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Electromagnetic and thermal responses in topological insulators and superconductors

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Work done in collaboration with

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Akira Furusaki (*RIKEN*)
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TOHOKU UNIVERSITY

References
Phys. Rev. Lett. 106, 166802 (2011)
Phys. Rev. Lett. 108, 026802 (2012)
arXiv:1211.0533

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Electromagnetic and thermal responses in topological insulators and superconductors

outline

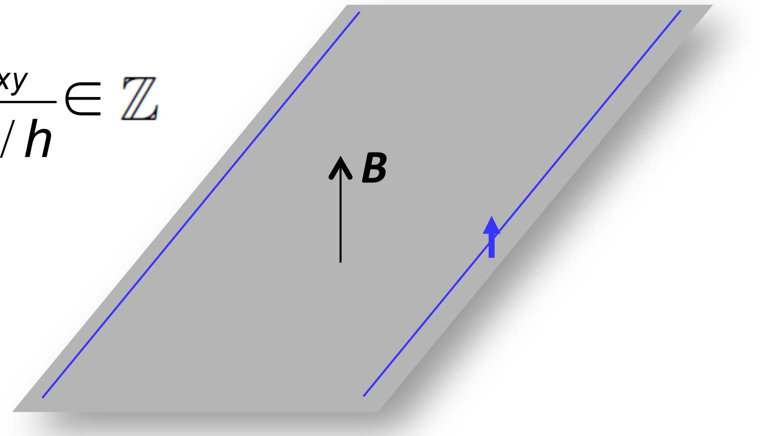
1. Introduction: Topologically nontrivial states
2. Responses in two dimensions
3. Responses in three dimensions

Topological states

- **Quantum Hall effect**

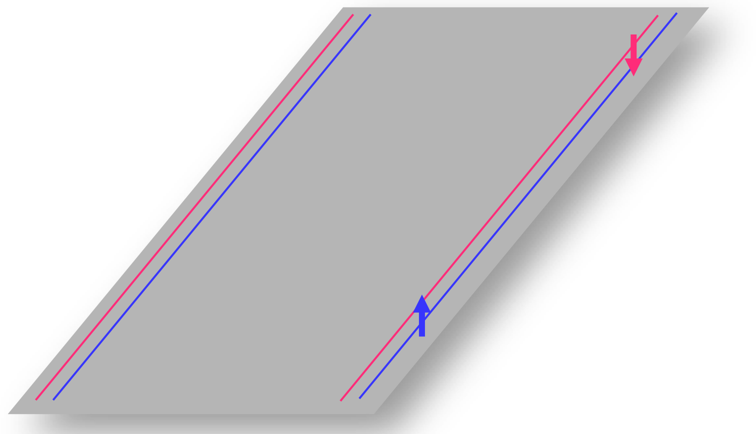
  Laughlin (1980)
  TKNN (1982)
  Kohmoto (1985)

$$\frac{\sigma_{xy}}{e^2/h} \in \mathbb{Z}$$



- **Quantum spin Hall effect (topological insulators)**

  2d Kane-Mele (2005), BHZ(2006)
  3d Fu et al., Moore-Balents, Roy



- **Chiral superconductors**

  Read-Green (2000)

- **Topological SC (3d)**

  Schnyder et al. (2008)
  Fu-Berg, Sato (2010)

Topological states

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  Read-Green (2000)

- **Topological SC (3d)**

  Schnyder et al. (2008)
  Fu-Berg, Sato (2010)

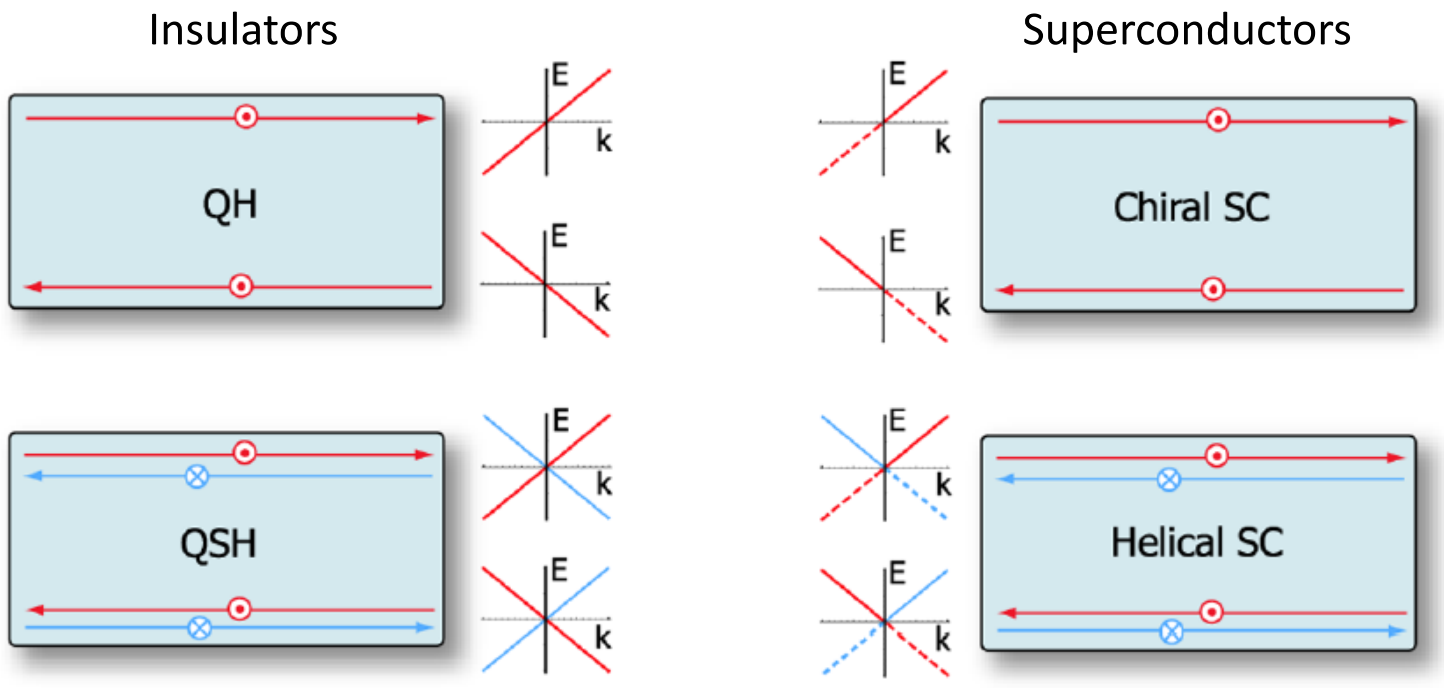
symmetry \ *d*	1	2	3
A	0	$\mathbb{Z}$	0
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$2\mathbb{Z}$	0
CI	0	0	$2\mathbb{Z}$

Topological SC

PRL **102,** 187001 (2009) PHYSICAL REVIEW LETTERS week ending 8 MAY 2009

Time-Reversal-Invariant Topological Superconductors and Superfluids in Two and Three Dimensions

Xiao-Liang Qi, Taylor L. Hughes, S. Raghu, and Shou-Cheng Zhang
Department of Physics, McCullough Building, Stanford University, Stanford, California 94305-4045, USA
(Received 3 April 2008; published 4 May 2009)



