Topological particle in magnets
- Skyrmion -

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Collaborators

Theory

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Experiment

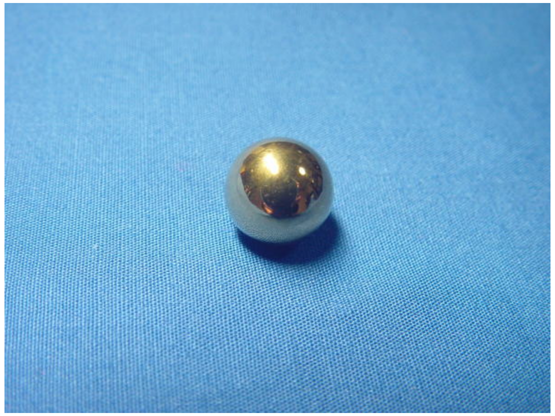
X. Z. Yu, Y. Matsui, Y. Onose, N. Kanazawa, T. Ideue,
Y. Shiomi, Y. Taguchi, Y. Tokura

"Particle" and "Field" in physics

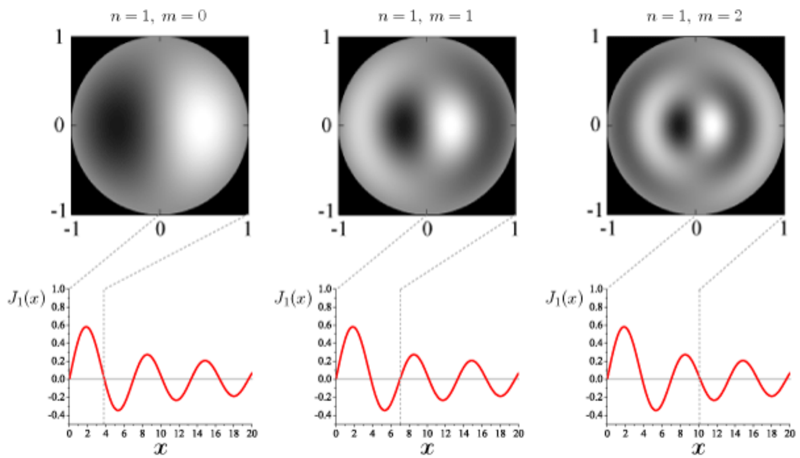
Particle is a lump of the field with higher energy and momentum.

Particle can be created and annihilated.

Particle has a finite lifetime → How to make the particle stable ?



Classical particle

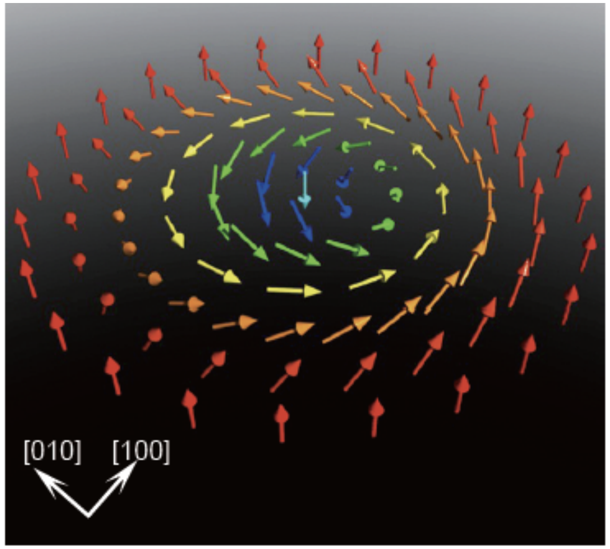


Particle in field theory

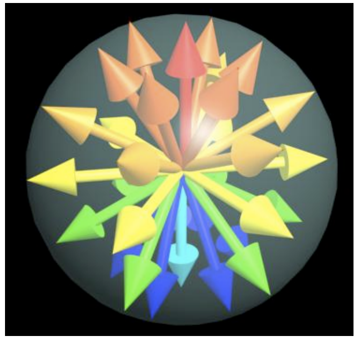
Skyrmion as a topologically protected particle in magnets

a model for hardon T.H.R. Skyrme 1962

Skyrmion spin configuration



Mapping to a sphere



$$\pi_2(S^2) = Z$$

$$N_{sk} = \frac{1}{4\pi}\int dr^2 \vec{n}\cdot\left(\frac{\partial\vec{n}}{\partial x}\times\frac{\partial\vec{n}}{\partial y}\right)$$

The integral of the solid angle
= topological number
How many times the mapping wrap the unit sphere

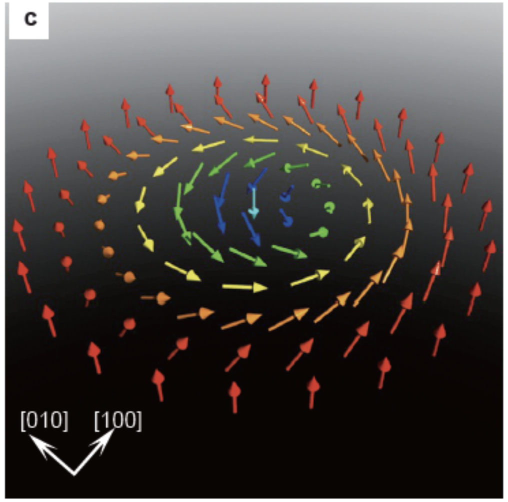
$$H_S = \int d^3x\left[\frac{J}{2a}(\nabla\mathbf{n})^2 + \frac{D}{a^2}\mathbf{n}\cdot[\nabla\times\mathbf{n}] - \frac{\mu}{a^3}\mathbf{H}\cdot\mathbf{n}\right]$$

DM interaction

Topological stability of skyrmion as long as spins vary slowly

Coupled dynamics of conduction electrons and Skyrmion

J.D.Zang, J.H. Han, M.Mostovoy, and N.N.



