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Berry curvature and topological phases for magnons

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Magnon thermal Hall effect for magnetostatic modes

- Matsumoto, Murakami, Phys. Rev. Lett. 106, 197202 (2011).
- Matsumoto, Murakami, Phys. Rev. B 84, 184406 (2011)

Topological Magnonic crystals

- Shindou, Matsumoto, Ohe, Murakami, Phys. Rev. B 87,174402 (2013),
- Shindou, Ohe, Matsumoto, Murakami, Saitoh, Phys. Rev. B 87,174427 (2013),

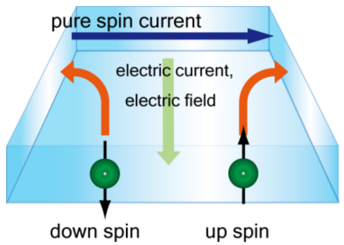
Phenomena due to Berry curvature of band structure

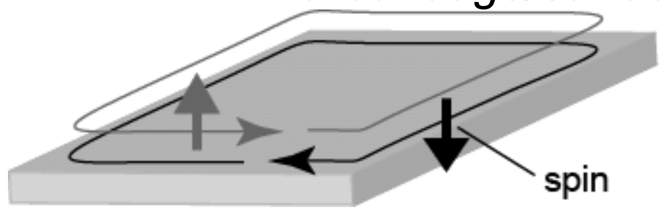
Gapless | Gapped | Topological edge/surface modes in gapped systems

Fermions

• Hall effect → Quantum Hall effect
chiral edge modes

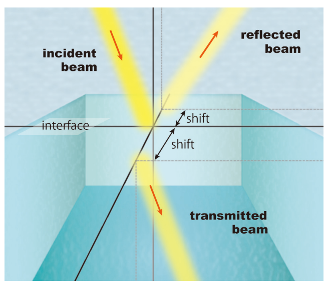
• Spin Hall effect (of electrons) → Topological insulators
helical edge/surface modes

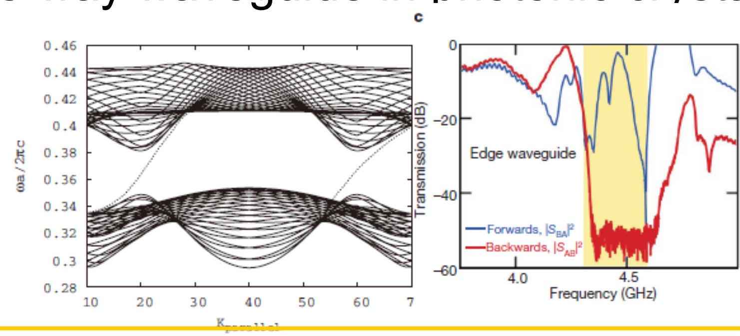




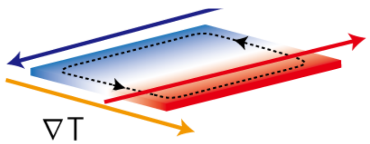
Bosons

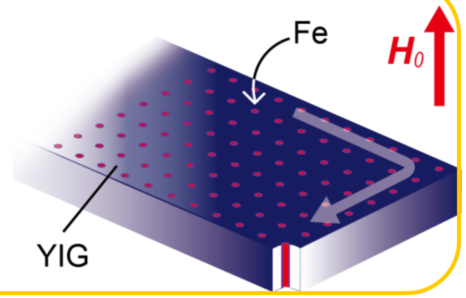
•Spin Hall effect of light → one-way waveguide in photonic crystal



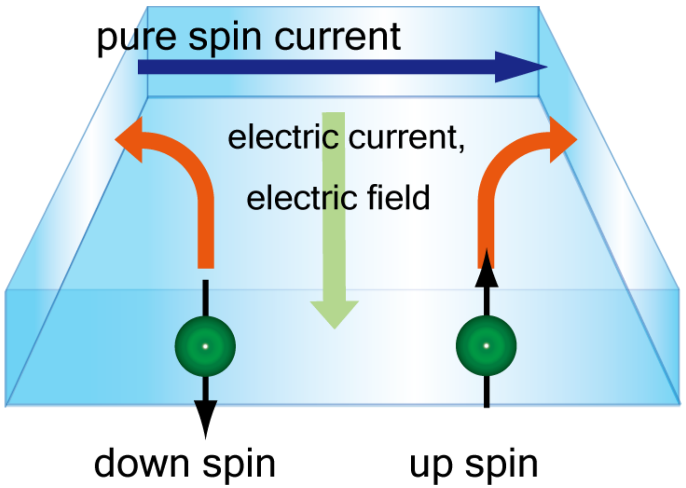


• Magnon thermal Hall effect → topological magnonic crystal





Intrinsic spin Hall effect in metals& semiconductors



- SM, Nagaosa, Zhang, Science (2003)
- Sinova et al., Phys. Rev. Lett. (2004)

semiclassical eq. of motion for wavepackets

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -e\vec{E} \end{cases}$$

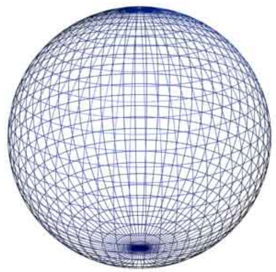
Force

Adams, Blount; Sundaram,Niu, …

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$ **: Berry curvature**

$u_{n\vec{k}}$: periodic part of the Bloch wf.

$\psi_{n\vec{k}}(\vec{x}) = u_{n\vec{k}}(\vec{x})e^{i\vec{k}\cdot\vec{x}}$ (n : band index)



It represents geometric structure of bands in k-space

Magnon thermal Hall effect

Magnon thermal Hall effect by Berry curvature – previous works --

Theory:

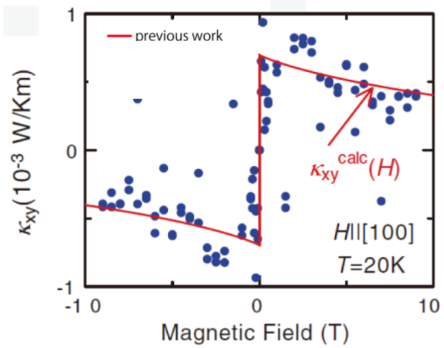
- S. Fujimoto, Phys. Rev. Lett. 103, 047203 (2009).
- H. Katsura, N. Nagaosa, and P. A. Lee, Phys. Rev. Lett.104, 066403 (2010).

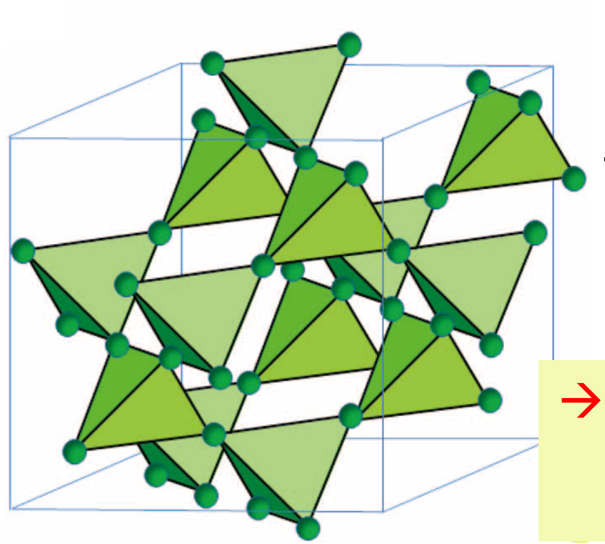
Experiment & theory:

- Y. Onose, et al., Science 329, 297 (2010);

$Lu_2V_2O_7$: Ferromagnet

$$H_{\rm eff} = \sum -J\boldsymbol{S}_i\cdot\boldsymbol{S}_j + \boldsymbol{D}_{ij}\cdot(\boldsymbol{S}_i\times\boldsymbol{S}_j) - g\mu_{\rm B}\boldsymbol{H}\cdot\sum_i \boldsymbol{S}_i.$$





Dyaloshinskii-Moriya interaction → Berry phase

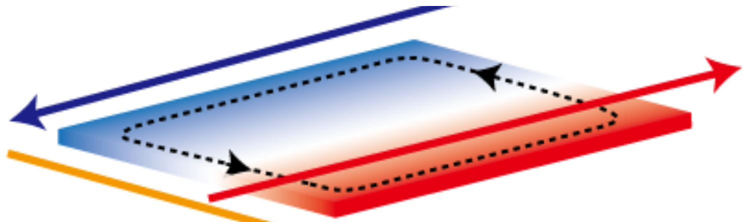
Thermal Hall conductivity

$$\kappa^{xy} = \frac{2}{\hbar V}\sum_{n,k}\rho\left(\varepsilon_{n\vec{k}}\right)\mathrm{Im}\left\langle \frac{\partial u_{n\vec{k}}}{\partial k_x}\right|\left(\frac{H+\varepsilon_{n\vec{k}}}{2}\right)^2\left|\frac{\partial u_{n\vec{k}}}{\partial k_y}\right\rangle$$

→ Correction term !

