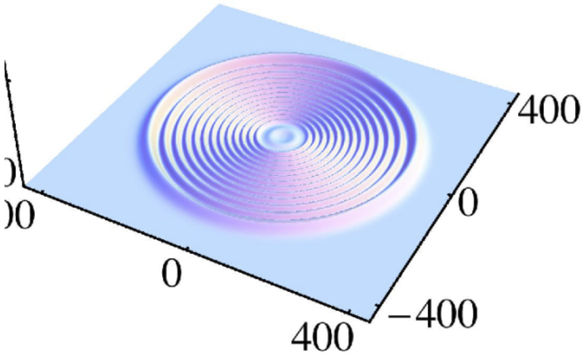
- Hole emission analogous to spontaneous positron emission.



- Probing strongly hybridized (Fano-) resonant states with STM/STS (at a few K).



- Direct measurement of vacuum charge from capacitance (*a la* Ashoori) or quantum point contact.

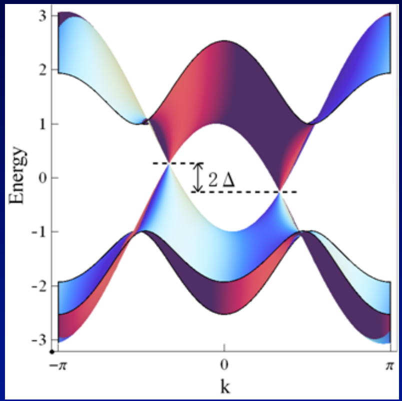


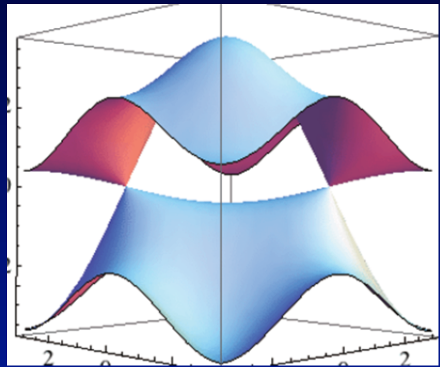
Maksym and Aoki, arXiv:1211.5552

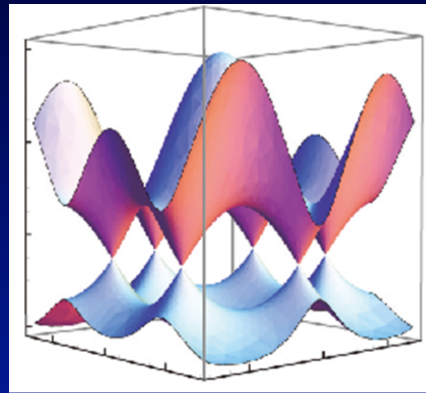
Summary

(a)Topological and chiral aspects in graphene

--- how general? unexpectedly robust

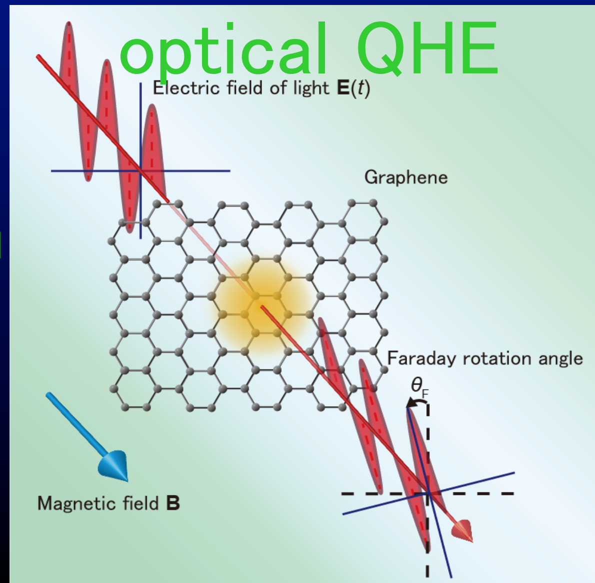






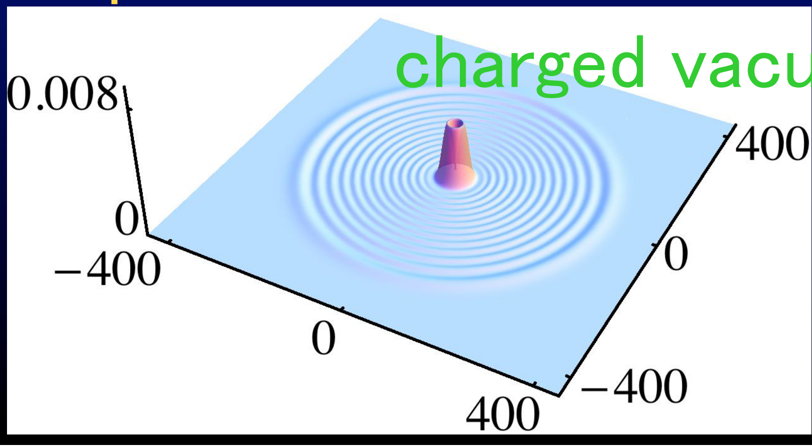
(b)Does graphene QHE appear in optics?

optical QHE



(c)Graphene quantum dot?

charged vacuum



Future problems

- ✔ Extension to wider phenomena
- ✔ Extension to topological insulators