http://www.topological-qp.jp/english/index.html

Theory of superconducting topological insulator

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ISSP June 13 (2013)

Main collaborators

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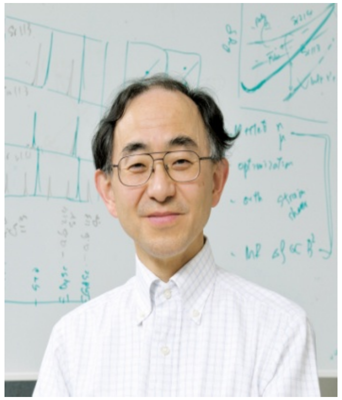
M. Sato（Nagoya）

K. Yada (Nagoya / Twente)

T. Hashimoto（Nagoya）

Main collaborators

N. Nagaosa (Univ. Tokyo / Riken）



Y. Ando (Osaka)

S. Nakosai (Univ. Tokyo）

K. Segawa (Osaka)

Contents of our talk

(1)What is superconducting topological insulator

(2)Andreev bound state and quasi particle tunneling

(3)Josephson current

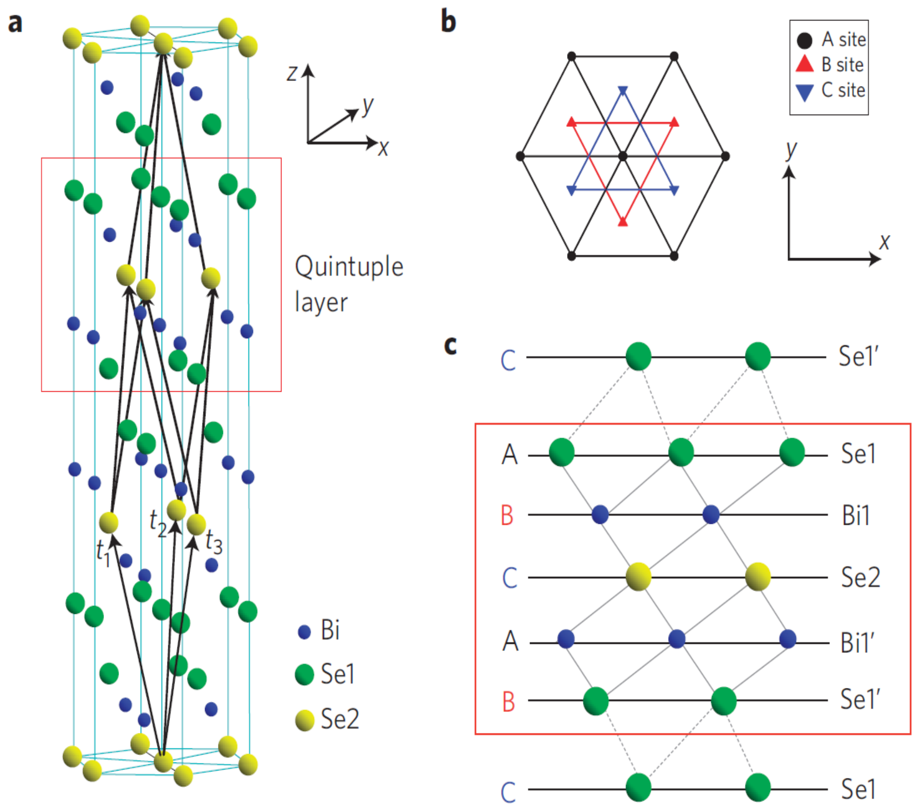
(4)Spin susceptibility

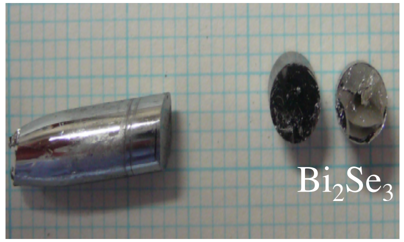
(5)Relevant Rashba superconductor system

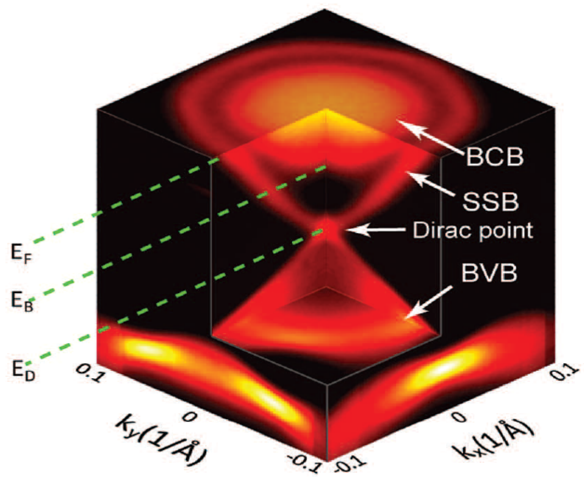
Topological insulator Bi_2Se_3

- Nonzero topological number Z_2
- Helical Dirac Cone as a surface state
- Strong spin-orbit coupling

Crystal structure Bi_2Se_3



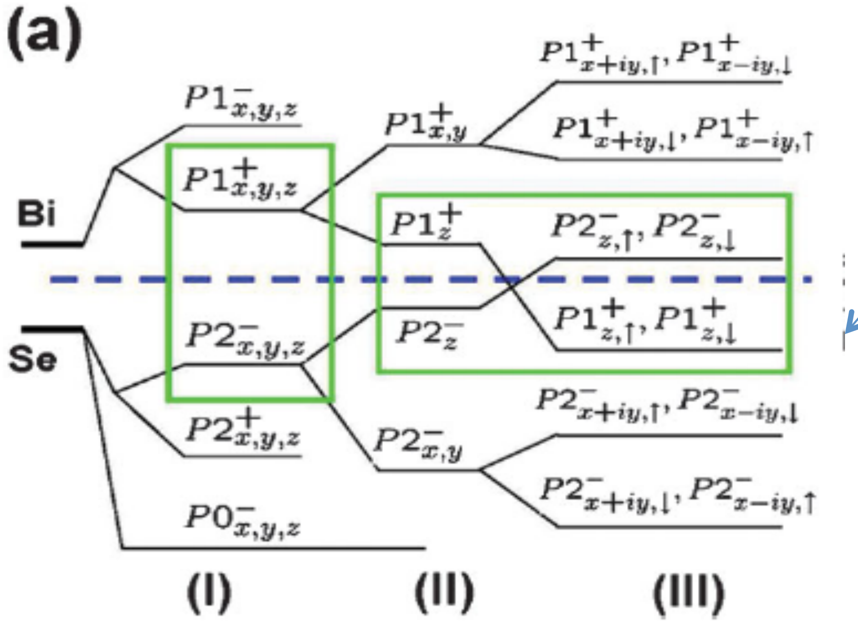




Electronic band structure of Bi_2Se_3 measured by ARPES

Y. L. Chen *et al*. Science 329, 659 (2010)

Electronic states of Bi_2Se_3

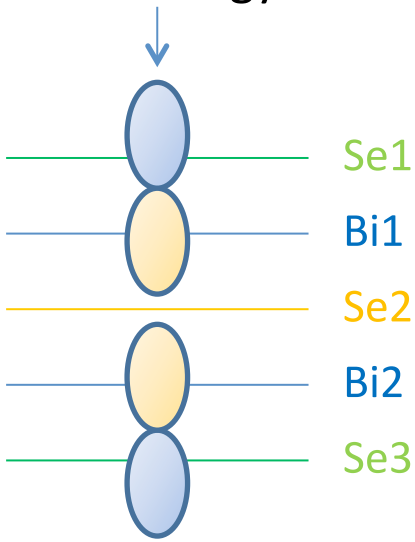


bonding

crystal field

SO coupling

two low-energy effective orbitals



unit cell of Bi_2Se_3

energy levels of the atomic orbitals in Bi_2Se_3

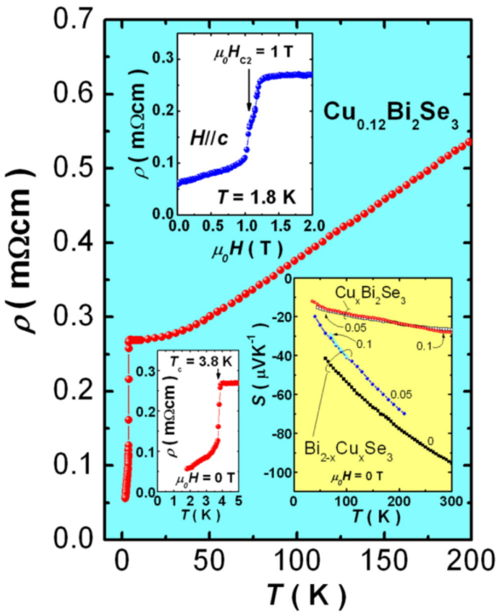
Zhang et al, Nature 09

Superconducting topological insulator

$Cu_xBi_2Se_3$

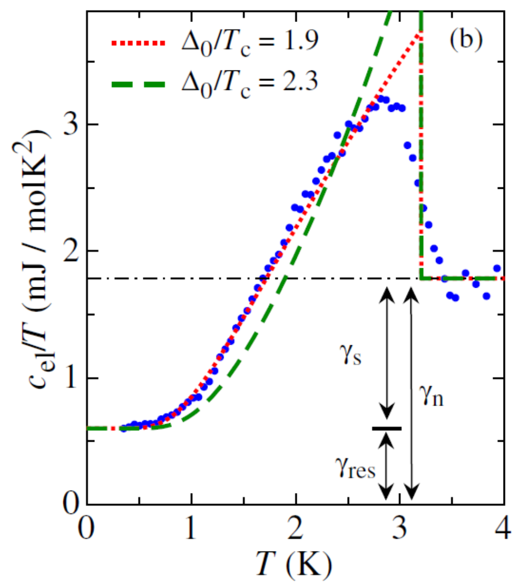
Cu doped topological insulator

Resistivity



Y.S.Hor *et al*, PRL 104, 057001 (2010)

Specific heat



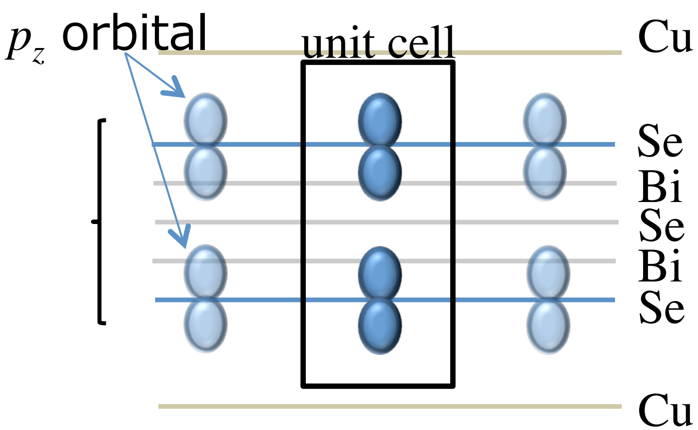
M. Kriener *et al.*, PRL **106**, 127001 (2011)

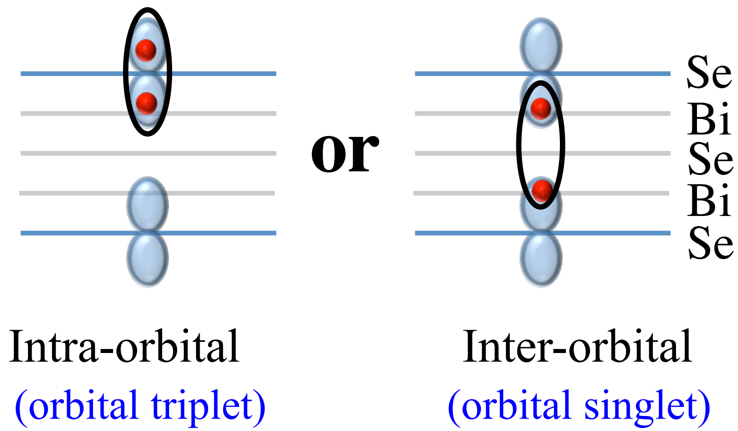
Tc 3.8K

Candidate of pair potentials

Liang Fu, Erez Berg, PRL,105, 097001 (2010)

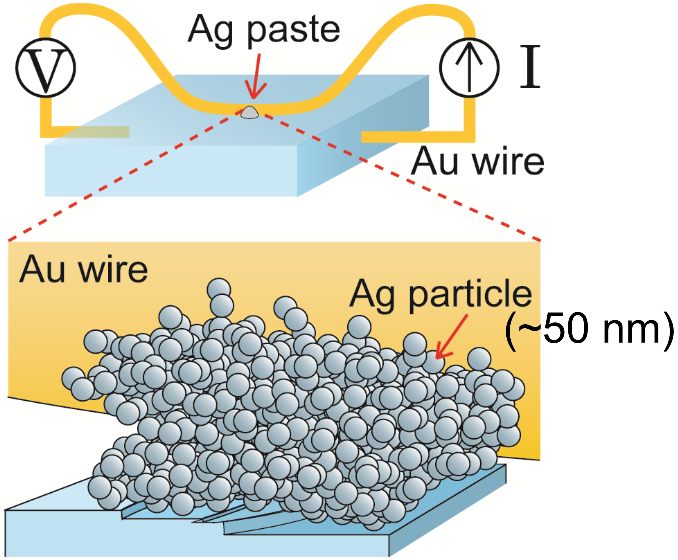
	Energy gap	irreducible representation	spin	Orbital	Inversion symmetry
Δ_1	full gap	A_{1g}	singlet	intra	even
Δ_2	full gap	A_{1u}	triplet	inter	odd
Δ_3	point node	A_{2u}	singlet	intra	odd
Δ_4	point node	E_u	triplet	inter	odd