- R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106, 197202 (2011).
- R. Matsumoto, S. Murakami, Phys. Rev. B 84, 184406 (2011)

Magnon Thermal Hall conductivity (Righi-Leduc effect)



∇T

$$(j_Q)_x = \kappa_{xy} (\nabla T)_y$$

$$\kappa^{xy} = \frac{2k_B^2 T}{\hbar V} \sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}})) \mathrm{Im} \left\langle \frac{\partial u_n}{\partial k_x} \middle| \frac{\partial u_n}{\partial k_y} \right\rangle$$

Berry curvature

$$c_2(\rho) = \int_0^\rho \left[\log\left(\frac{1+t}{t}\right)\right]^2 dt = (1+\rho)\left[\log\left(\frac{1+\rho}{\rho}\right)\right]^2 - (\log\rho)^2 - 2\mathrm{Li}_2(-\rho)$$

ρ: Bose distribution

R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106, 197202 (2011)
T. Qin, Q. Niu and J. Shi,Phys. Rev. Lett. 107, 236601 (2011)

(1) Semiclassical theory

Eq. of motion

$$\begin{cases} \dot{\vec{x}} = \frac{1}{\hbar}\frac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -\nabla U \end{cases}$$

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \middle| \times \middle| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

: Berry curvature

(2) Linear response theory

Density matrix

$$g(H) = f_0(H) + f_1(H)$$

equilibrium

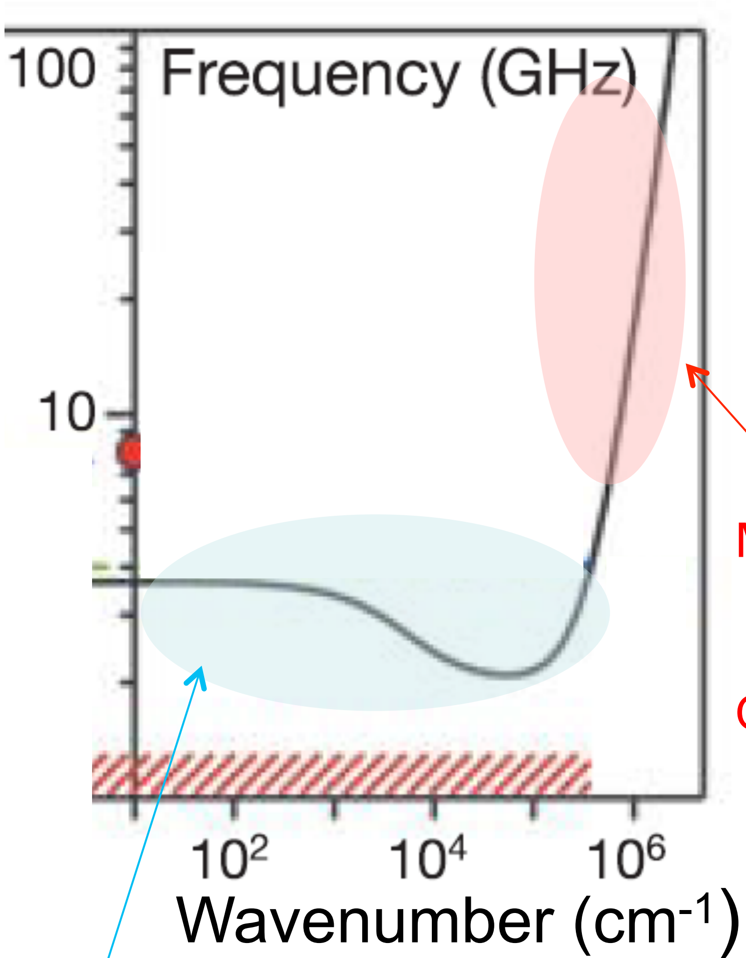
deviation by external field

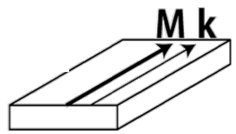
Current

$$\boldsymbol{j}(\boldsymbol{r}) = \boldsymbol{j}^{(0)}(\boldsymbol{r}) + \boldsymbol{j}^{(1)}(\boldsymbol{r})$$

$$\boldsymbol{j}_E(\boldsymbol{r}) = \boldsymbol{j}_E^{(0)}(\boldsymbol{r}) + \boldsymbol{j}_E^{(1)}(\boldsymbol{r})$$

Magnetostatic modes in ferromagnetic films (e.g. in YIG)





MSBVW (magnetostatic backward volume mode)

Magnon (← exchange) $E \propto k^2$

No anisotropy in YIG

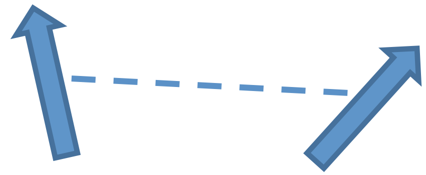
Quantum mechnical, short ranged

Magnetostatic mode

(← dipolar interaction ← film)

Classical, long-ranged

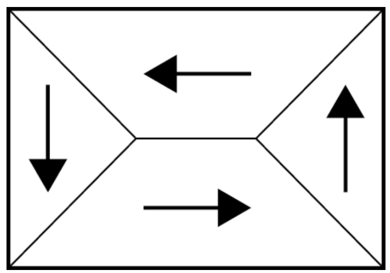
Magnetic dipole interaction



$$H_{\text{dipole}} = \frac{\mu_0}{4\pi\,|\boldsymbol{r}-\boldsymbol{r}'|^3}\left\{3\,\frac{\boldsymbol{S}_{\boldsymbol{r}}\cdot(\boldsymbol{r}-\boldsymbol{r}')\;\boldsymbol{S}_{\boldsymbol{r}'}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^2} - \boldsymbol{S}_{\boldsymbol{r}}\cdot\boldsymbol{S}_{\boldsymbol{r}'}\right\}.$$

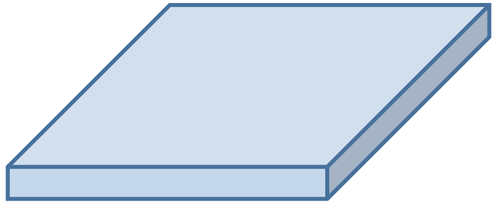
- Dominant in long length scale (microns)
- Similar to spin-orbit int.
  → Berry curvature

- Long-ranged → nontrivial, controlled by shape



Magnetic domains

Magnetostatic modes in ferromagnetic films



M: magnetization, γ: gyromagnetic ratio, ***H***: external magnetic field

- Landau-Lifshitz (LL) equation $\quad \dfrac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H})$
- Maxwell equation $\quad \nabla\cdot\mathbf{B} = 0 \;,\quad \nabla\times\mathbf{H} = 0$ (magnetostatic limit)
- Boundary conditions $\quad \mathbf{B}_{1\perp} = \mathbf{B}_{2\perp} \;,\quad \mathbf{H}_{1//} = \mathbf{H}_{2//}$

Generalized eigenvalue eq.

B. A. Kalinikos and A. N. Slavin, *J. Phys. C* **19**, 7013 (1986)

$$\hat{H}\mathbf{m}(z) = \omega\sigma_z\mathbf{m}(z) \qquad \left(\hat{H}\mathbf{m}(z) \equiv \omega_H\mathbf{m}(z) - \omega_M\int_{-L/2}^{L/2} dz'\,\hat{G}(z,z')\mathbf{m}(z')\right)$$

$\omega_H = \gamma H_0$, $\omega_M = \gamma M_0$, L: thickness of the film, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\mathbf{m}(z) = \begin{pmatrix} m_x + i m_y \\ m_x - i m_y \end{pmatrix}$

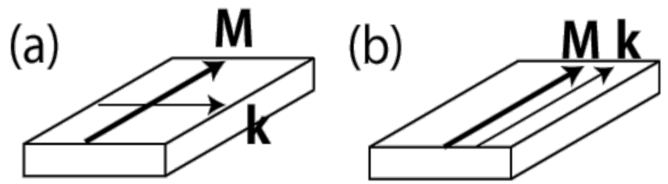
M_0: saturation magnetization, H_0: static magnetic field, $z \perp$ film,

$\hat{G}$: 2×2 matrix of the Green's function, ω: frequency of the spin wave

Magnetostatic modes in ferromagnetic films

Berry curvature

$$\Omega_n^{\gamma}(\mathbf{k}) = -\varepsilon_{\alpha\beta\gamma}\,\mathrm{Im}\left\langle \frac{\partial \mathbf{m}_{n,\mathbf{k}}}{\partial k_\alpha} \middle| \sigma_z \middle| \frac{\partial \mathbf{m}_{n,\mathbf{k}}}{\partial k_\beta} \right\rangle$$



(a) MagnetoStatic Surface Wave (MSSW)