Topological SC

PRL 102, 187001 (2009) PHYSICAL REVIEW LETTERS week ending 8 MAY 2009

Time-Reversal-Invariant Topological
in Two and Thre

Xiao-Liang Qi, Taylor L. Hughes, S.
Department of Physics, McCullough Building, Stanford
(Received 3 April 2008; pu

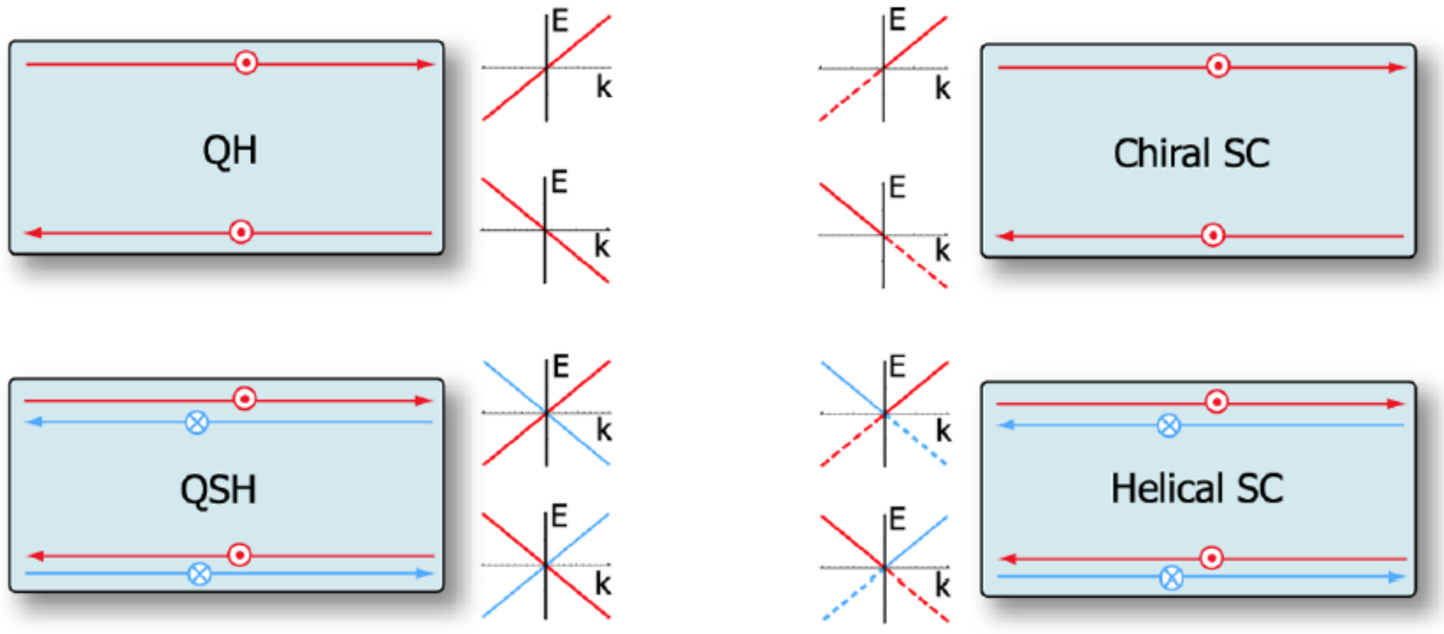
Majorana fermions

$$\psi(x) = \psi^{+}(x)$$

particle = antiparticle

Insulators

Superconductors

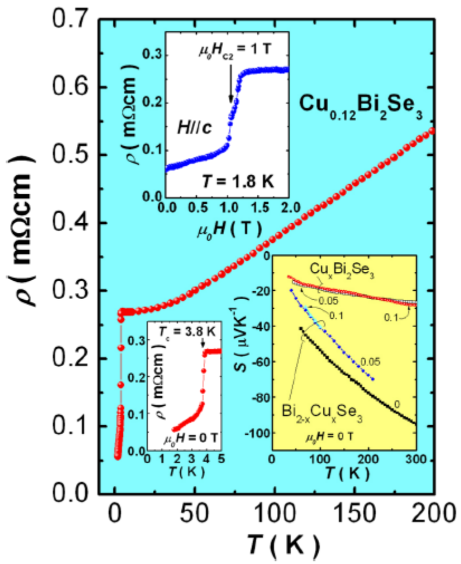


Topological SC

$Cu_xBi_2Se_3$ and $Sn_{1-x}In_xTe$ (3D TSC)

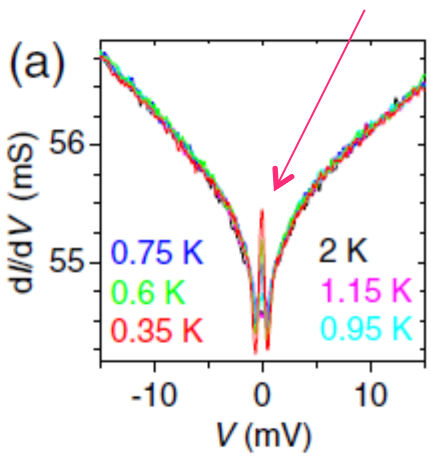
Experiments:
- Hor et al., PRL 104, 057001 (2010)
- Kriener et al., PRL 106, 127004 (2011)
- Sasaki et al., PRL 107, 217001 (2011)
- Sasaki et al., PRL 109, 217004 (2012)

Theory:
- Fu and Berg, PRL 105, 097001 (2010)
- Sato, PRB 81, 220504 (2010)
- Hsieh and Fu, PRL 108, 107005 (2012)
- Michaeli and Fu, PRL 109, 187003 (2012)

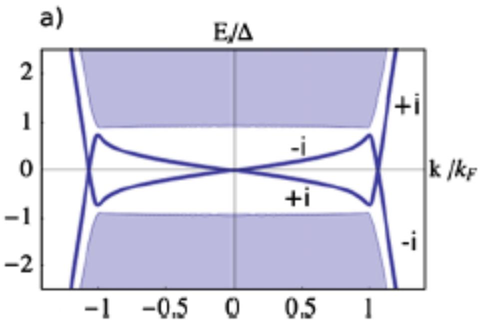


[Hor et al. 2010]

in-gap states (Majorana modes?)



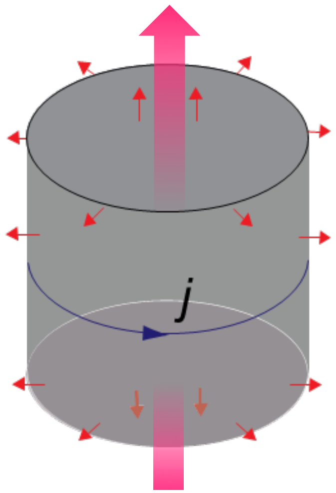
[Sasaki et al. 2012]



[Hsieh-Fu 2012]

Electric/Thermal responses

Response to an electric field

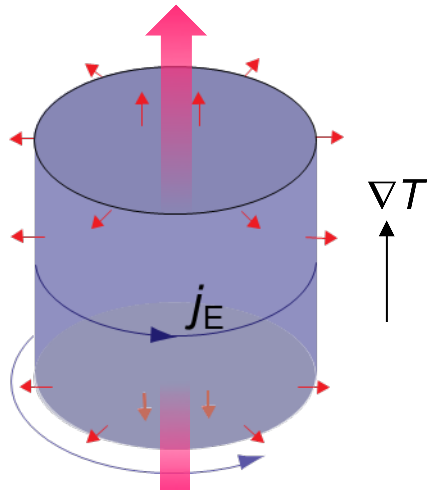


topological ME effect

$$\boldsymbol{M} = \frac{e^2}{2hc} \boldsymbol{E}$$

Qi, Hughes, Zhang (2008)

