



# The topological insulator with strong correlation effects

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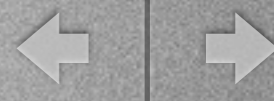
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Feng Lu, Jian Zhou Zhao, HongMing Weng, Zhong Fang

Lu et al, PRL 110, 096401 (2013)



# outline

- Band inversion and TI
- Mix valence compound: Band inversion between d and f bands; the important role of e-e interaction
- SmB<sub>6</sub> and PuTe
- Conclusion



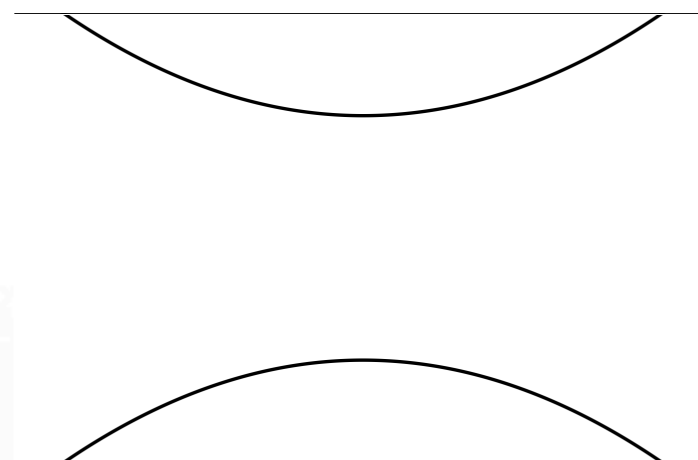
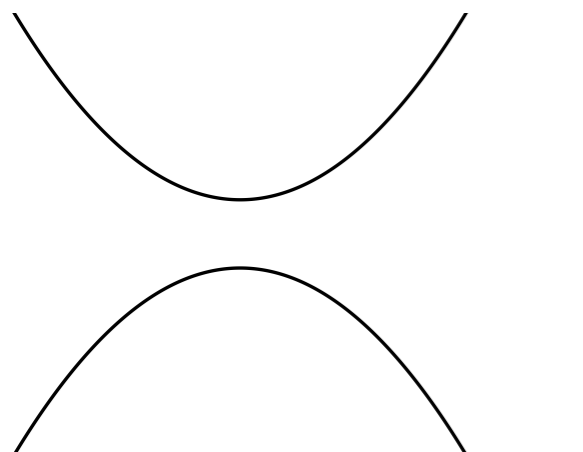


## Topological classification of band insulators

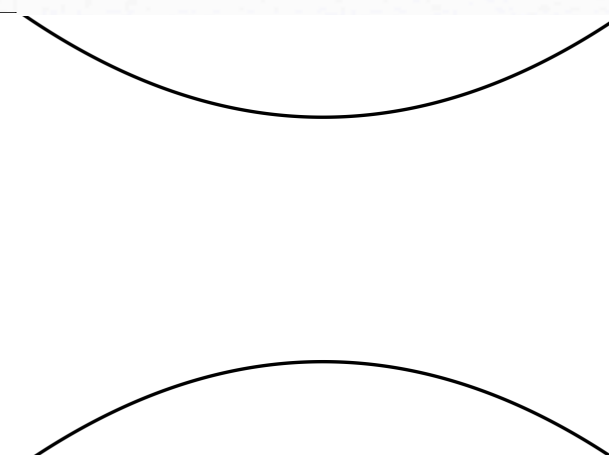
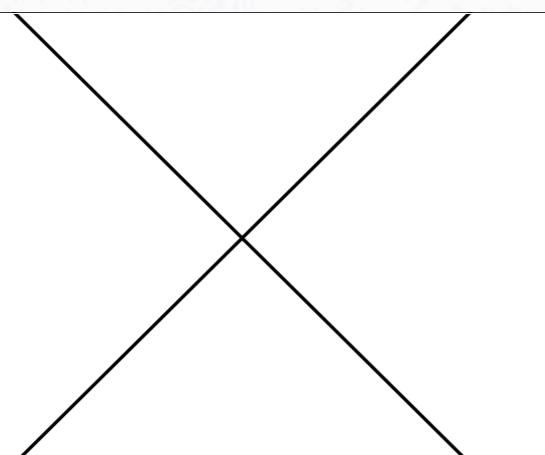
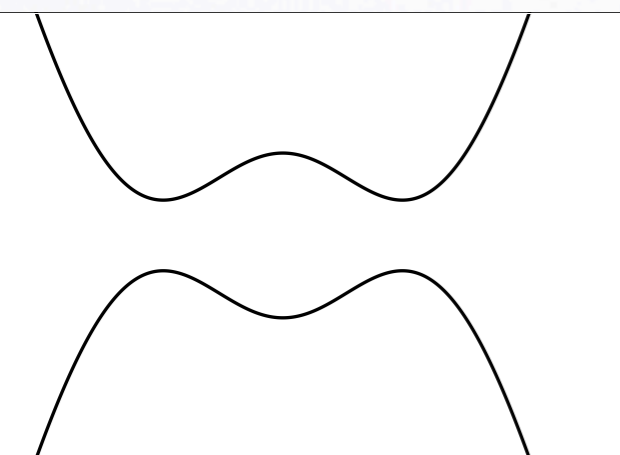
- normal insulator: the electronic structure can be smoothly transformed to isolated atoms
- can not be smoothly transformed to isolated atoms without going through a phase transition,  $Z_2$  invariance defined for non-interacting TI
- TI with interaction can be defined by the Theta angle of of topological magneto-electric effect



# Evolution of band structure from solid to isolated atoms



Normal Insulator



Topological Insulator



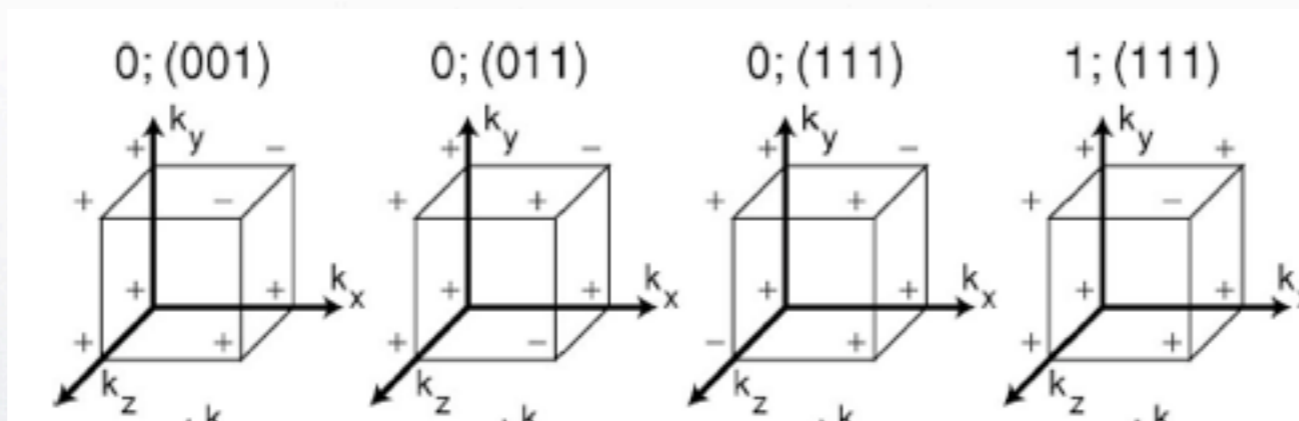


# Z2 invariance for non-interacting TI with inversion symmetry

- **Simple rules for TI with inversion symmetry:  
strong index and weak indices;  $\mathbf{K}=-\mathbf{K}$  high symmetry points**

$$(-1)^{\nu_0} = \prod_{i=1}^8 \delta_i.$$

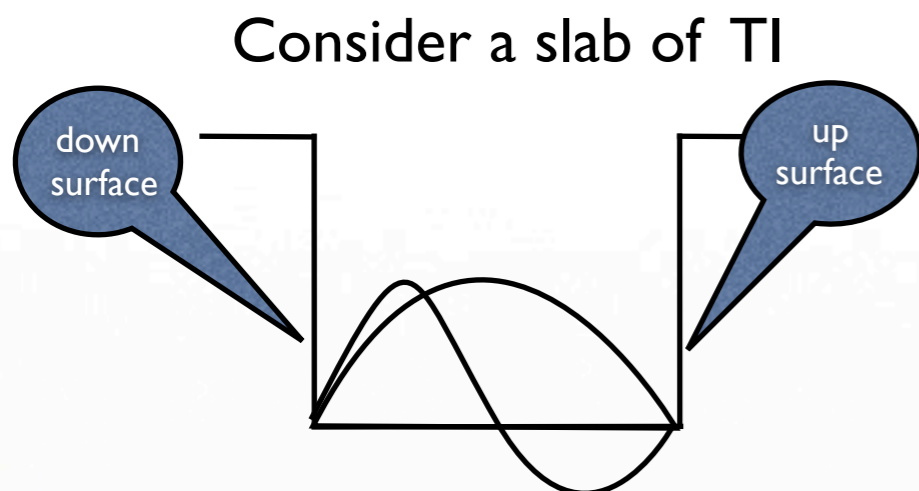
$$\delta_i = \prod_{m=1}^N \xi_{2m}(\Gamma_i).$$



