Symmetry Protected and Symmetry Enriched Topological Phases

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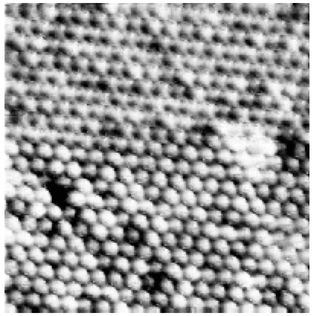
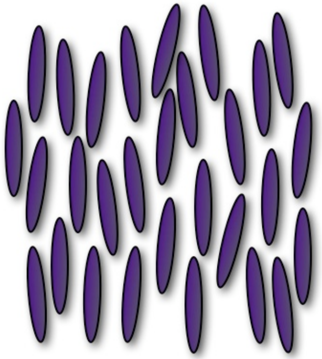
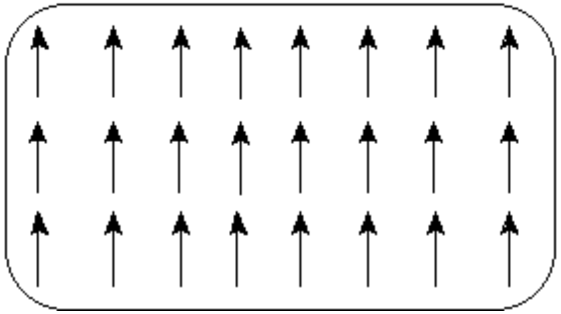
Symmetry breaking classification:





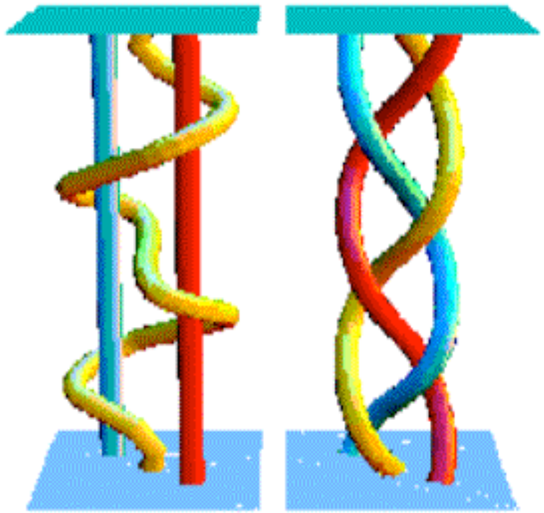
Nematic



- Ginzburg-Landau-Wilson
- local order parameters

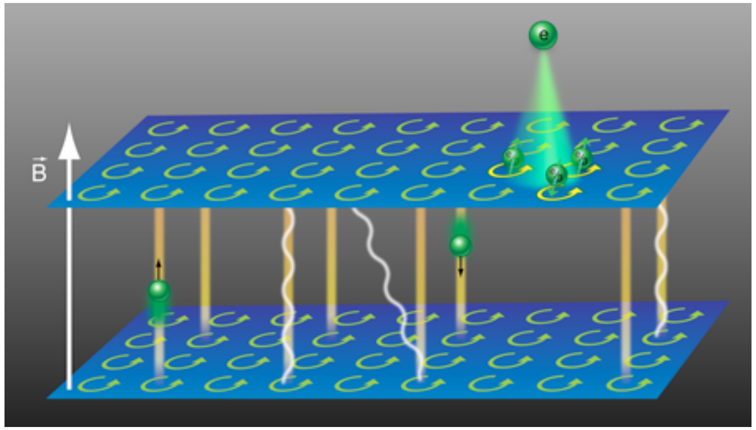
Topological phases:

2d gapped

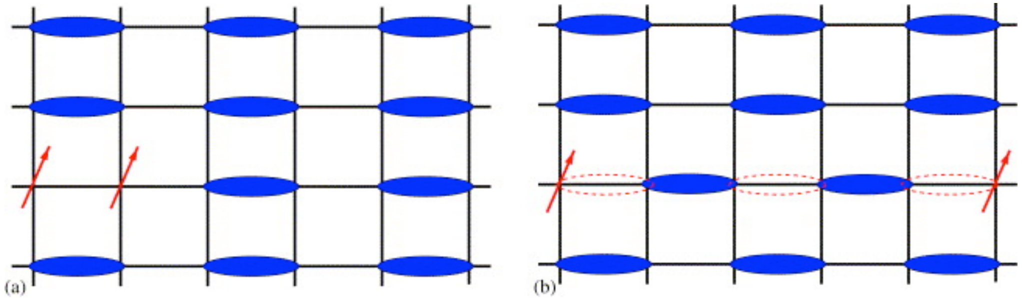


- deconfined fractionalized quasiparticles

Fractionalized:

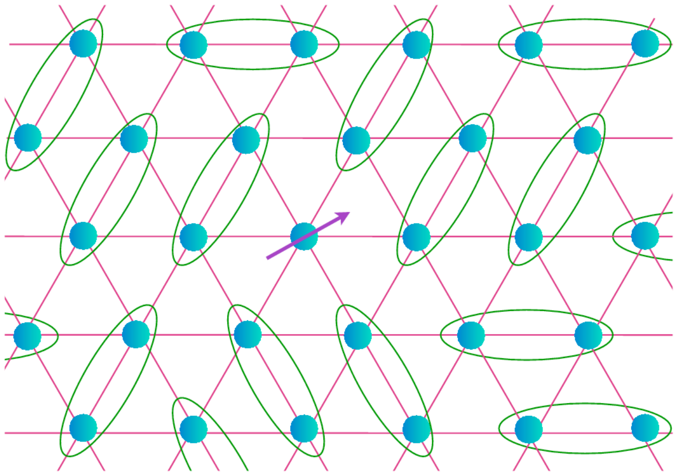


Laughlin quasiparticles



spinons (not quite)

Deconfined:



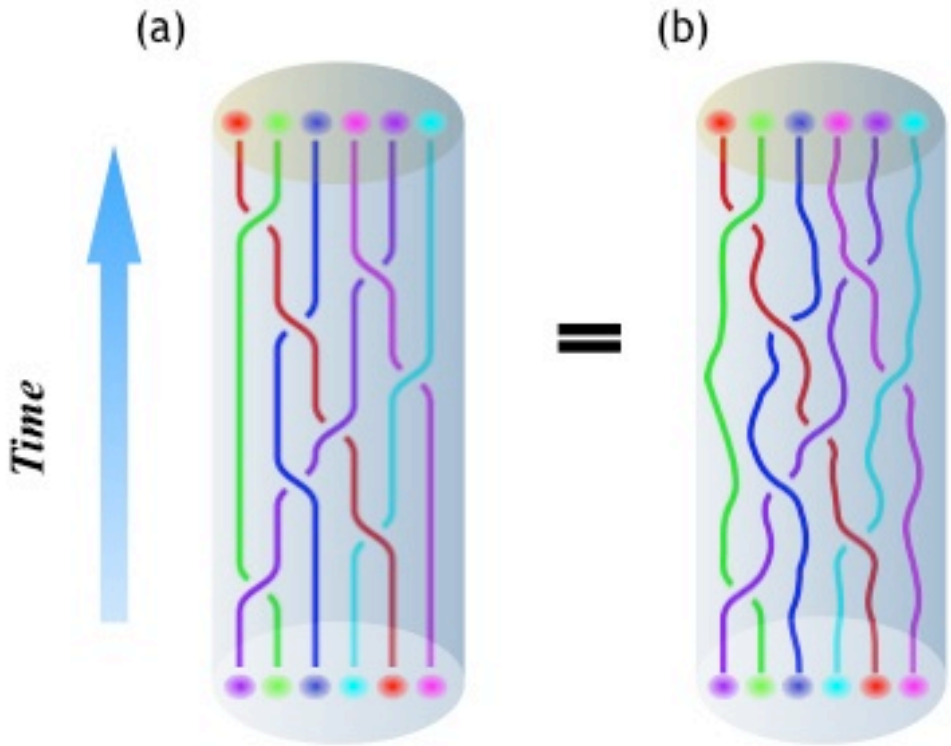
deconfined spinon

Braiding statistics:

- more than fermions / bosons

- statistics can be a more general complex phase, or even matrix

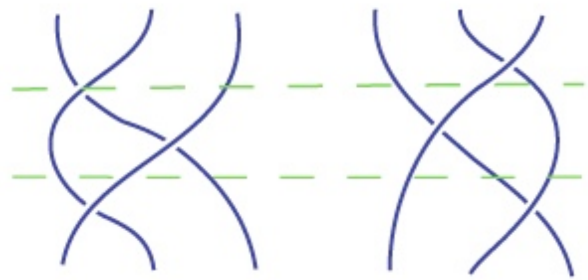
- "anyons" (Wilczek)



Rigidity:

-"anyon" is a bit of a misnomer: quasiparticle braiding structure is very rigid.