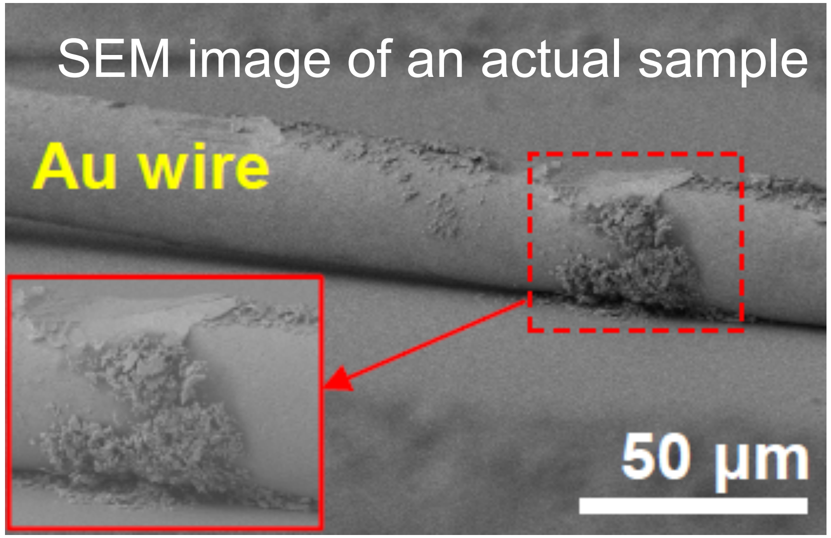


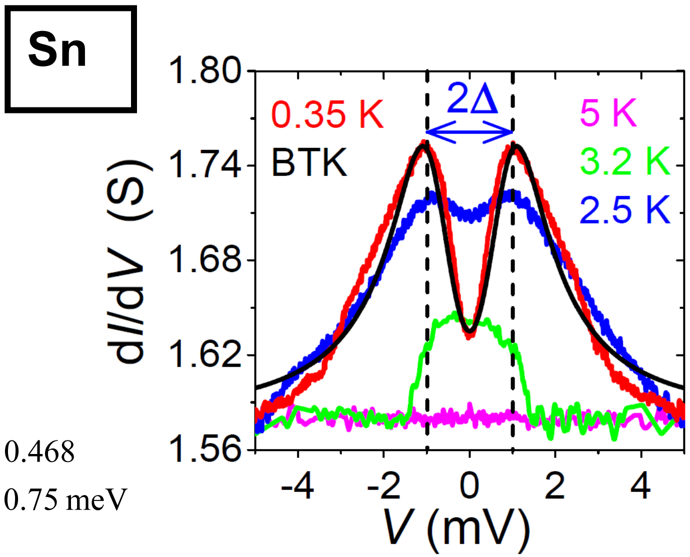


$Cu_xBi_2Se_3$ Effective orbital p_z orbital **(No momentum dependence)**

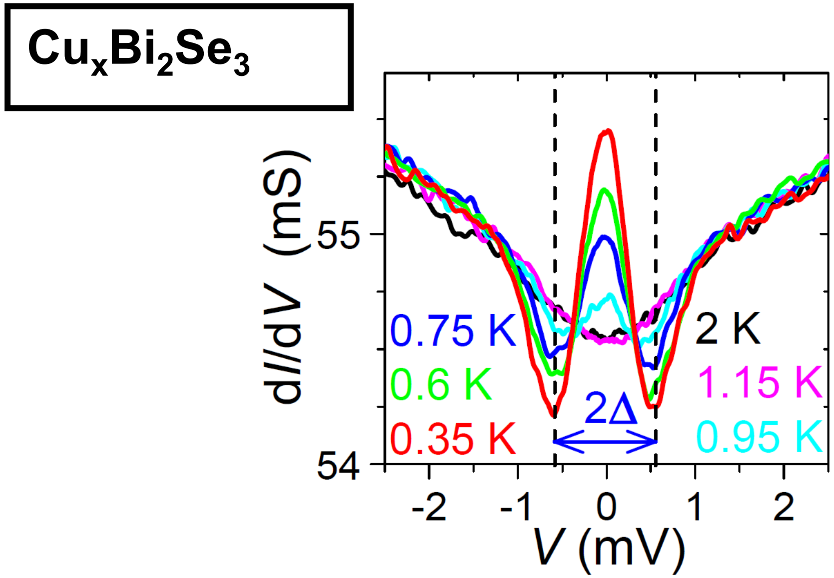
Tunneling spectroscopy







Z = 0.468
Γ = 0.75 meV



Ando's group (Osaka)

S. Sasaki et al PRL 107 217001 (2011)

Contents of our talk

(1)What is superconducting topological insulator

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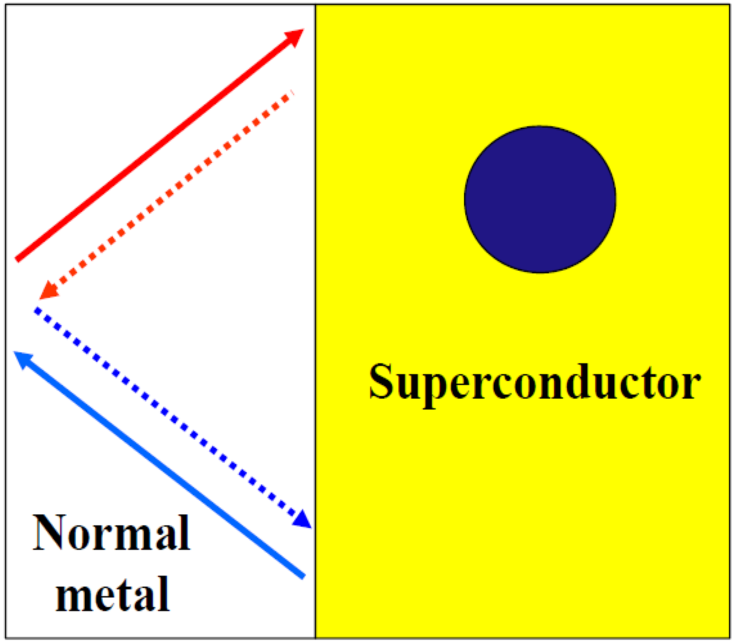
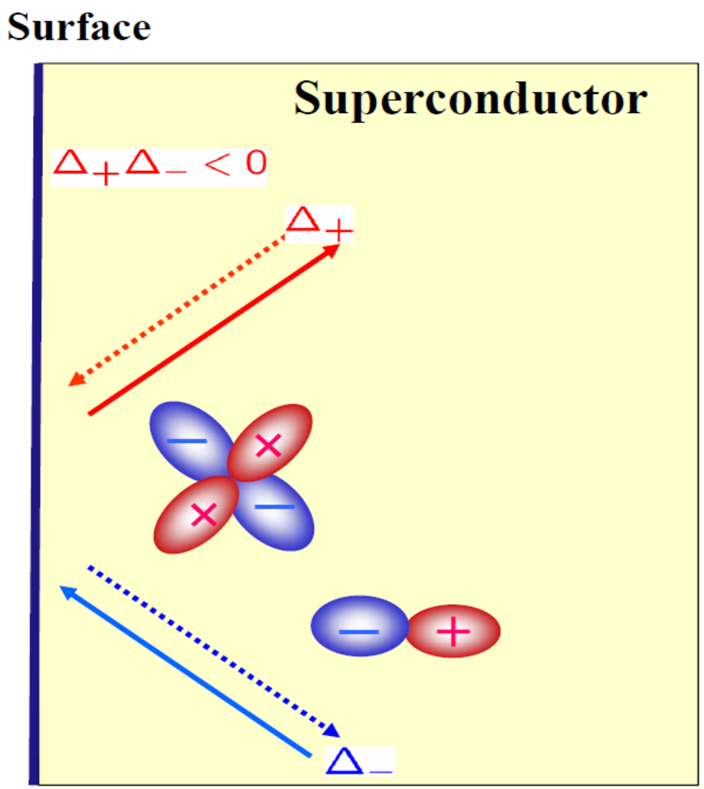
Andreev bound state (non-topological and topological)



Andreev bound state with non zero energy (de Gennes, Saint James)

Not edge state

Non topological



Mid gap (zero energy) Andreev bound state

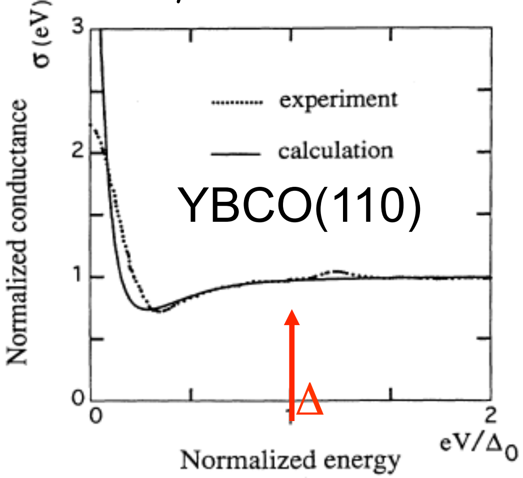
Surface Andreev bound state

Edge state Topological

L. Buchholtz & G. Zwicknagl (81):, J. Hara & K. Nagai : Prog. Theor. Phys. 74 (86)
C.R. Hu : (94) Tanaka Kashiwaya (95), ...

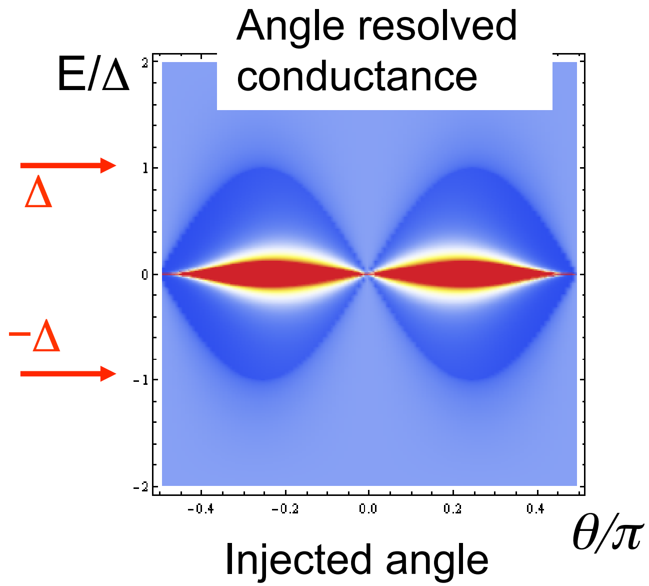
Tunneling spectrum in two-dimensional topological superconductors

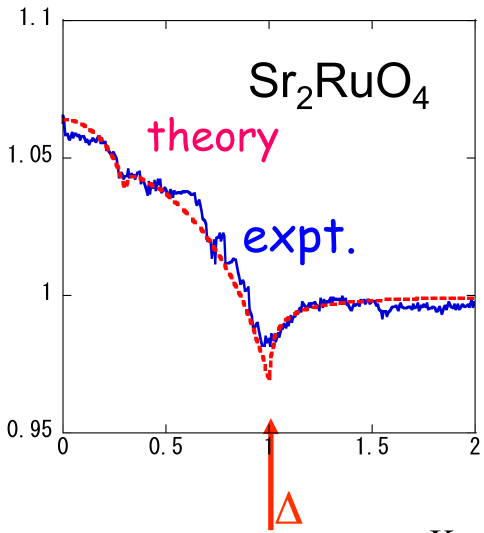
S.Kashiwaya, Tanaka, 1995



$d_{x2\text{-}y2}$-wave
nodal gap

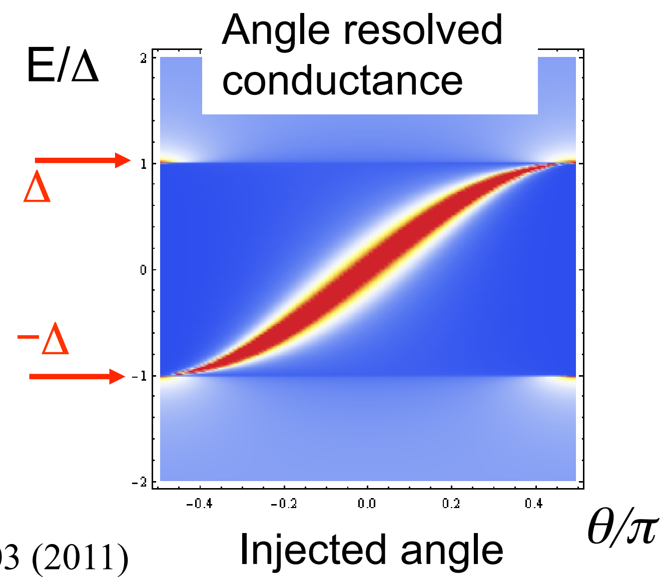
zero energy flat band
of surface ABS





chiral *p*-wave
full gap
chiral edge state (ABS)

broad zero-bias peak
due to linear dispersion



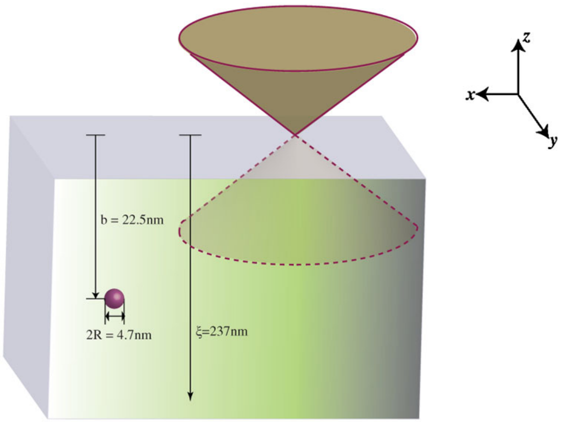
Kashiwaya *et al*, Phys. Rev. Lett. **107**, 077003 (2011)