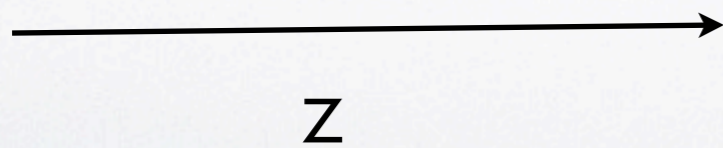
L. Fu and C. Kane, PRB 76,045302



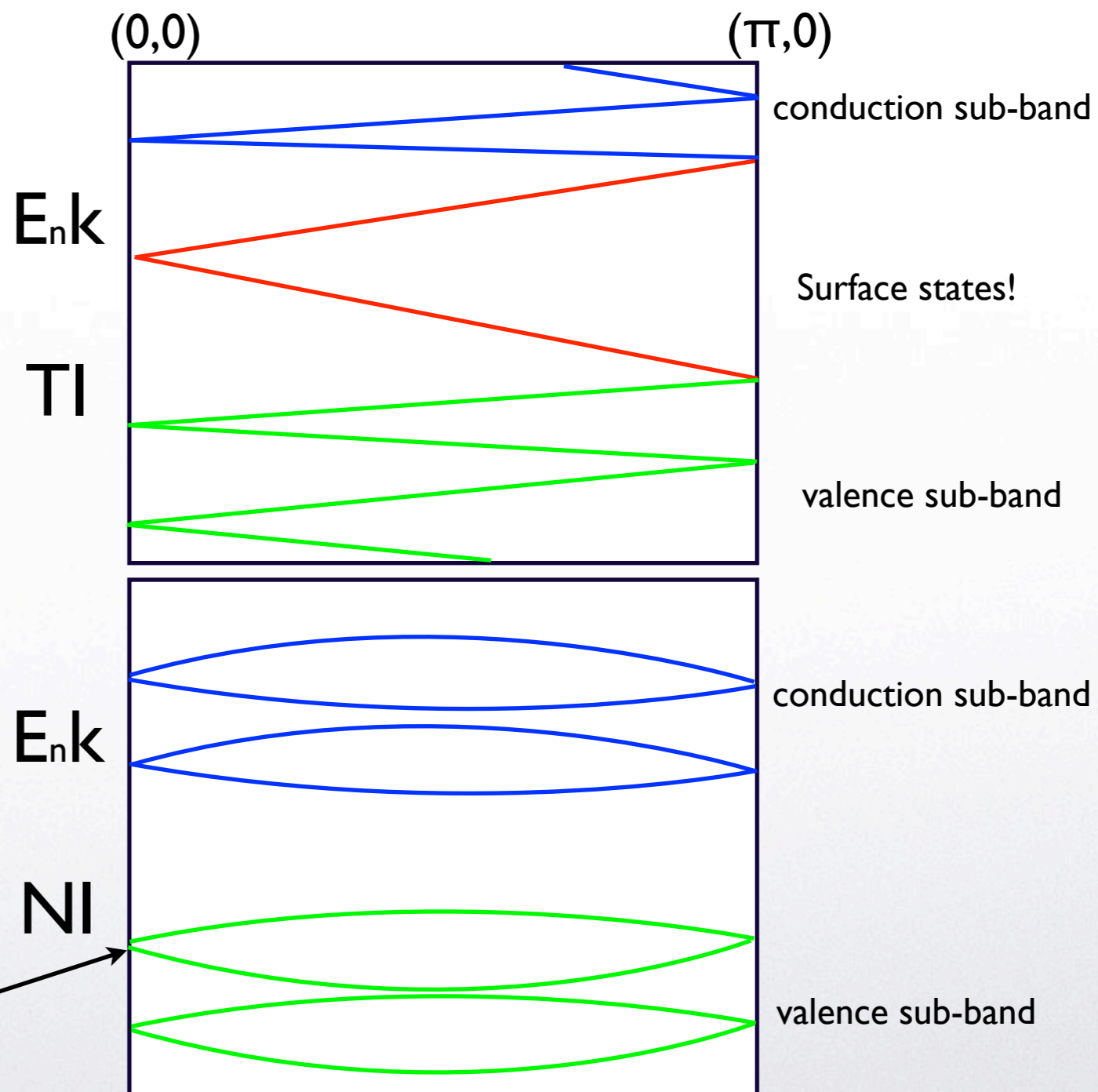
# TI and surface states



$$\psi_n(k_x, k_y)$$



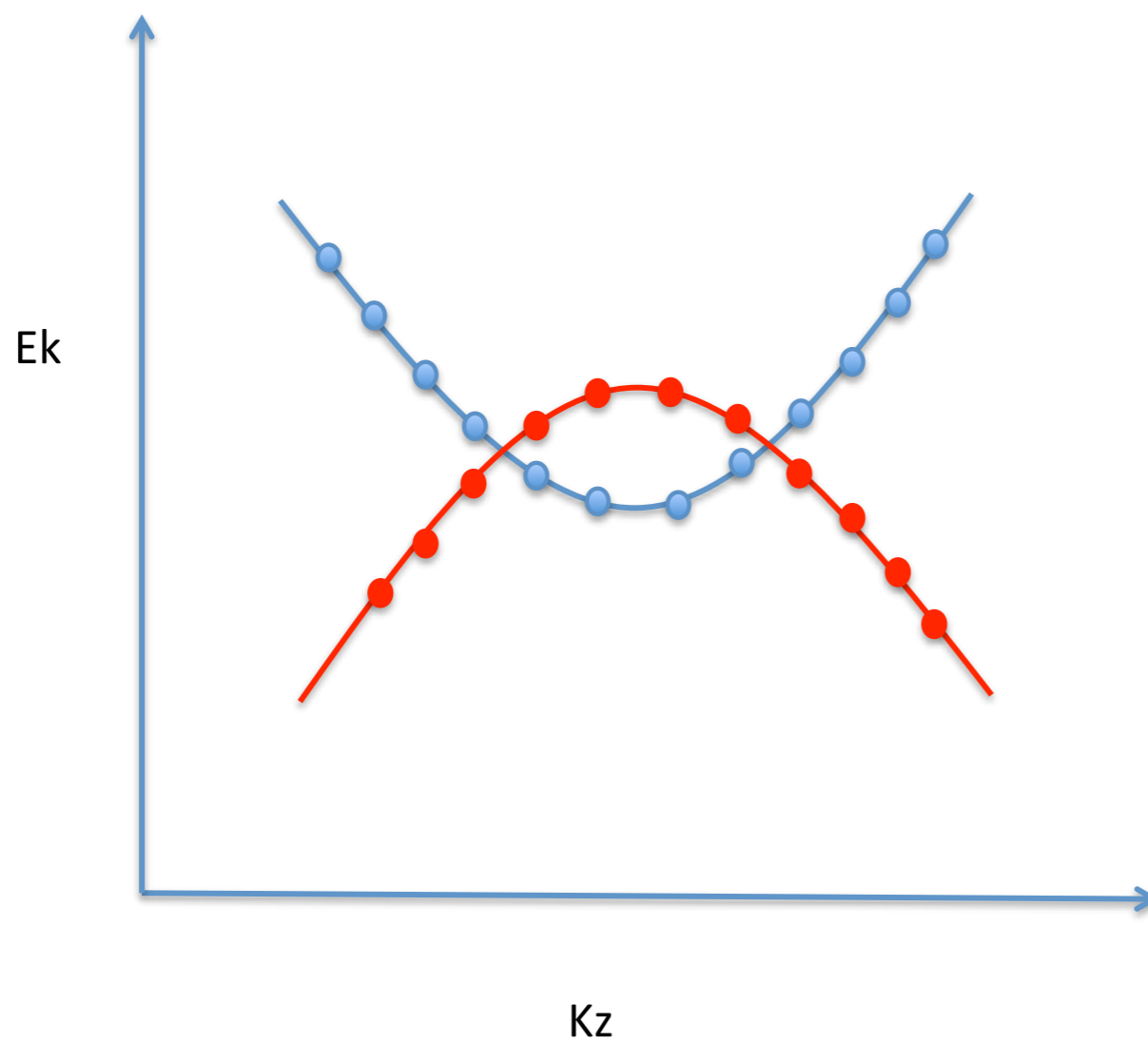
$k = -k + G$ ; Kramers degeneracy







## Introduction: Band Inversion and TI





# The different types of band inversion

- Band inversion between s and p bands: HgTe
- Band inversion between bonding and anti-bonding p-bands: Bi<sub>2</sub>Se<sub>3</sub>, Sb
- Band inversion generated by valence fluctuation of 4f/5f bands
- Very strong correlation effects in f-electron materials:
- Does these materials topologically non-trivial?



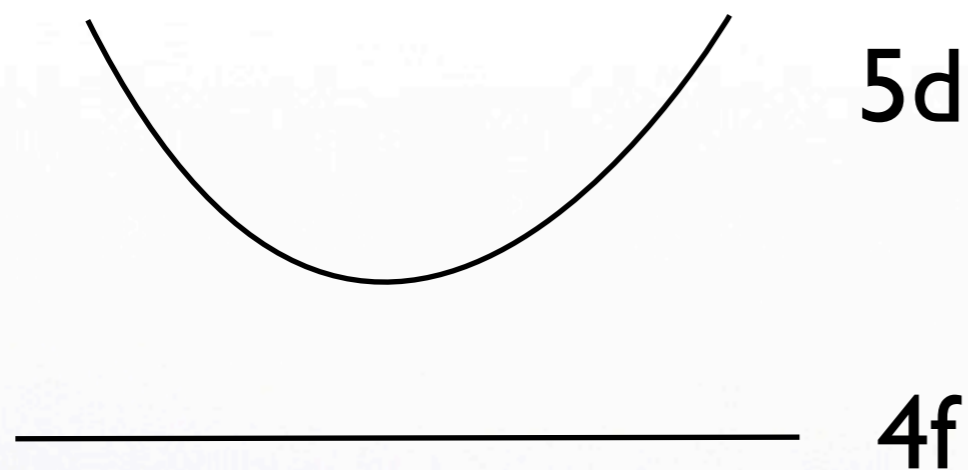


## 4f/5f compounds with intermediate valence

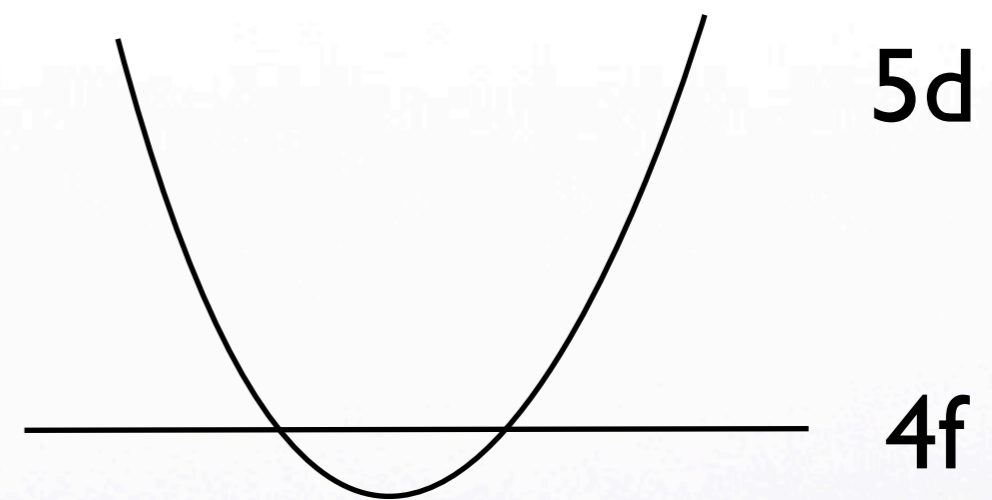
- golden phase of SmS
- SmB<sub>6</sub>
- PuTe and PuSe
- Yb
- Are these materials topological Kondo insulator? [M. Dzero et al, Phys. Rev. Lett. 104, 119901,2010](#)



# Intermediate Valence and band inversion: from the band theory point of view



Rare earth compounds  
with divalence (**not very stable**),  
Sm, Eu, Yb.....



intermediate valence state  
band theory point of view





# Is the band description correct for these compounds?

- from non-interacting band insulator to strong coupling “Kondo insulator”
- how to capture the correct electronic structure?
- how to describe its topological nature?



# How to compute the Z2 invariance for interacting system?

Formula derived from the topological field theory:  
[PRL.105,256803 \(2010\)](#)

$$P_3 = \frac{\pi}{6} \int_0^1 du \int \frac{d^4k}{(2\pi)^4} \text{Tr} \epsilon^{\mu\nu\rho\sigma} [G\partial_\mu G^{-1} G\partial_\nu G^{-1} \times G\partial_\rho G^{-1} G\partial_\sigma G^{-1} G\partial_u G^{-1}] \quad (1)$$

- Pole expansion of the self energy without k dependence [PRB 85, 235135 \(2012\)](#); [EPL98 \(2012\) 57001](#)
- Using the eigenstate of  $H_0 + \Sigma(0)$ , condition: there is no singularity along the imaginary axis of self energy, [PRB 85, 165126 \(2012\)](#)





## Our method: LDA+Gutzwiller

- Gutzwiller type of ground state wave function
- Combined with first principle code
- Suitable for the study of f-electrons
- Has been applied to many correlated system:  
LaOFeAs;  $\text{NaxCoO}_2$ ; Ce; Pu....PHYSICAL REVIEW B 79, 075114 ,2009



## Main difficulties for the electronic structure calculation for f-electron systems

- Strong interactions among f-electrons, which can be expressed in terms of Slater integrals:  $F_0, F_2, F_4, F_6$
- for SmB<sub>6</sub>  $F_0=5.8\text{eV}$ ,  $F_2=9.9\text{eV}$ ,  $F_4=7.09\text{eV}$ ,  $F_6=4.99\text{eV}$
- Typical interaction strength is one order bigger than the f-band width
- Multiplet state VS the band state





# The Gutzwiller Trial wave function used in this study

## Rotational Invariant Gutzwiller Approximation

Gutzwiller variational wavefunction:

$$|\Psi_G\rangle = \mathcal{P}|\Psi_0\rangle = \prod_{\mathbf{R}} \mathcal{P}_{\mathbf{R}}|\Psi_0\rangle$$

$$\mathcal{P}_{\mathbf{R}} = \sum_{\Gamma'} \lambda(\mathbf{R})_{\Gamma\Gamma'} |\Gamma, \mathbf{R}\rangle \langle \Gamma', \mathbf{R}|$$

$|\Gamma\rangle$ : eigenstates of atomic hamiltonian  $H_U$

$\Psi_0$ : uncorrelated wave function (Wick's Theorem holds)

$\mathcal{P}_{\mathbf{R}}$ : projector operator modify weight of local configuration

Gutzwiller Constraints:

$$\langle \Psi_0 | \mathcal{P}^\dagger \mathcal{P} | \Psi_0 \rangle = 1$$

$$\langle \Psi_0 | \mathcal{P}^\dagger \mathcal{P} n_{i\alpha} | \Psi_0 \rangle = \langle \Psi_0 | n_{i\alpha} | \Psi_0 \rangle$$

We only apply truncation respect to the occupation number, for SmB6, we keep all the atomic states with  $n_f=5,6,7$ , about 8000 variational parameters!