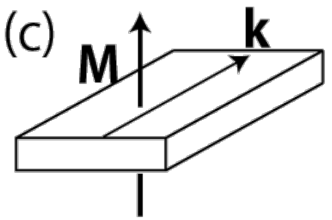
(b) MagnetoStatic Backward Volume Wave (MSBVW)

Zero Berry curvature

$\Omega_n^{z}(\mathbf{k}) = 0$ ← symmetry

(2-fold in-plane rotation + time reversal)

(c) MagnetoStatic Forward Volume Wave (MSFVW)



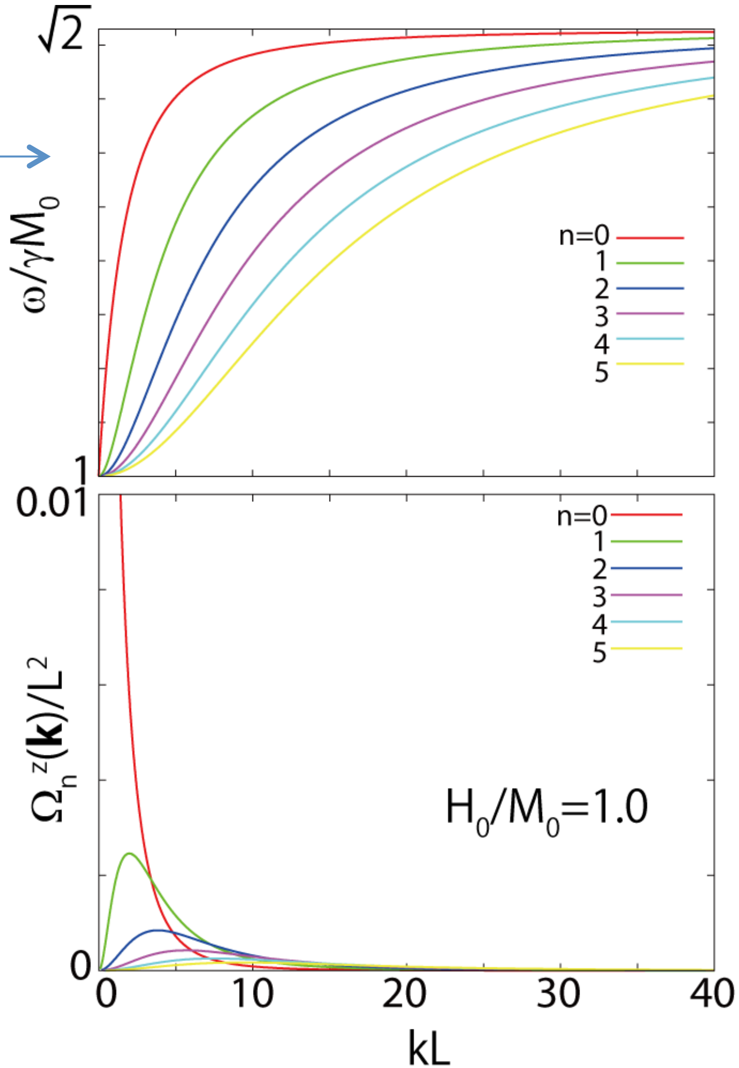
We can expect the Berry curvature to be nonzero !

Berry curvature for MSFVW mode

Dispersion for n=0～5. (H_0/M_0=1.0)

Berry curvature

$$\Omega_n^z(\mathbf{k}) = \frac{1}{2\omega_H k}\frac{\partial \omega_n}{\partial k}\left(1-\frac{\omega_H^2}{\omega_n^2}\right)$$



- R. Matsumoto, S. Murakami, PRL 106,197202 (2011), PRB84, 184406 (2011)

Bosonic BdG eq. and Berry curvature

Cf: Phonons: Qin, Zhou, Shi, PRB 86, 104305 (2012)
Electrons: Sumiyoshi, Fujimoto, JPSJ 82, 023602 (2013)

Generalized eigenvalue eq. $H_k y = w_k s_z y$

→ Bogoliubov-de Gennes Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}} \begin{pmatrix} \boldsymbol{\beta}_{\boldsymbol{k}}^\dagger & \boldsymbol{\beta}_{-\boldsymbol{k}} \end{pmatrix} H_{\boldsymbol{k}} \begin{pmatrix} \boldsymbol{\beta}_{\boldsymbol{k}} \\ \boldsymbol{\beta}_{-\boldsymbol{k}}^\dagger \end{pmatrix}$$

Diagonalization

$$\mathcal{E}_{\boldsymbol{k}} = T_{\boldsymbol{k}}^\dagger H_{\boldsymbol{k}} T_{\boldsymbol{k}} = \begin{pmatrix} E_{\boldsymbol{k}} & \\ & E_{-\boldsymbol{k}} \end{pmatrix}$$

T: paraunitary matrix

$$T_{\boldsymbol{k}}^\dagger \sigma_3 T_{\boldsymbol{k}} = \sigma_3, \qquad T_{\boldsymbol{k}} \sigma_3 T_{\boldsymbol{k}}^\dagger = \sigma_3.$$

Berry curvature for *n*-th band

$$\Omega_{n\boldsymbol{k}} \equiv i\epsilon_{\mu\nu} \left[\sigma_3 \frac{\partial T_{\boldsymbol{k}}^\dagger}{\partial k_\mu} \sigma_3 \frac{\partial T_{\boldsymbol{k}}}{\partial k_\nu} \right]_{nn} \qquad \Omega_{n\boldsymbol{k}} = -\Omega_{n+N,-\boldsymbol{k}}$$

Thermal Hall conductivity for bosonic BdG eq.

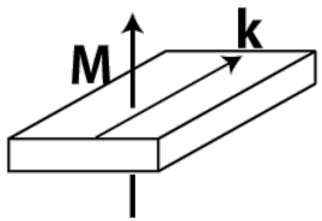
Linear response theory →

$$\kappa_{\mu\nu} = -\frac{k_B^2 T}{\hbar V}\sum_{\boldsymbol{k}}\sum_{n=1}^{N}\left(c_2\left(g\left(\varepsilon_{n\boldsymbol{k}}\right)\right) - \frac{\pi^2}{3}\right)\Omega_{n\boldsymbol{k}}.$$

Berry curvature

$$c_2(x) = (1+x)\left(\log\frac{1+x}{x}\right)^2 - (\log x)^2 - 2\mathrm{Li}_2(-x)$$

(e.g.) MSFVW(Magnetostatic forward volume wave) mode



higher T (room temp.)

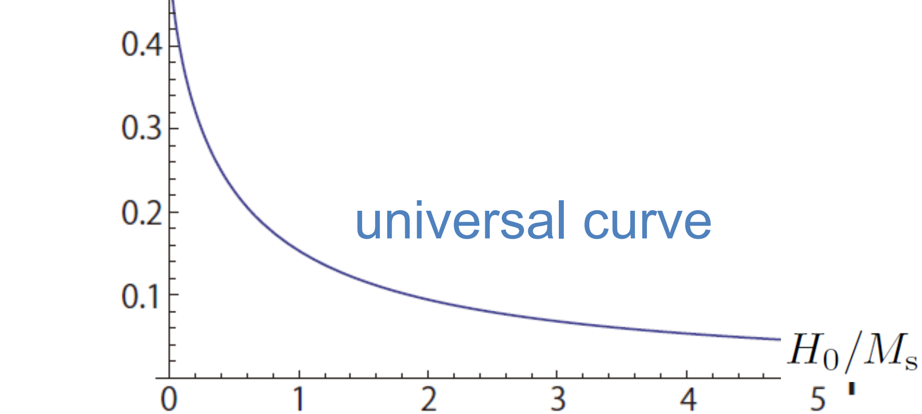
$\frac{k_B T}{\gamma\hbar H} \gg 1$ → κ_{xy} T-indep.