Andreev bound state (topological edge state) and topological invariant

Andreev bound state	Topological invariant	Time reversal symmetry	Materials	Theory of tunneling	Insulator (semi-metal)
Flat	1d winding Number Z for fixed k_y AIII (BDI)class	○	Cuprate p_x-wave	PRL (1995) JPSJ(1998)	Graphene (zigzag edge)
Chiral	2d winding Number Z D class	×	Sr_2RuO_4 ^{3}He A	PRB (1997)	QHS QAHS
Helical	Z_2 DIII class	○	$s+p$-wave (NCS)	PRB (2007)	QSHS (2D Topological insulator)
Cone	3d winding Number Z DIII class	○	^{3}He B	PRB (2003)	Topological insulator

Sato, Kashiwaya, Maeno (Kotai Butsuri 2011) Tanaka, Sato, Nagaosa (JPSJ Review)

ABS in B-phase of superfluid ^{3}He

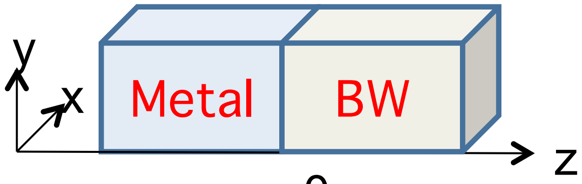


Cone type ABS

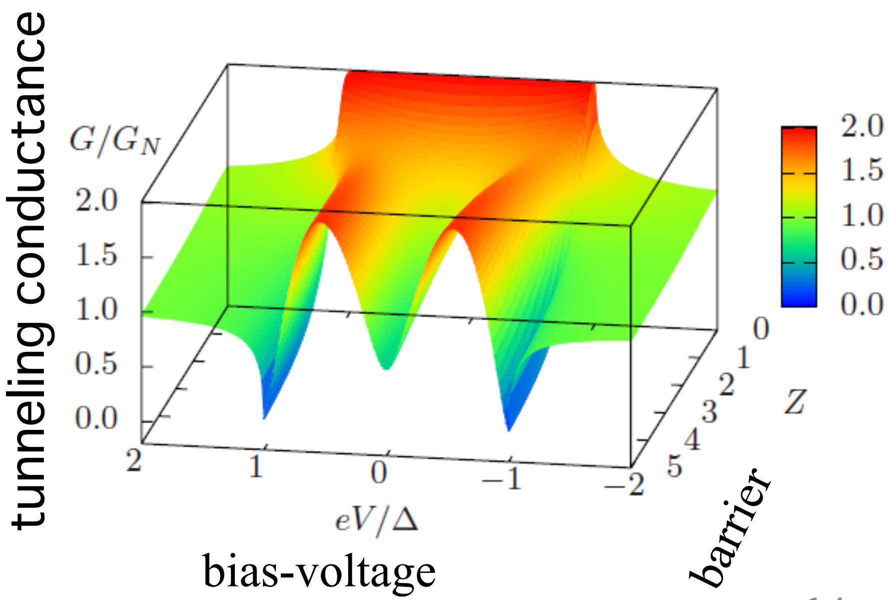
Salomaa Volovik (1988)
Schnyder (2008)
Roy (2008) Nagai (2009)
Qi (2009)
Kitaev(2009)
Chung, S.C. Zhang (2009)
Volovik (2009)

perpendicular injection ZES: Buchholtz and Zwicknagle (1981)

BW state (B-phase in ^{3}He)
full gap superconductor

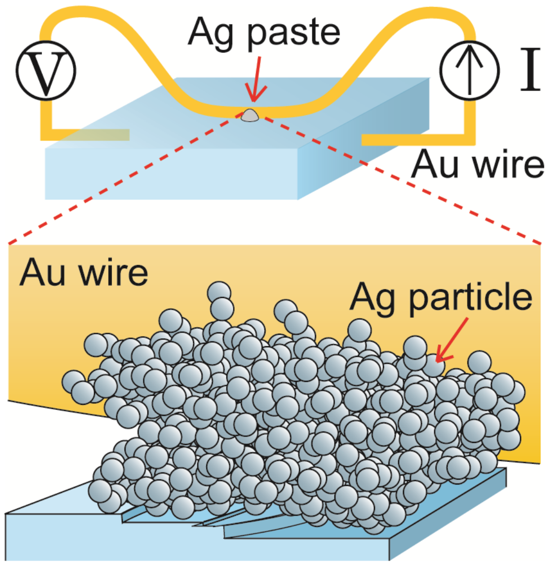


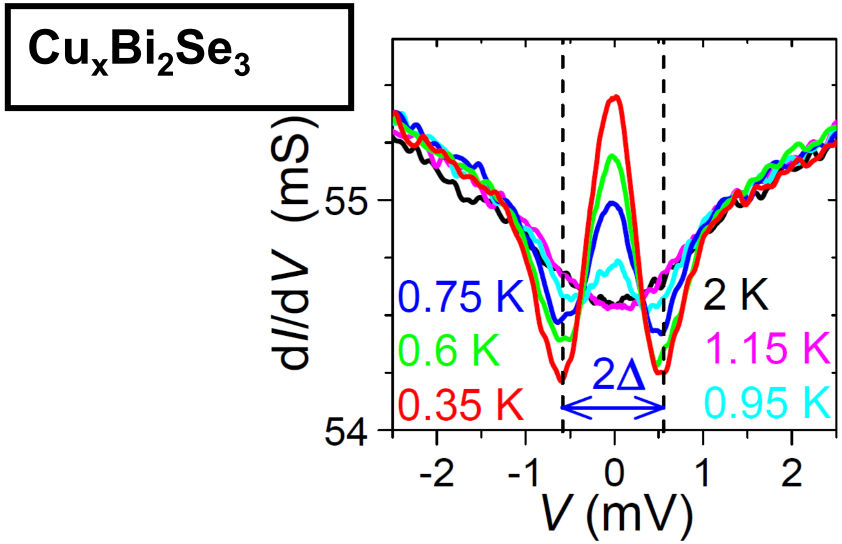
no zero-bias peak
due to linear dispersion
of surface ABS



Y. Asano *et al*, PRB '03

Tunneling experiment

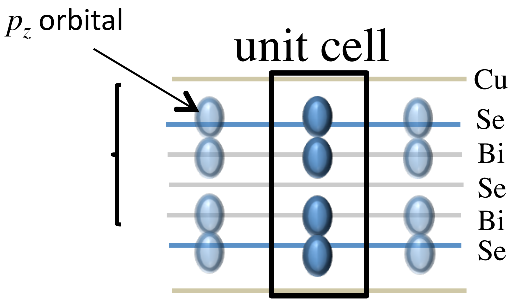




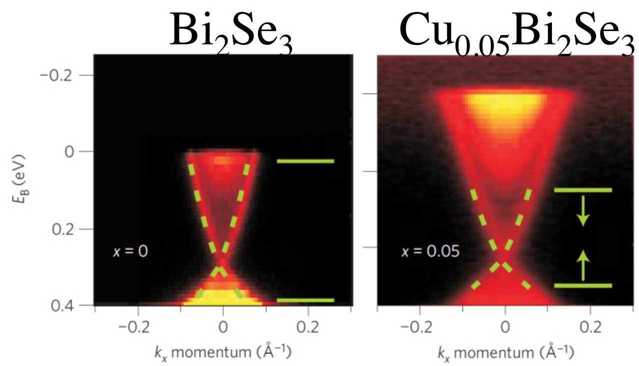
S. Sasaki et al PRL 107 217001 (2011)

If $Cu_xBi_2Se_3$ is a 3D topological superconductor with odd-parity, Tunneling spectroscopy can not be explained by pair potential realized in B-phase in ^3He, which is a typical example of 3d full gap superconductor.

Effective Hamiltonian of $Cu_xBi_2Se_3$



ARPES



Model Hamiltonian (Normal state) H.Zhang *et al*, Nature Phys. 5, 438 (2009)

$$H_0(k) = m\sigma_x + v(k_x\sigma_z s_y - k_y\sigma_z s_x) + v_z k_z \sigma_y$$

Model Hamiltonian (superconducting state) BdG Hamiltonian

$$H(k) = [H_0(k) - \mu]\tau_z + \hat{\Delta}\tau_x$$

8×8 matrix

Pauli matrix σ : orbital, s : spin, τ : particle hole

Possible pairings

		Matrix representation	Parity (spatial inversion)
$\hat{\Delta}_{1a}$	$c_{1\uparrow}c_{1\downarrow} + c_{2\uparrow}c_{2\downarrow}$	Δ	Even
$\hat{\Delta}_{1b}$	$c_{1\uparrow}c_{2\downarrow} - c_{1\downarrow}c_{2\uparrow}$	$\Delta\sigma_x$	Even
$\hat{\Delta}_{2}$	$c_{1\uparrow}c_{2\downarrow} + c_{1\downarrow}c_{2\uparrow}$	$\Delta\sigma_y s_z$	Odd
$\hat{\Delta}_{3}$	$c_{1\uparrow}c_{1\downarrow} - c_{2\uparrow}c_{2\downarrow}$	$\Delta\sigma_z$	Odd
$\hat{\Delta}_{4}$	$c_{1\uparrow}c_{2\uparrow} \mp c_{1\downarrow}c_{2\downarrow}$	$\Delta\sigma_y s_x$ $(\Delta\sigma_y s_y)$	Odd

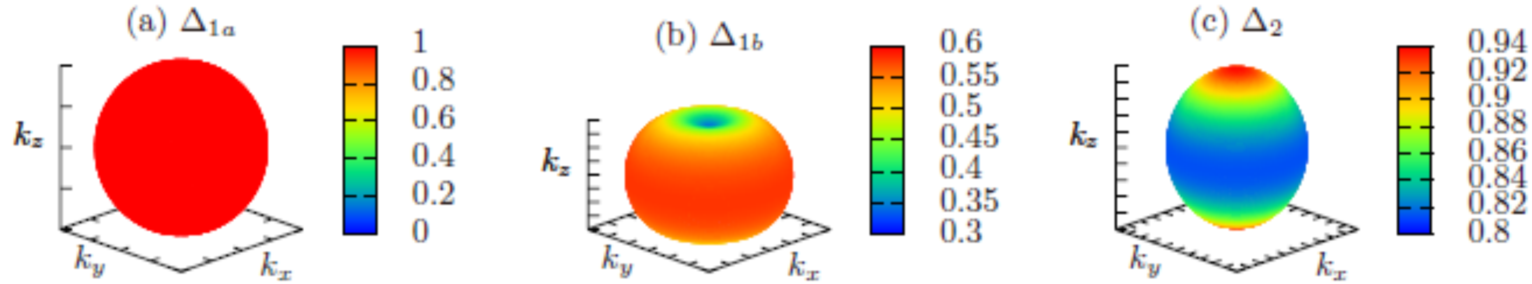
Fu and Berg, Phys. Rev. Lett. 105 097001(2010)