## Effective Hamiltonian in Gutzwiller approximation

- Under Gutzwiller approximation, we can define

$$E_G = \langle 0 | P H_{LDA} P | 0 \rangle + E_{int} \approx \langle 0 | H_{eff} | 0 \rangle + \sum_{\Gamma} \lambda_{\Gamma, \Gamma} E_{\Gamma}$$

It can be easily proved that  $H_{eff}$  is equivalent to  $H_0 + \Sigma(0)$  by comparing the Green's function in low frequency limit

$$G(i\omega) = \frac{z}{i\omega - H_{eff}} = \frac{1}{i\omega/z - H_{eff}/z}$$

$$H_0 + \Sigma(0) = -G^{-1}(0) = H_{eff}/z$$





## The double counting problem

- The total Hamiltonian treated in LDA+Gutzwiller

$$H_{total} = H_{LDA} + H_U + H_{DC}$$

$$H_{DC} = V_{DC} \sum_{k\sigma} f_{k\sigma}^\dagger f_{k\sigma}$$

- A commonly used form for DC term:

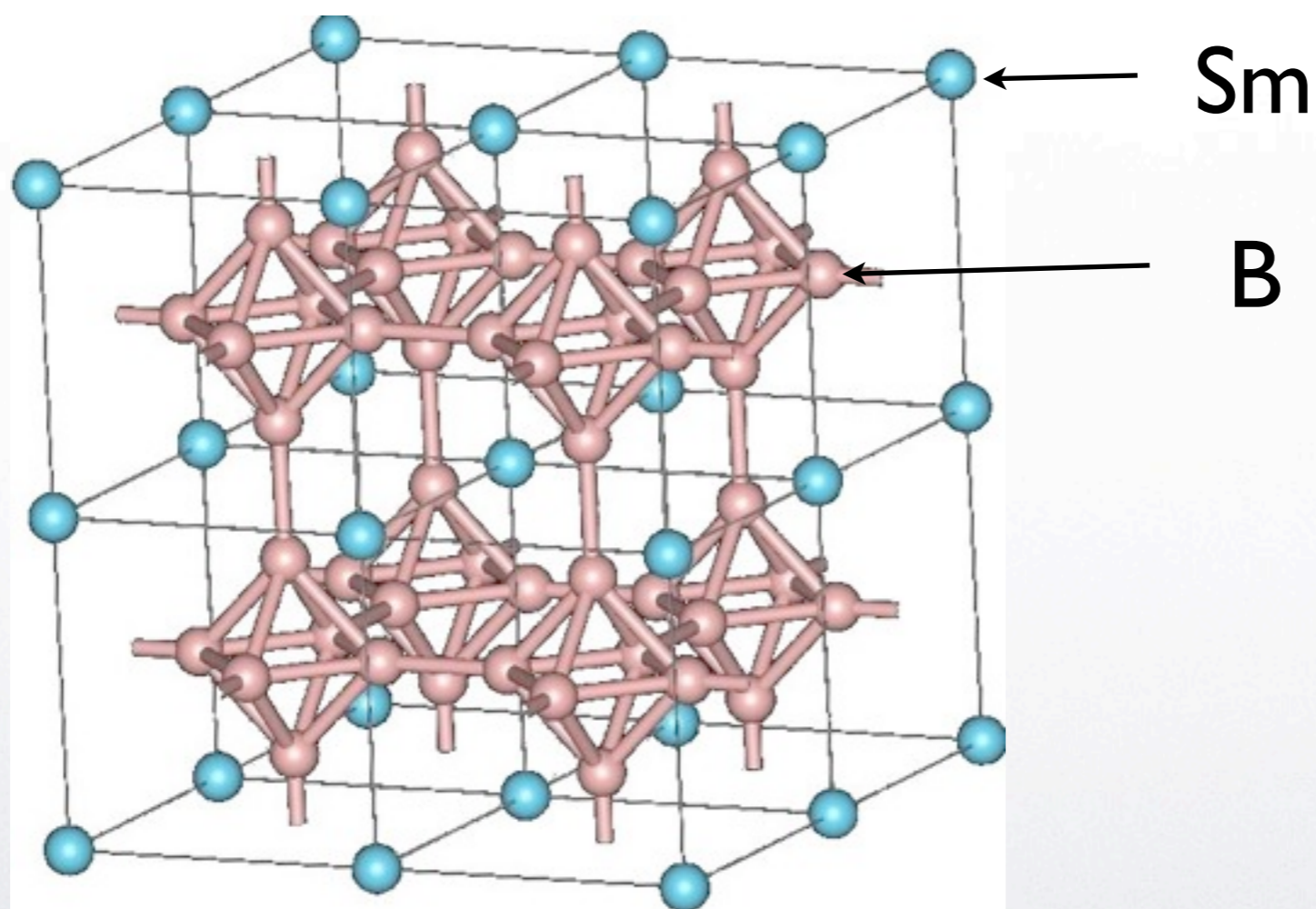
$$V_{ab}^{DC} = \delta_{ab} \left[ \bar{U} \left( \bar{n}_c - \frac{1}{2} \right) - \bar{J} \left( \bar{n}_c^\sigma - \frac{1}{2} \right) \right].$$

- Kotliar et al, RMP78, 865,2006





# The structure of SmB<sub>6</sub>







from PHYSICAL REVIEW B 66, 165209, 2002

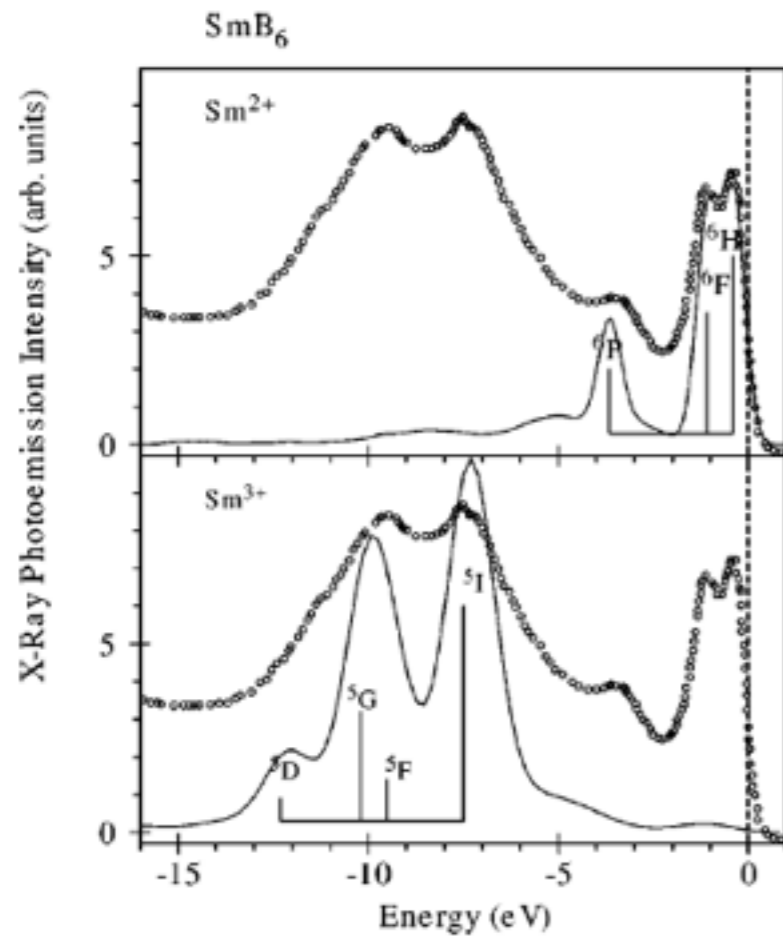
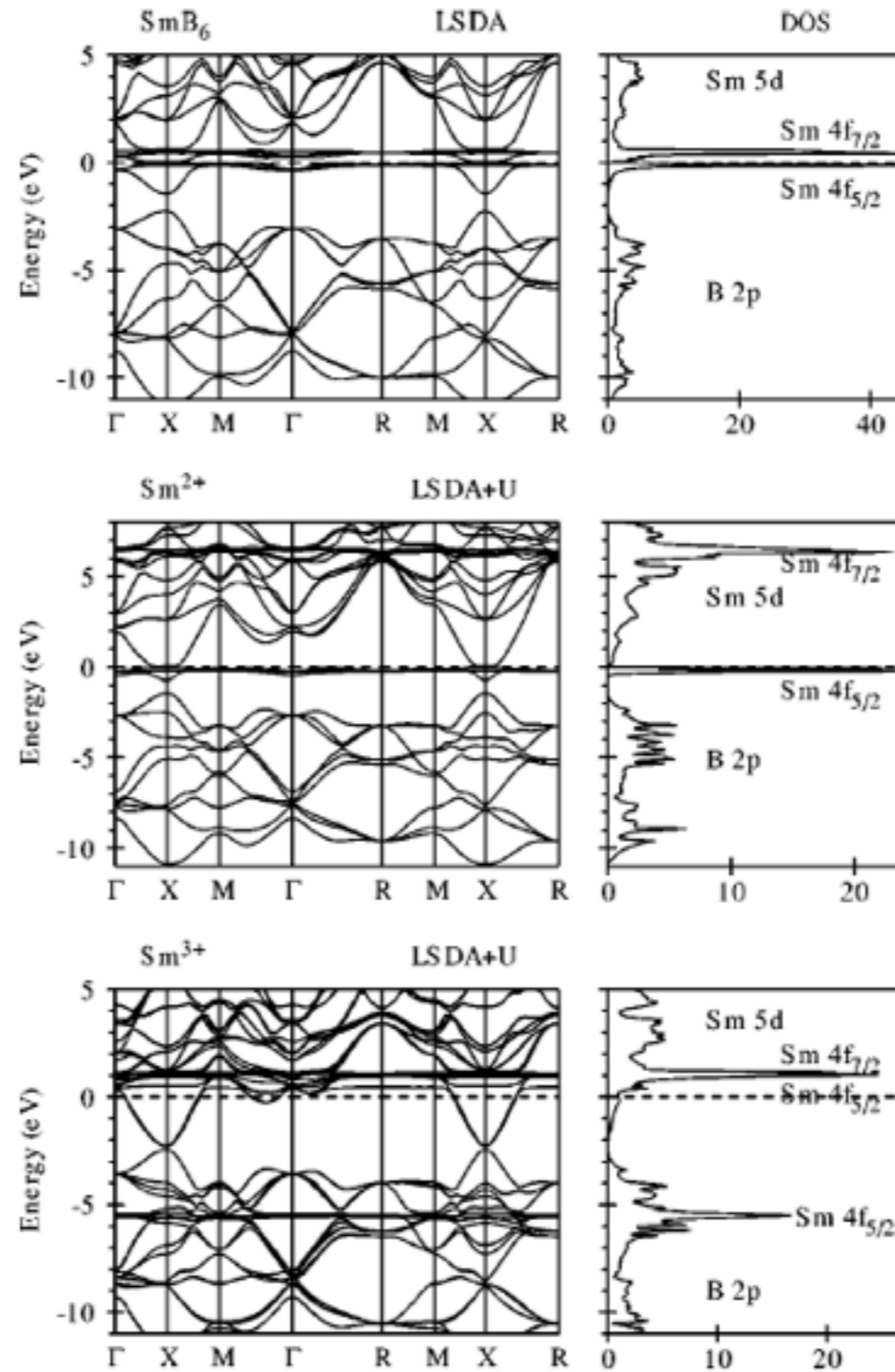


