Topological insulators: interaction effects and new states of matter

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Emergent Quantum Phases in Condensed Matter
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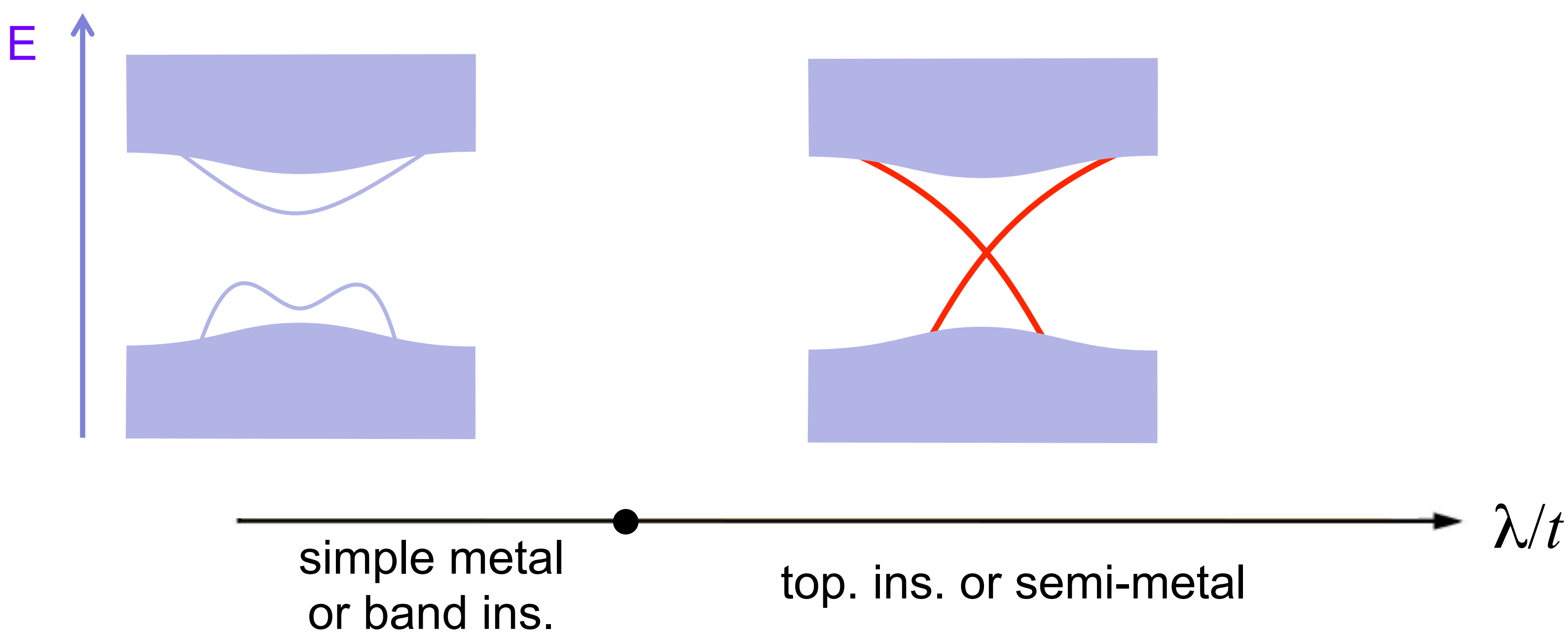
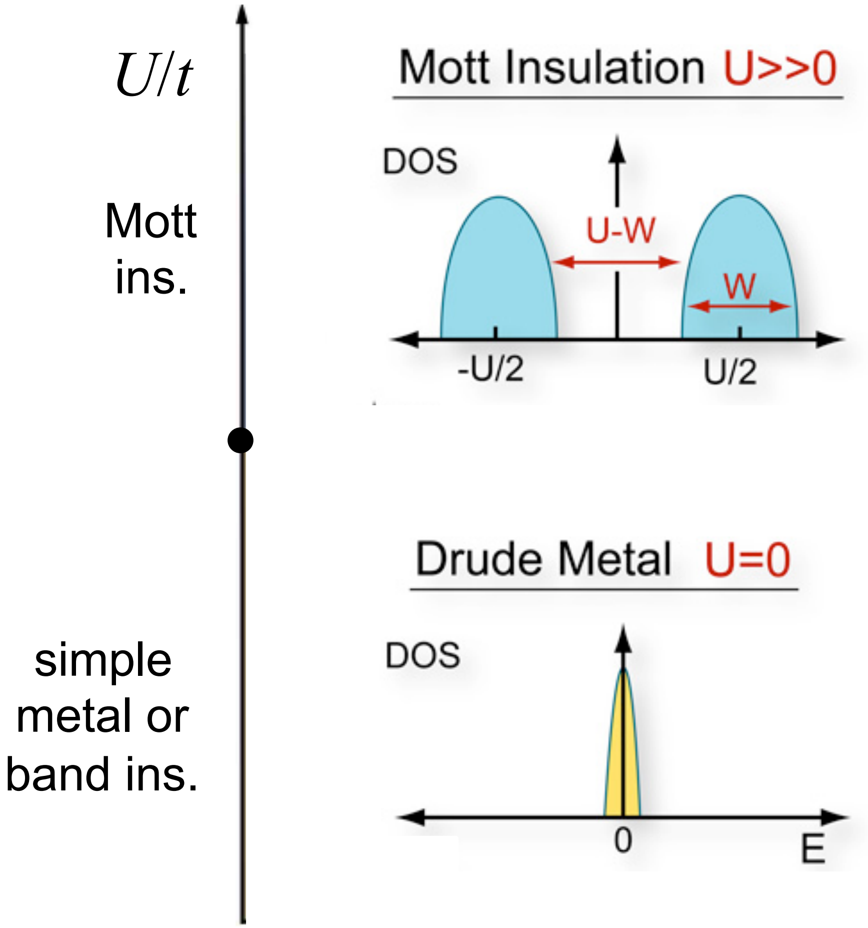
Band topology

$$H = \sum_{i,j;\alpha\beta} t_{ij,\alpha\beta}\, c^\dagger_{i\alpha} c_{j\beta} + \mathrm{h.c.} + \lambda \sum_i \boldsymbol{L}_i \cdot \boldsymbol{S}_i$$