Response to a temperature gradient



$$\boldsymbol{L} = \frac{(\pi k_B)^2 T}{6hv} \nabla T$$

Present work

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Electromagnetic and thermal responses in topological insulators and superconductors

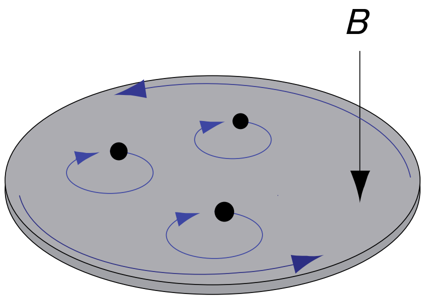
outline

1. Introduction: Topologically nontrivial states
2. Responses in two dimensions
3. Responses in three dimensions

QHE and chiral SC

QHE

2DEG

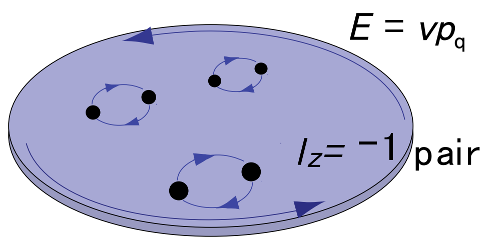


Bulk: ***gapped*** (Landau levels)

Edge: ***chiral Dirac modes***
(edge states)

2d chiral SC

^{3}He-A, Sr_2RuO_4, …



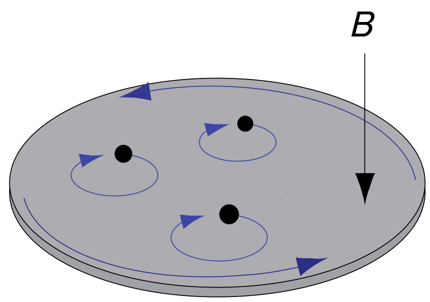
Bulk: ***gapped*** (SC gap)

Edge: ***chiral Majorana modes***
(Andreev bound states)

QHE and chiral SC

QHE

2DEG



gauge symmetry *preserved*

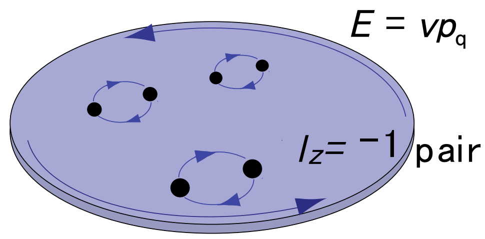
total *charge* is conserved

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = 0$$

Charge transport

2d chiral SC

^3He-A, Sr_2RuO_4, …



gauge symmetry *broken*

but total *energy* is still conserved

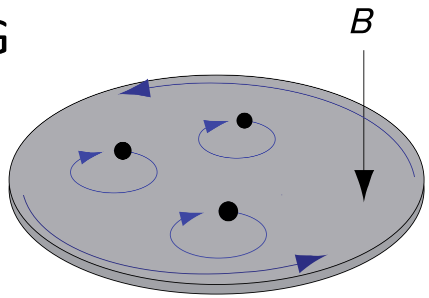
$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \boldsymbol{j}_E = 0$$

Thermal transport

Quantum Hall effect

QHE

2DEG



$$\begin{cases} \boldsymbol{j} = \sigma_H \boldsymbol{E} \times \hat{\boldsymbol{z}} \\ \boldsymbol{j} = c \nabla \times \boldsymbol{M} = -c \dfrac{\partial \boldsymbol{M}}{\partial \mu} \times \nabla \mu \end{cases}$$

Streda formula :
$$\sigma_H = ec \frac{\partial M^z}{\partial \mu} = ec \frac{\partial N}{\partial B^z}$$

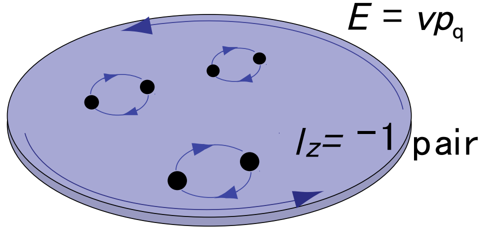
$$dF = -N d\mu - \boldsymbol{M} \cdot d\boldsymbol{B}$$

Cross-correlated responses:

$$\delta M_z = (\sigma_H / ec) \delta \mu$$
$$\delta N = (\sigma_H / ec) \delta B_z$$

Streda formula: $\sigma_H \Rightarrow \kappa_H$

We consider the BdG theory of SCs



Energy current

$$\begin{cases} \boldsymbol{j}_E = -\kappa_H \nabla T \times \hat{\boldsymbol{z}} \\ \boldsymbol{j}_E = \nabla \times \boldsymbol{M}_E = -\dfrac{\partial \boldsymbol{M}_E}{\partial T} \times \nabla T \end{cases}$$

Qin et al. PRL (2011), Nomura et al. PRL (2012)

Streda-like formula : $\kappa_H = \dfrac{\partial M_E^z}{\partial T}$

Streda formula: $\sigma_H \Rightarrow \kappa_H$

Hamiltonian　　Energy current

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & j_E^x & j_E^y & j_E^z \\ p^x & & & \\ p^y & & & \\ p^z & & & \end{pmatrix}$$

Momentum

- $L^{\mu\nu} = \frac{1}{c}\langle x^\mu p^\nu - x^\nu p^\mu \rangle$

Orbital angular momentum $\boldsymbol{L}$

- $M_E^{\mu\nu} = \frac{1}{2}\langle x^\mu j_E^\nu - x^\nu j_E^\mu \rangle$

$\longrightarrow \quad \boldsymbol{j}_E = \nabla \times \boldsymbol{M}_E$

Energy current

$$\begin{cases} \boldsymbol{j}_E = -\kappa_H \nabla T \times \hat{\boldsymbol{z}} \\ \boldsymbol{j}_E = \nabla \times \boldsymbol{M}_E = -\dfrac{\partial \boldsymbol{M}_E}{\partial T} \times \nabla T \end{cases}$$

Qin et al. PRL (2011),　Nomura et al. PRL (2012)

$$\kappa_H = \frac{\partial M_E^z}{\partial T}$$

Streda formula: $\sigma_H \Rightarrow \kappa_H$

Hamiltonian　　Energy current

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Momentum

- $L^{\mu\nu} = \frac{1}{c}\langle x^\mu p^\nu - x^\nu p^\mu \rangle$

  Orbital angular momentum ***L***

- $M_E^{\mu\nu} = \frac{1}{2}\langle x^\mu j_E^\nu - x^\nu j_E^\mu \rangle$

  $\longrightarrow \quad \boldsymbol{j}_E = \nabla \times \boldsymbol{M}_E$

In (pseudo) Lorentz invariant cases

(e.g. BdG Hamiltonian)

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\downarrow$$

$$\boldsymbol{M}_E \propto \boldsymbol{L}$$

$$\kappa_H = \frac{\partial M_E^z}{\partial T} = \frac{v^2}{2}\frac{\partial L^z}{\partial T}$$