FIG. 3. Comparison of the calculated 4*f* DOS of SmB<sub>6</sub> using the LSDA+*U* approximation with the experimental XPS spectra from Ref. 18, taking into account the multiplet structure of the 4*f*<sup>5</sup> and 4*f*<sup>4</sup> final states (see explanations in the text).

compare to XPS

The valence determined by XPS is 2.54



band structure  
obtained by LDA  
and LDA+U

Assuming some  
kind of orbital  
ordering from Sm3+  
phase



# Valence of Sm determined by exp

	paper	experiment	nf	Average valence
SmB6	PRB: 1976 14, 4586	XPS	5.3	2.7
SmB6	JAP: 1970 41, 898	Mossbauer effect	5.4	2.6
SmB6	Physica B: 1995 215, 99	neutron experiments	5.44	2.56
SmB6	JPCS: 2009 176, 012034	XAS	5.47	2.53

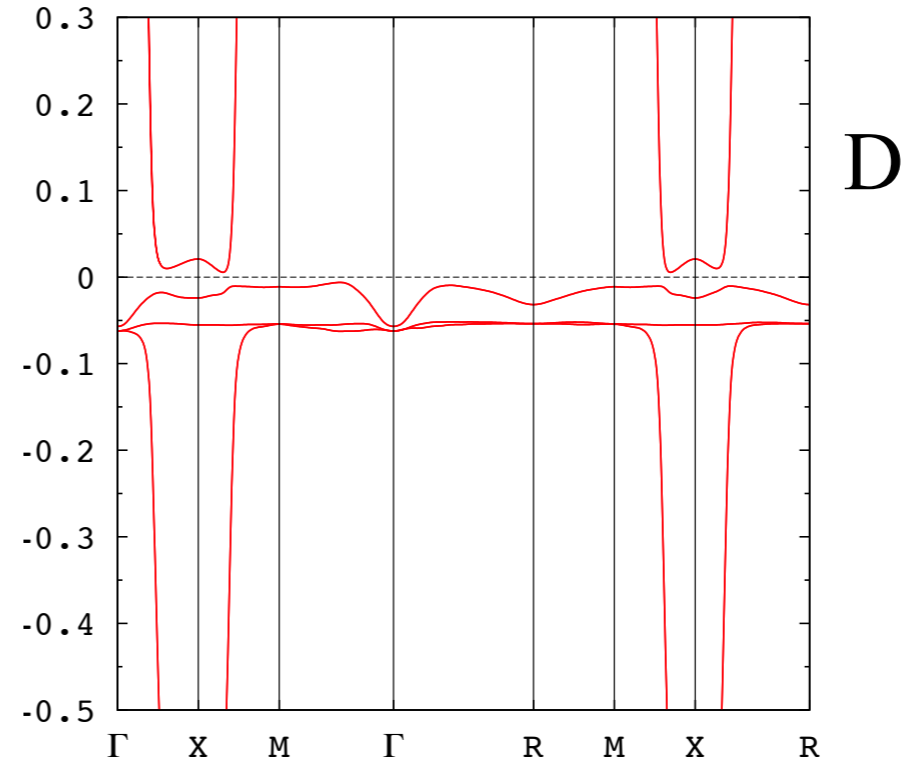
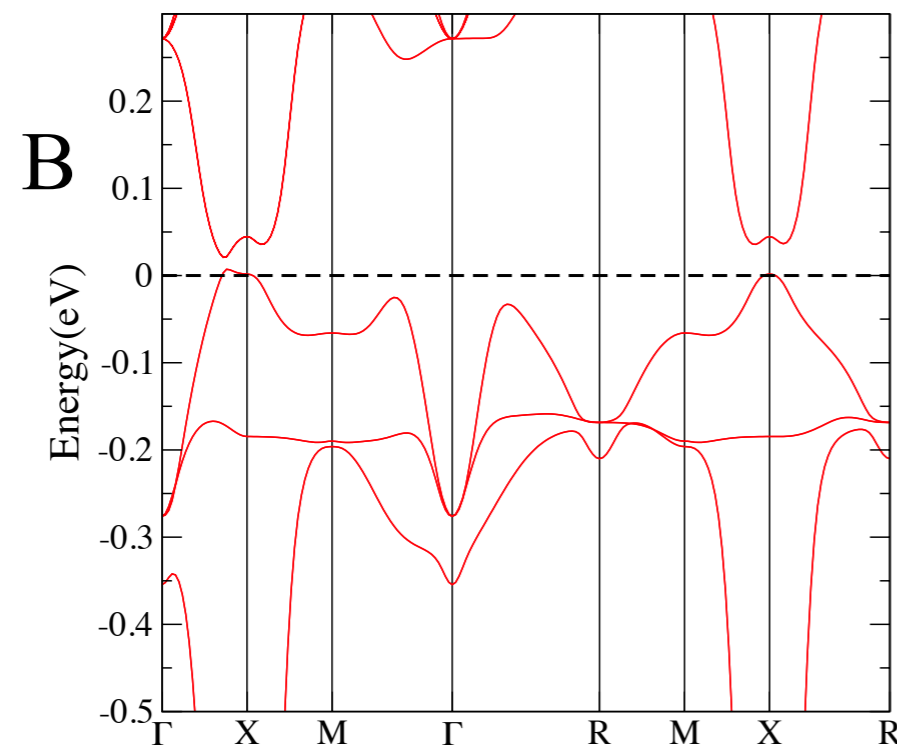
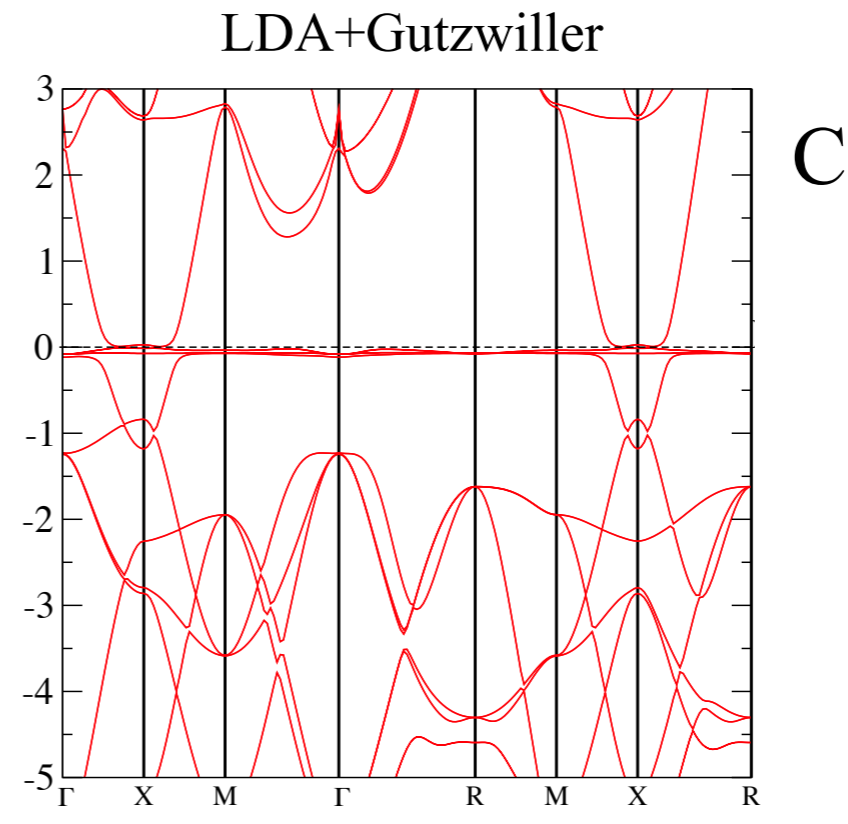
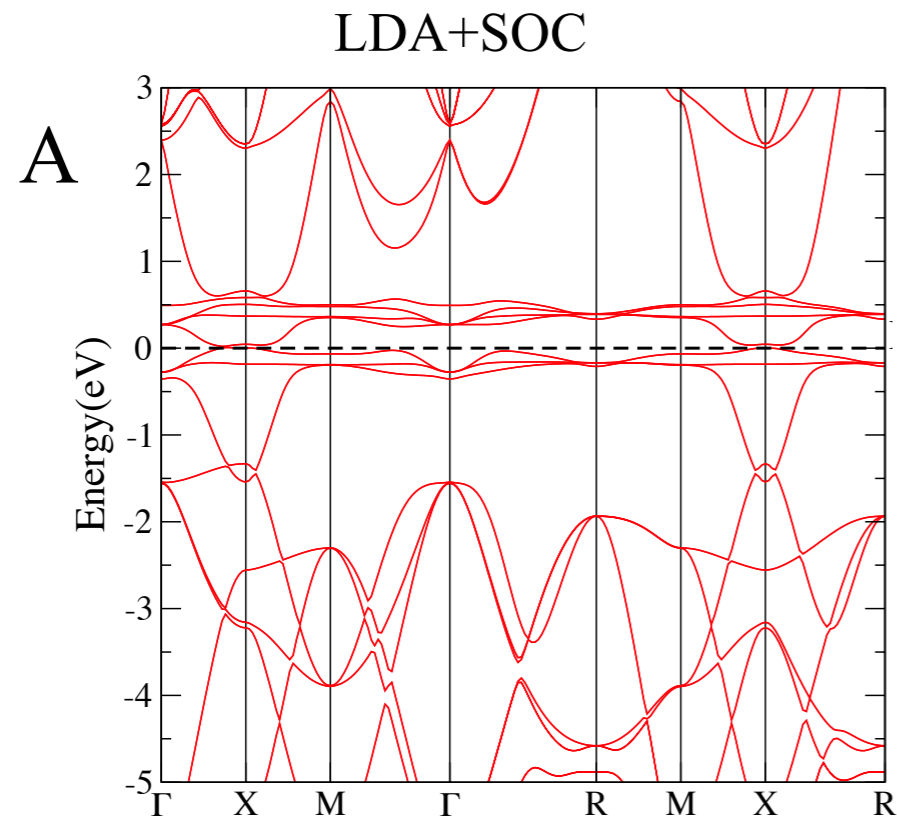


# SmB6: Z, nf, Gap vs Parity

		$E_g$	$\Gamma$	3X	R	3M	Tol
SmB6	LDA	25mev	+	-	+	+	-
<b>SmB6</b>	<b>LDA + G</b>	<b>10mev</b>	<b>+</b>	<b>-</b>	<b>+</b>	<b>+</b>	<b>-</b>

Vdc	$n_f$	$Z(R^2_{mat})$	
		5/2	7/2
	5.35		
<b>26.4</b>	<b>5.45</b>	<b>0.18</b>	<b>0.59</b>