- consistency conditions, eg.:

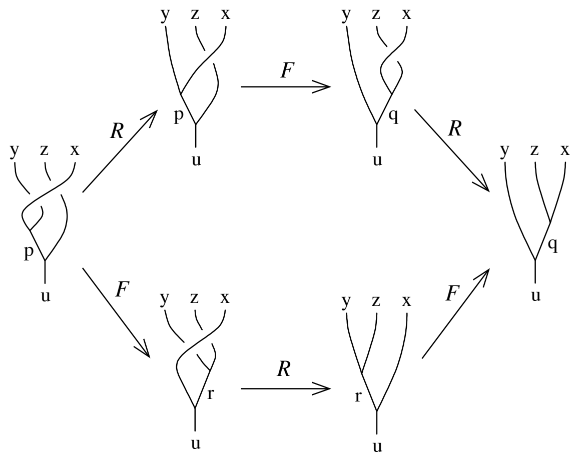


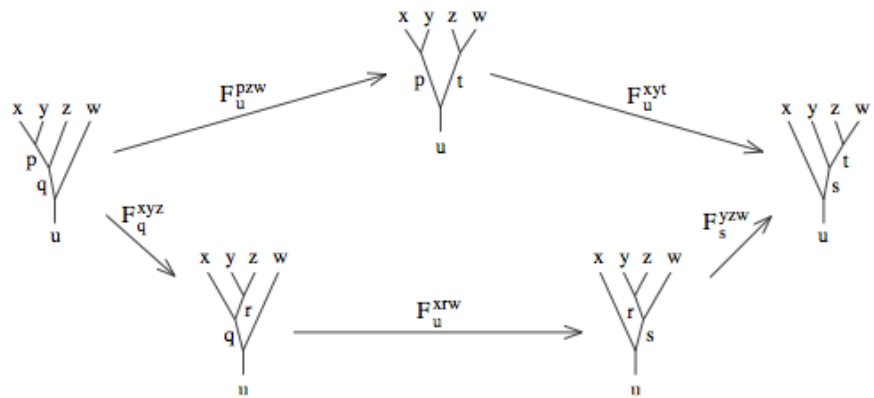
$$R_{12}\ R_{13}\ R_{23} = R_{23}\ R_{13}\ R_{12},$$

- structure encoded in numbers (or matrices):

R_{ij} and F_{ijk}

- resulting structure: “Unitary Modular Tensor Category” (UMTC):





- the solutions are discrete, so the particular solution is itself a discrete invariant of the gapped phase

- works for “bosonic” systems - i.e. spin systems (can be extended to fermions)

- Hence, can classify bosonic “intrinsic topological order” by UMTCs. (Moore & Read, Wen, etc.)

Symmetries:

- symmetry group G: $[H, U_g] = 0$

- "Symmetry Protected Topological Order": no symmetry breaking, no deconfined anyons

- classified by group cohomology $H^{d+1}(G, U(1))$

(1d: AKLT; Chen, Gu, Wen; Fidkowski, Kitaev; Pollman, Berg, Turner, Oshikawa; Hatsugai et al.; 2d: Chen, Gu, Liu, Wen; Levin, Gu)

Remainder of talk:

- only gapped 2d bosonic (i.e. spin) systems

- intrinsic topological order + symmetries = "symmetry enriched topological order"

    - classification and physical picture

    (Essin, Hermele; Mesaros, Ran; Lu, Vishwanath, etc.)