Energy Gap function

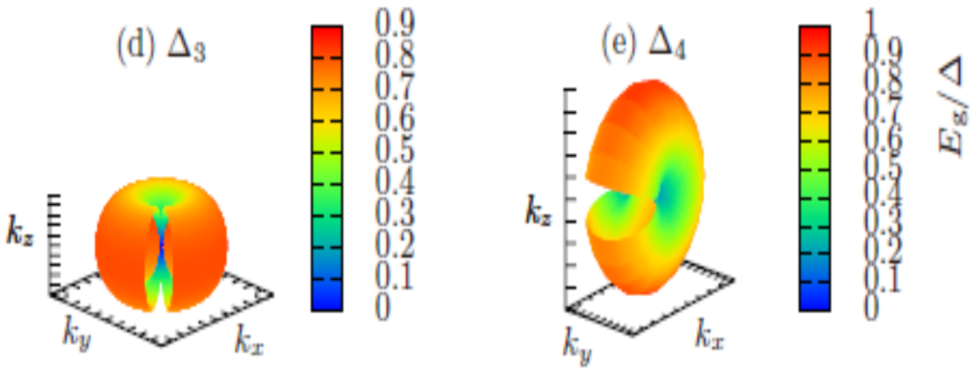
Full Gap

spin-singlet intra-orbital

spin-triplet inter-orbital
spatial inversion odd



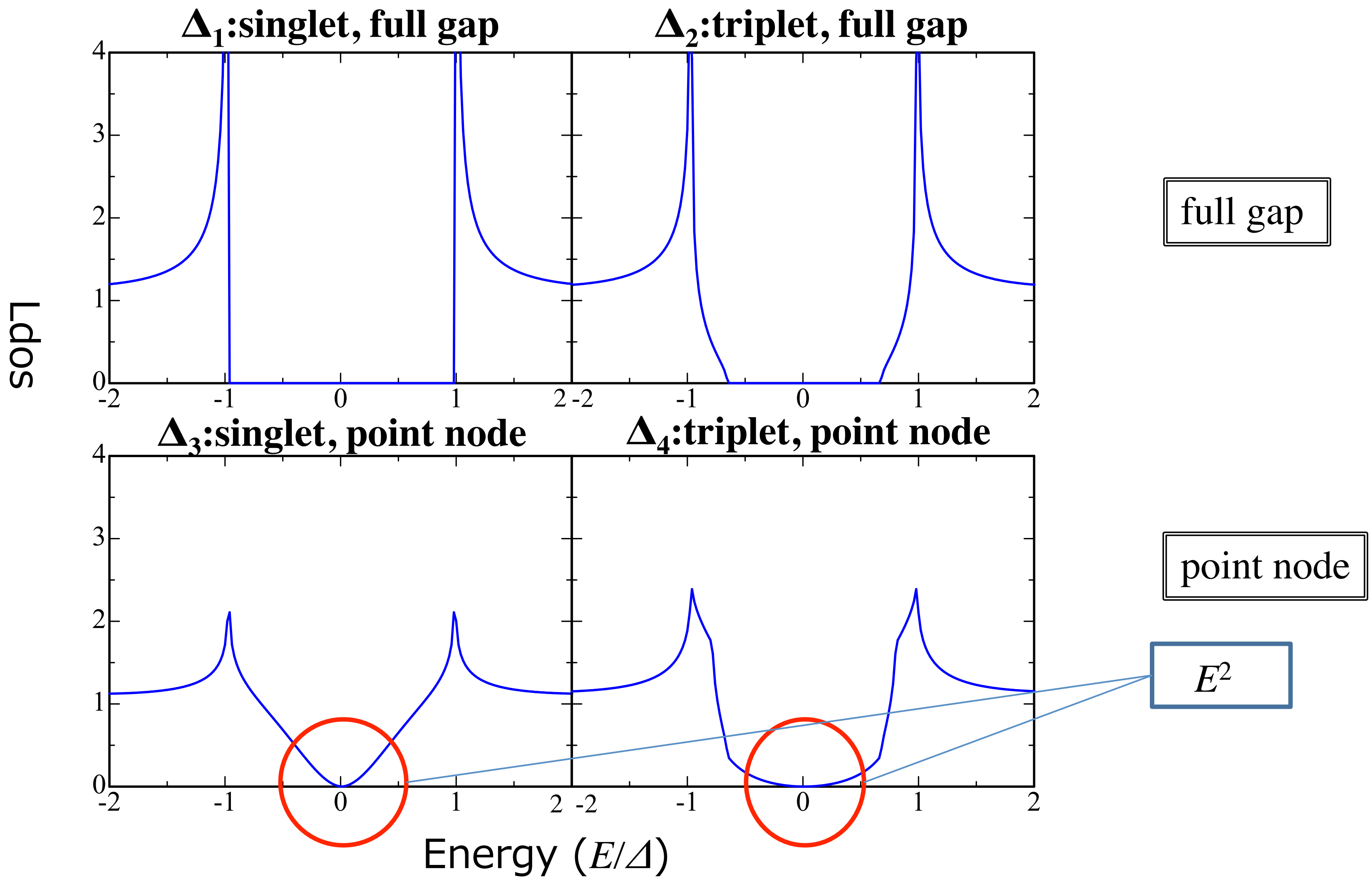
Point Node spatial inversion odd



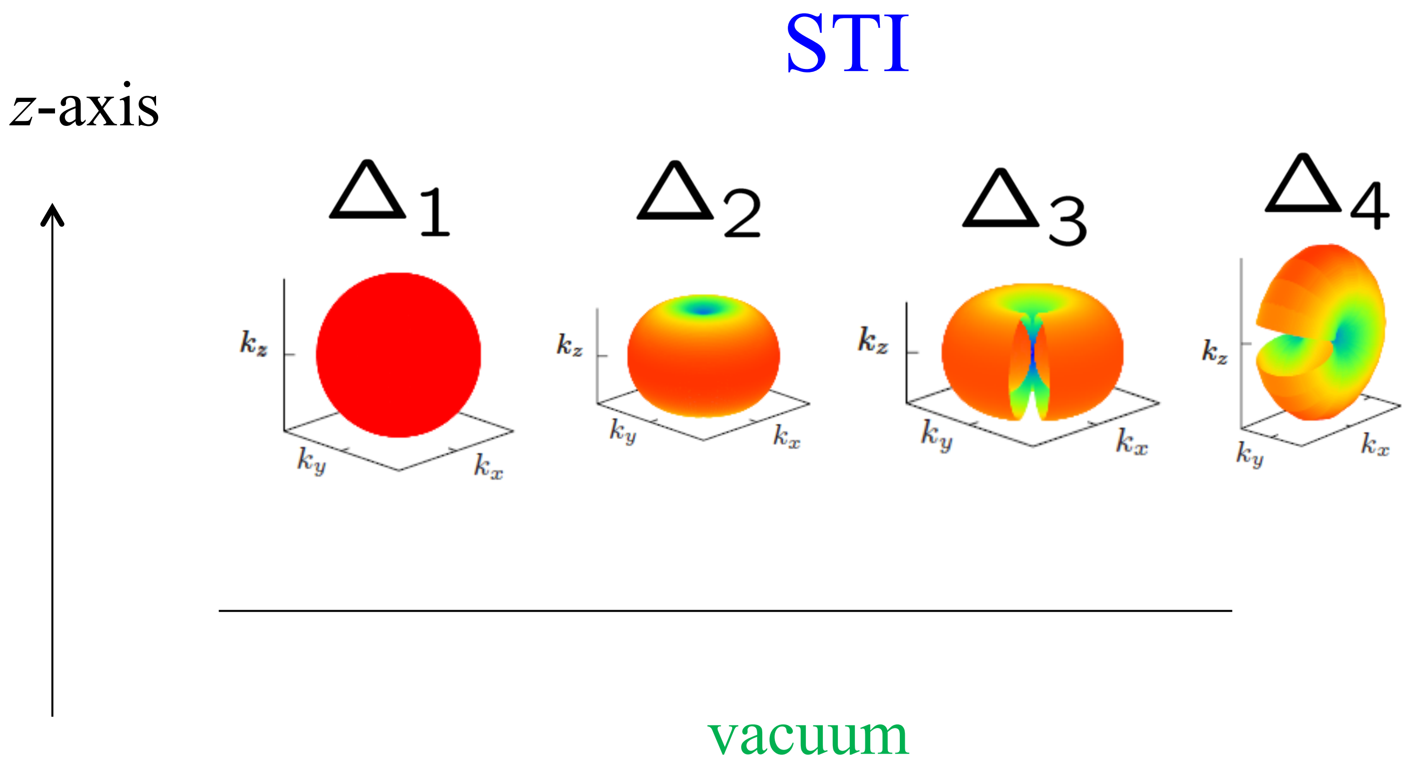
Fu and Berg, Phys. Rev. Lett. 105 097001(2010)

Yamakage et al., PRB Rapid (2011)

Bulk local density of state



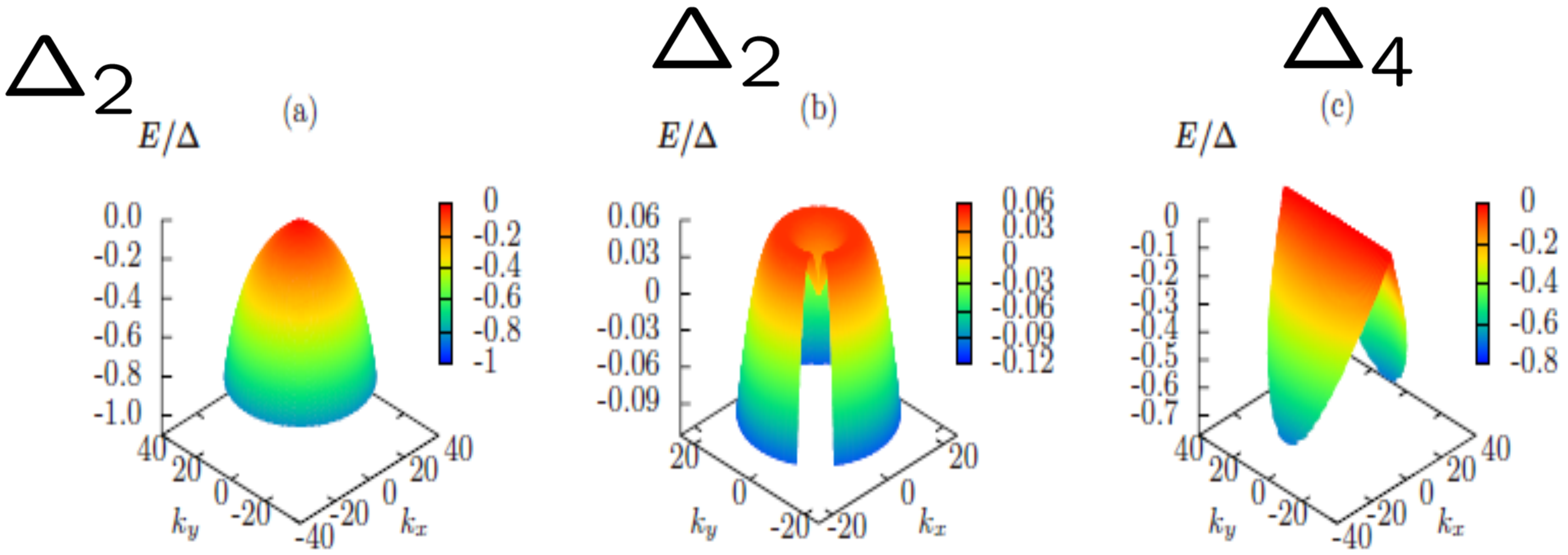
Surface state generated at z=0



STI (Superconducting topological insulator)

Dispersions of Andreev bound state

spin-triplet inter-orbital spatial inversion odd-parity



Normal Cone

Caldera Cone

Deformed Cone

(Only positive spin helicity
$k_x s_y - k_y s_x = +k$ states
are shown.)

(Only negative energy
states are shown.)

(solution of confinement condition $\psi(z=0)=0$)

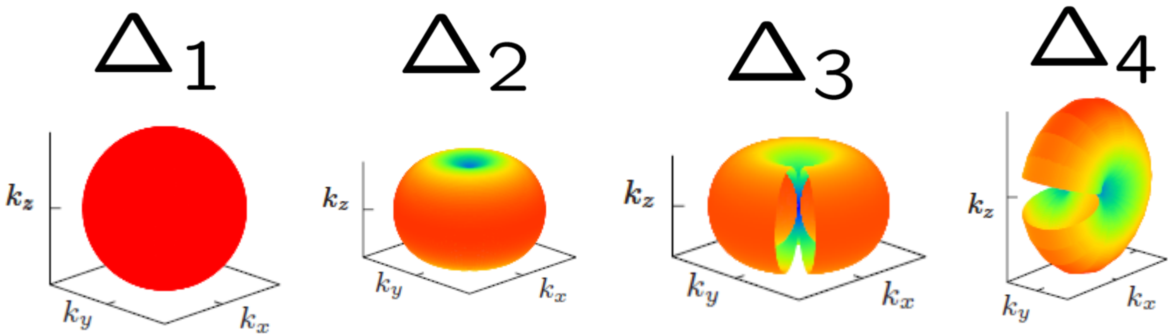
Hsieh and Fu PRL 108 107005(2012); arXiv: 1109.3464

A. Yamakage, PRB, **85**, 180509(R) (2012)

Charge transport in normal metal / STI junctions

STI

z-axis

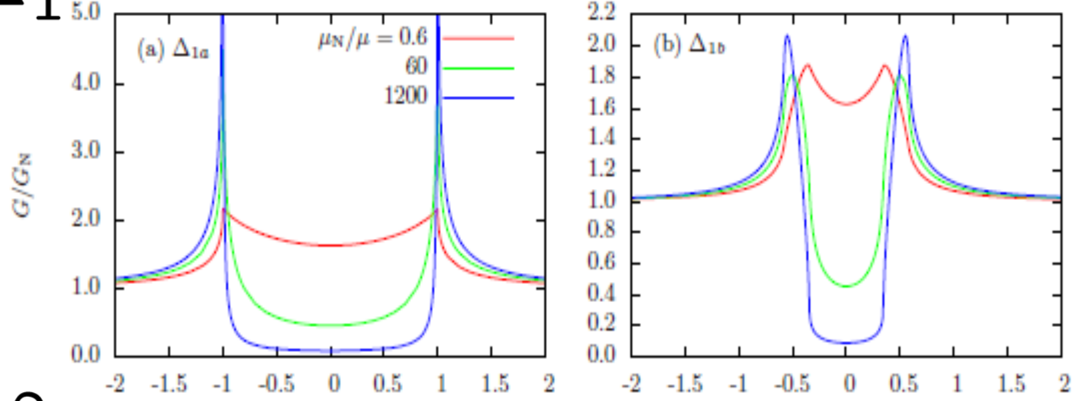


Normal metal

STI (Superconducting topological insulator)