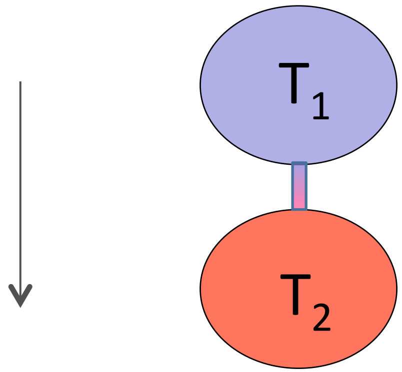
Temperature gradient = gravity

Equivalence Principle:

The laws of physics in a *gravitational field* are identical to those in a local *accelerating frame* and in a *temperature gradient.*

Tolman-Ehrenfest , Luttinger

gravitational field



$$T_1 \neq T_2 = T_1\left(1 + \Delta\phi_g / c^2\right)$$

$$\frac{1}{c^2}\nabla\phi_g = \frac{\nabla T}{T}$$

$$\boldsymbol{j}_E = -\hat{\kappa}\nabla T = -\hat{\kappa}T\nabla\phi_g$$

Newly found formulae

Streda (1982)

$$\sigma_H = c\frac{\partial M^z}{\partial \phi} = c\frac{\partial \rho}{\partial B^z}$$

$$F = e\left(E + \frac{1}{c} v \times B\right)$$

Lorentz force

Present work

$$\kappa_H = \frac{1}{T}\frac{\partial M_E^z}{\partial \phi_g} = \frac{1}{T}\frac{\partial Q}{\partial B_g^z}$$

$d\phi_g = T^{-1}dT$, $B_g^z = (2/v^2)\Omega^z$

$$F = m\left(E_g + 2v \times B_g\right)$$

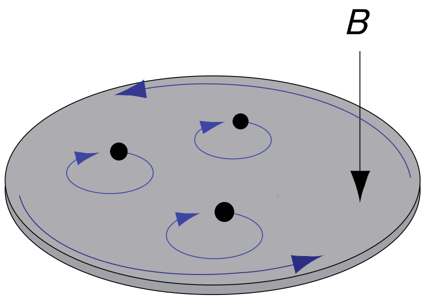
Coriolis force

Electromagnetism-gravity analogy

Responses of 2d-TSC

Streda (1982)

$$\sigma_H = c\frac{\partial M^z}{\partial \phi} = c\frac{\partial \rho}{\partial B^z}$$

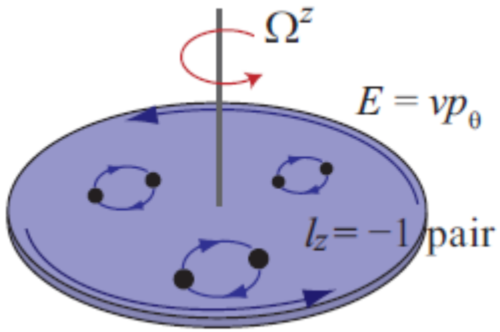


Charge density changes as

$$\Delta\rho = (\sigma_H / c)\Delta B^z$$

Present work

$$\kappa_H = \frac{1}{T}\frac{\partial M_E^z}{\partial \phi_g} = \frac{1}{T}\frac{\partial Q}{\partial B_g^z}$$



Thermal energy changes as

$$\Delta Q = (2\kappa_H T/v^2)\Delta\Omega^z$$

Intrinsic angular momentum

Combine

$$\begin{cases} \kappa_H = \dfrac{v^2}{2}\dfrac{\partial L^z}{\partial T} \\ \kappa_H = \dfrac{\pi^2 k_B^2}{6h} T \end{cases}$$

for chiral *p*-wave SC

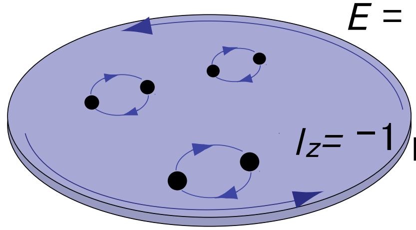
$$L^z(T) = L^z(0) + \frac{\pi^2 \hbar k_F^2}{6}\left(\frac{k_B T}{\Delta}\right)^2 + \cdots$$

This follows a crude argument for intrinsic angular momentum

$$L^z(T_c) = 0$$

$$\downarrow$$

$$L^z(0) \approx -\hbar\pi k_F^2 \left(\frac{k_B T_c}{\Delta}\right)^2 \approx -\hbar\frac{n}{2}$$



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Electromagnetic and thermal responses in topological insulators and superconductors

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1. Introduction: Topologically nontrivial states
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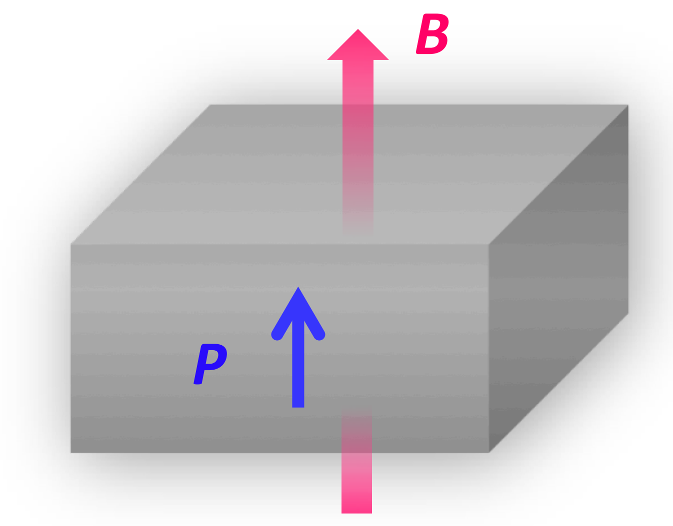
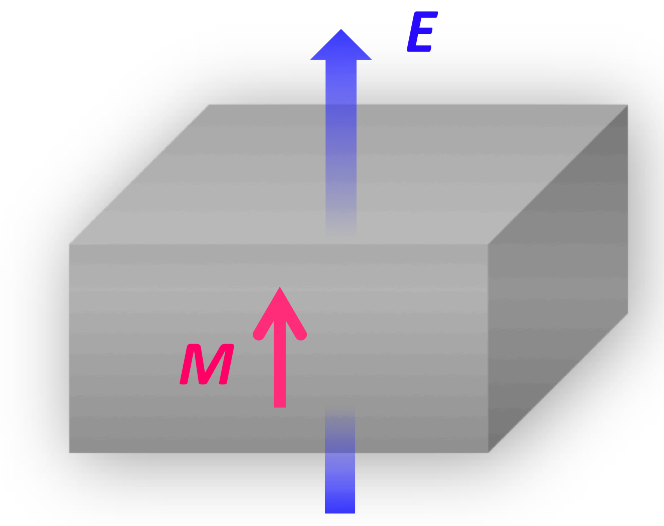
Responses to B and E in 3d-TI

Cross-correlated responses

(magnetization)	$M = \alpha_m E$	(electric field)
(electric polarization)	$P = \alpha_e B$	(magnetic field)