Effective EMF due to spin texture acting on conduction electrons

$$\begin{cases} e_i = -\partial_i a_0 - \frac{1}{c}\dot{a}_i = \frac{\hbar}{2e}\left(\mathbf{n}\cdot\partial_i\mathbf{n}\times\dot{\mathbf{n}}\right), \\ h_i = [\boldsymbol{\nabla}\times\mathbf{a}]_i = \frac{\hbar c}{2e}\delta_{iz}\left(\mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right), \end{cases}$$

$$H_{\mathrm{int}} = -\frac{1}{c}\int d^3x\,\mathbf{j}\cdot\mathbf{a}$$ Coupling term

Lorentz force

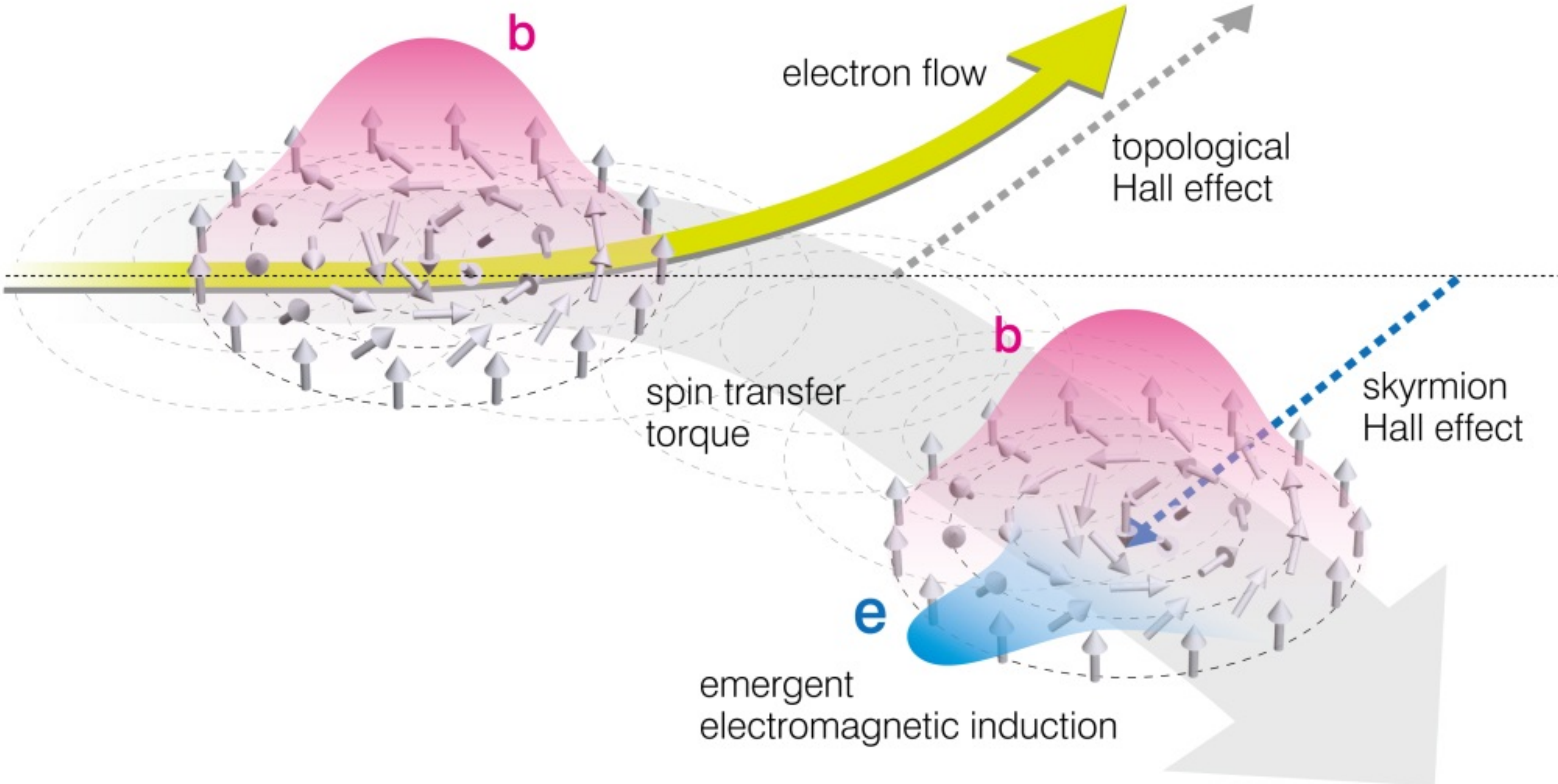
$$\frac{\partial n}{\partial t} + \mathbf{v}\cdot\frac{\partial n}{\partial \mathbf{x}} - e\left(\mathbf{E}+\mathbf{e}+\frac{1}{c}[\mathbf{v}\times(\mathbf{H}+\mathbf{h})]\right)\cdot\frac{\partial n}{\partial \mathbf{P}} = -\frac{\delta n}{\tau}$$

Boltzmann equation

$$\dot{\mathbf{n}} = \frac{\hbar\gamma}{2e}(\mathbf{j}\cdot\boldsymbol{\nabla})\mathbf{n} - \gamma\left[\mathbf{n}\times\frac{\delta H_S}{\delta\mathbf{n}}\right] + \alpha\left[\dot{\mathbf{n}}\times\mathbf{n}\right]$$