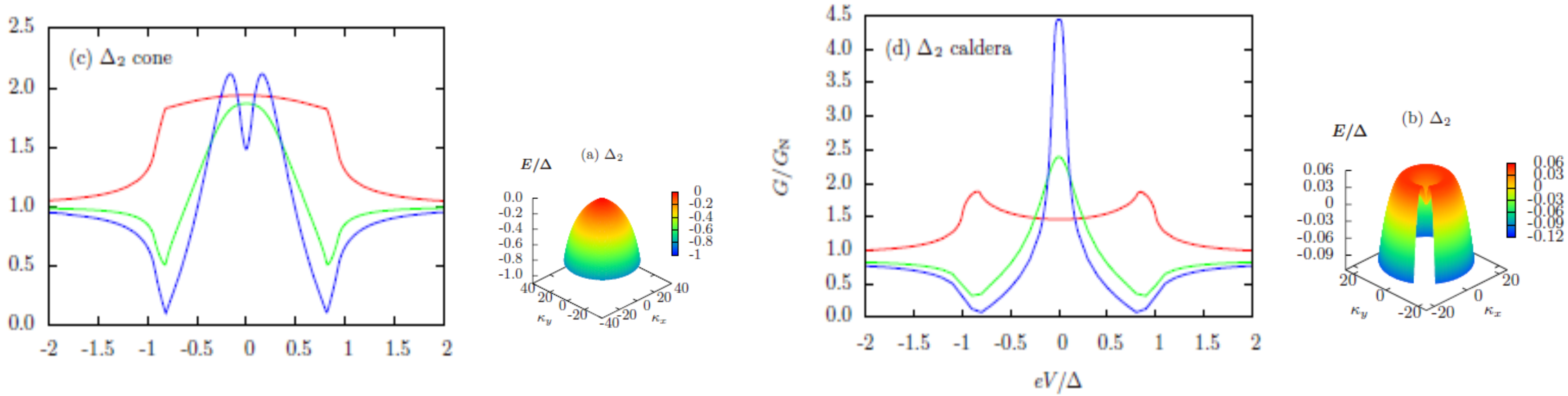
Tunneling conductance between normal metal / superconducting topological insulator junction

Δ_1



Similar to conventional spin-singlet s-wave superconductor

Δ_2



Zero bias conductance peak is possible even for Δ_2 case with full gap

Hsieh and Fu PRL 108 107005(2012); arXiv: 1109.3464

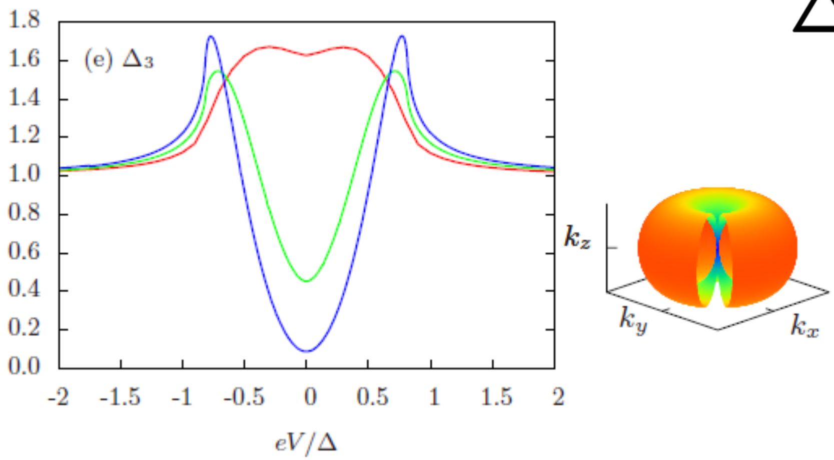
A. Yamakage, PRB, **85**, 180509(R) (2012)

Conductance between normal metal / STI junction

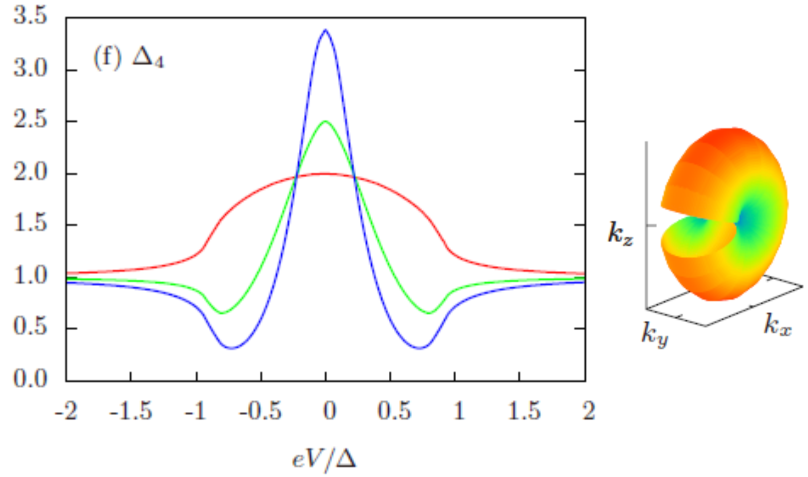
(Spatial inversion odd-parity)

Point node case

Δ_3



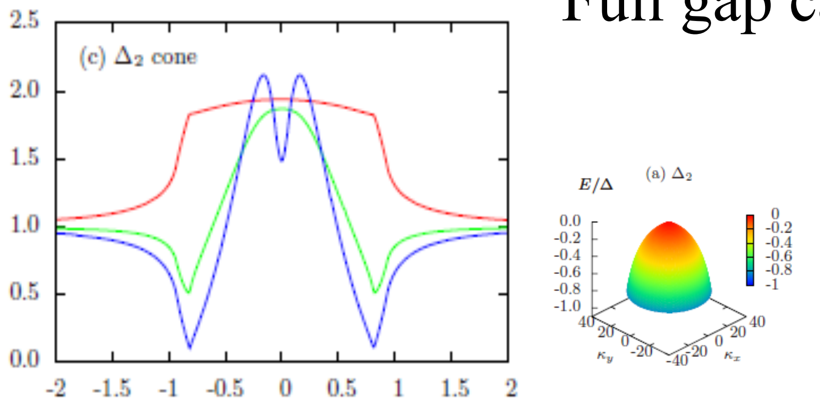
Δ_4



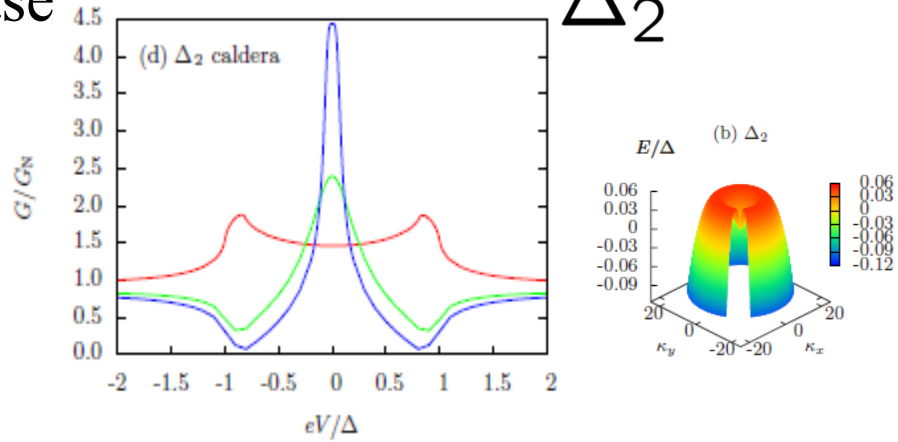
Tunneling conductance strongly depends on the direction of nodes.

Full gap case

Δ_2



Δ_2



A. Yamakage, PRB, **85**, 180509(R) (2012)

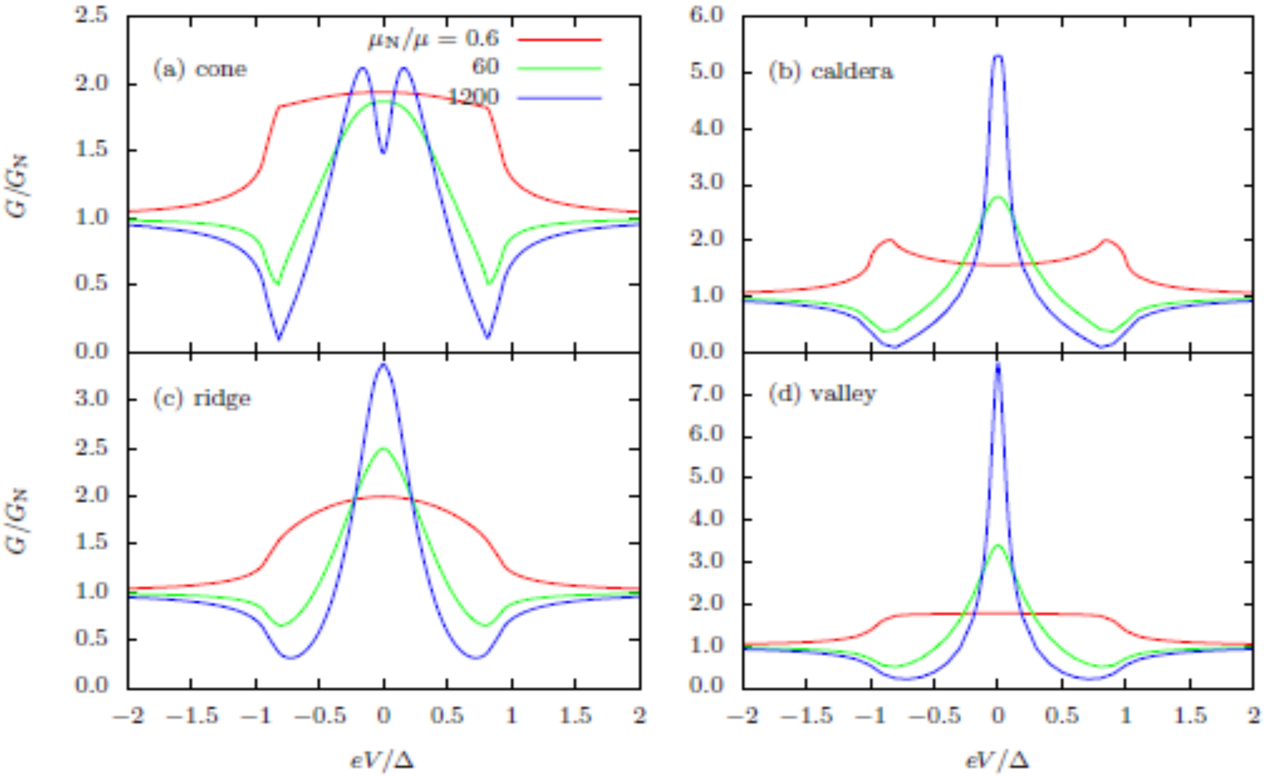
Tunneling conductance with ABS

Andreev bound state (Majorana Fermion)