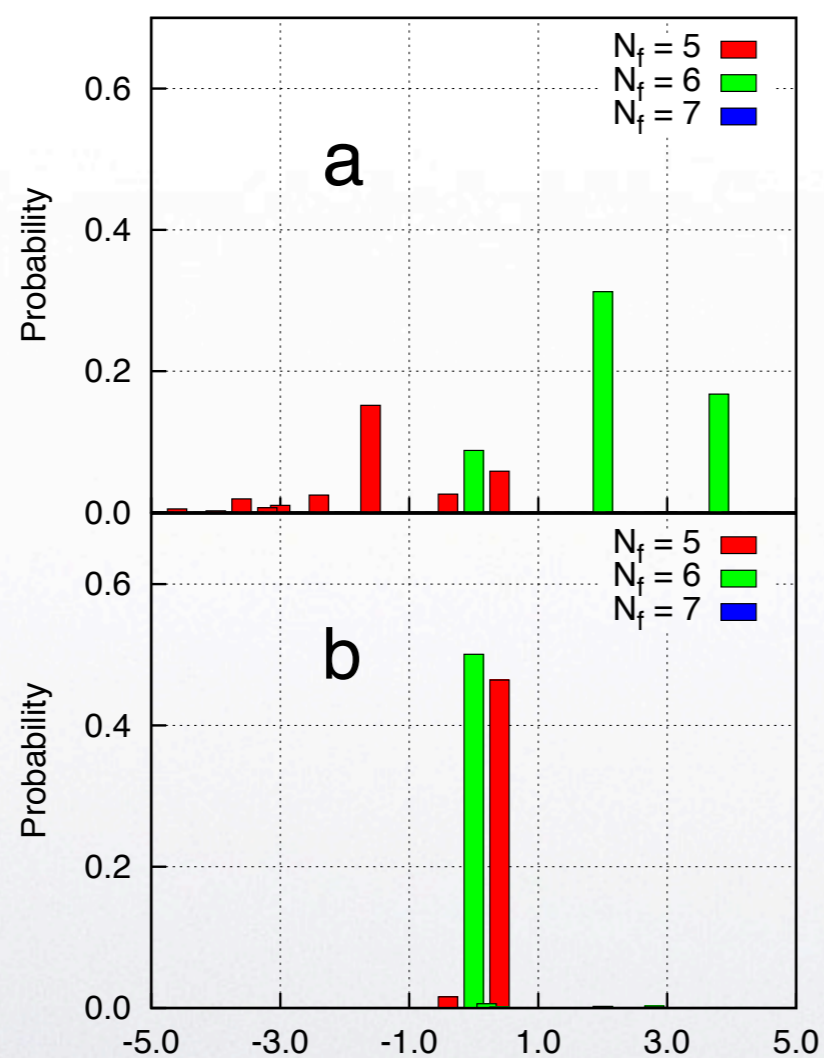
# Band structure obtained by LDA and LDA+G







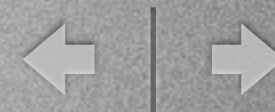
# The probability of atomic eigenstates for SmB6



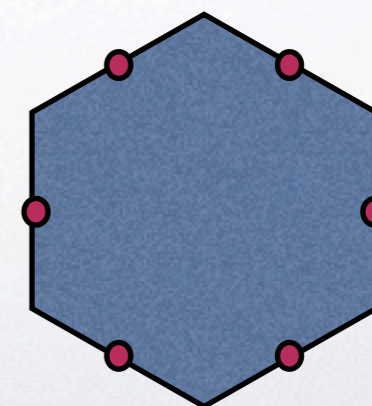
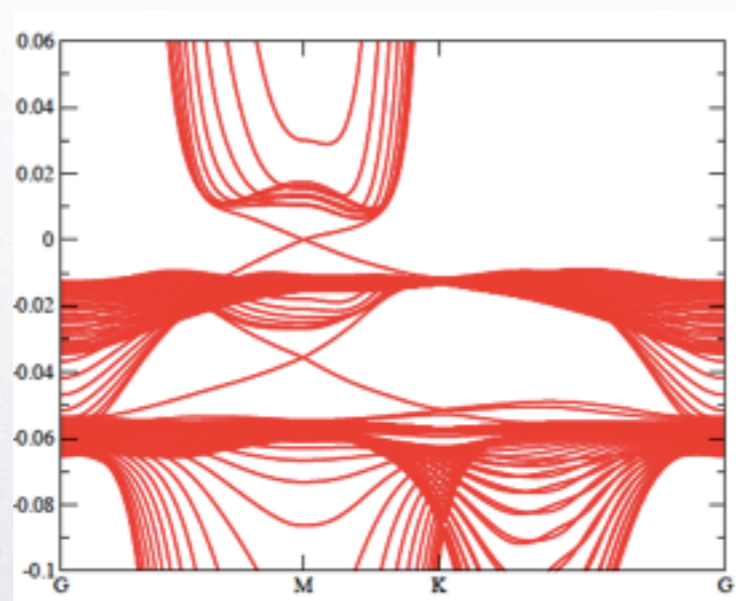
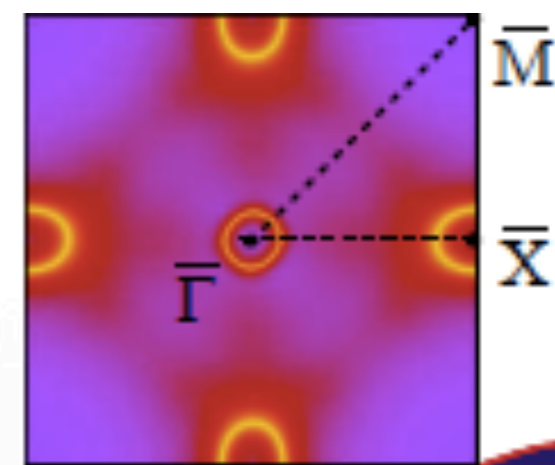
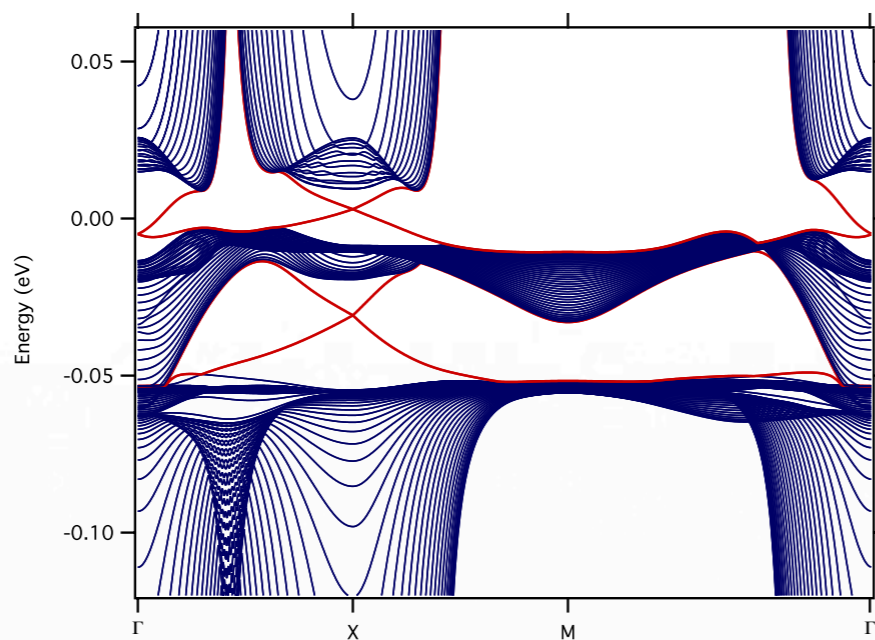
LDA

LDA+G





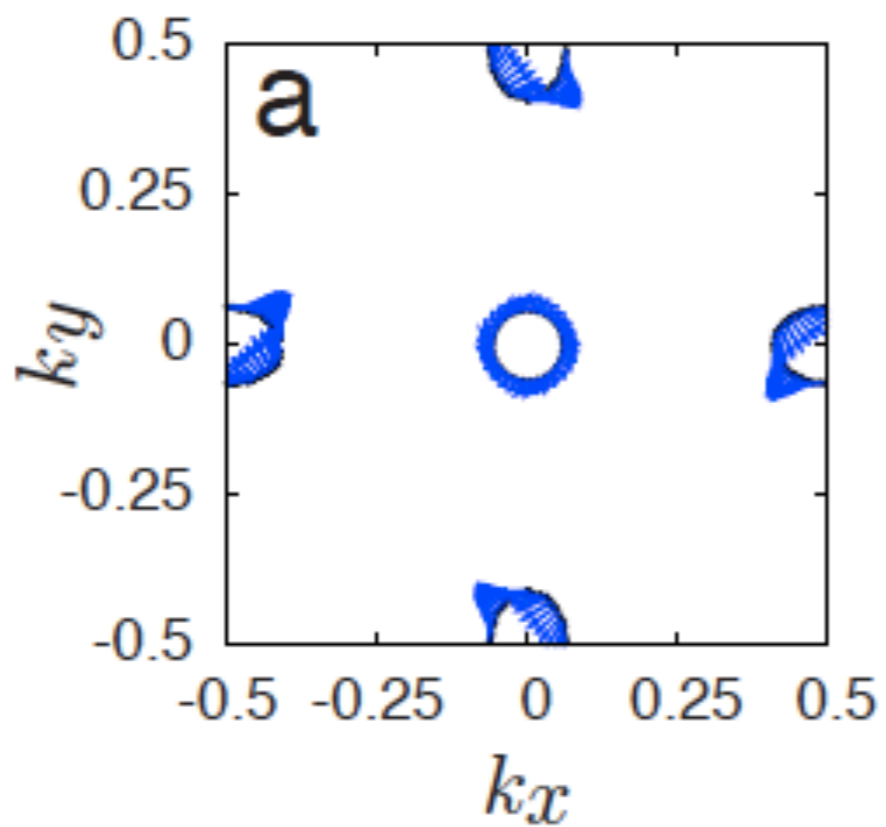
# The unique surface states of SmB6 on (001) surface