Spin transfer torque

LLG equation


b
electron flow
topological
Hall effect
b
spin transfer
torque
skyrmion
Hall effect
e
emergent
electromagnetic induction

Spin Transfer Torques in MnSi at Ultralow Current Densities

F. Jonietz,[1] S. Mühlbauer,[1,2] C. Pfleiderer,[1*] A. Neubauer,[1] W. Münzer,[1] A. Bauer,[1] T. Adams,[1] R. Georgii,[1,2] P. Böni,[1] R. A. Duine,[3] K. Everschor,[4] M. Garst,[4] A. Rosch[4]

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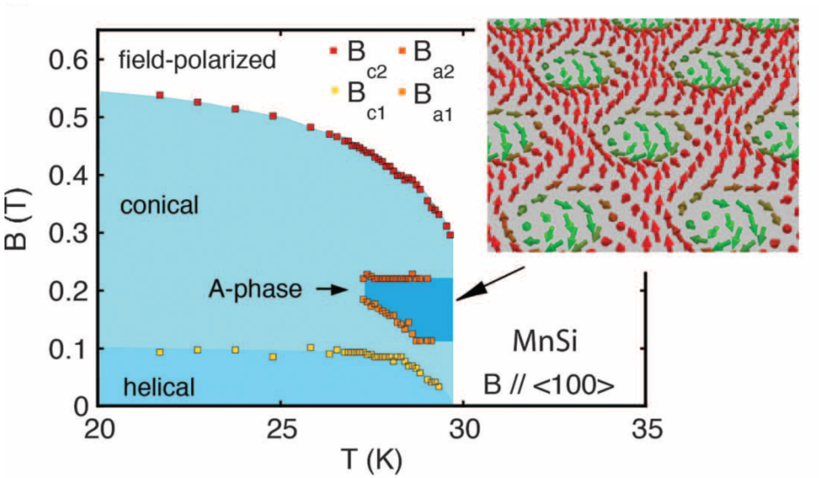
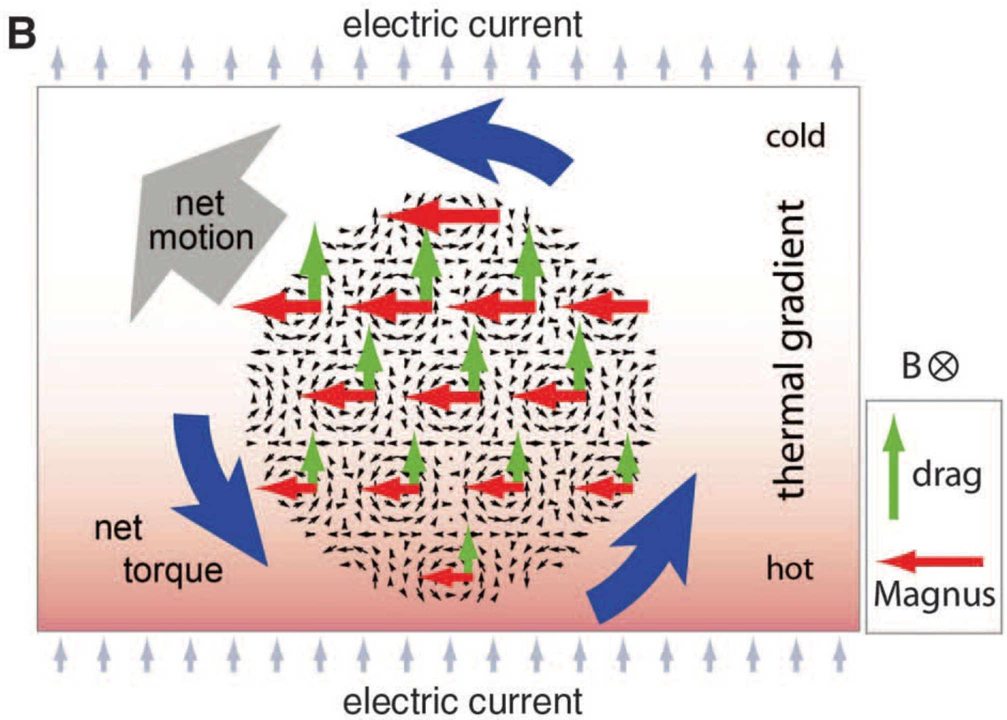