spin-triplet inter-orbital spatial inversion odd-parity

Δ_2

Δ_4

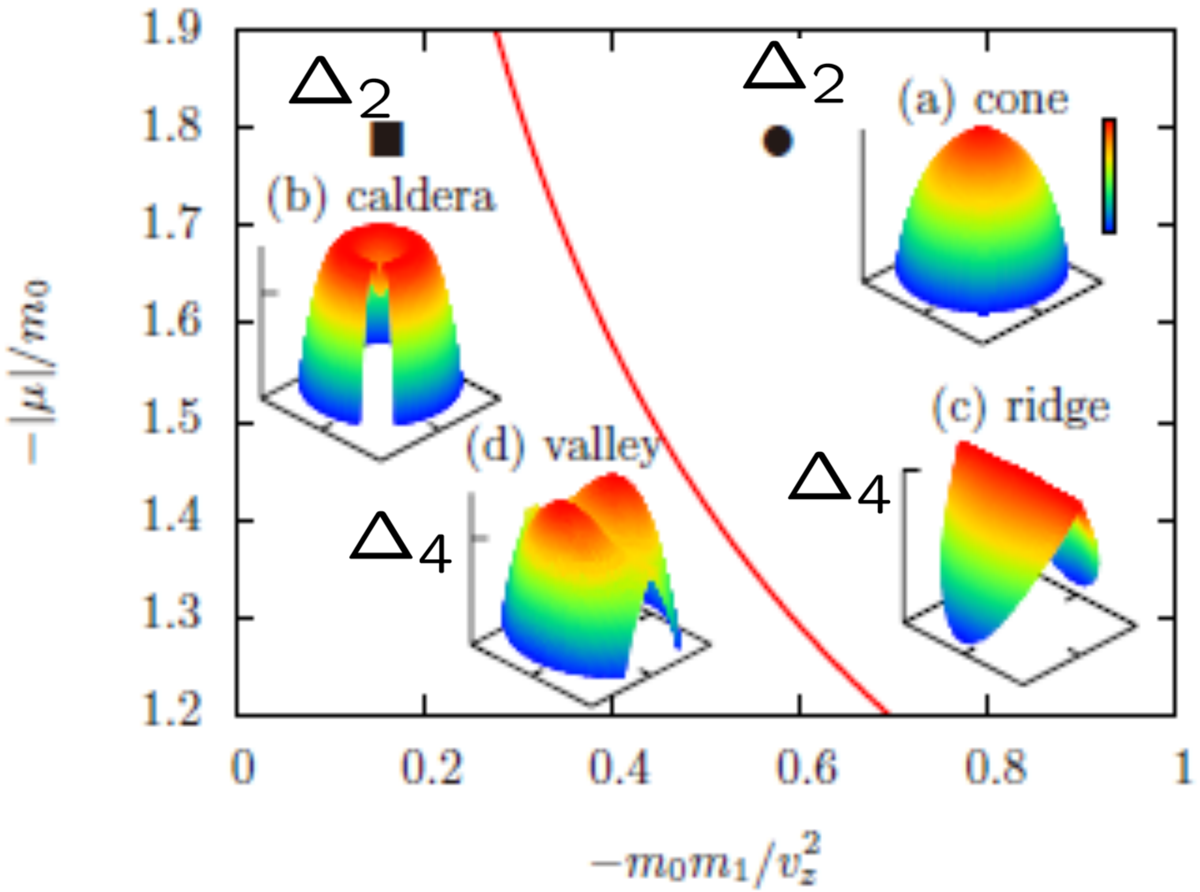


large μ

small μ

A. Yamakage, PRB, **85**, 180509(R) (2012)

Structural transition of the energy dispersion of Andreev bound state

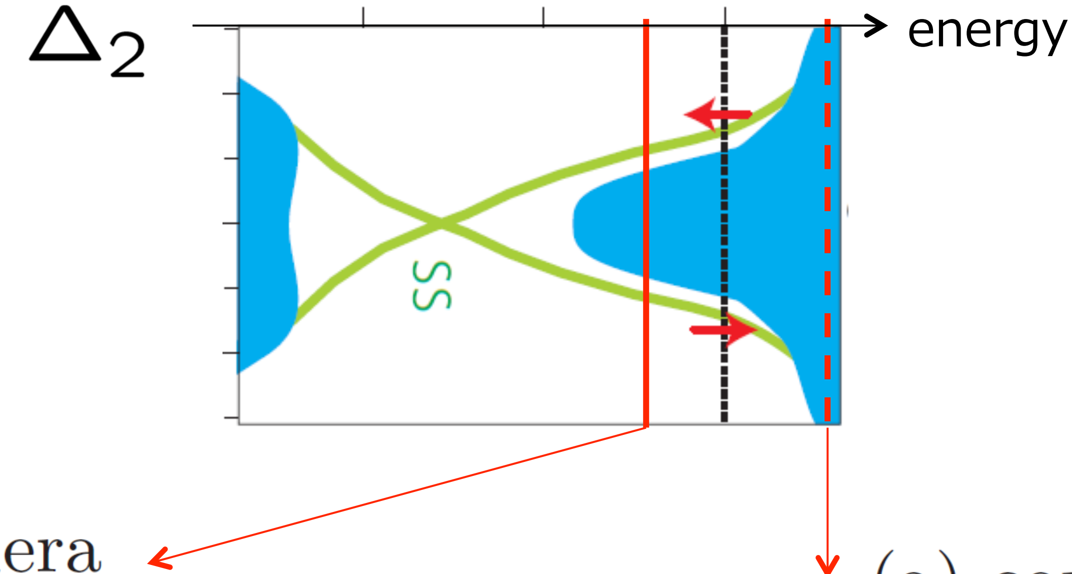


Yamakage

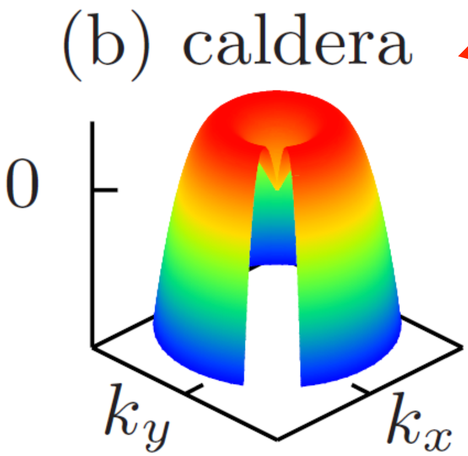
Transition line $\mu^2 = -m_0 v_z^2/m_1$

A. Yamakage, PRB, **85**, 180509(R) (2012)

Structural transition of the dispersion of ABS

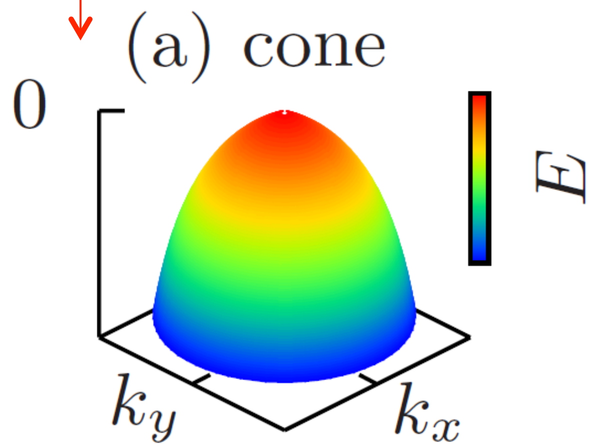


A. Yamakage



transition

transition point:
group velocity = 0
$\mu = v_z^2/m_1$



L. Hao and T. K. Lee, PRB ’11 T. H. Hsieh and L. Fu, PRL ’12
A. Yamakage, K. Yada, M. Sato, and Y. Tanaka, PRB 2012

Summary (1)
Theory of tunneling spectroscopy of superconducting topological insulators

1. Δ_2 and Δ_4 are consistent with point-contact experiment by Ando's group.

2. Zero-bias conductance peak is possible even in full-gap topological 3d superconductors, differently from the case of BW states.

3. This originates from the structural transition of energy dispersion of ABS.

Yamakage, Yada, Sato, and Tanaka, Physical Review B 85 180509(R) 2012

Summary of the Topological natures of four pairings

Pair potential	Irreducible representation	spin	orbital	Gap structure	Parity (spatial inversion)	Topological
$\Delta_1=\Delta$	A_{1g}	Singlet	intra	isotropic full gap	even	No
$\Delta_2=\Delta\sigma_y s_z$	A_{1u}	triplet	inter	anisotropic full gap	odd	DIII Z
$\Delta_3=\Delta\sigma_z$	A_{2u}	singlet	intra	Point node (z-direction)	odd	DIII Z_2
$\Delta_4=\Delta\sigma_y s_x$	E_u	triplet	inter	Point node (z-direction)	odd	DIII Z_2

Supplementary materials in
S. Sasaki et al PRL 107 217001 (2011)

Current status of tunneling experiments

Consistent with Ando's group with ZBCP

- G. Koren, et al, Phys. Rev. B 84, 224521 (2011).
- T. Kirzhner, et. al, Phys. Rev. B 86, 064517 (2012).
- G. Koren and T. Kirzhner, Phys. Rev. B 86, 144508 (2012).

Contradict with Ando's group with full gap (STM)

- N. Levy, et al, Phys. Rev. Lett. 110 117001 (2013)

Composition and crystal structures of the actual samples have not fully clarified yet.

We must need further experimental research.
Theoretical works in bulk properties become important.

Contents of our talk

(1)What is superconducting topological insulator

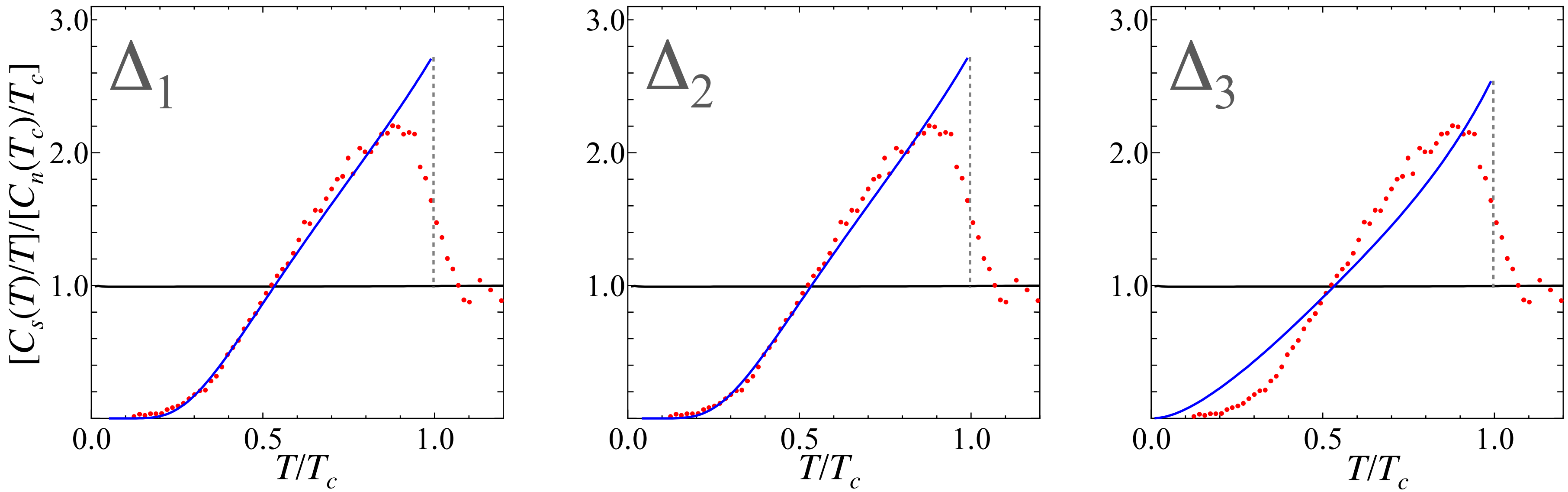
(2)Andreev bound state and quasi particle tunneling

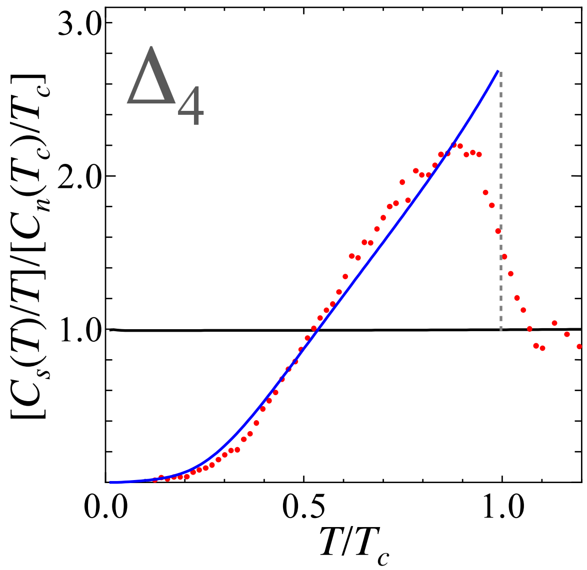
(3)Josephson current

(4)Spin susceptibility and specific heat

(5)Relevant Rashba superconductor system

Temperature dependence of specific heat





calculation

Experimental results in Ando's group

M. Kriener *et al.*, PRL **106**, 127001 (2011).

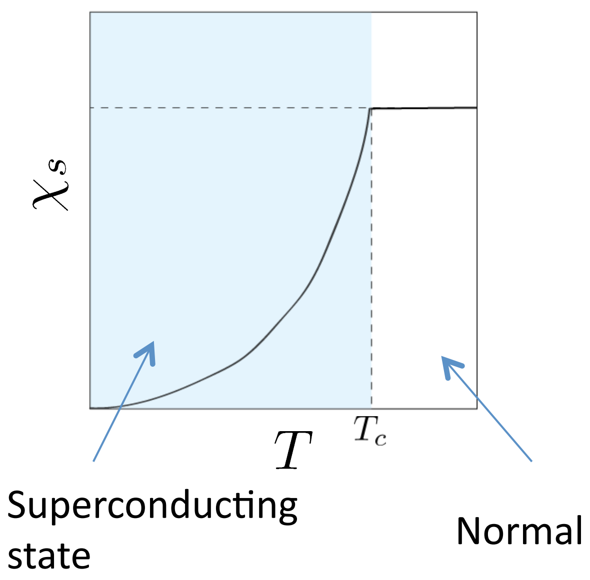
Experimental data can be fitted for Δ_1, Δ_2, and Δ_4 pairings.
Δ_3 is not consistent with experiment.

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

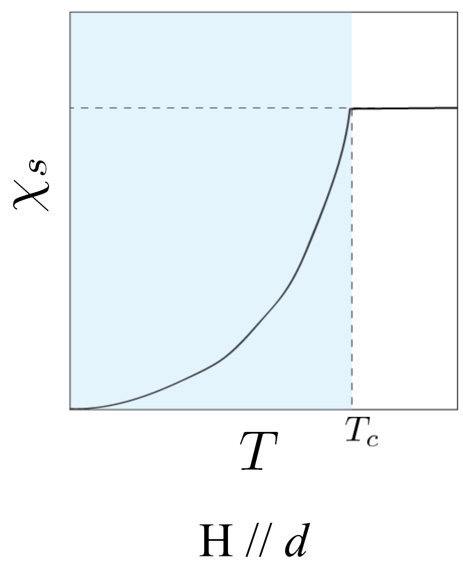
Temperature dependence of spin-susceptibility

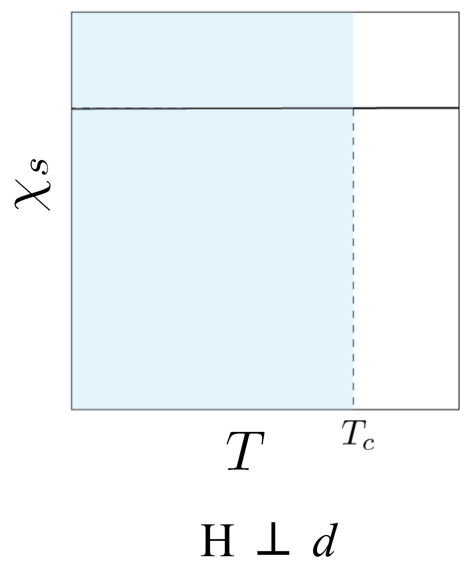
Standard case

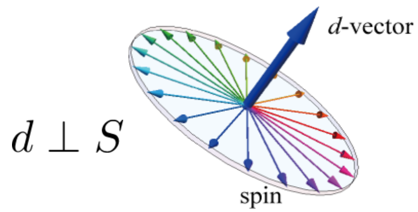
Spin-singlet pairing



Spin-triplet pairing







In the actual $Cu_xBi_2Se_3$, the situation becomes complex due to strong spin-orbit coupling.

Spin susceptibility of superconducting topological insulator $Cu_xBi_2Se_3$

$$\chi_i = -\mu_B^2 \lim_{\boldsymbol{q}\to 0} \frac{1}{V} \sum_{\boldsymbol{k}\alpha\beta\mu} \frac{f(E_\alpha(\boldsymbol{k})) - f(E_\beta(\boldsymbol{k}+\boldsymbol{q}))}{E_\alpha(\boldsymbol{k}) - E_\beta(\boldsymbol{k}+\boldsymbol{q}) + i0} \times \langle\alpha|s_i|\beta\rangle\langle\beta|\frac{g_{i\mu}}{2} s_i\sigma_\mu|\alpha\rangle.$$

We calculate spin susceptibility for four possible pairing states.