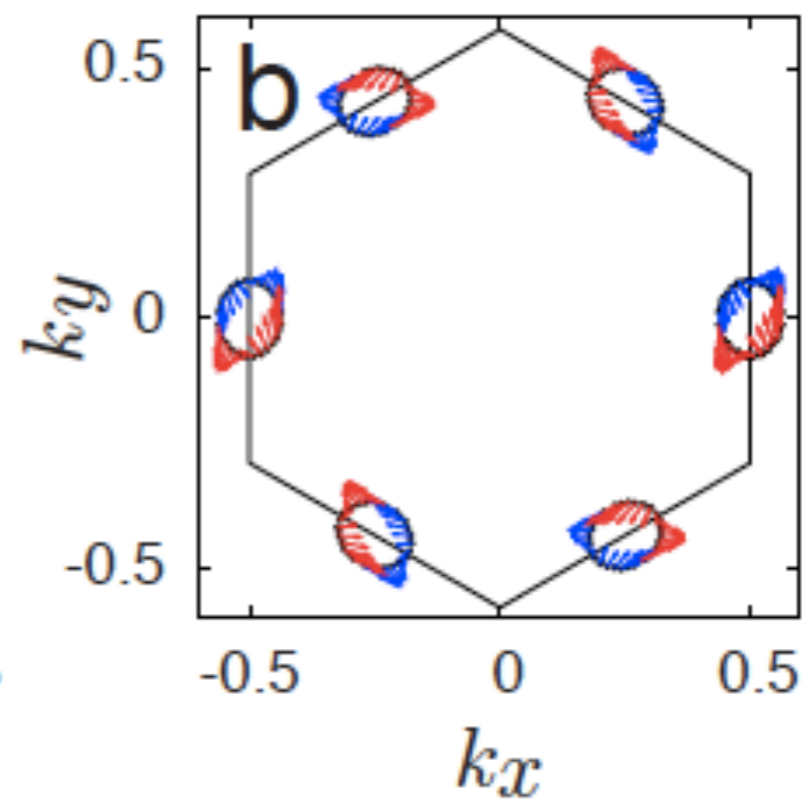




## Spin texture for surface states



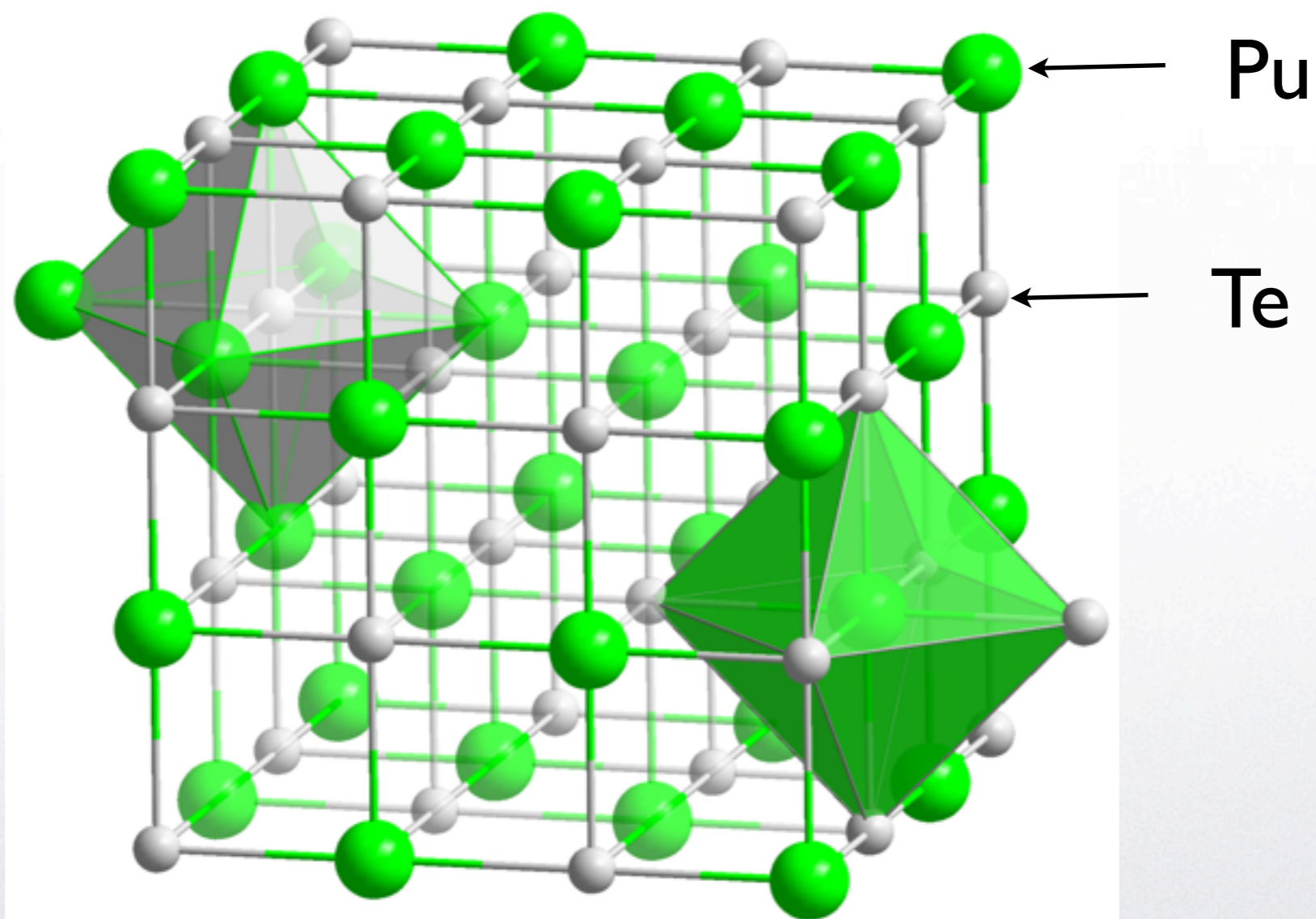
001 surface



111 surface



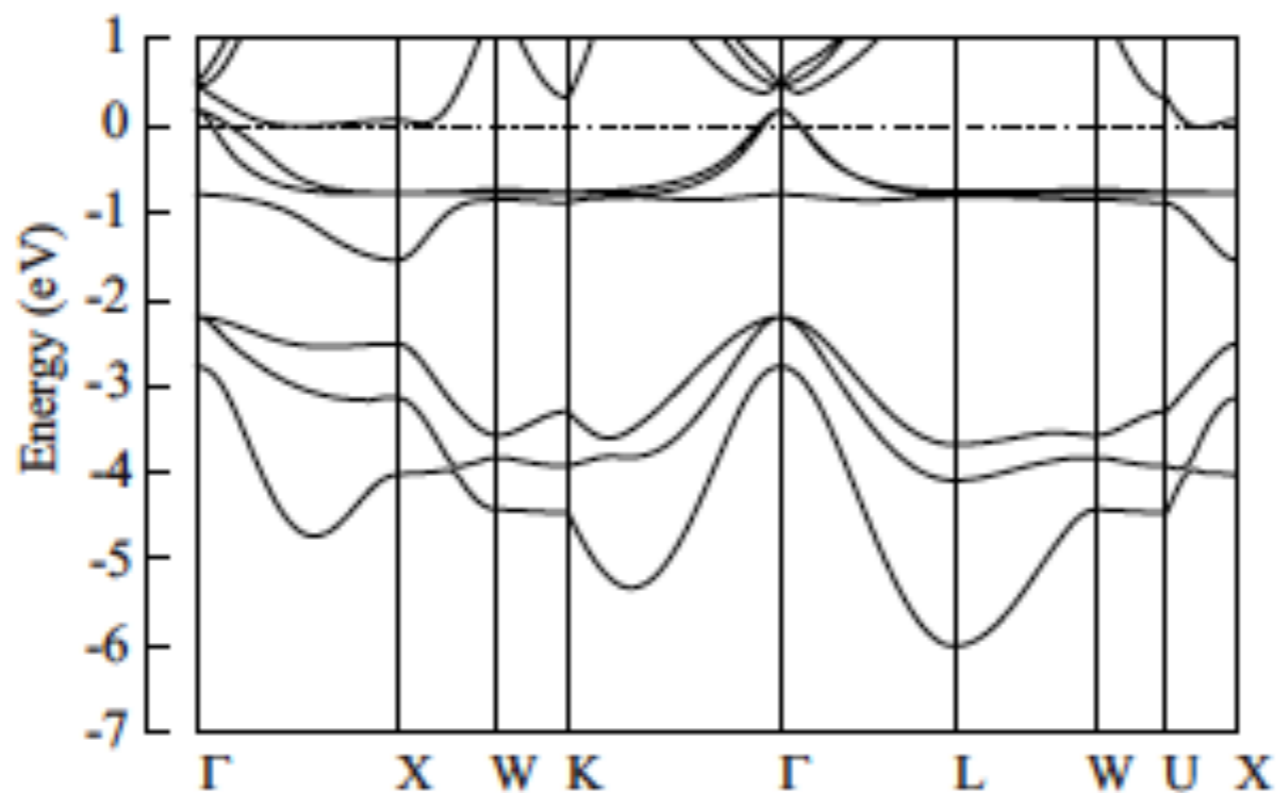
# PuTe







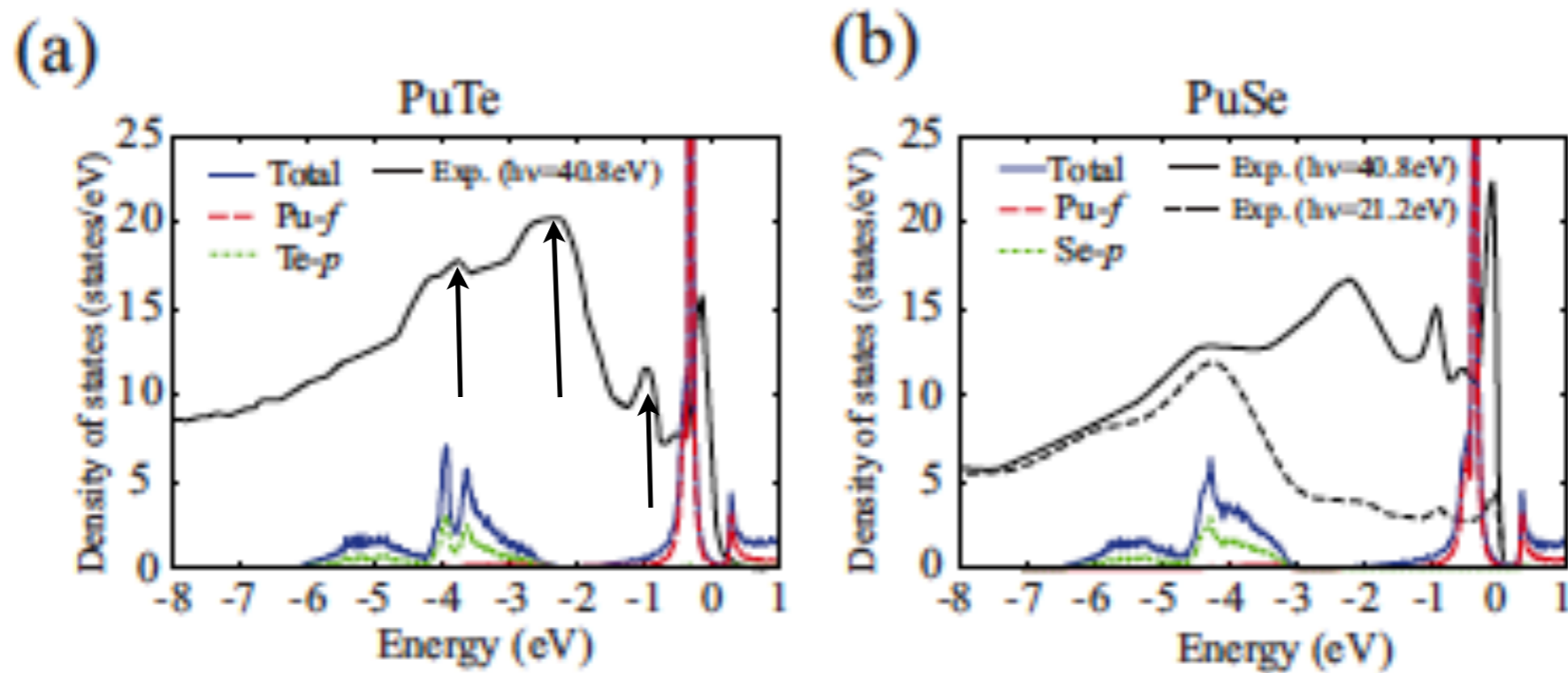
# The results obtained by LDA+U



PHYSICAL REVIEW B **80**, 161103, 2009



# Clear multiplet structure in XPS



PHYSICAL REVIEW B 80, 161103(R) (2009)