$$j_{critical} \approx 10^{6}\, A/m^{2}$$

c.f. $$j_{critical} \approx 10^{10} - 10^{12}\, A/m^{2}$$

for domain wall motion

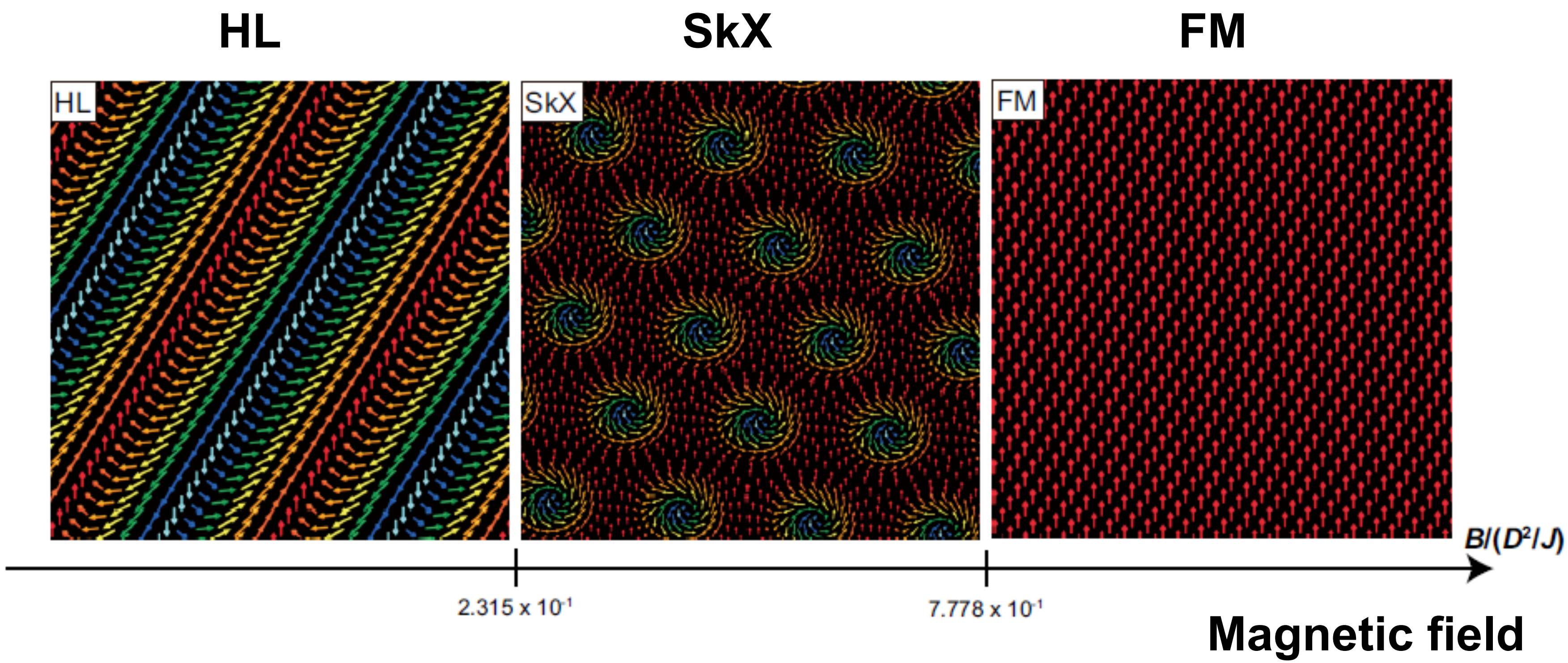


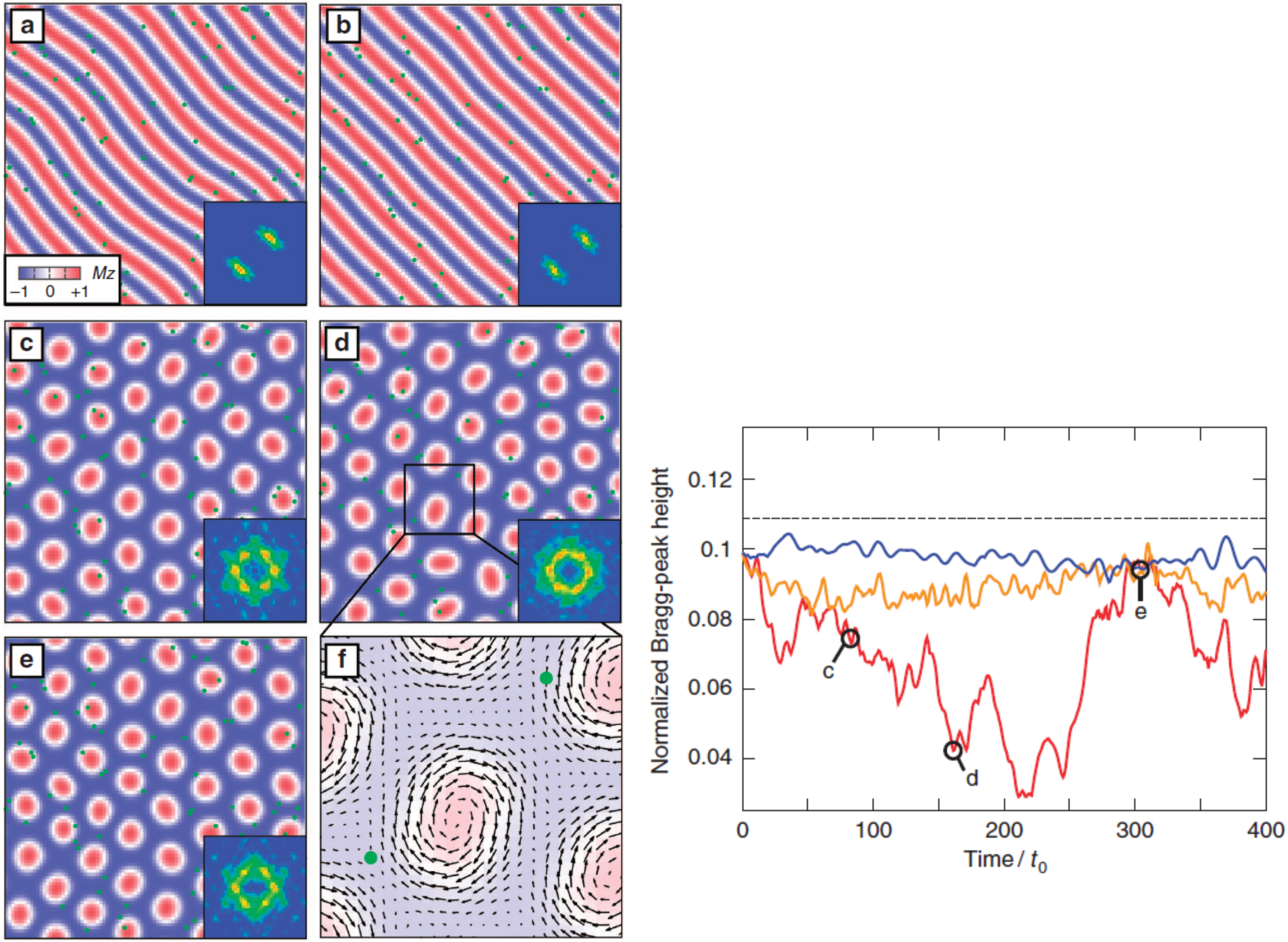
Simulation of Skyrmion motion

$$\frac{\mathrm{d}\vec{M}_{\vec{r}}}{\mathrm{d}t} = \gamma \vec{M}_{\vec{r}} \times B^{\mathrm{eff}}_{\vec{r}} - \frac{\alpha}{M} \vec{M}_{\vec{r}} \times \frac{\mathrm{d}\vec{M}_{\vec{r}}}{\mathrm{d}t} - \frac{pa^3}{2eM}\left(\vec{j}\cdot\vec{\nabla}\right)\vec{M}_{\vec{r}} - \frac{pa^3\beta}{2eM^2}\left[\vec{M}_{\vec{r}} \times \left(\vec{j}\cdot\vec{\nabla}\right)\vec{M}_{\vec{r}}\right],$$

spin transfer torque

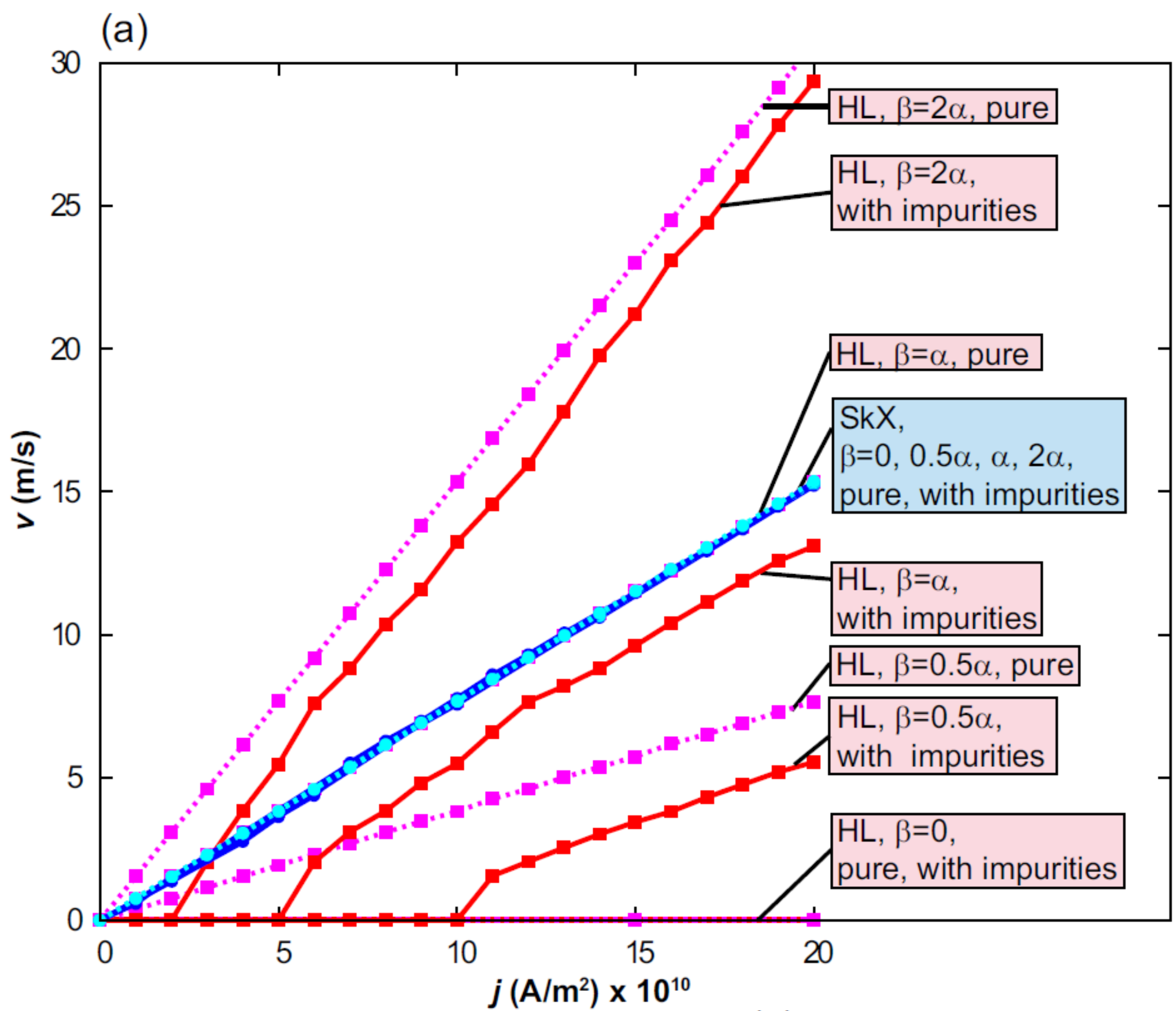
Impurity pinning due to single spin easy axis anisotropy