(Example):

$\gamma = 2.8\mathrm{MHz/Oe},\ M_s = 1750\mathrm{gauss},\ T = 300\mathrm{K}$

$H_{ex} = 3000\mathrm{Oe},\ l_{ex} = 17.2\mathrm{nm}$ for YIG,

$$\kappa_{xy} \approx 5.8\times 10^{-8}\ \mathrm{W/Km}$$



Topological chiral modes in magnonic crystals

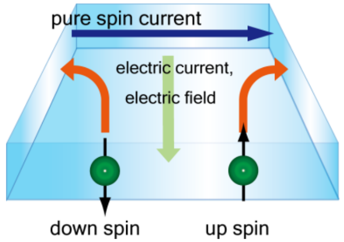
- Shindou, Matsumoto, Ohe, Murakami, Phys. Rev. B 87,174402 (2013),
- Shindou, Ohe, Matsumoto, Murakami, Saitoh, Phys. Rev. B 87,174427 (2013),

Phenomena due to Berry curvature of band structure

Gapless

- Hall effect

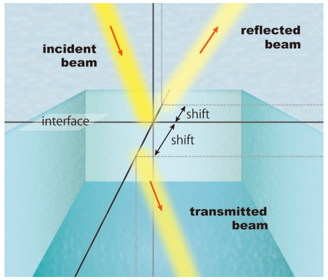
- Spin Hall effect (of electrons)



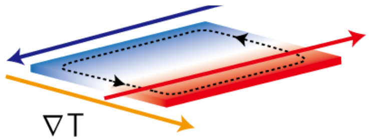
Fermions

Bosons

- Spin Hall effect of light



- Magnon thermal Hall effect

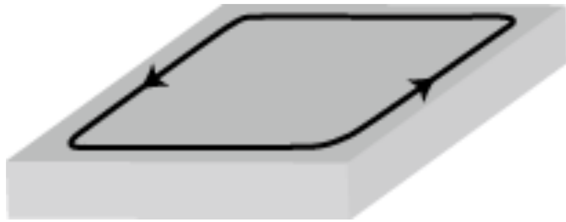


Chern number & topological chiral modes

Band gap → Chern number for n-th band = integer

$$\mathrm{Ch}_n = \int_{BZ} \frac{d^2k}{2\pi}\Omega_n(\vec{k})$$

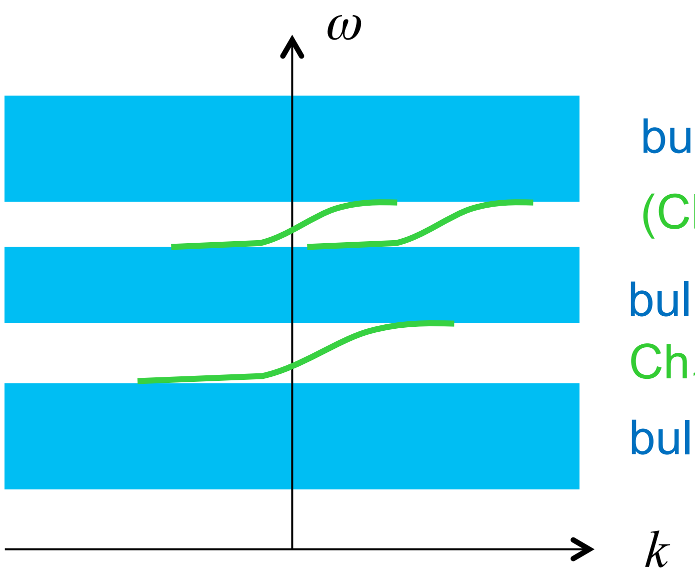
Berry curvature



tooological chiral edge modes

$$\sum_{n \in \text{bands below } E} \mathrm{Ch}_n = \#(\text{chiral edge states in the gap at } E)$$

- Analogous to chiral edge states of quantum Hall effect.



bulk mode: Chern number= Ch_3

$(\mathrm{Ch}_1+\mathrm{Ch}_2)$ topological edge modes

bulk mode: Chern number= Ch_2

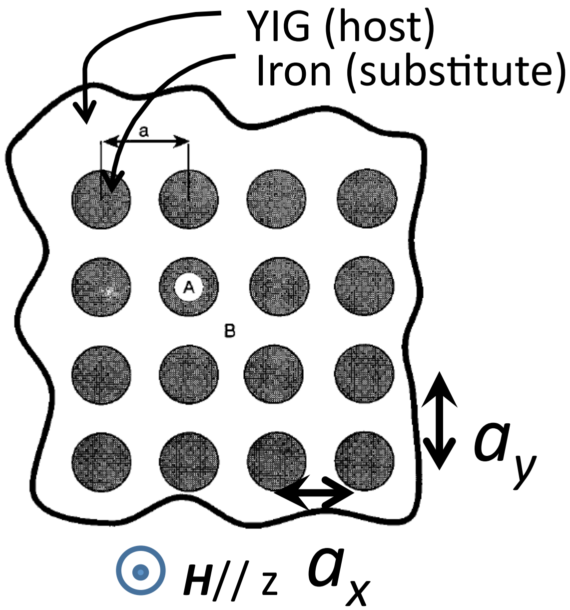
Ch_1 topological edge modes

bulk mode: Chern number= Ch_1

2D Magnonic Crystal : periodically modulated magnetic materials

- Landau-Lifshitz equation $\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\text{eff}}$
- Maxwell equation (magnetostatic approx.)

$$\nabla \times \boldsymbol{H} = \boldsymbol{0},$$
$$\nabla \cdot (\boldsymbol{H} + 4\pi \boldsymbol{M}) = 0.$$



- Saturation magnetization M_s
- exchange interaction length Q

modulated

- Linearized EOM

$$\frac{1}{|\gamma|\mu_0}\frac{\partial m_\pm}{\partial t} = \mp iH_0 m_\pm \pm 2iM_s(\nabla \cdot Q\nabla)m_\pm \mp 2im_\pm(\nabla \cdot Q\nabla)M_s \pm ih_\pm M_s.$$

$$m_\pm \equiv m_x \pm im_y.$$
$$h_\pm \equiv h_x \pm ih_y.$$

External field

exchange field (quantum mechanical short-range)

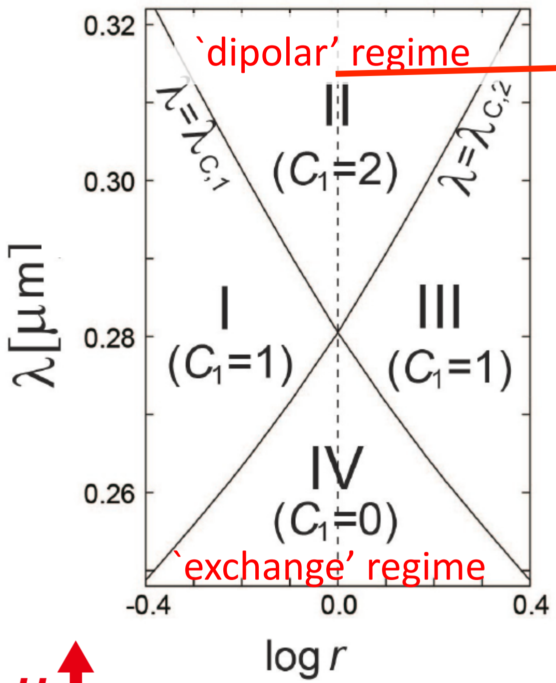
Dipolar field (classical, long range)

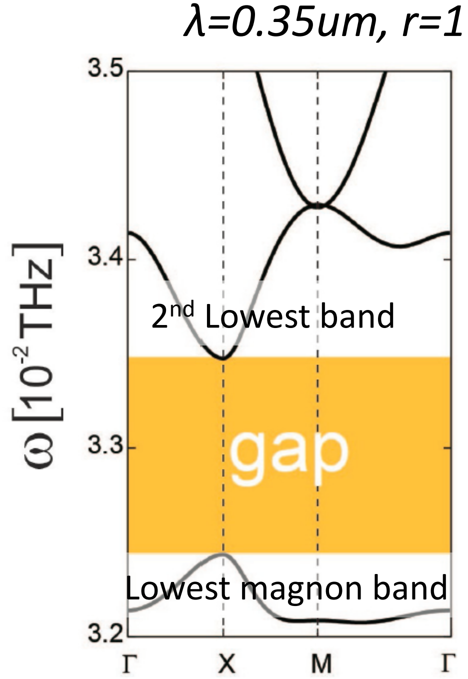
$$\Rightarrow \mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}} [\,\beta_{\boldsymbol{k}}^\dagger \;\; \beta_{-\boldsymbol{k}}\,] \cdot \boldsymbol{H}_{\boldsymbol{k}} \cdot \begin{bmatrix} \beta_{\boldsymbol{k}} \\ \beta_{-\boldsymbol{k}}^\dagger \end{bmatrix}.$$

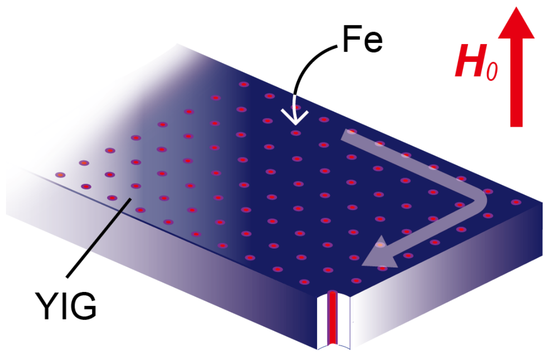
bosonic Bogoliubov – de Gennes eq.