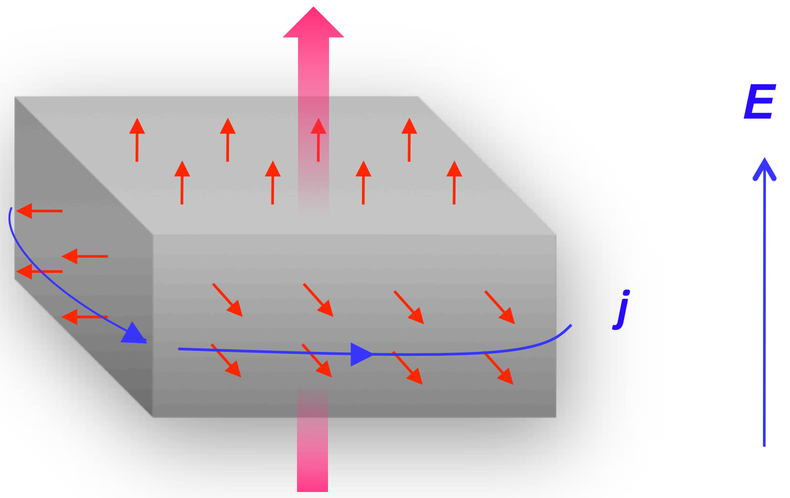




Responses to B and E in 3d-TI

$\boldsymbol{B} = (4\pi/c)\,\boldsymbol{j}$

3D TI + magnetic impurities



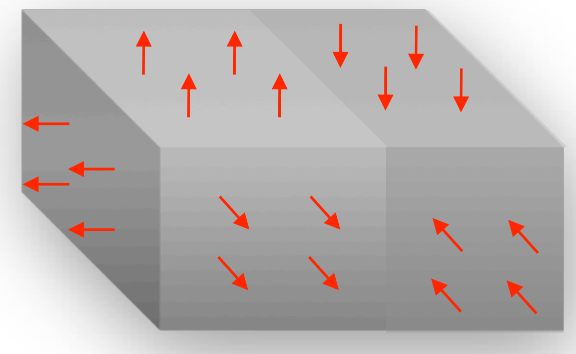
$$\boldsymbol{M} = \frac{e^2}{2hc}\boldsymbol{E}$$

Qi, Hughes, Zhang '08

Needs magnetic impurities

with the magnetization all pointing out (or in)

Responses to B and E in 3d-TI



$\sigma_{xy} > 0$ $\sigma_{xy} < 0$

Surface QH states

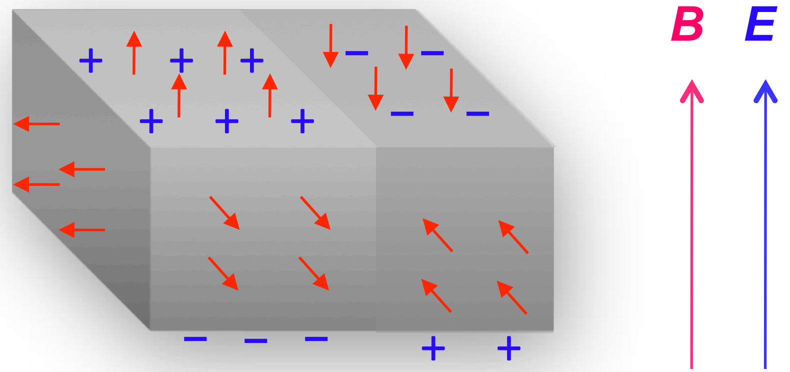
$$j^{\mu} = \tfrac{1}{2}\sigma_{xy}\varepsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

$$\begin{cases} \rho = \sigma_{xy} B_z \\ \vec{j} = \sigma_{xy}\vec{E}\times\hat{z} \end{cases}$$

Responses to B and E in 3d-TI

$$E_{ME} = -\int d^3x \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\theta}{\pi}\, \mathbf{E}\cdot\mathbf{B}$$

Apply *B*-field
Apply *E*-field



$\sigma_{xy} > 0$ $\sigma_{xy} < 0$

Surface QH states

$$j^{\mu} = \tfrac{1}{2}\sigma_{xy}\varepsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

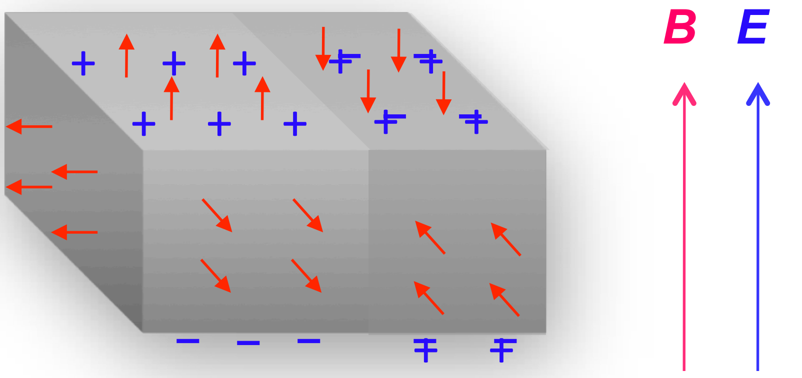
$$\rho = \sigma_{xy} B_z$$

$$\mathbf{j} = \sigma_{xy}\mathbf{E}\times\hat{z}$$

Responses to B and E in 3d-TI

$$E_{ME} = -\int d^3x \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\theta}{\pi}\, \mathbf{E}\cdot\mathbf{B}$$

Apply *B*-field
Apply *E*-field



$\sigma_{xy} > 0$ $\sigma_{xy} < 0$

Surface QH states

$$j^{\mu} = \tfrac{1}{2}\sigma_{xy}\varepsilon^{\mu\nu\lambda}F_{\nu\lambda} \quad \begin{cases} \rho = \sigma_{xy} B_z \\ j = \sigma_{xy} E \times \hat{z} \end{cases}$$

Responses to B and E in 3d-TI

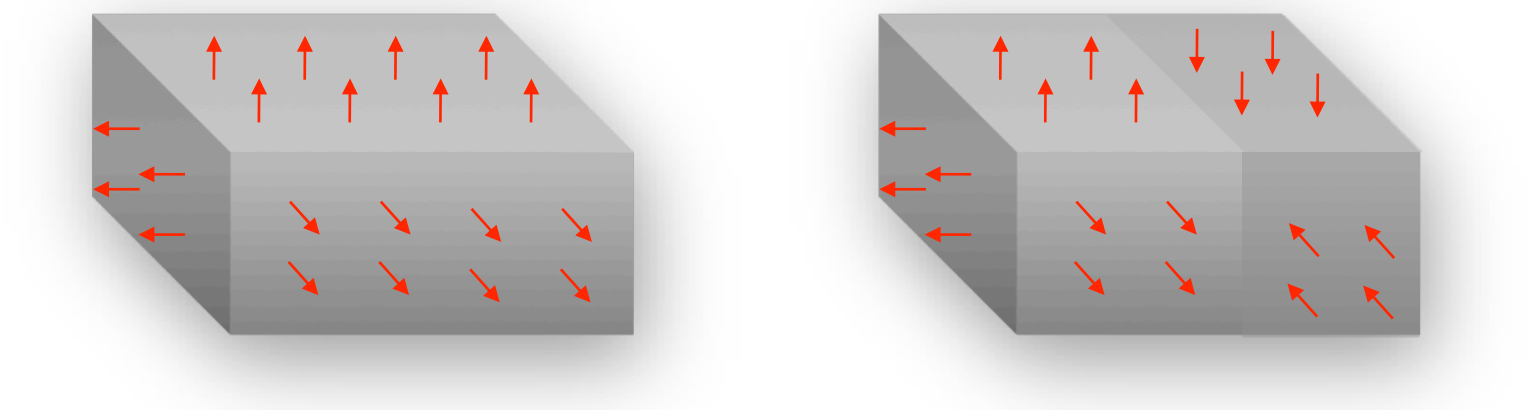
$$E_{ME} = -\int d^3x \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\theta}{\pi}\, \mathbf{E}\cdot\mathbf{B}$$

$$\mathbf{M} = -\frac{\partial E_{ME}}{\partial \mathbf{B}} = \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\theta}{\pi}\, \mathbf{E}$$

$$\mathbf{P} = -\frac{\partial E_{ME}}{\partial \mathbf{E}} = \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\theta}{\pi}\, \mathbf{B}$$