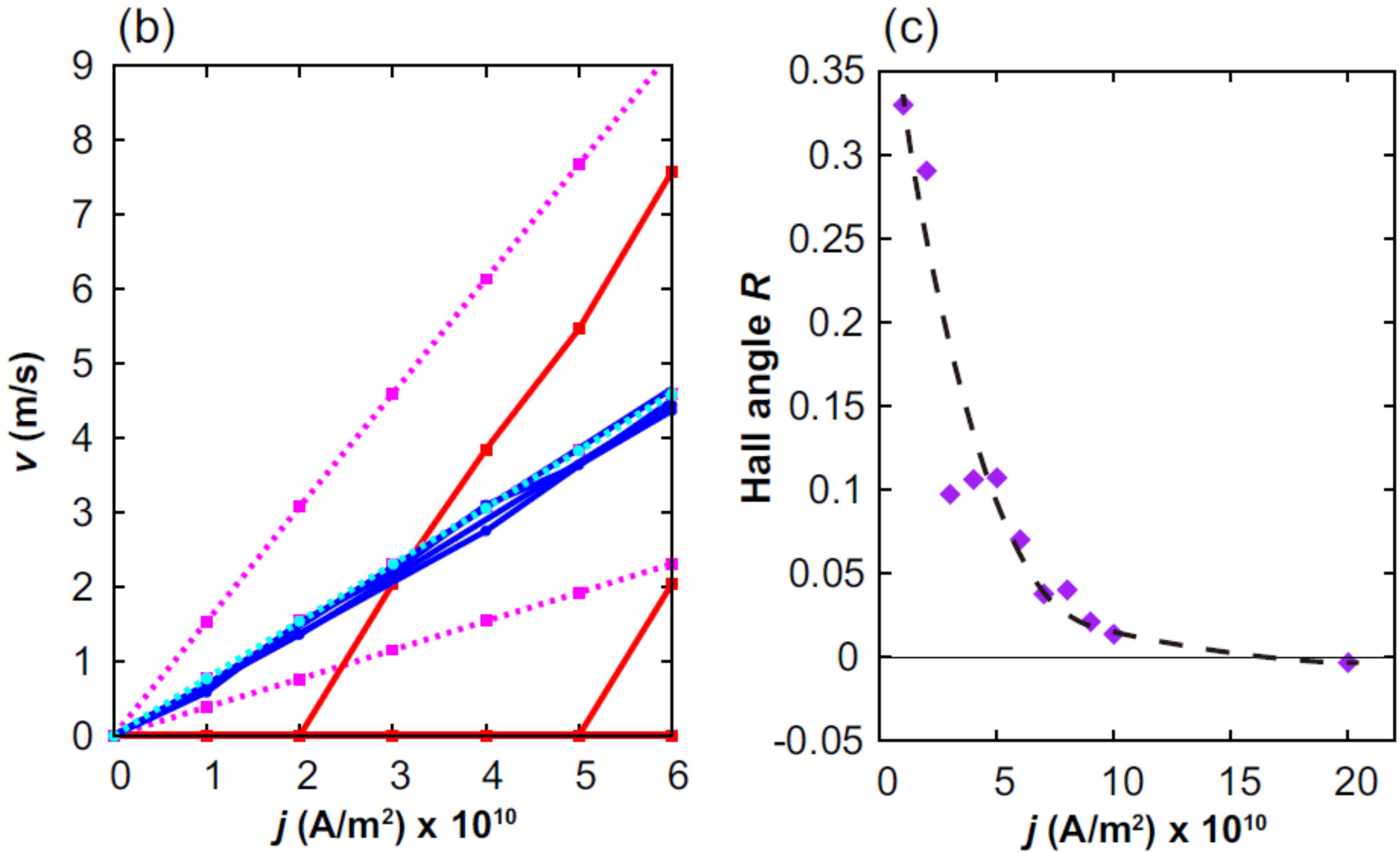




a
b
c
d
e
f
Mz
−1 0 +1
Normalized Bragg-peak height
0.12
0.1
0.08
0.06
0.04
0
100
200
300
400
Time / t_0
c
d
e

J. Iwasaki, M. Mchizuki, and N.N. Nature Commun. 2013





(b)
v (m/s)
j (A/m²) x 10¹⁰
(c)
Hall angle R
j (A/m²) x 10¹⁰

Analysis of the Skyrmion motion by Thiele equation

What is the equation of motion of skyrmion ?

current Sk velocity

$$\boldsymbol{G}\times(\boldsymbol{v}_{\mathrm{s}}-\boldsymbol{v}_{\mathrm{d}})+\boldsymbol{\mathcal{D}}(\beta\boldsymbol{v}_{\mathrm{s}}-\alpha\boldsymbol{v}_{\mathrm{d}})+\boldsymbol{F}_{\mathrm{pin}}=\boldsymbol{0}$$

Topological number

$$\mathbf{G}=(0,\ 0,\ \mathcal{G})\qquad \mathcal{G}=\int_{\text{unit cell}} \mathrm{d}^2 r\left(\frac{\partial\hat{\Omega}}{\partial x}\times\frac{\partial\hat{\Omega}}{\partial y}\right)\cdot\hat{\Omega}=4\pi Q$$