magnonic crystal

Chern number for the 1st band

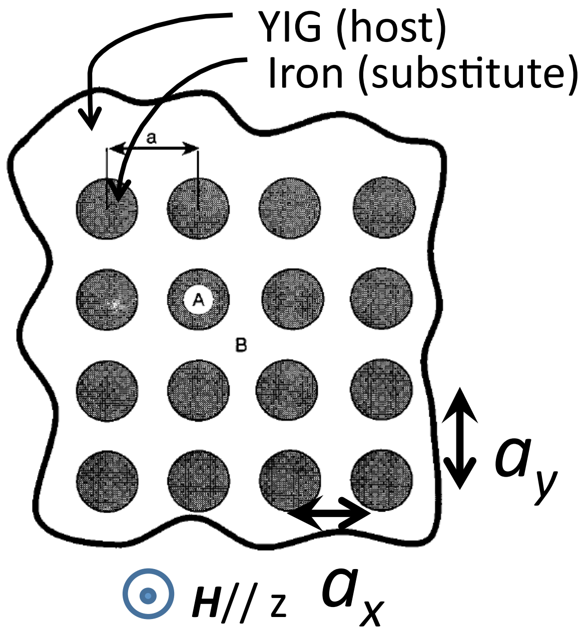






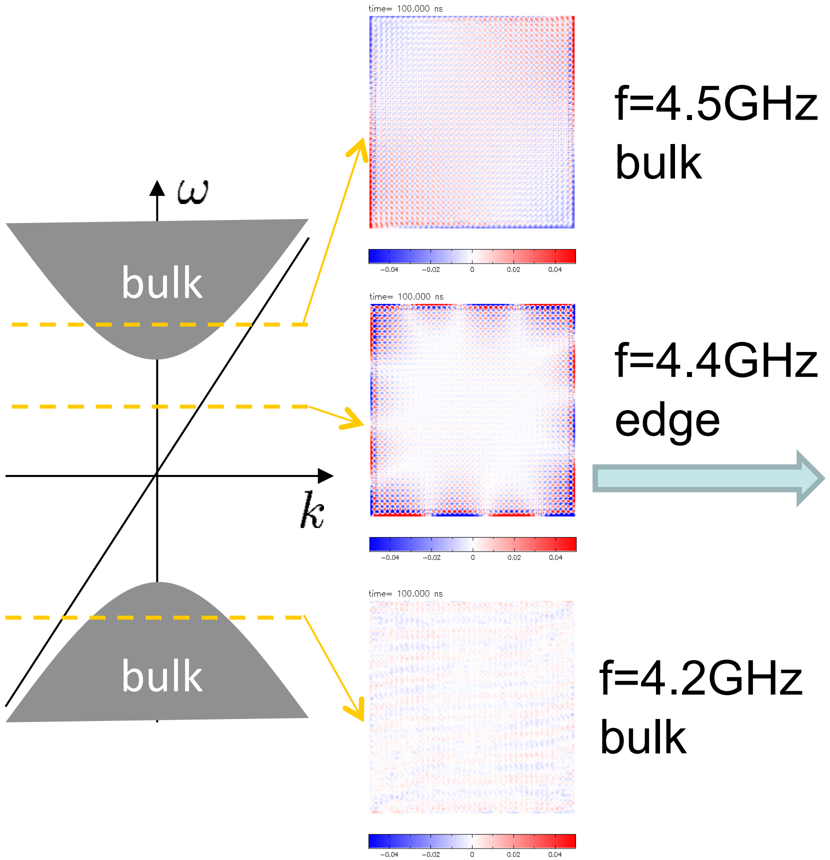
$\lambda = \sqrt{a_x a_y}$: unit cell size

$r = a_y / a_x$: aspect ratio of unit cell

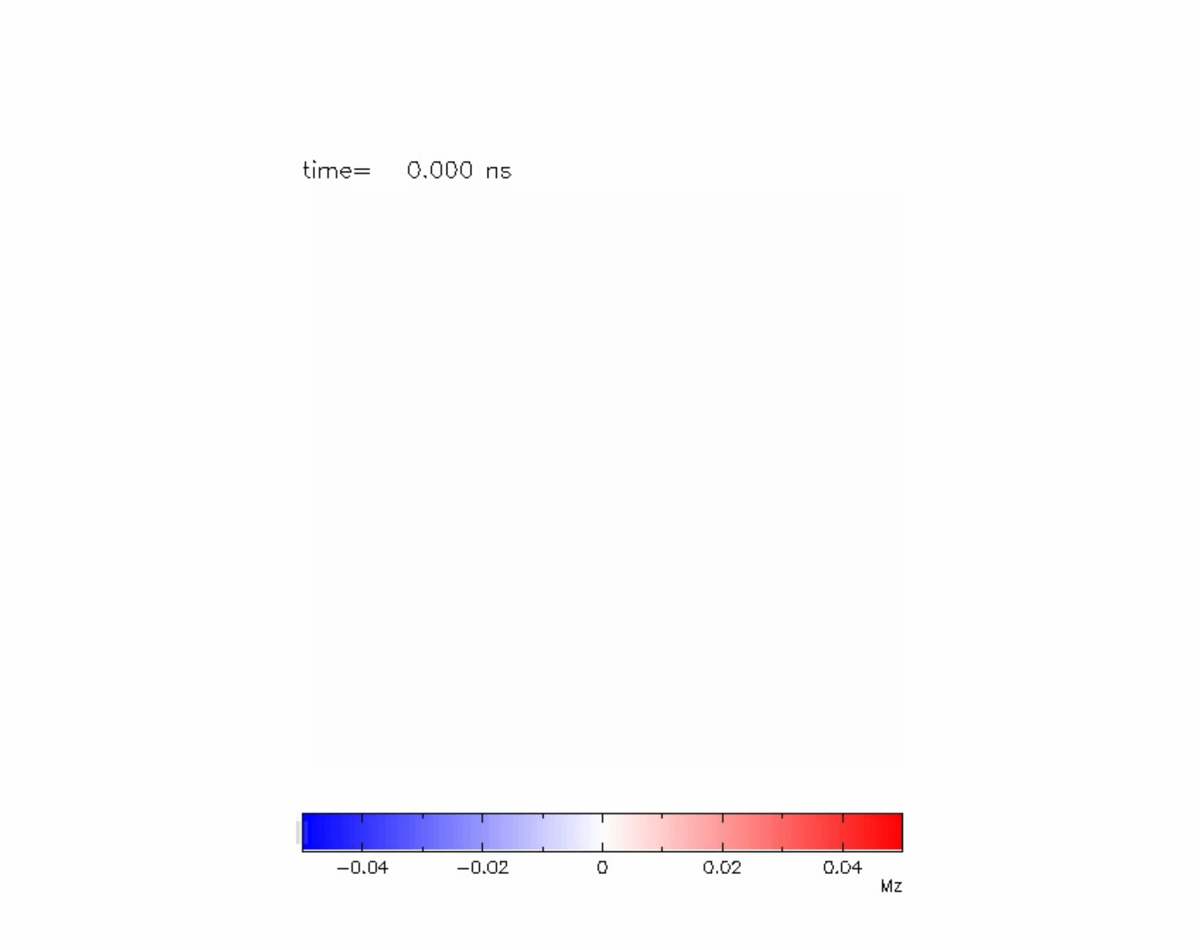


- Larger lattice const. → dipolar interaction is dominant
  → non-trivial Chern integer (like spin-orbit interaction,

Simulation (by Dr. Ohe)



External AC magnetic field applied



External field:
dc field: out-of-plane
ac field: in-plane

Magnonic crystals with ferromagnetic dot array

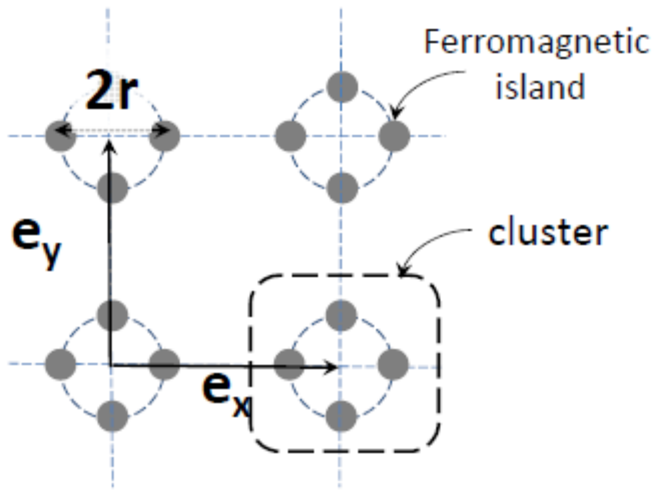
R. Shindou, J. Ohe, R. Matsumoto, S. Murakami, E. Saitoh, arXiv: 1304.1630

dot (=thin magnetic disc) → cluster: forming "atomic orbitals"