Qi, Hughes, Zhang '08

Responses to B and E in 3d-TI



-Polarization energy (Axion term)

$$E_{ME} = -\int d^3x \left(\frac{\alpha}{4\pi}\right)\frac{\theta}{\pi} \mathbf{E}\cdot\mathbf{B}$$

$$\simeq -10^{10}\left(E[\mathrm{V/cm}]\right)\left(B[\mathrm{T}]\right)\left(L[\mathrm{cm}]\right)^3 \quad [\mathrm{eV}]$$

-Zeeman energy

$$U_{\mathrm{aniso}}/L^2 = 5\times 10^{10} \quad [\mathrm{eV/cm^2}]$$

-Anisotropy energy

$$U_{\mathrm{Zeeman}}/L^2 \sim 10^9 (B[\mathrm{T}]) \quad [\mathrm{eV/cm^2}]$$

Newly found formulae

2 dimensions

QHE

$$\sigma_H = c\frac{\partial M^z}{\partial \phi} = c\frac{\partial \rho}{\partial B^z}$$

2d chiral SC

$$\kappa_H = \frac{1}{T}\frac{\partial M_E^z}{\partial \phi_g} = \frac{1}{T}\frac{\partial Q}{\partial B_g^z}$$

Previous section

3 dimensions

3d-TI

$$\chi_{ij}^{ME} = \frac{\partial M^i}{\partial E^j} = \frac{\partial P^i}{\partial B^j}$$

3d-TSC

? ?

This section

Newly found formulae

2 dimensions

QHE

$$\sigma_H = c\frac{\partial M^z}{\partial \phi} = c\frac{\partial \rho}{\partial B^z}$$

2d chiral SC

$$\kappa_H = \frac{1}{T}\frac{\partial M_E^z}{\partial \phi_g} = \frac{1}{T}\frac{\partial Q}{\partial B_g^z}$$

Previous section

3 dimensions

3d-TI

$$\chi_{ij}^{ME} = \frac{\partial M^i}{\partial E^j} = \frac{\partial P^i}{\partial B^j}$$

3d-TSC

$$\chi_{ij}^{TM} = \frac{\partial M_E^i}{\partial E_g^j} = \frac{\partial P_E^i}{\partial B_g^j}$$

This section

Response to thermal gradient

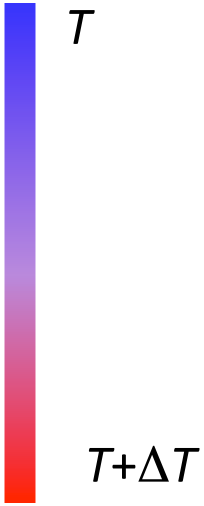
Cross-correlation between Electric field ***E*** and Magnetic field ***B***

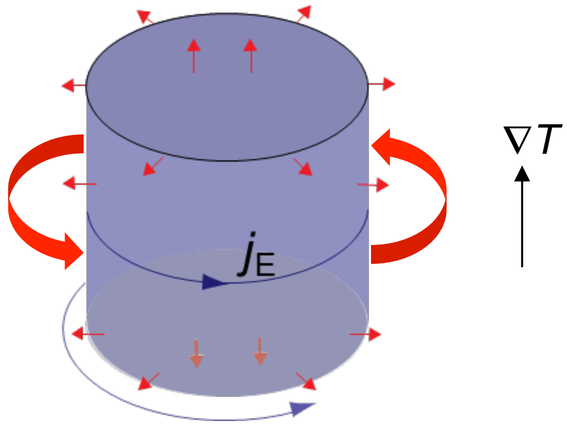
$$\boldsymbol{M} = \alpha_m \boldsymbol{E}$$

Cross-correlation between Thermal gradient and Rotational motion

$$\boldsymbol{L} \propto \nabla \boldsymbol{T}$$

angular momentum





Electromagnetism and gravity

$$\boldsymbol{F} = e\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)$$

$$\boldsymbol{F} = m\left(\boldsymbol{E}_g + 2\boldsymbol{v}\times\boldsymbol{B}_g\right)$$

Maxwell equation

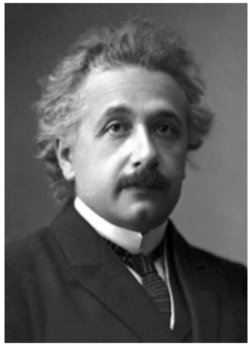
$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\nu$$

$$j^\nu = (\rho c, \boldsymbol{j})$$

Einstein equation

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}$$

$$T^{0\nu} = (\rho_m c^2, \boldsymbol{j}_E c)$$



In the static case

$$\nabla\cdot\boldsymbol{E} = 4\pi\rho$$

$$\nabla\times\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j}$$

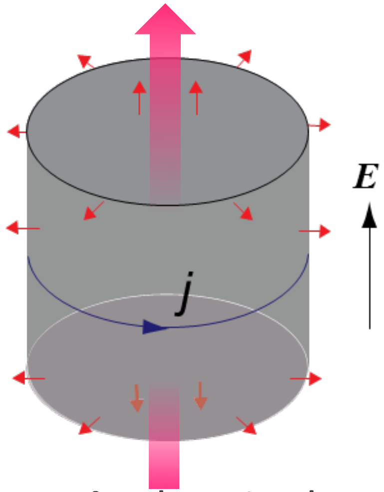
In the linearized approximation

$$\nabla\cdot\boldsymbol{E}_g = -4\pi G\rho_m$$

$$\nabla\times\boldsymbol{B}_g = -\frac{4\pi G}{c}\boldsymbol{j}_E$$

Electric/Thermal responses

Quantum Hall effect
at the surface



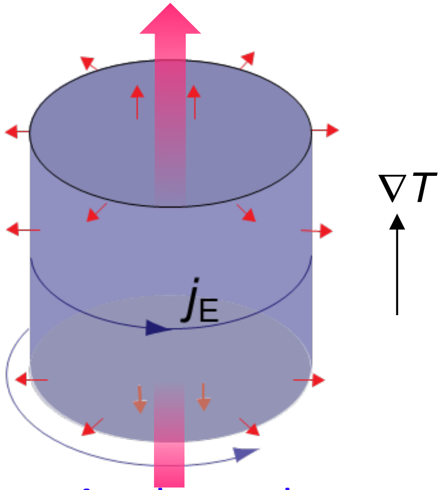
$\boldsymbol{j}$: electric charge current

Magnetization is generated electrically

Qi, Hughes, Zhang (2008)