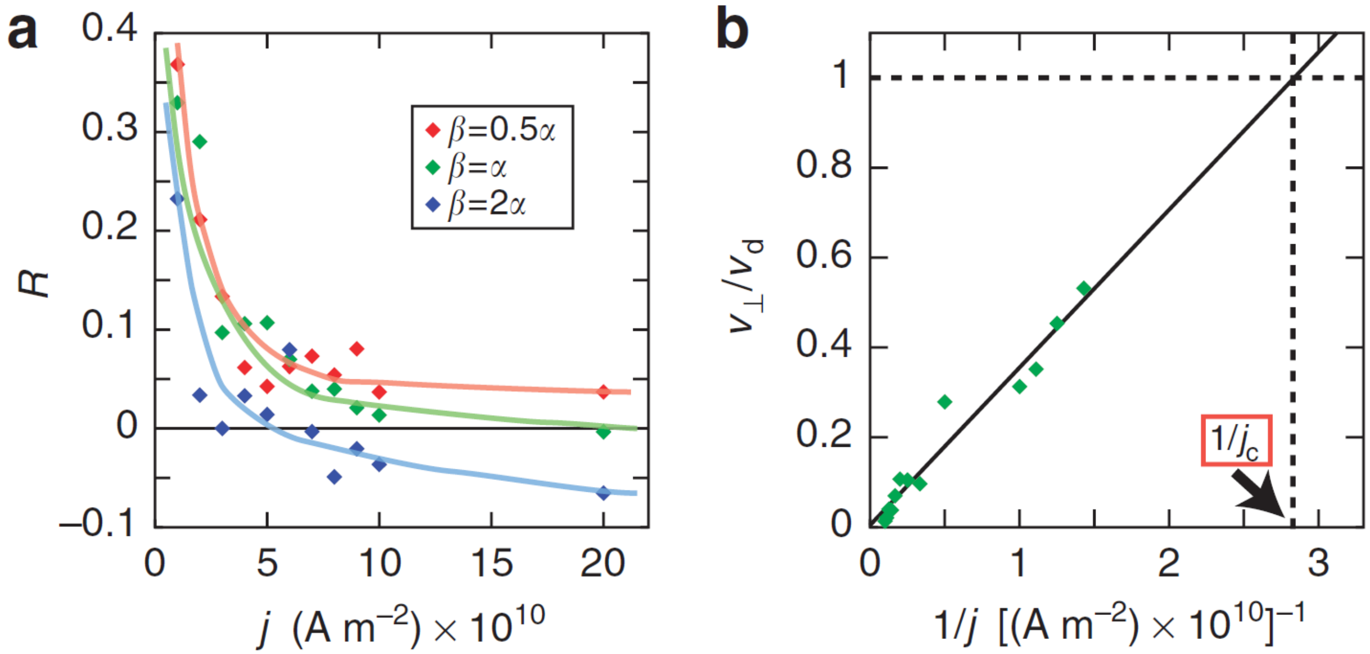
G=0 for domain wall !

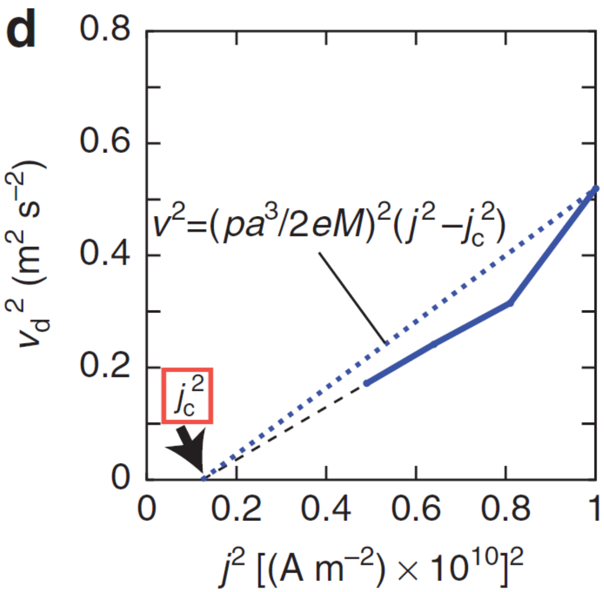
$$\mathcal{D}_{ij}=\int_{\text{unit cell}} \mathrm{d}^2 r\,\partial_i\hat{\Omega}\cdot\partial_j\hat{\Omega}=\begin{cases}\mathcal{D} \text{ for } (i,j)=(x,x),(y,y)\\ 0 \text{ otherwise}\end{cases}$$

K. Everschor et al., PRB 2012
T. Schulz et al, Nat. Phys. 2012
Shi-Zeng Lin et al., 2013

$$\frac{v_\perp}{v_\mathrm{d}} = \frac{R}{\sqrt{1+R^2}} = \frac{j_\mathrm{c}}{j}$$