With spin-orbit (SO) Coupling

$$H_0(\mathbf{k}) = m\sigma_x + v(k_x\sigma_z s_y - k_y\sigma_z s_x) + v_z k_z \sigma_y$$

Without spin-orbit (SO) coupling

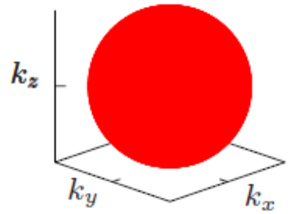
$$H_0(\mathbf{k}) = m\sigma_x + \cancel{v(k_x\sigma_z s_y - k_y\sigma_z s_x)} + v_z k_z \sigma_y$$

Spin orbit coupling term → 0

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Calculated spin-susceptibility (1)

Δ_1

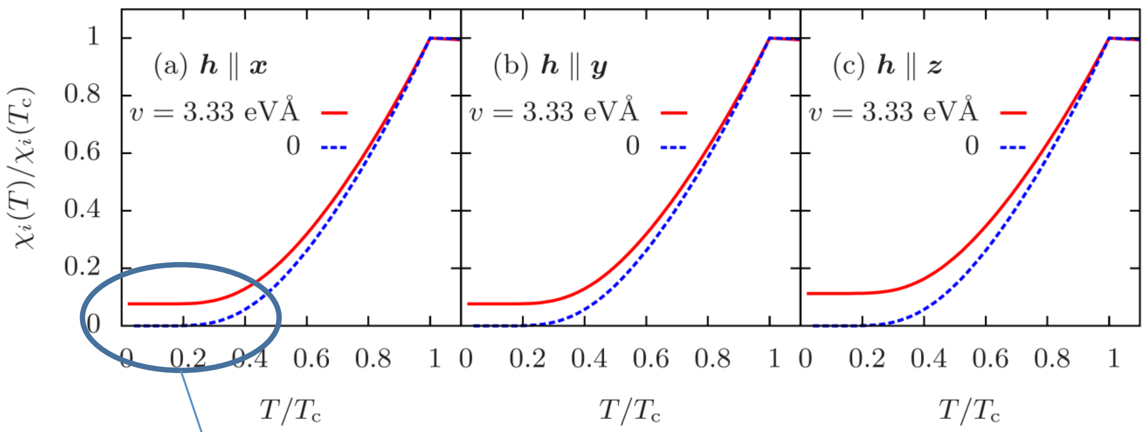


Spin-singlet intra-orbital spatial inversion even

Similar to conventional spin-singlet pairing

with SO

without SO

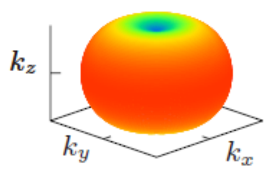


Due to the Van Vleck susceptibility, χ does not become zero even at T=0.

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Calculated spin-susceptibility (2)

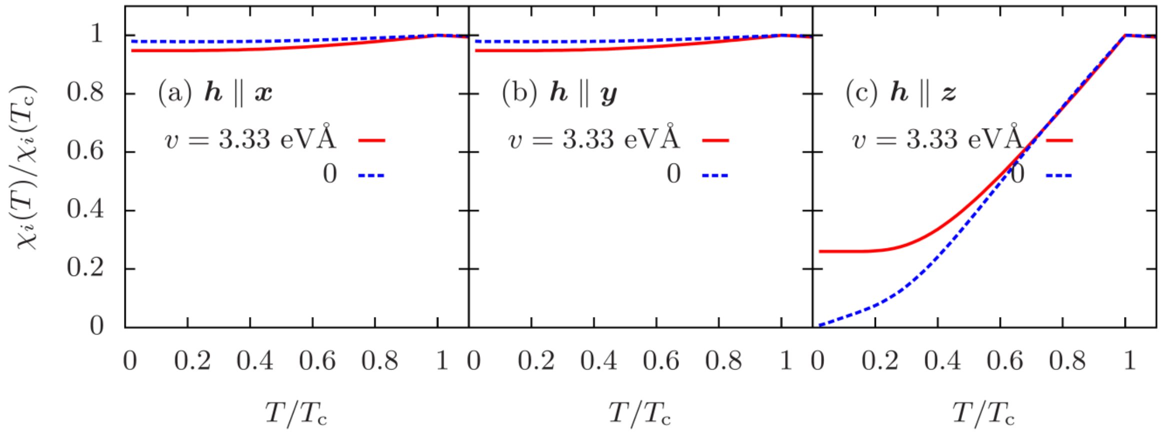
Δ_2



with SO

without SO

Spin-triplet inter-orbital spatial inversion odd



Susceptibility decreases when the magnetic field is along the z-direction.

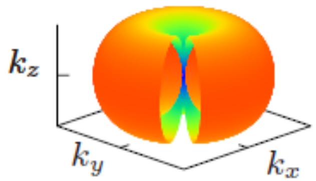
$$\boldsymbol{d} = \Delta\,(0, 0, \sigma_y)$$ (orbital basis)

$$\tilde{\boldsymbol{d}}(\boldsymbol{k}) = \Delta \left(\frac{v k_x}{m_0}, \frac{v k_y}{m_0}, \frac{v_z k_z}{|m_0|} \tilde{\sigma}_z - \mathrm{sgn}(m_0) \tilde{\sigma}_y \right)$$ band basis

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Calculated spin-susceptibility (3)

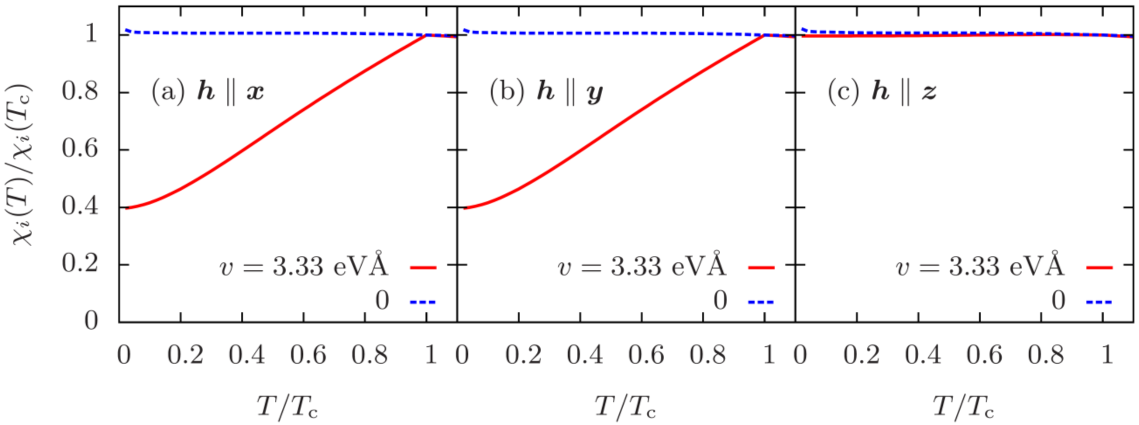
Δ_3



with SO

without SO

Spin-singlet intra-orbital spatial inversion odd



Susceptibility decreases when the magnetic field is along the xy-plane consistent with the direction of d-vector in the band basis.

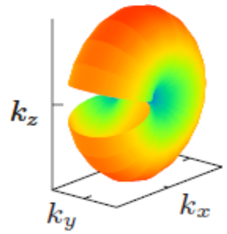
spin-singlet (orbital basis)

band basis $$\tilde{\boldsymbol{d}}(\boldsymbol{k}) = \Delta \tilde{\sigma}_z \left(-\frac{v k_y}{|m_0|}, \frac{v k_x}{|m_0|}, 0 \right)$$

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Calculated spin-susceptibility (4)

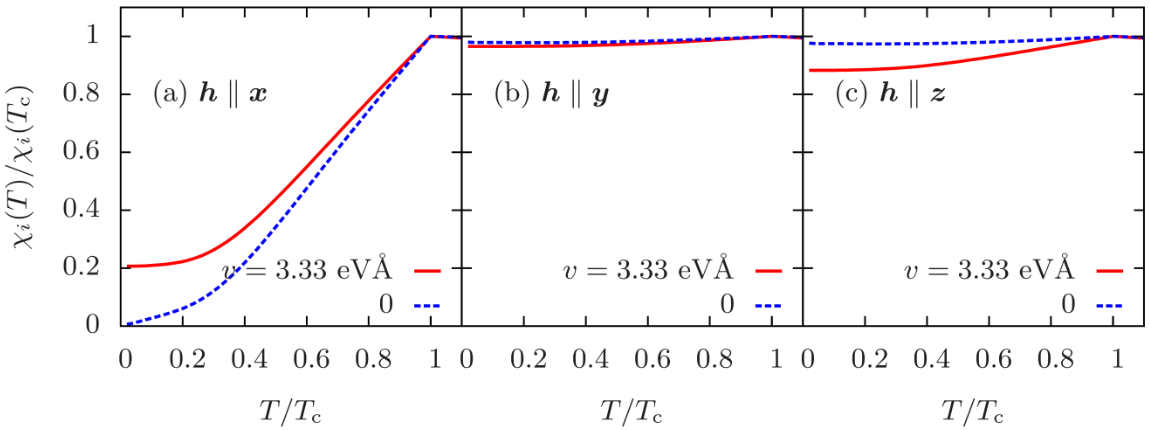
Δ_4



with SO

without SO

Spin-triplet inter-orbital spatial inversion odd



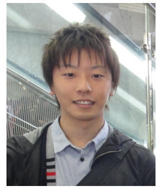
Susceptibility decreases seriously when the magnetic field is along the x-direction.

orbital basis $\boldsymbol{d} = \Delta\left(\sigma_y, 0, 0\right)$

band basis $\tilde{\boldsymbol{d}}(\boldsymbol{k}) = \Delta\left(\frac{v_z k_z}{|m_0|}\tilde{\sigma}_z - \tilde{\sigma}_y, 0, -\frac{v k_x}{m_0}\right)$

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Summary (2)



Hashimoto

	Rep.	Gap structure	Specific heat	Andreev bound state (xy-plane)	Spin susceptibility		
					χ_x	χ_y	χ_z
Δ_1	A_{1g}	Isotropic full gap	Yes	No	↙	↙	↙
Δ_2	A_{1u}	Anisotropic full gap	Yes	Yes	—	—	↙
Δ_3	A_{2u}	Point nodes at pole	No	No	↙	↙	—
Δ_4	E_u	Point nodes on equator	Yes	Yes	↙	—	—

- We find that the temperature dependence of specific heat and the susceptibility are different in each pairing symmetry.
- It is possible to determine pairing symmetry only from bulk quantities.
- We think Δ_2 and Δ_4 are most probable candidates consistent with specific heat and point contact experiments by Ando's group.

T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

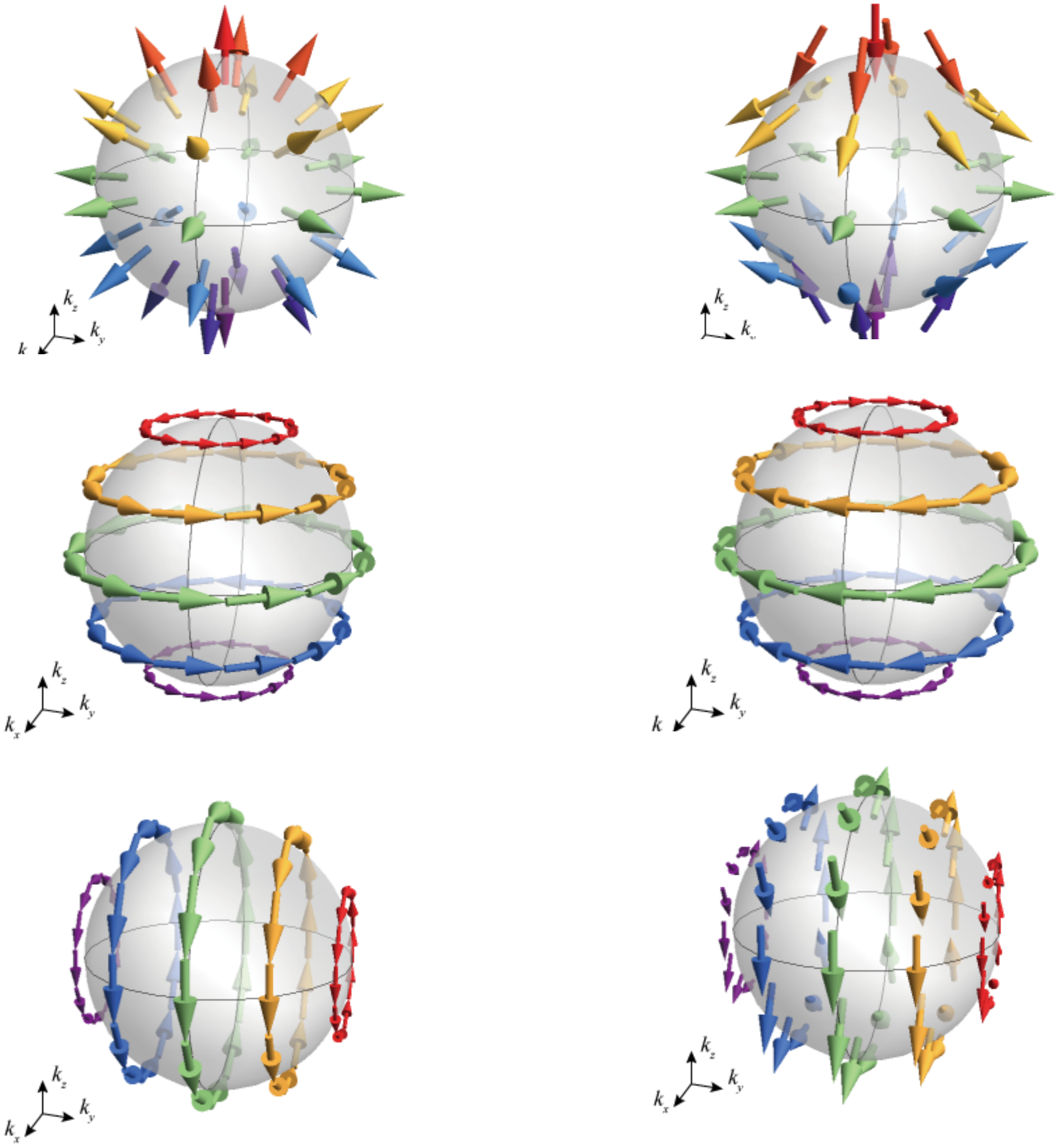
Direction of d-vector in the band basis

d -vector (valence band) | d -vector (conduction band)