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{j}$$

Quantum thermal Hall effect
at the surface



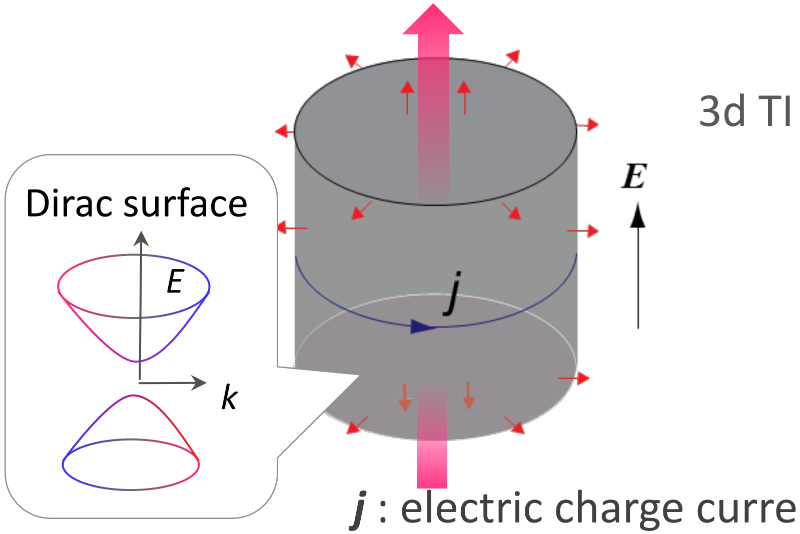
$\boldsymbol{j}_E$: thermal energy current

Present work

$$\nabla \times \boldsymbol{B}_g = -\frac{4\pi G}{c} \boldsymbol{j}_E$$

Electric/Thermal responses

Quantum Hall effect
at the surface



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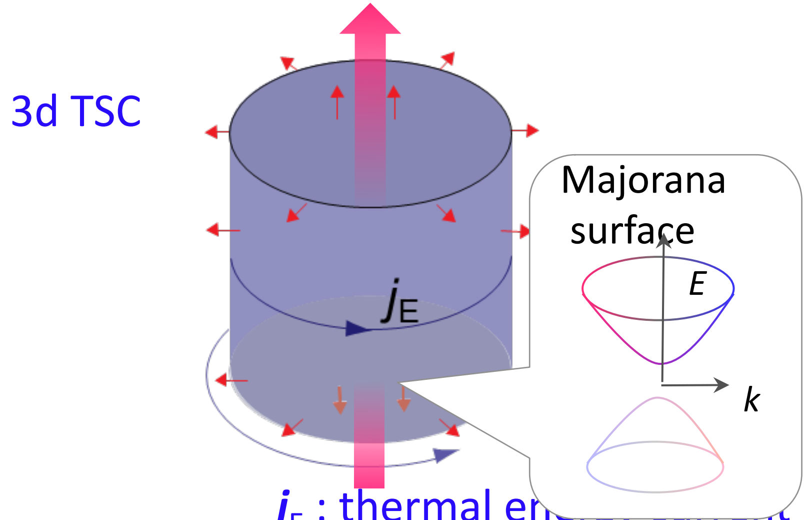

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QTHE of surface Majorana fermions

Majorana fermions at the surface

$$\mathcal{H}_{\rm surf} = -i\hbar v\left(\sigma_z\partial_x + \sigma_x\partial_y\right) + m\sigma_y$$

$$\mathcal{L}_{\rm surf} = \tfrac{1}{2}\psi^T\left[i\hbar\partial_t - \mathcal{H} - \tfrac{1}{2}\{\phi_g, \mathcal{H}\}\right]\psi$$

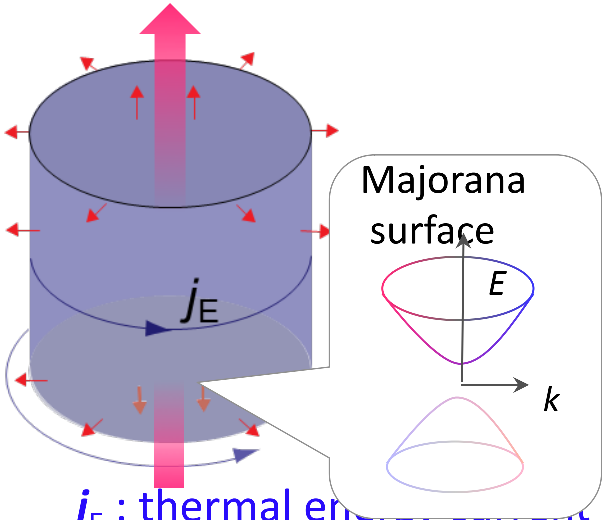
ϕ_g: gravitational potential [Luttinger (1964)]

$$\boldsymbol{j}_E = \hat{\kappa}T\left(-\frac{\nabla T}{T} - \nabla\phi_g\right)$$

$$\kappa_H = \frac{\partial M_E^z}{\partial T} = \frac{v^2}{2}\frac{\partial L^z}{\partial T}$$

$$\kappa_H = \mathrm{sgn}(m)T\frac{\pi^2}{6}\frac{k_B^2}{2h}$$

Quantum thermal Hall effect at the surface



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$j_E/v^2 = P_\phi$: momentum

Orbital angular momentum per unit volume

$$L^z = \frac{r\times P_\phi}{(\pi r^2\ell)} = \frac{2}{v^2}\kappa_H\partial_z T$$

coupling energy $\boldsymbol{L}\cdot\boldsymbol{\Omega}$

$$U_\theta = -\int d^3\boldsymbol{x}\, \underbrace{\frac{2}{v^2}\kappa_H\nabla T\cdot\boldsymbol{\Omega}}_{\boldsymbol{L}\cdot\boldsymbol{\Omega}}$$

$$= -\int d^3\boldsymbol{x}\left(\frac{k_B^2T^2}{24\hbar v}\right)\theta\,\boldsymbol{E}_g\cdot\boldsymbol{B}_g$$

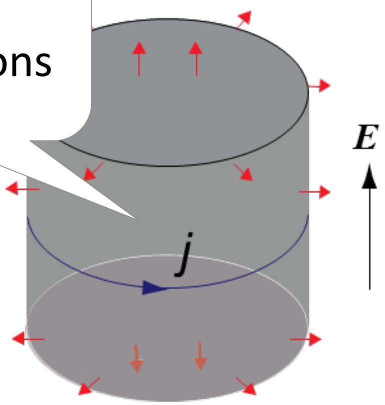
$$\left(\boldsymbol{E}_g = \tfrac{1}{T}\nabla T,\quad \boldsymbol{B}_g = \tfrac{2}{v}\Omega\right)$$

Thermal responses in 3d TSC

Topological insulator

Bi_2Se_3 , Bi_2Te_3 ,…

QHE of
Dirac fermions



$$S_\theta = -\int dt d^3\boldsymbol{x} \left(\frac{\theta\alpha}{16\pi^2}\right)\varepsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

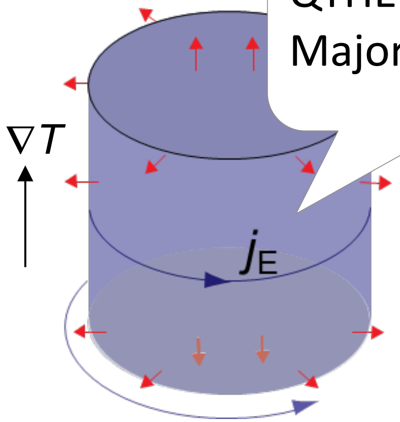
$$= -\int dt d^3\boldsymbol{x} \left(\frac{e^2}{4\pi^2\hbar c}\right) \theta\, \boldsymbol{E}\cdot\boldsymbol{B}$$

Qi, Hughes, Zhang (2008)

Topological superconductor

^{3}He-B, $Cu_xBi_2Se_3$ (?)