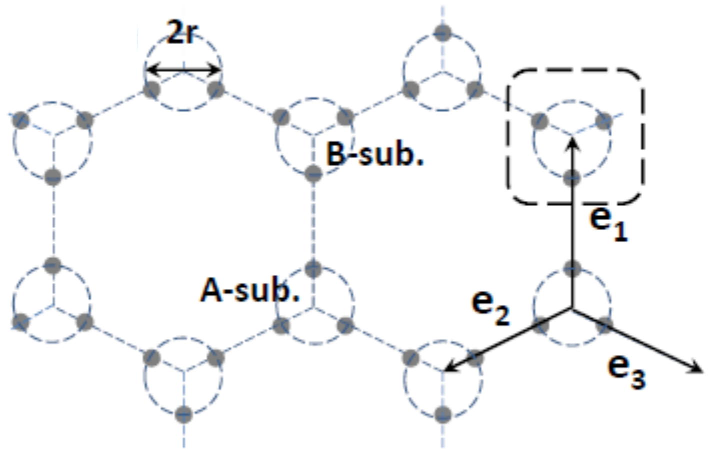
➔ convenient for (1) understanding how the topological phases appear

(2) designing topological phases

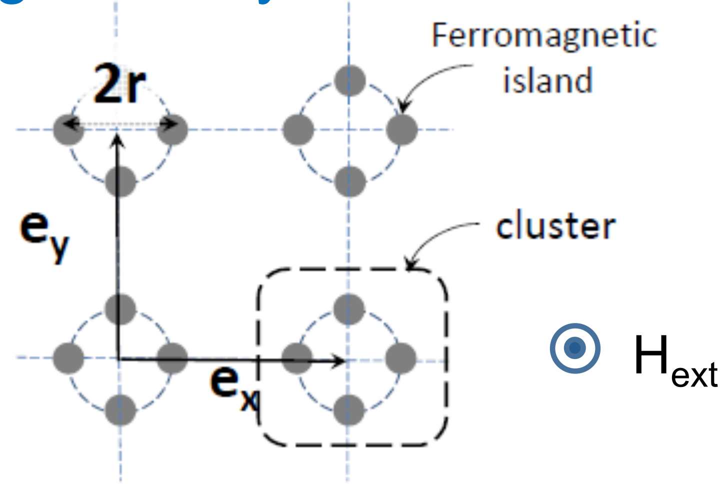
decorated square lattice



decorated honeycomb lattice



Magnonic crystals: decorated square lattice



$|\vec{e}_x| = |\vec{e}_y| = 2.4,\ 2r = 1.2,\ \Delta V = 1.70,\ M_s = 1.0$

Magnetostatic energy

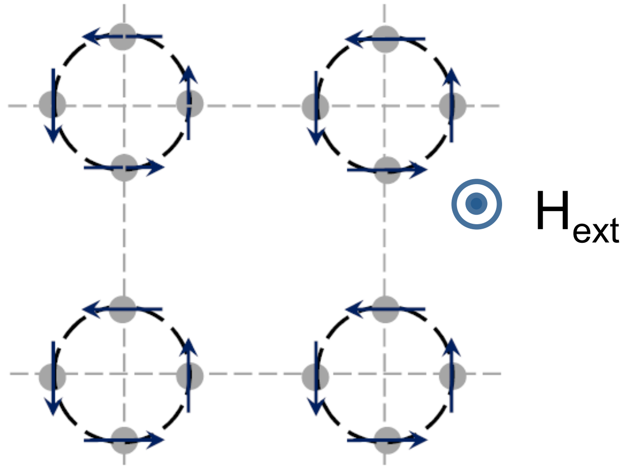
$$E = -\frac{1}{2}(\Delta V)^2 \sum_{i,j}^{i \neq j} M_a(\boldsymbol{r}_i) f_{ab}(\boldsymbol{r}_i - \boldsymbol{r}_j) M_b(\boldsymbol{r}_j)$$

$$+ H \Delta V \sum M_z(\boldsymbol{r}_j).$$

$$f_{ab}(\boldsymbol{r}) = \frac{1}{4\pi}\left(\frac{\delta_{a,b}}{|\boldsymbol{r}|^3} - \frac{3r_a r_b}{|\boldsymbol{r}|^5}\right)$$

Equilibrium spin configuration

$H_{ext} < H_c = 1.71$



Tilted along H_{ext}

$H_{ext} > H_c$

Collinear // H_{ext}

Magnonic crystals: calculation of spin-wave bands

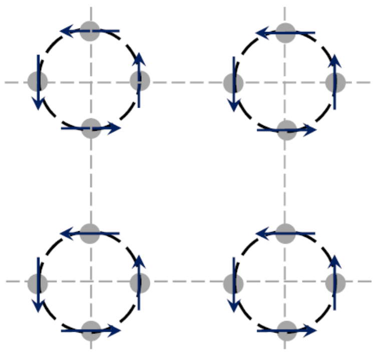
Magnetostatic energy

$$E = -\frac{1}{2}(\Delta V)^2 \sum_{i,j}^{i\neq j} M_a(\boldsymbol{r}_i) f_{ab}(\boldsymbol{r}_i - \boldsymbol{r}_j) M_b(\boldsymbol{r}_j) + H\Delta V \sum_i M_z(\boldsymbol{r}_j).$$

$$f_{ab}(\boldsymbol{r}) = \frac{1}{4\pi}\left(\frac{\delta_{a,b}}{|\boldsymbol{r}|^3} - \frac{3r_a r_b}{|\boldsymbol{r}|^5}\right)$$

Landau-Lifshitz eq. $\quad M(\boldsymbol{r}) \equiv M_0(\boldsymbol{r}) + \boldsymbol{m}_\perp(\boldsymbol{r})$

$$-\partial_t m_\mu(\boldsymbol{r}_i) = \epsilon_{\mu\nu}\alpha(\boldsymbol{r}_i) m_\nu(\boldsymbol{r}_i) + M_s \Delta V\, \epsilon_{\mu\nu} \sum_{j\neq i} f_{\nu\lambda}(\boldsymbol{r}_i, \boldsymbol{r}_j) m_\lambda(\boldsymbol{r}_j)$$



Rotated frame (equilibrium spin direction → *z'* axis)

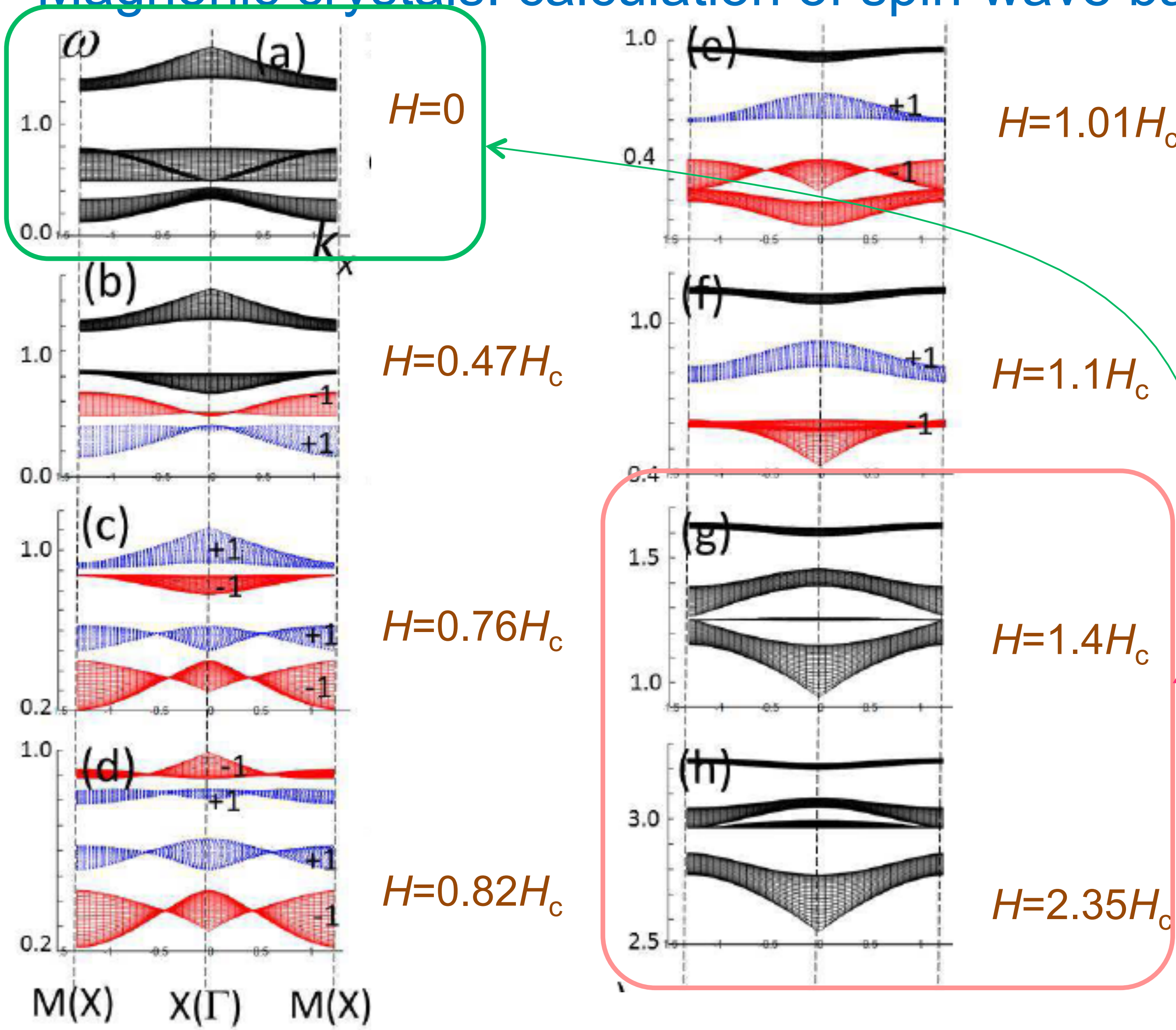
$$R(\boldsymbol{r}) M_0(\boldsymbol{r}) \equiv M_s \boldsymbol{e}_z$$