- Subtle obstructions: some seemingly OK states are physically inconsistent

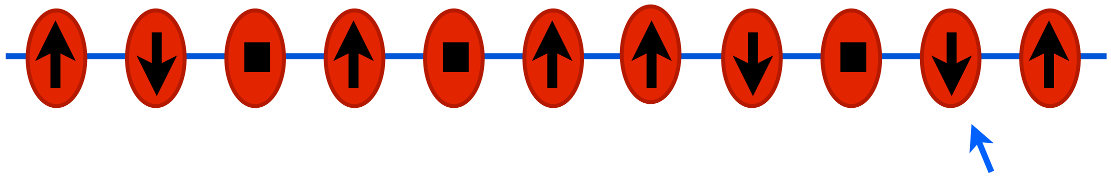
Eg. G=SO(3) (i.e. integral spins)

- $\mathbb{Z}_2$ gauge theory with spin-1/2 spinon OK

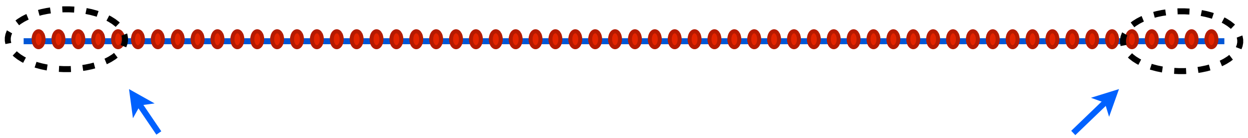
- Chiral spin liquid ($\nu = 1/2$ bosonic FQH) with spin-1/2 semion. INCONSISTENT

1 dimension:

G=SO(3), Haldane chain: $H = J \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$



spin-1 sites



effective spin 1/2's at endpoints: "fractionalization"

Projective representations of G

$$f = gh \quad \to \quad U_f = U_g U_h \quad \leftarrow \text{ordinary representation}$$

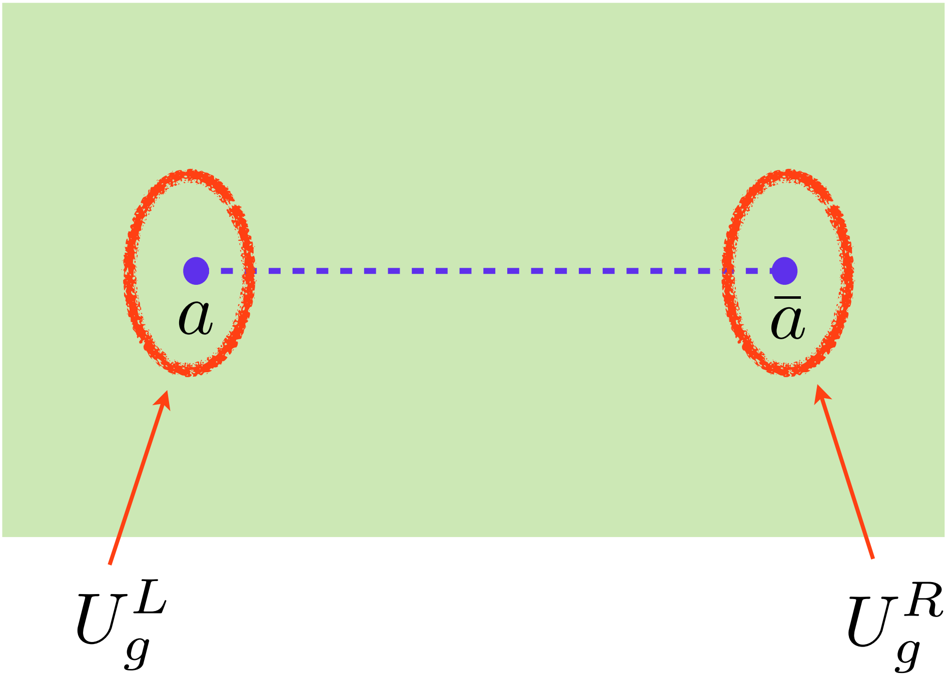
$$f = gh \quad \to \quad U_f = e^{i\phi(g,h)} U_g U_h \quad \leftarrow \text{projective representation}$$

$\phi(g, h)$: function of 2 group variables =>

$$H^2(G, U(1))$$

2 dimensions:

- anyons can carry fractional/projective quantum numbers of G:

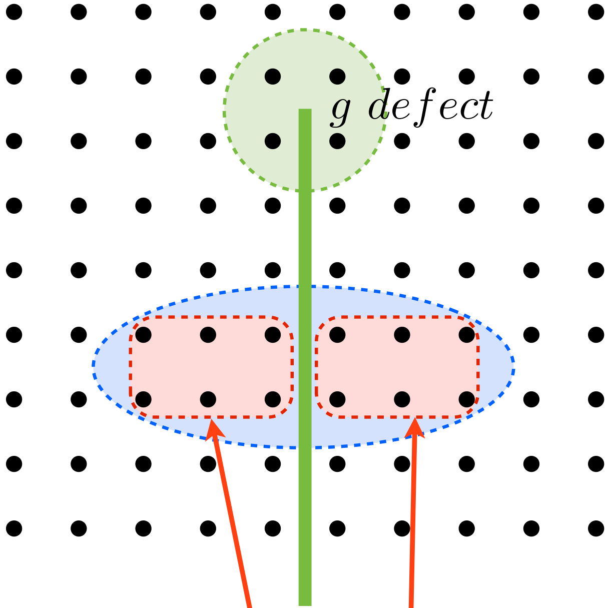


$$U^a_{gh} = e^{i\phi_a(g,h)} U^a_g U^a_h$$

constraint: $\phi_a(g,h) + \phi_b(g,h) = \phi_c(g,h)$ **=>** $H^2(G, \text{abelian anyons})$

$$a \times b = c$$

G-defects:



$$H_i = \sum_\alpha H_i^{L,\alpha} H_i^{R,\alpha}$$

$$\downarrow$$

$$H_i^{defect} = \sum H_i^{L,\alpha} \left(\mathcal{U}_g^{-1} H_i^{R,\alpha} \mathcal{U}_g \right)$$

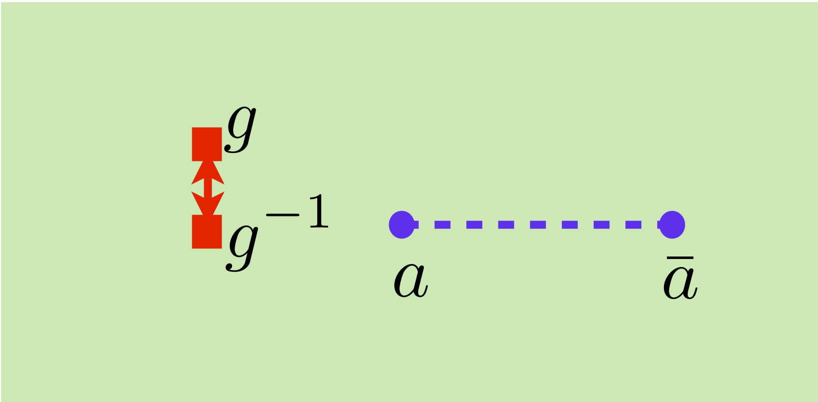
$$U_{fh}^{(g)} = e^{i\phi_g(f,h)} U_f^{(g)} U_h^{(g)}$$

$\downarrow$ leads to

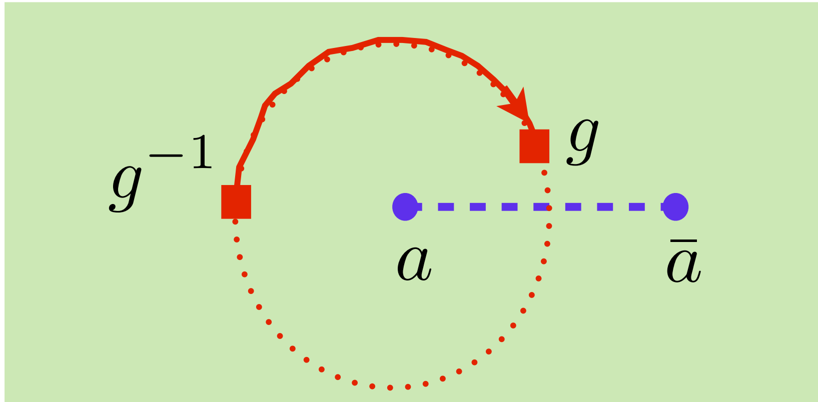
$$H^3(G, U(1))$$

Local action of G via adiabatic process:

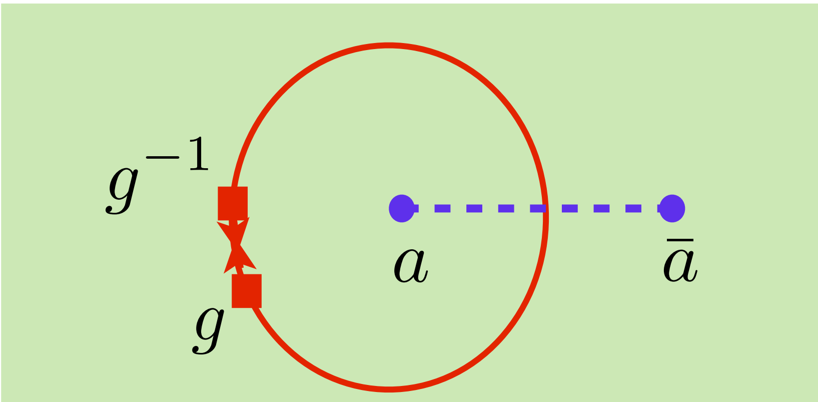
1. nucleate defect / anti-defect pair:



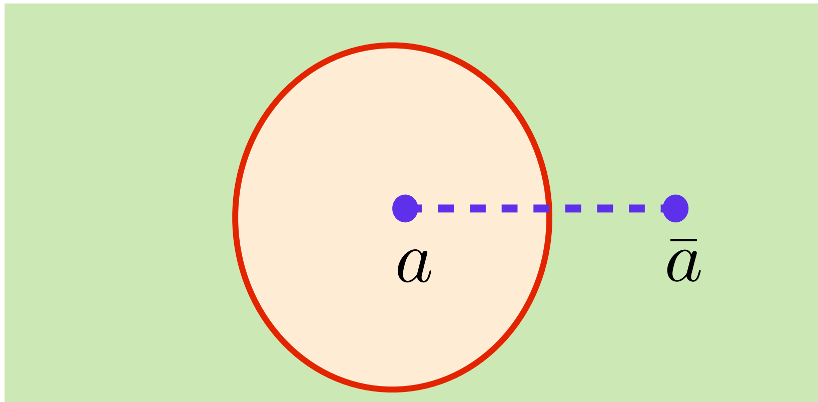
2. take one defect around anyon:



3. Annihilate defects:

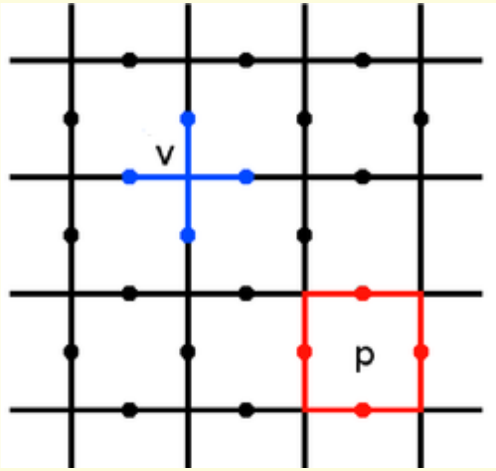


3. Act with g on spins inside defect loop to remove branch cut



Example: "deformed" toric code:

(= $\mathbb{Z}_2$ gauge theory)



$$A_v = \prod_{i \in v} \sigma_i^x, \;\; B_p = \prod_{i \in p} \sigma_i^z$$

$$H = -J \sum_v A_v + J \sum_p B_p, \;\; J > 0$$

note plus sign

G = spatial translations = $\mathbb{Z} \times \mathbb{Z}$

generators: T_x, T_y

Ground state of "deformed" toric code:

- ordinary toric code:

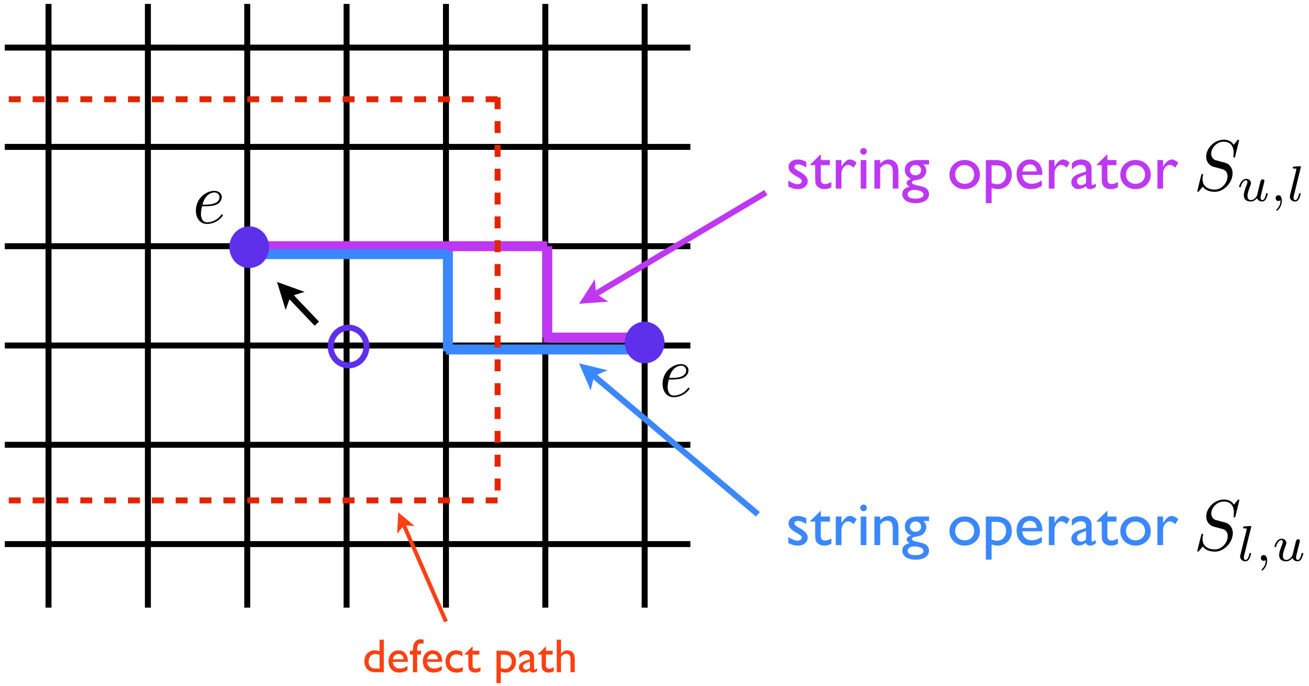
$$|\Psi\rangle = \sum_{\text{loop configs L}} |L\rangle$$

- "deformed" toric code:

$$|\Psi\rangle = \sum_{\text{loop configs L}} (-1)^{|L|} |L\rangle$$

area of loops in L (well defined with periodic boundary conditions and even # of sites)

Electric charge is projective under $\mathbb{Z} \times \mathbb{Z}$



$$U^L_{T_x} U^L_{T_y} |\Psi\rangle = S_{l,u}|0\rangle$$

$$U^L_{T_y} U^L_{T_x} |\Psi\rangle = S_{u,l}|0\rangle$$

but $\boxed{S_{u,l}|0\rangle = -S_{l,u}|0\rangle}$

Example 2:

$G = \mathbb{Z}_n$: fractional charges

$$U_{\frac{[k]}{n}} |a\rangle = e^{2\pi i \frac{[k]}{n} q} |a\rangle$$

a has charge q

[k] means (k mod n)

$$U_{\frac{[k+l]}{n}} |a\rangle = e^{2\pi i \frac{[k+l]}{n} q} |a\rangle$$

$$U_{\frac{[k]}{n}} U_{\frac{[l]}{n}} |a\rangle = e^{2\pi i \frac{[k]+[l]}{n} q} |a\rangle$$

phase mismatch when q is fractional

Gauge G:

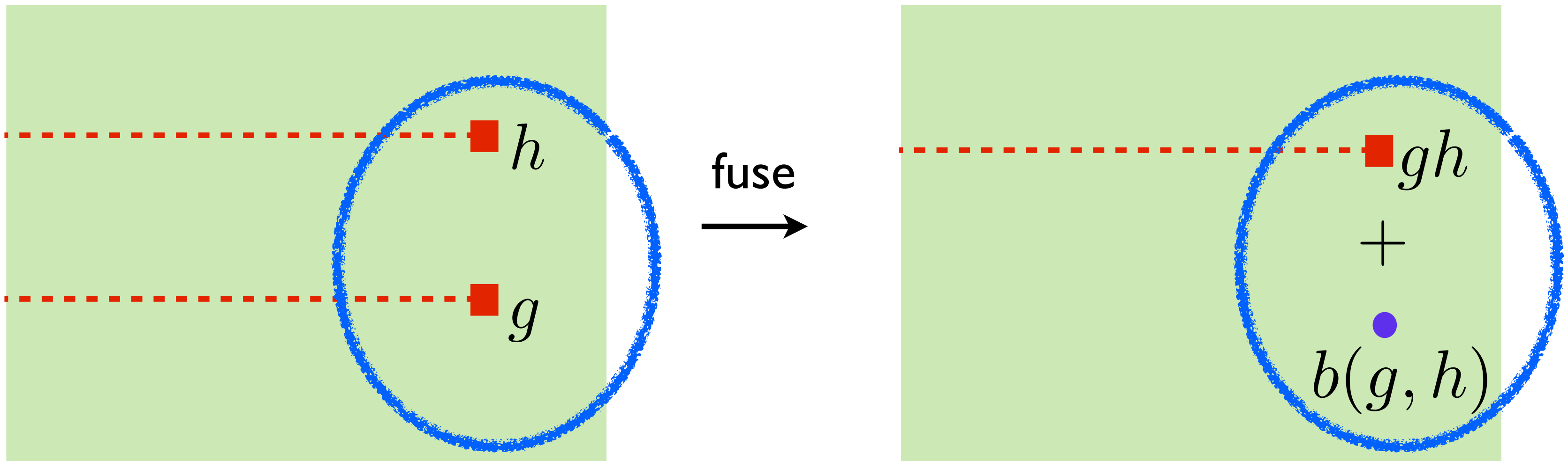
G-fluxes, G-charges become deconfined excitations:

quasiparticles = {original anyons, G-fluxes, G-charges}

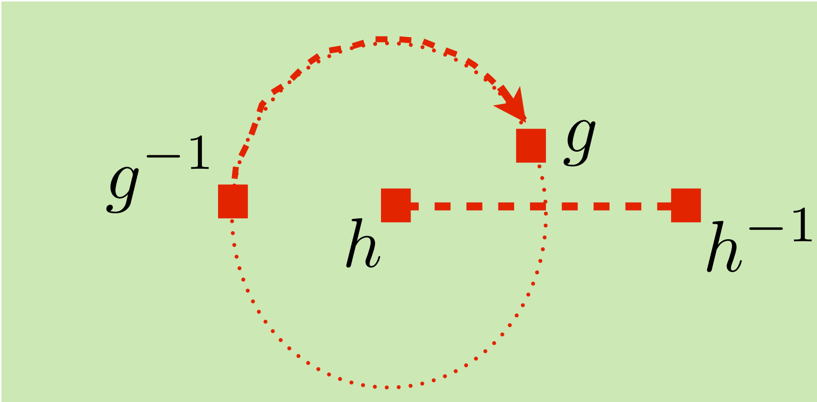
Then the projective character of the anyons and defects is reflected in the fusion/braiding structure of this enlarged theory.

(see also Levin & Gu, arXiv:1202:3120)

1) projective/fractional anyons => defect fusion rules



2) projective/fractional defects => defect braiding rules



Any consistent theory for fusion/braiding in enlarged theory must come from some some choice of these two invariants

(Etingof et al., Kirillov, Mueger)

(can also handle case when G changes anyon superselection sectors)

Connection to 3D SPT's:

Is every choice of $H^2(G, A)$ and $H^3(G, U(1))$ realized in some 2d theory?

No: sometimes there is an obstruction to a consistent choice of braiding and fusion rules in the enlarged theory

Example:

anyons = {1,a} (a=semion)

abelian theory, K=(2)

$\nu = 1/2$ bosonic FQHE

G = $\mathbb{Z}_2 \times \mathbb{Z}_2$, a is projective under G

$G = SO(3)$ works too

Although this theory cannot be realized in 2d, we have an exactly solvable model which realizes it at the surface of a 3d SPT.

(with F. Burnell, X. Chen, A. Vishwanath)

In general, the obstruction is in $H^4(G, U(1))$, which classifies 3d SPT's.

Conclusions:

- Classified symmetry enriched topological phases (SETs), given data of UMTC and symmetry group G

- Resulting SETs are in one to one correspondence with fusion/braiding structures for anyons + G-defects

- These SETs are parametrized by data that describes fractional/projective character of anyons and defects

- Not all choices of this data lead to consistent SETs: interpretation as "anomalous" realization of G at the surface of 3d SPT.