QTHE of
Majorana fermions



$$U_\theta = -\int d^3\boldsymbol{x}\ \frac{2}{v^2}\kappa_H \nabla T\cdot\boldsymbol{\Omega}$$

$$= -\int d^3\boldsymbol{x} \left(\frac{k_B^2T^2}{24\hbar v}\right) \theta\, \boldsymbol{E}_g\cdot\boldsymbol{B}_g$$

Thermal responses in 3d TSC

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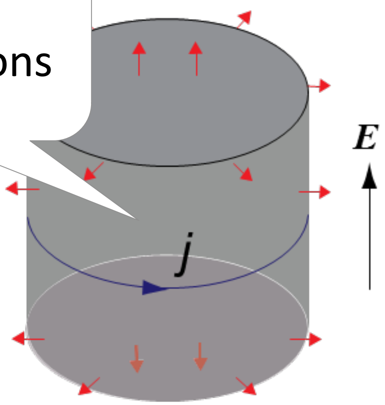

$$\boldsymbol{M} = \frac{e^2}{2hc}\boldsymbol{E}$$

$$\boldsymbol{P} = \frac{e^2}{2hc}\boldsymbol{B}$$

Qi, Hughes, Zhang (2008)

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QTHE of
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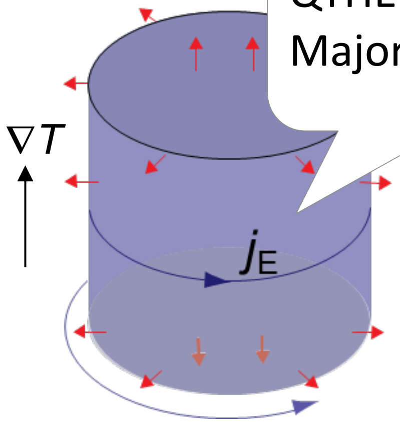


$$\boldsymbol{L} = \frac{(\pi k_B T)^2}{6hv}\boldsymbol{E}_g$$

$$\boldsymbol{P}_E = \frac{(\pi k_B T)^2}{6hv}\boldsymbol{\Omega}$$

$$Q = -\nabla\cdot\boldsymbol{P}_E$$

Present work

Summary

Topological properties of insulators and superconductors are characterized by

Cross-correlated responses

	EM responses	Thermal responses
2D	$\delta M_z = (\sigma_H/ec)\delta\mu$ $\delta N = (\sigma_H/ec)\delta B_z$	$\delta L_z = (2\kappa_H/v^2)\,\delta T$ $\delta Q = (2\kappa_H T/v^2)\delta\Omega_z$
3D	$\boldsymbol{M} = \frac{e^2}{2hc}\boldsymbol{E}$ $\boldsymbol{P} = \frac{e^2}{2hc}\boldsymbol{B}$	$\boldsymbol{L} = \frac{(\pi k_B T)^2}{6hv}\boldsymbol{E}_g$ $\boldsymbol{P}_E = \frac{(\pi k_B T)^2}{6hv}\boldsymbol{\Omega}$

$Q = -\nabla\cdot\boldsymbol{P}_E$

Topological field theories

Topological insulator

Qi, Hughes, Zhang (2008)
Essin, Moore, Vanderbuilt (2009)

$$S_{\theta}^{EM} = \int d^4x \left(\frac{\alpha}{16\pi^2}\right)\theta\varepsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

$$= \int d^4x \left(\frac{\alpha}{4\pi^2}\right)\theta\, \boldsymbol{E}\cdot\boldsymbol{B}$$

⬇

- Surface QHE
- Topological ME effect

Topological superconductor

Ryu, Moore, Ludwig (2011)
Wong, Qi, Zhang (2011)

$$S_{\theta}^{GR} = \int d^4x \left(\frac{1}{1536\pi^2}\right)\theta\varepsilon^{\mu\nu\rho\lambda}R^{\alpha}{}_{\beta\mu\nu}R^{\beta}{}_{\alpha\rho\lambda}$$

R: Riemann tensor

Physical consequence is not obvious

Thermal responses?

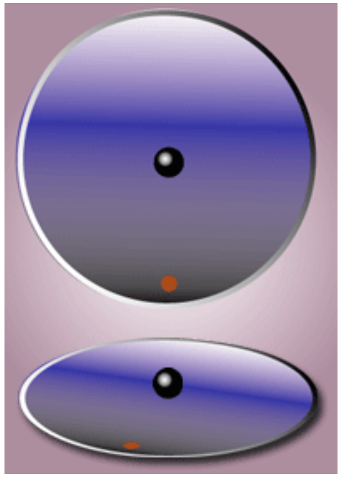
$$\boldsymbol{F} = m\,(\boldsymbol{E}_g + 2\boldsymbol{v} \times \boldsymbol{B}_g)$$

$\boldsymbol{B}_g$

In the rotating flame

↓

Coriolis force



gravitomagnetic field
(Barnet field)

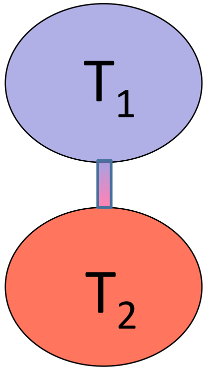
$$\boldsymbol{B}_g = \boldsymbol{\Omega}$$

$\boldsymbol{E}_g$

gravitational field

$$\boldsymbol{E}_g = -\nabla\phi$$

↓



$$T_1 \neq T_2 = T_1\left(1 + \Delta\phi / c^2\right)$$

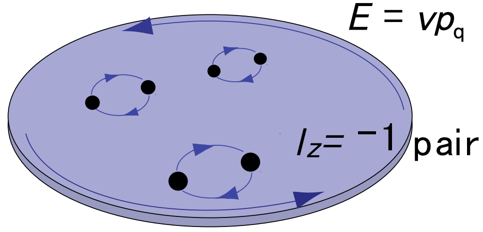
$$\frac{1}{c^2}\boldsymbol{E}_g = \frac{\nabla T}{T}$$

Tolman-Ehrenfest , Luttinger

Edge modes in chiral SC

chiral Majorana edge modes

$$H_{edge} = \frac{1}{2}\int_0^L dx\, \psi(-i\hbar v \partial_x)\psi$$



$T^{10}=T^{00}=T^{01}$

$$\langle j_E \rangle = v\frac{\langle H_{edge}\rangle}{L} = \frac{v^2}{2}\langle L^z_{edge}\rangle$$

↑ Orbital angular momentum

$$\frac{\langle H_{edge}\rangle}{L} = \frac{1}{2}\int\frac{dk}{2\pi}\, v\hbar k\frac{1}{e^{v\hbar k/k_BT}+1} = \frac{\pi^2 k_B^2 T^2}{12hv}$$

$$\kappa_H \equiv \frac{\pi^2 k_B^2}{6h}T$$

$$\kappa_H \equiv \frac{\partial\langle j_E\rangle}{\partial T} = \frac{v^2}{2}\frac{\partial\langle L^z_{edge}\rangle}{\partial T}$$