Generalized eigenvalue eq.

$$\sum_j (H)_{\boldsymbol{r}_i,\boldsymbol{r}_j} \begin{pmatrix} m_+(\boldsymbol{r}_j) \\ m_-(\boldsymbol{r}_j) \end{pmatrix} = \sigma_3 \begin{pmatrix} m_+(\boldsymbol{r}_i) \\ m_-(\boldsymbol{r}_i) \end{pmatrix} \overline{E} \qquad m_\pm \equiv m_x \pm i m_y$$

where $$(H)_{\boldsymbol{r}_i,\boldsymbol{r}_j} = -M_s \alpha(\boldsymbol{r}_i)\delta_{\boldsymbol{r}_i,\boldsymbol{r}_j} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} - M_s \Delta V (1 - \delta_{\boldsymbol{r}_i,\boldsymbol{r}_j}) \begin{pmatrix} f_{++}(\boldsymbol{r}_i,\boldsymbol{r}_j) & f_{+-}(\boldsymbol{r}_i,\boldsymbol{r}_j) \\ f_{-+}(\boldsymbol{r}_i,\boldsymbol{r}_j) & f_{--}(\boldsymbol{r}_i,\boldsymbol{r}_j) \end{pmatrix}$$

Magnonic crystals: calculation of spin-wave bands



Red: Ch=-1
Blue: Ch=+1

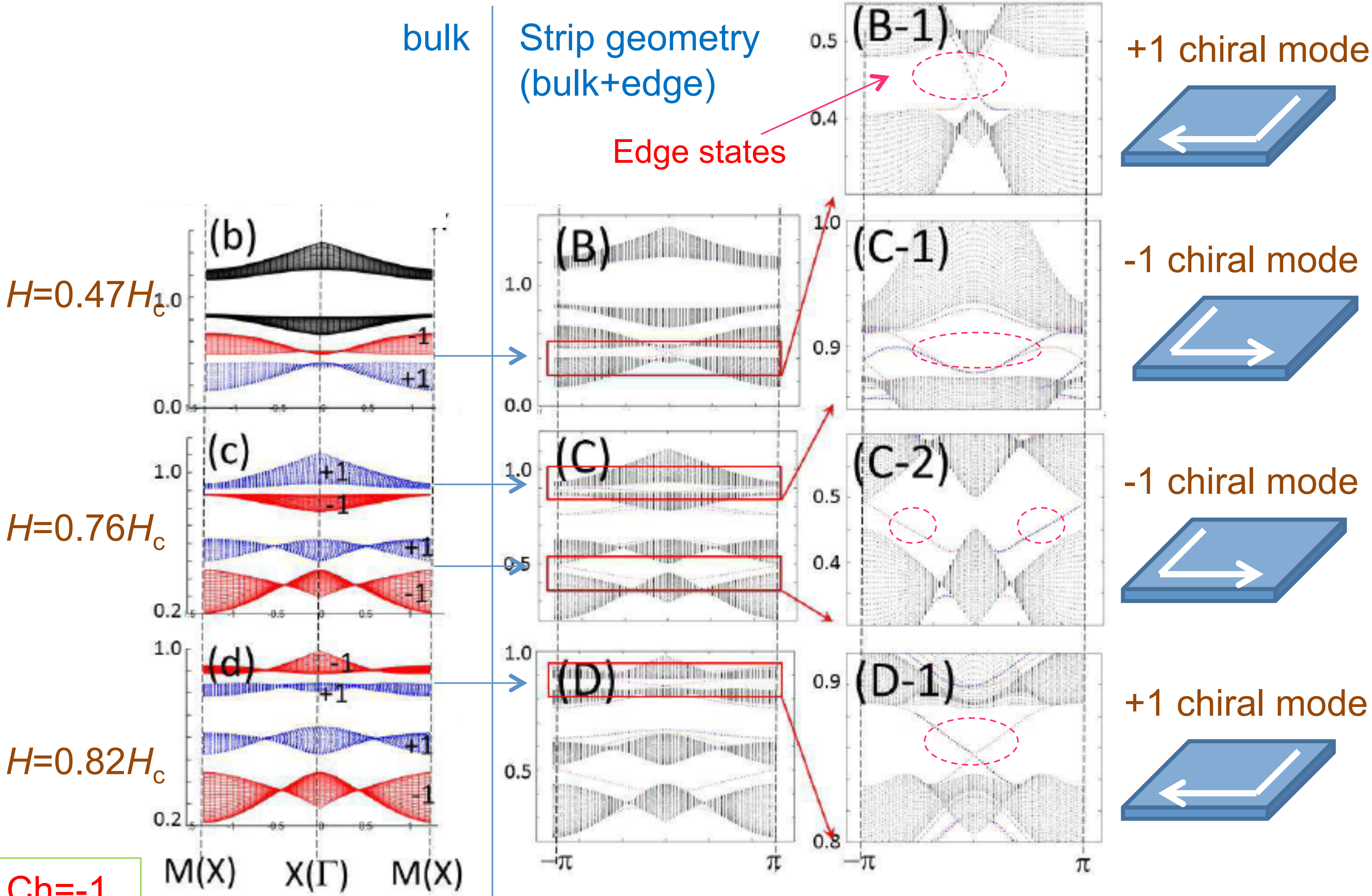
Time-reversal symmetry

- Small $H \ll H_c$
- Large $H \gg H_c$

Topologically trivial

Weak dipolar interaction compared with H

Magnonic crystals: edge states and Chern numbers (1)

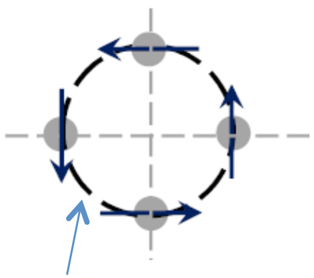


Red: Ch=-1
Blue: Ch=+1

“atomic orbitals”

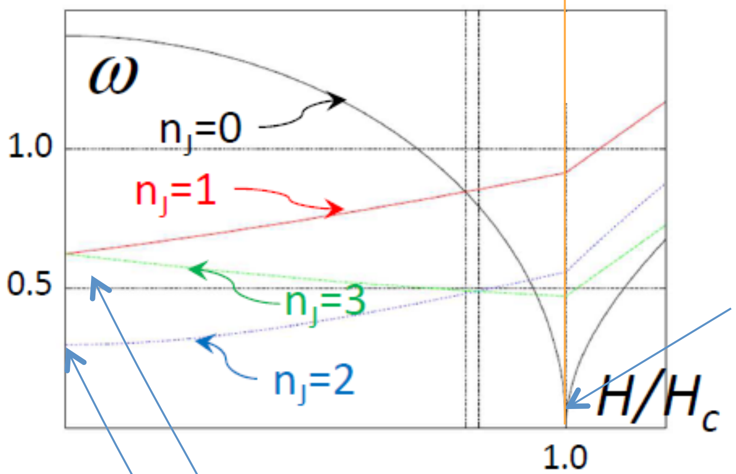
One cluster = atom

Equilibrium configuration



H<Hc: noncollinear → $H<H_c$: noncollinear

$H>H_c$: collinear // z



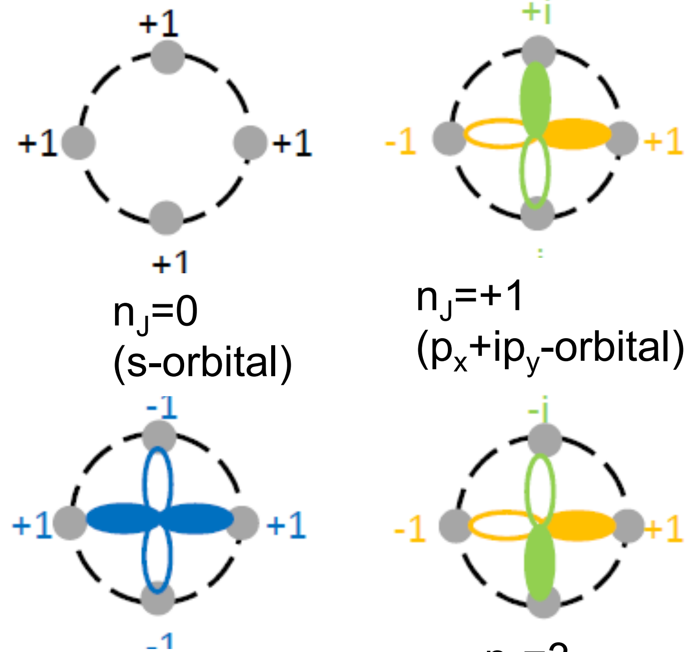
n_J=0 softens at $H=H_c$

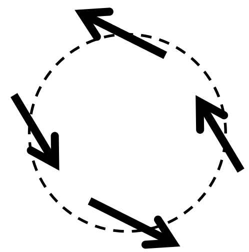
n_J=1 and n_J=3 degenerate at H=0

n_J=2 is lowest at H=0: favorable for dipolar int.