$$v_d = \sqrt{v_\parallel^2 + v_\perp^2}$$

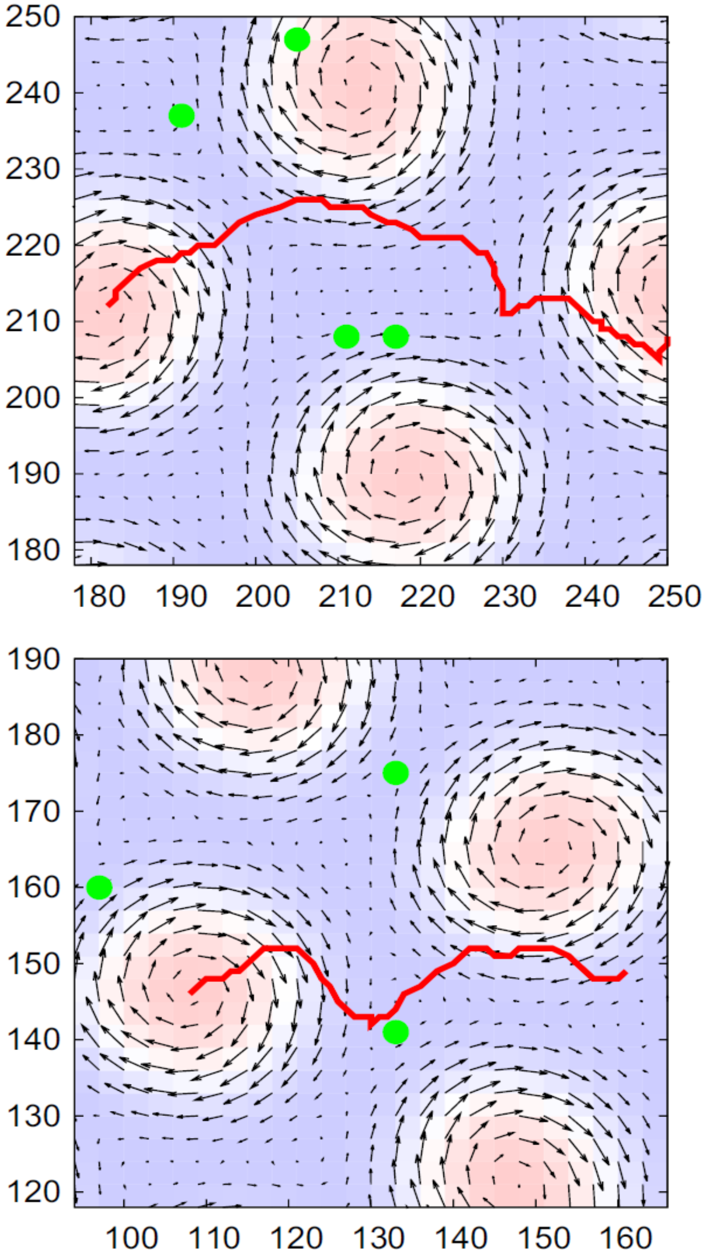




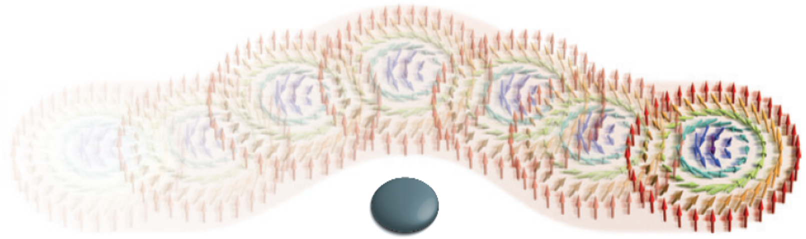
$$v_\mathrm{d}^2 = \left(\frac{pa^3}{2eM}\right)^2 (j^2 - j_\mathrm{c}^2)$$

$$\frac{F_\mathrm{pin}^\mathrm{SkX}}{F_\mathrm{pin}^\mathrm{HL}} = \frac{1}{\pi\beta}\frac{j_\mathrm{c}^\mathrm{SkX}}{j_\mathrm{c}^\mathrm{HL}} = 0.56$$

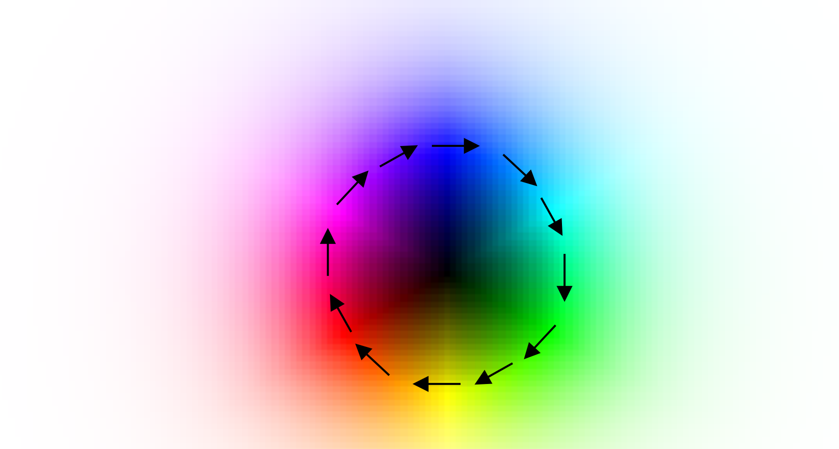
reduced pinning force for Sk



$$\dot{X} = +\frac{1}{\gamma}\frac{\partial V}{\partial Y}, \quad \dot{Y} = -\frac{1}{\gamma}\frac{\partial V}{\partial X}$$

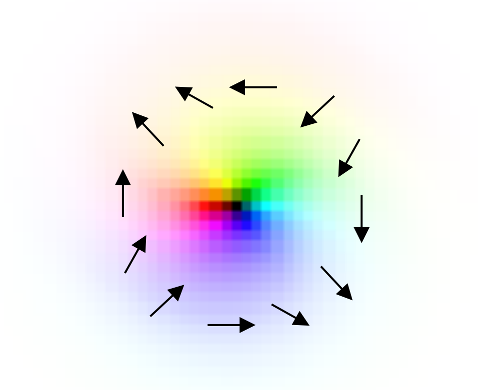


A.Rosch Nature Nanotechnology



$m = 1, \quad N_{sk} = -1$

skyrmion
= particle



$m = -1, \quad N_{sk} = 1$

anti-skyrmion
= anti particle

$$\vec{n}(\vec{r}) = (\cos\Phi(\phi)\sin\Theta(r), \sin\Phi(\phi)\sin\Theta(r), \cos\Theta(r))$$

$$\Phi(\phi) = m\phi + \gamma$$

m: vorticity　　γ: helicity

$$H_{DM} = D\vec{n}\cdot(\nabla\times\vec{n}) = D\sin[(m-1)\phi+\gamma]\left(\frac{d\Theta}{dr} + \frac{m}{2r}\sin 2\Theta\right)$$

Conclusions

1. Skyrmion as topological particle in magnets

$$\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) \approx \nabla \times \vec{a}$$

2. Skyrmion physics in magnets

$$\bar{H}_{\rm int} = -\frac{1}{c}\int d^3x \mathbf{j} \cdot \mathbf{a}$$

- conduction electron

  topological Hall effects, electromagnetic induction

- current-driven motion of skyrmion

  novel dynamics of skyrmion non-Newtonian particle

  skyrmion circuits and "skyrmionics"

  hadron-meson physics and simulation for universe

Emergent electromagnetism in solids