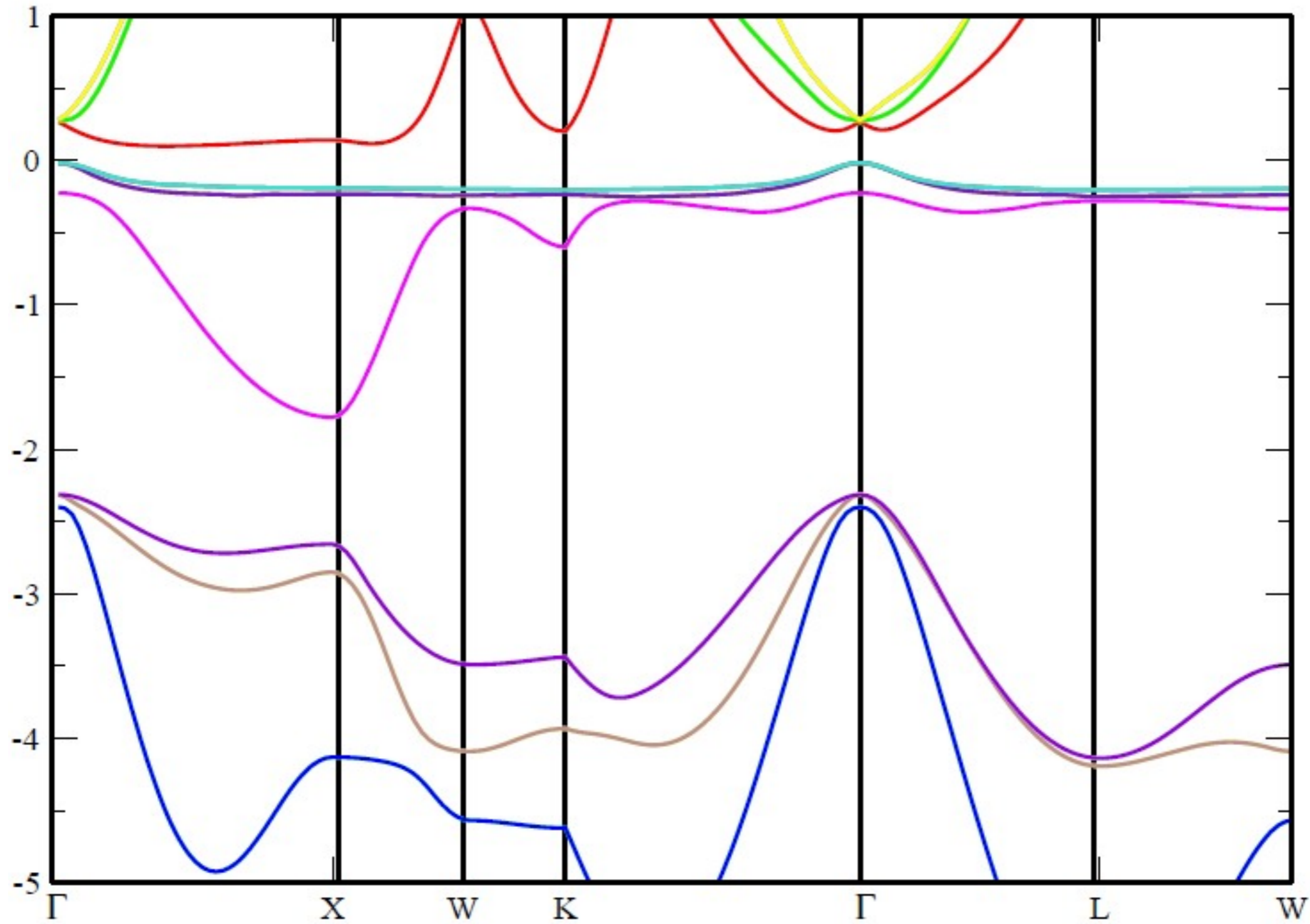
# PuTe: Z, nf, Gap vs Parity

Udc	$n_f$	$E_g$	$Z(R_{mat}^2)$	
			5/2	7/2
<b>20.5eV</b>	<b>5.11</b>	<b>0.12ev</b>	<b>0.51</b>	<b>0.67</b>

		$E_g$	$\Gamma$	3X	4L	Tol
PuTe	LDA		-	+	-	-
<b>PuTe</b>	<b>GW</b>	<b>0.12ev</b>	-	+	-	-