Spin wave excitations: “atomic orbitals”

relative phase for precessions





Magnonic crystals: tight-binding model with atomic orbitals

(example) *:*

H=0.47H_c → H=0.82H_c

gap between 3rd and 4th bands

⬇

retain only n_J=0 and n_J=1 orbitals

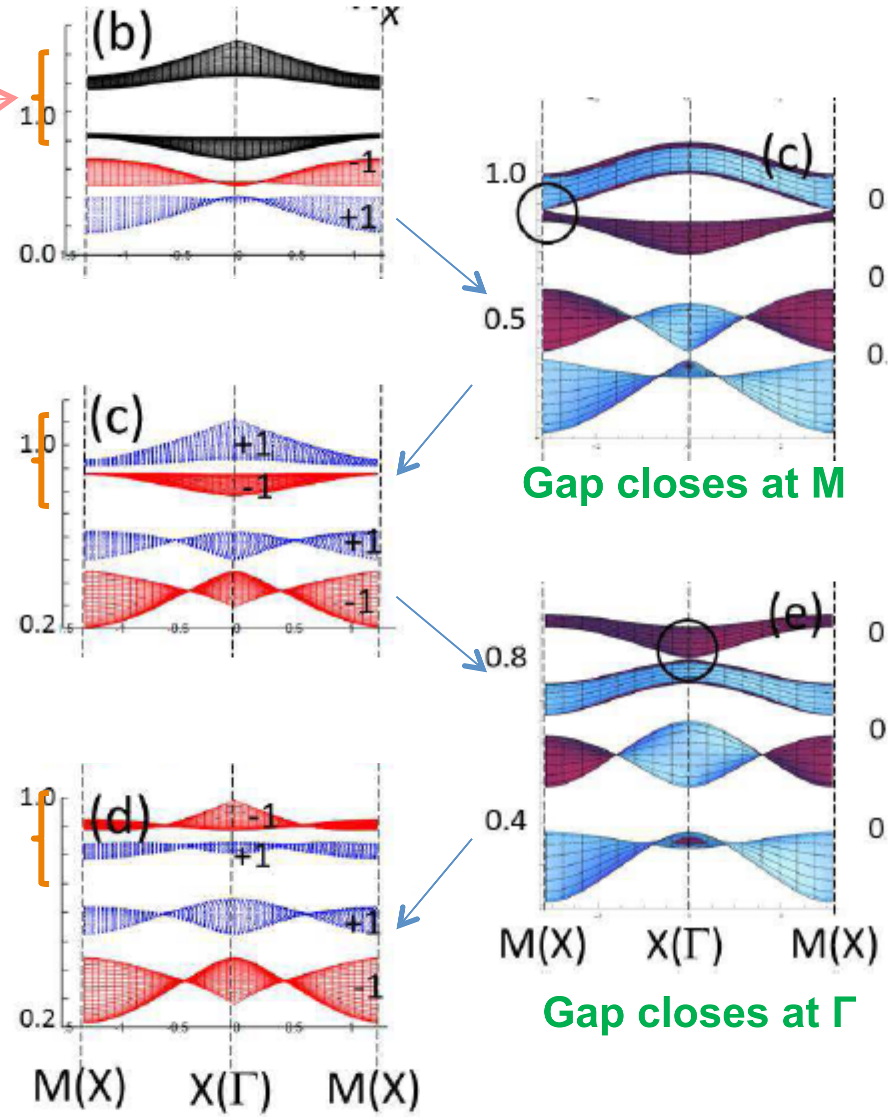
→ tight binding model

$$H_{01,\boldsymbol{k}} = \begin{pmatrix} \varepsilon_0 + 2a_{00}(c_{k_x} + c_{k_y}) & -2ib_{01}(s_{k_x} - is_{k_y}) \\ 2ib_{01}(s_{k_x} + is_{k_y}) & \varepsilon_1 + 2a_{11}(c_{k_x} + c_{k_y}) \end{pmatrix}$$

2´ 2 Hamiltonian

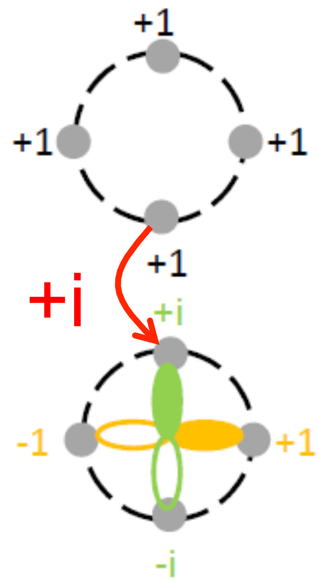
(parameters – dependent on H_{ext})

→Gap closing + topological transition



$$H_{01,\boldsymbol{k}} = \begin{pmatrix} \varepsilon_0 + 2a_{00}(c_{k_x} + c_{k_y}) & -2ib_{01}(s_{k_x} - is_{k_y}) \\ 2ib_{01}(s_{k_x} + is_{k_y}) & \varepsilon_1 + 2a_{11}(c_{k_x} + c_{k_y}) \end{pmatrix}$$

complex phase for hopping
← p_x+ip_y orbitals



$n_J=0$
(s-orbital)

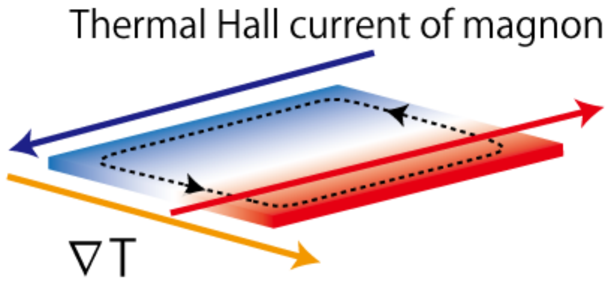
$n_J=+1$
(p_x+ip_y-orbital)

= Model for quantum anomalous Hall effect

$$H = \begin{pmatrix} \epsilon_k + M(k) & Ak_- \\ Ak_+ & \epsilon_k - M(k) \end{pmatrix} \quad \begin{matrix} |s,-\rangle \\ |p_x + ip_y,-\rangle \end{matrix}$$

$$k_\pm = k_x \pm ik_y,$$
$$\epsilon_k = C - D(k_x^2 + k_y^2),\ \mathcal{M}(k) = M - B(k_x^2 + k_y^2)$$

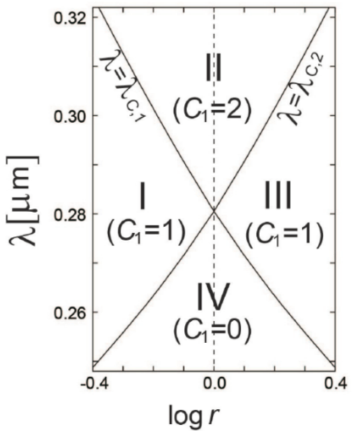
e.g. Bernevig et al., Science 314, 1757 (2006);



Summary

- Magnon thermal Hall effect (Righi-Leduc effect)

$$\kappa^{xy} = \frac{2k_B^2 T}{\hbar V} \sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}})) \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial k_x} \middle| \frac{\partial u_{n\boldsymbol{k}}}{\partial k_y} \right\rangle \quad c_2(\rho) = \int_0^\rho \left(\log \frac{1+t}{t} \right)^2 dt$$



- Topological chiral modes in magnonic crystals
  - ✓_magnonic crystal with dipolar int.
    → bosonic BdG → Berry curvature & Chern number

Thin film

phases with different Chern numbers by changing lattice constant

Array of disks

non-zero Chern numbers

atomic orbitals → tight-binding model → reproduce spin-wave bands

- Matsumoto, Murakami, Phys. Rev. Lett. 106,197202 (2011)
- Matsumoto, Murakami, Phys. Rev. B 84, 184406 (2011)
- Shindou, Matsumoto, Ohe, Murakami, Phys. Rev. B 87,174402 (2013),
- Shindou, Ohe, Matsumoto, Murakami, Saitoh, Phys. Rev. B 87,174427 (2013),