Δ_2

Δ_3

Δ_4



T. Hashimoto, J. Phys. Soc. Jpn. 82 044704 (2013)

Contents of our talk

(1)What is superconducting topological insulator

(2)Andreev bound state and quasi particle tunneling

(3)Josephson current

(4)Spin susceptibility and specific heat

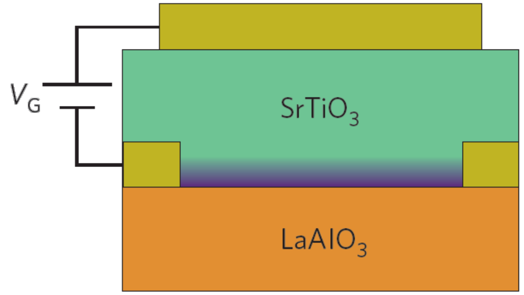
(5)Relevant Rashba superconductor system

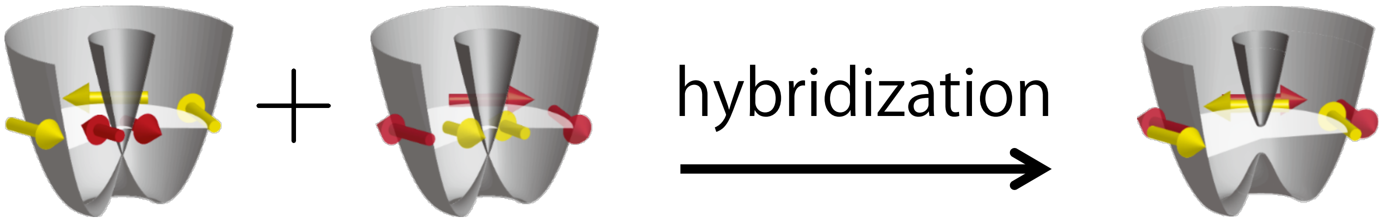
DIII superconductor from conventional systems

Using Interface superconductivity

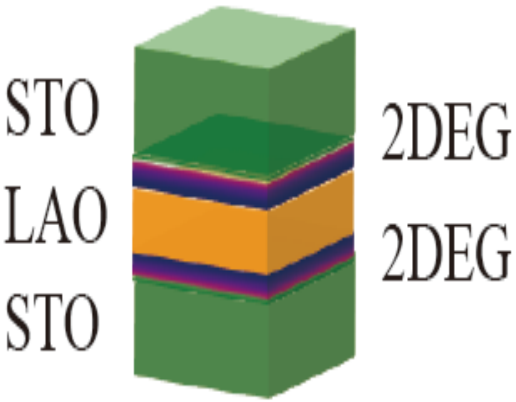
Interface of transition metal oxides

- 2d electron gas — Ohtomo & Hwang Nature 2004
- superconductivity — Reyren *et al.* Science 2007
- tunable Rashba SOI — Caviglia *et al.* PRL 2010





One-dimensional Majorana (Helical)



$$\Delta_1 = -\Delta_2$$

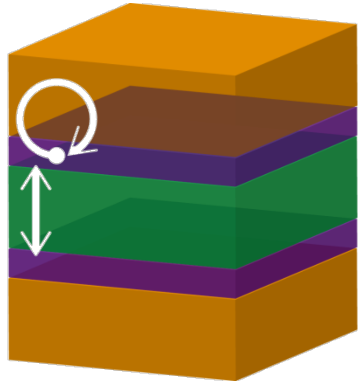
Intra-layer pairing with different sign

Nakosai, Tanaka Nagaosa, PRL(2012)

Model construction

kinetic Hamiltonian

$$\mathcal{H}_0(\boldsymbol{k}) = \frac{k^2}{2m} \boxed{-\varepsilon\sigma_x} + \boxed{\alpha\left(k_x s_y - k_y s_x\right)\sigma_z}$$



hybridize : transfer SOI : Rashba SOI

s : spin σ : layer

electron density-density interaction

$$\mathcal{H}_{\rm int}\left(\boldsymbol{x}\right) = \boxed{-U\left(n_1^2(\boldsymbol{x}) + n_2^2(\boldsymbol{x})\right)} \boxed{-2Vn_1(\boldsymbol{x})\,n_2(\boldsymbol{x})}$$

intra-layer inter-layer

Bogoliubov de-Gennes Hamiltonian

$$\mathcal{H}_{\rm BdG} = \begin{pmatrix} \mathcal{H}_0 - \mu & \Delta \\ \Delta & -\mathcal{H}_0 + \mu \end{pmatrix}$$

cf. Fu and Berg PRL 2010

S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Pair potentials

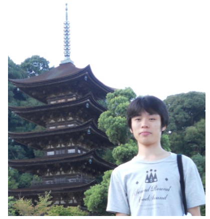
As compared to 3-d superconducting topological insulator $Cu_xBi_2Se_3$, the orbital index changes into layer index.

parity under an inversion operation

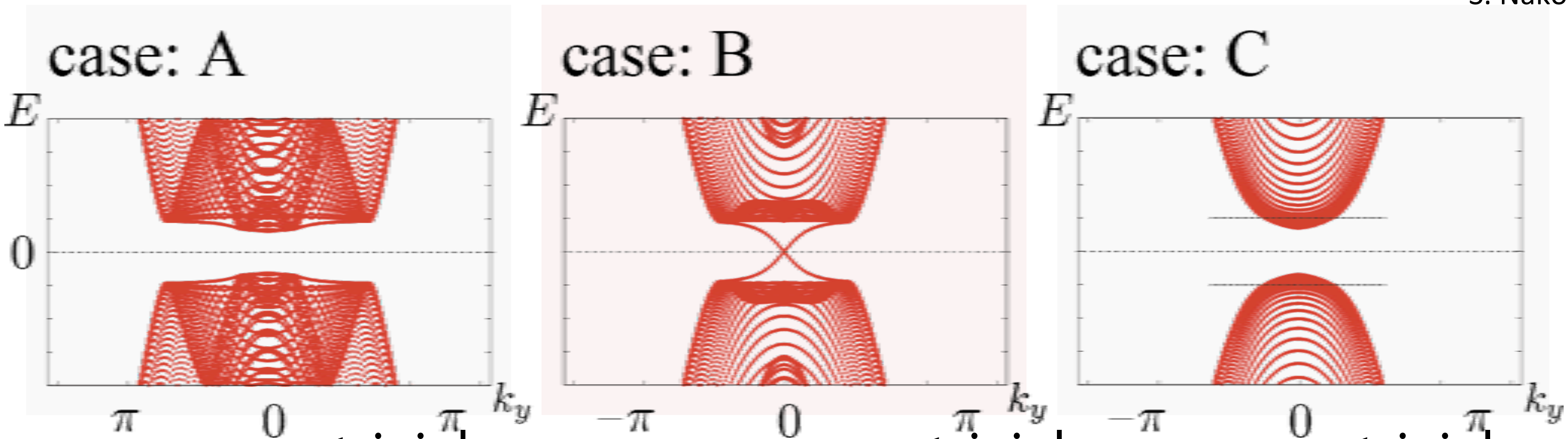
	irreps	matrix	spin	orbital	inversion	gap	topological
$\hat{\Delta}_1$	A_{1g}	I σ_x	singlet	inter	+	full	no
$\hat{\Delta}_2$	A_{1u}	$s_z\sigma_y$	triplet	inter	−	full	DIII Z_2
$\hat{\Delta}_3$	A_{2u}	σ_z	singlet	intra	−	full	DIII Z_2
$\hat{\Delta}_4$	E_u	$\begin{pmatrix} s_x\sigma_y \\ s_y\sigma_y \end{pmatrix}$	triplet	inter	−	point node	DIII Z_2

S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Topological superconducting state with Δ_3 pairing (intra-site inversion symmetry odd) is realized by choosing chemical potential.

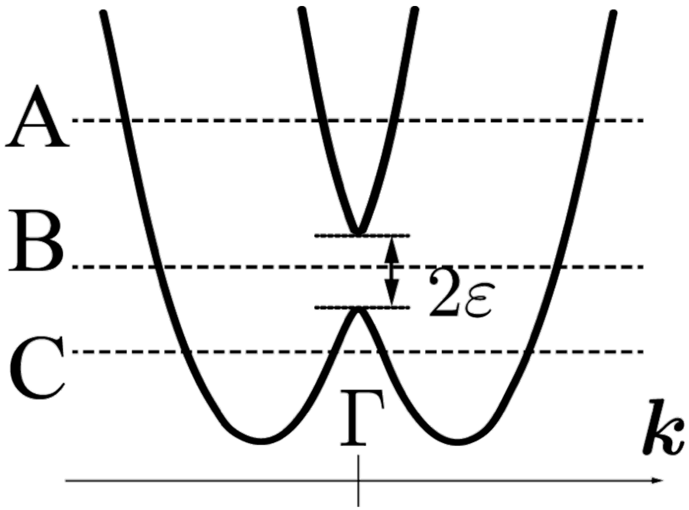


S. Nakosai

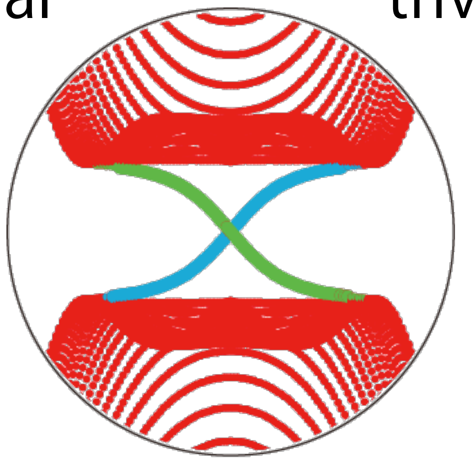


trivial non-trivial trivial

Bulk state



Fermi energy



helical Majorana edge states

S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Summary (3)
Topological superconductivity from Rashba system

1. We have proposed a new way to design DIII superconductor in 2D systems.

(Bilayer Rashba system realized at the interface of transition metal oxides.)

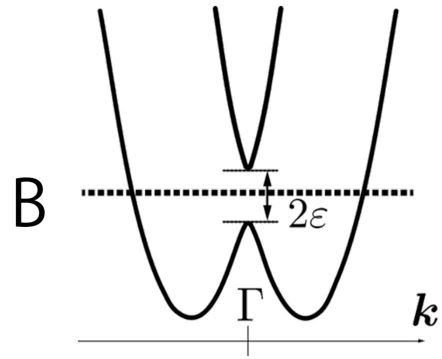
2. Andreev bound state appears as a helical edge modes without anisotropic pairing.

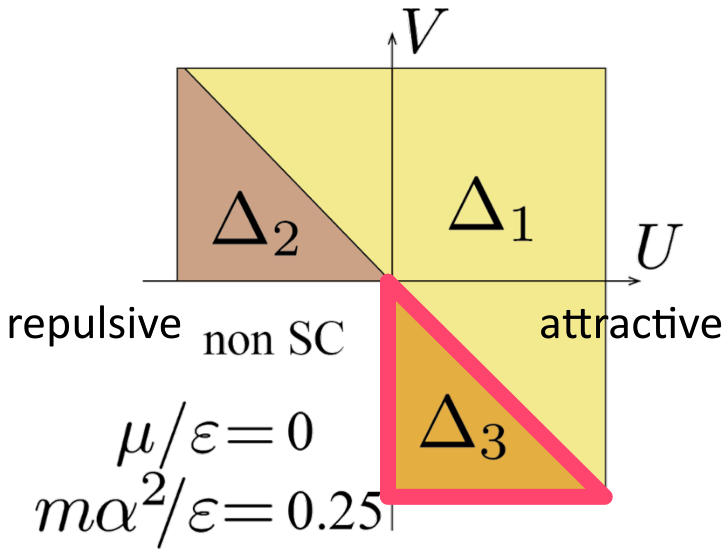
Yamakage, Yada, Sato, and Tanaka, Physical Review B 85 180509(R) 2012

Topological SC ?

We set the Fermi energy
within the hybridization gap.

1. [Fermi level] OK





2. [odd parity pairing potential] OK

NOTE:

Pairing amplitudes for Δ_2 and Δ_3 are proportional to α .

SOI-induced SC phases

Unconventional SC phase appears
in a feasible parameter region.

intra-layer : attractive (phonon mechanism)

inter-layer : repulsive (Coulomb interaction)