# PuTe

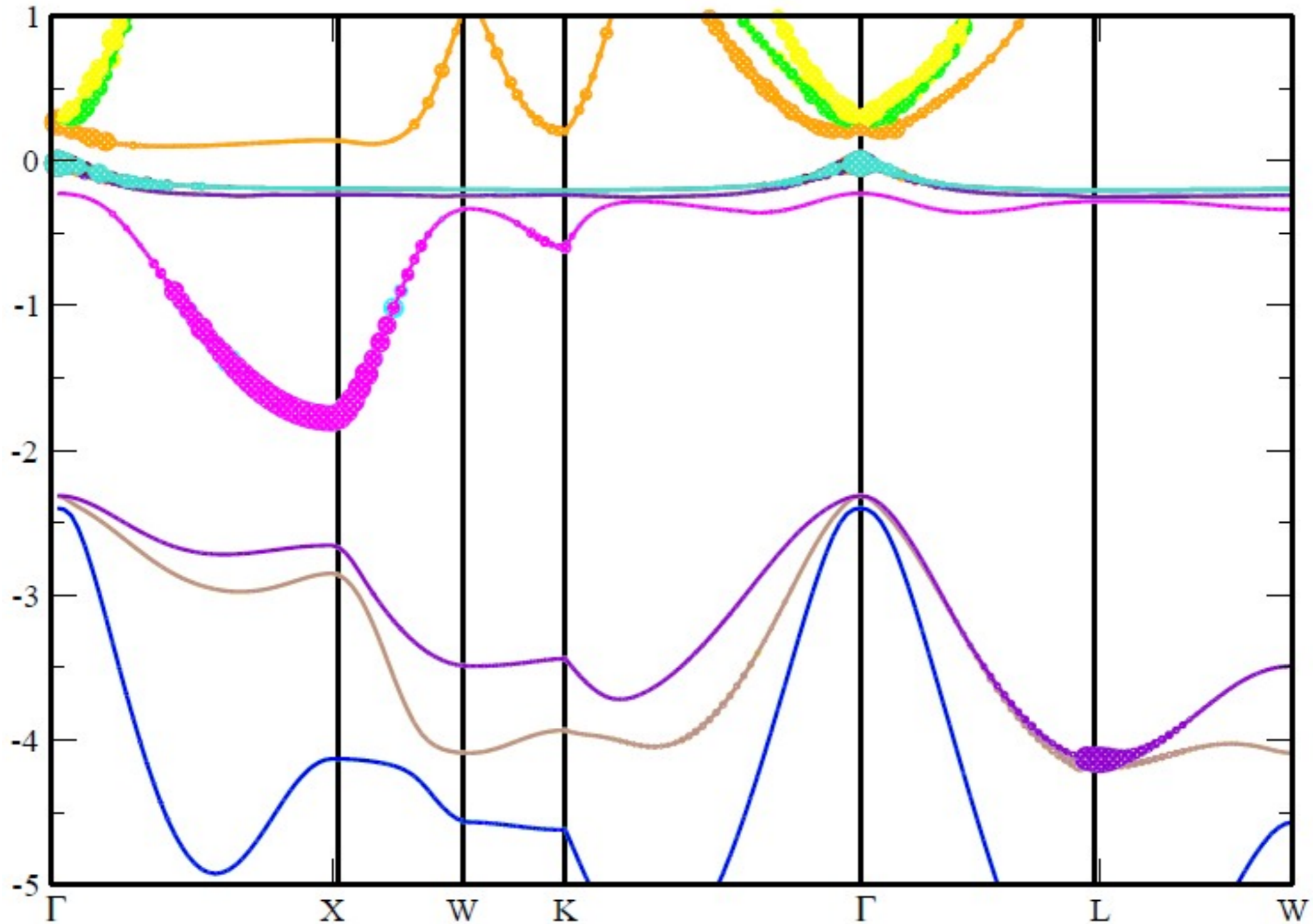
## Band structure: GW method





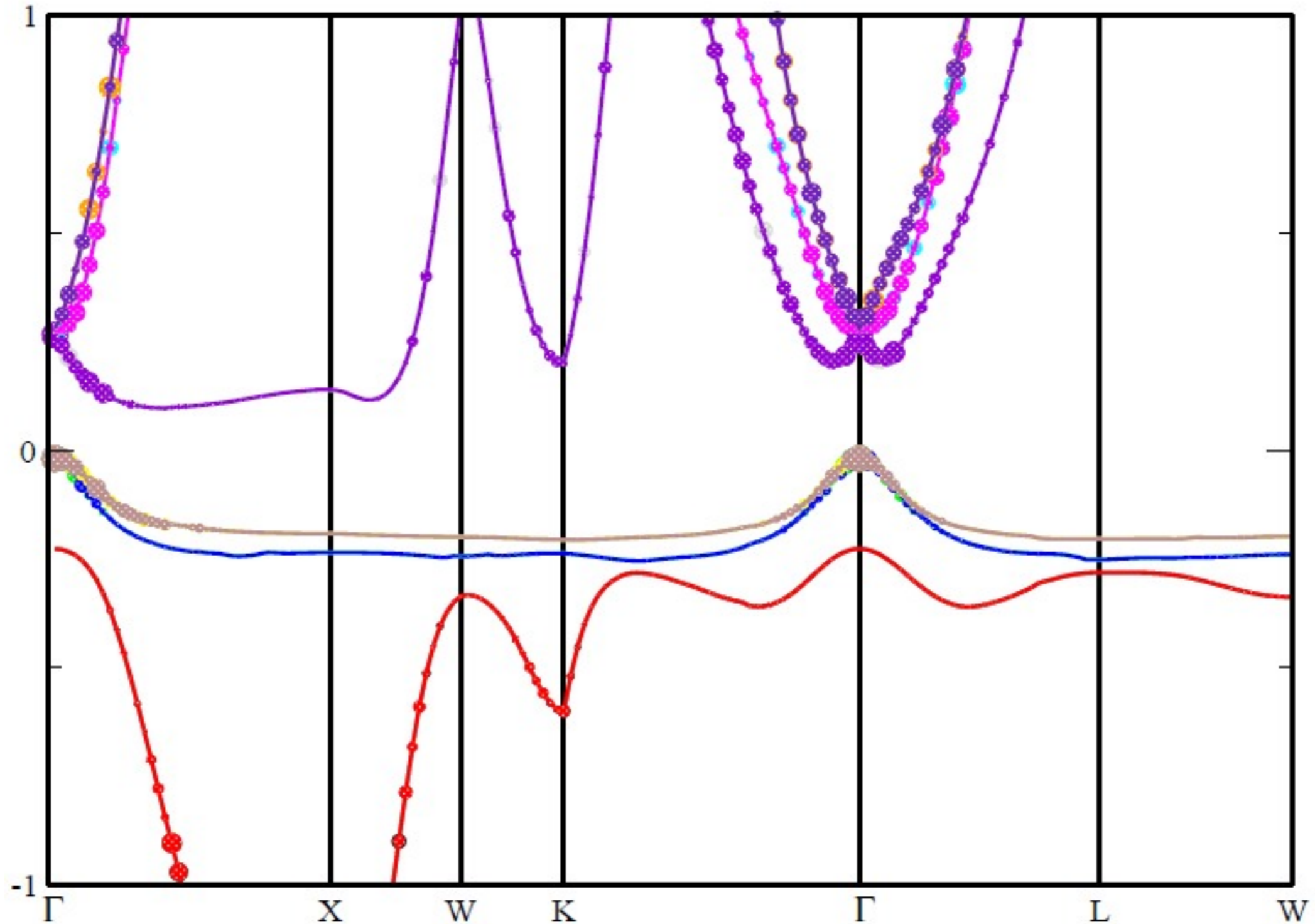
# PuTe

## Fat Band structure: GW method



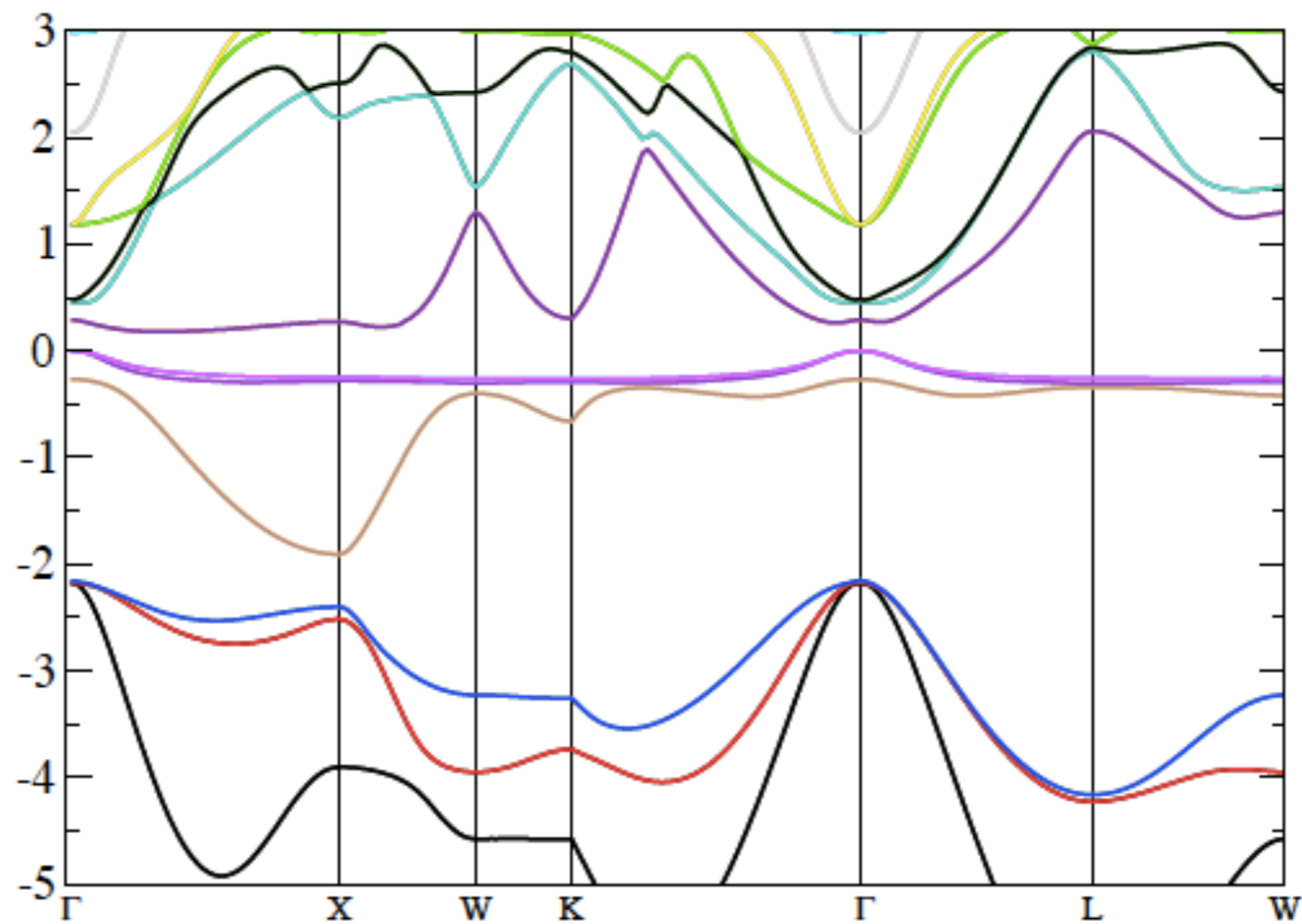
# PuTe

## Fat Band structure: GW method



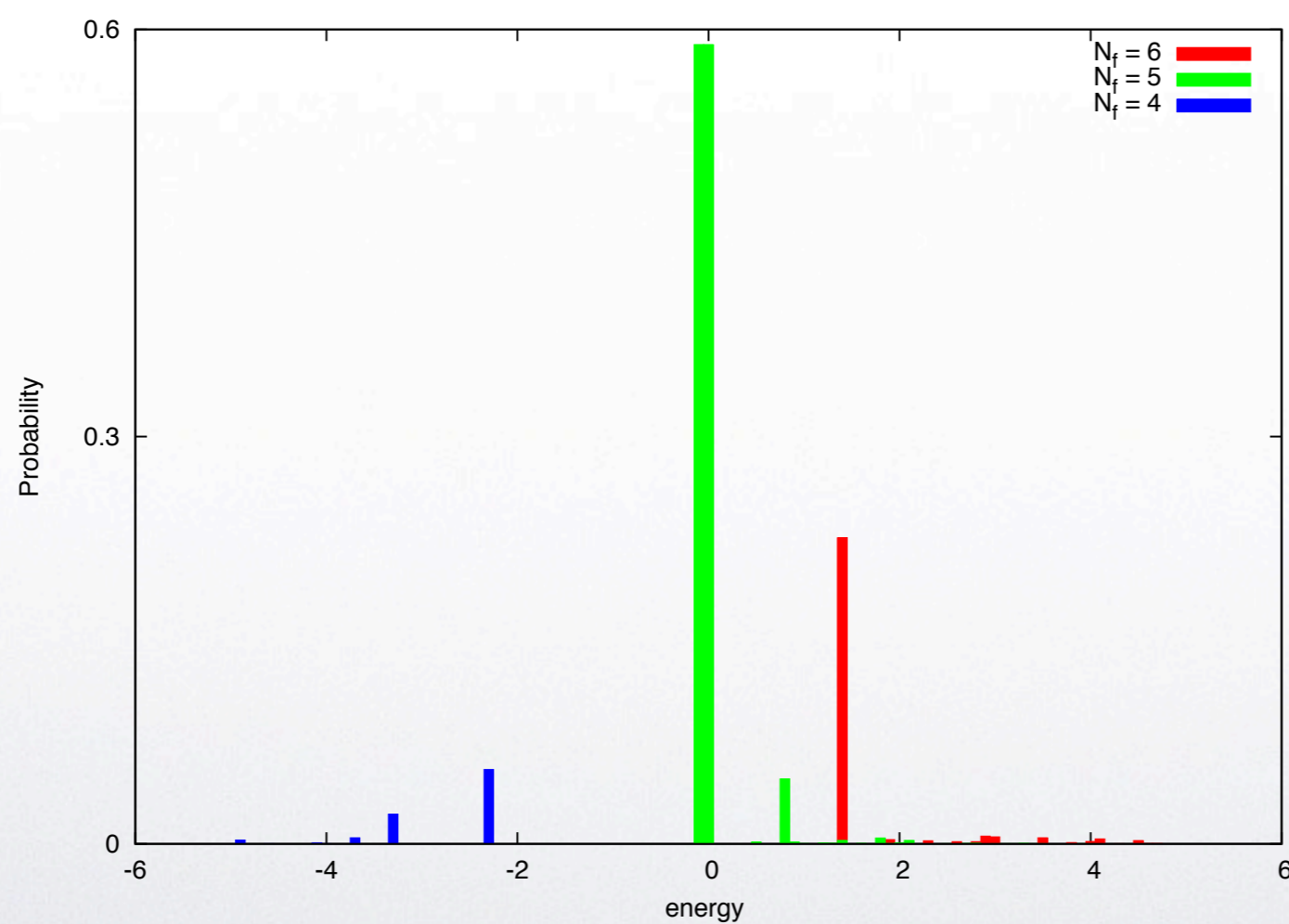


# Electronic structure of PuTe at 5.5GPa gap=0.21eV





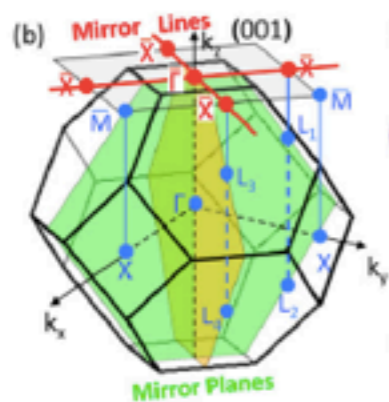
# The probability of atomic eigenstates for PuTe





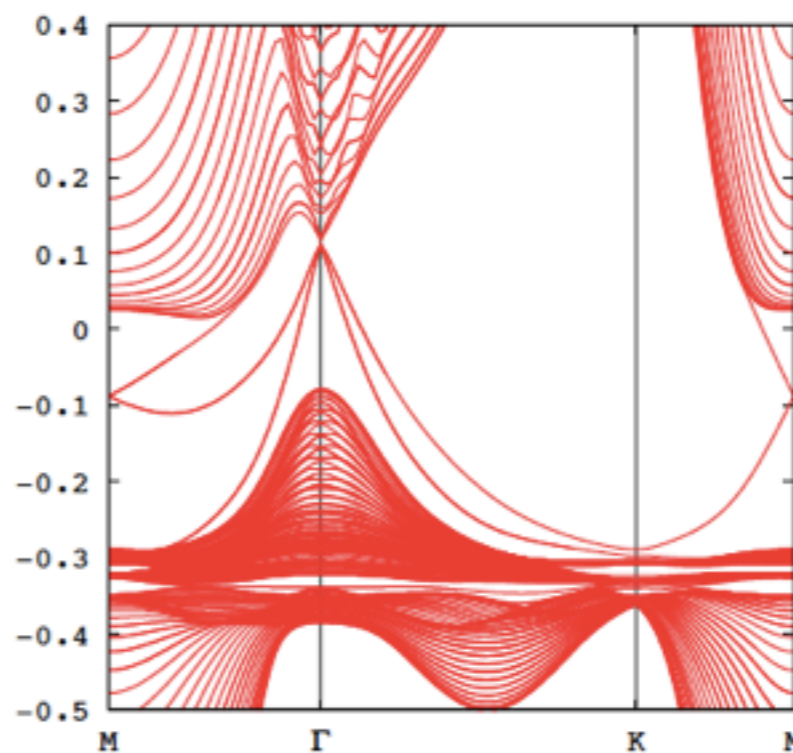
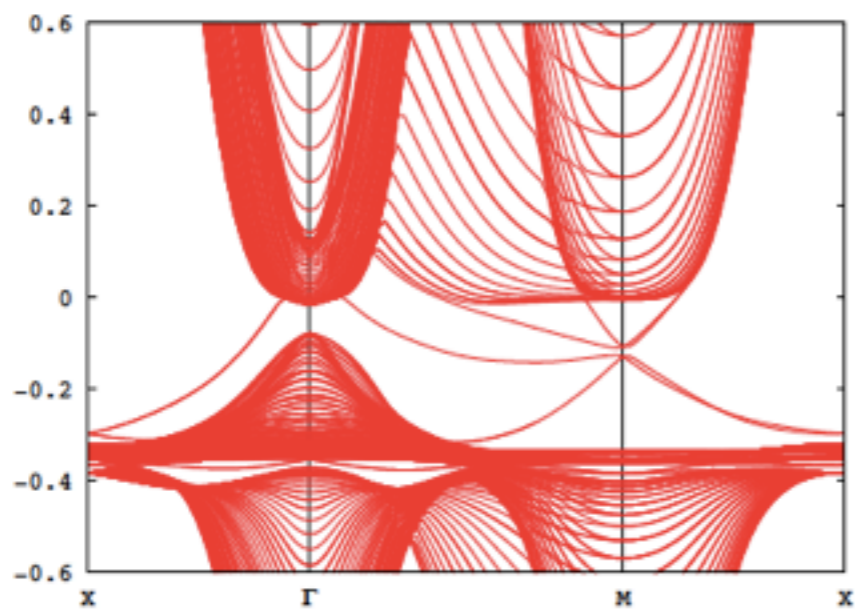
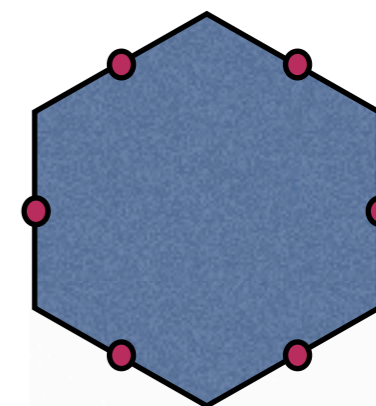


# Surface state



( 0 0 1 )

( 1 1 1 )







# Conclusions

- Mix valence TIs, SmB6 and PuTe
- The topological phase survives from the Strong e-e interaction among f electrons
- The  $Z_2$  invariance can be computed by the green's function in low frequency
- More similar materials!