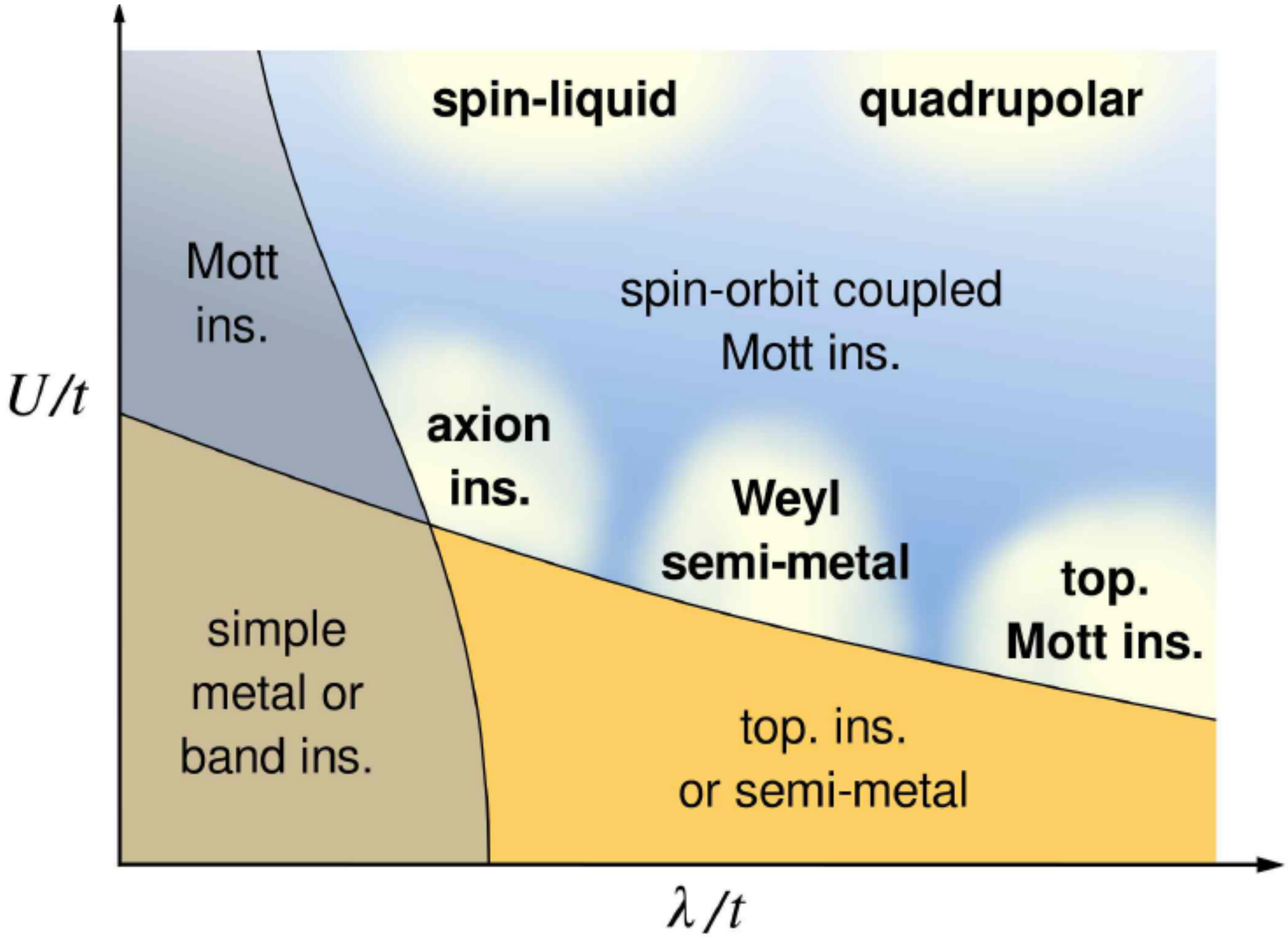
Electron correlations

$$H = \sum_{i,j;\alpha\beta} t_{ij,\alpha\beta}\; c^{\dagger}_{i\alpha} c_{j\beta} + \mathrm{h.c.} + U \sum_{i,\alpha} n_{i\alpha}(n_{i\alpha} - 1)$$



Band topology and correlations

$$H = \sum_{i,j;\alpha\beta} t_{ij,\alpha\beta}\, c^\dagger_{i\alpha} c_{j\beta} + \mathrm{h.c.} + U \sum_{i,\alpha} n_{i\alpha}(n_{i\alpha} - 1) \; + \lambda \sum_i \boldsymbol{L}_i \cdot \boldsymbol{S}_i$$



Witczak-Krempa, Chen, Kim, Balents, arXiv:1305.2193

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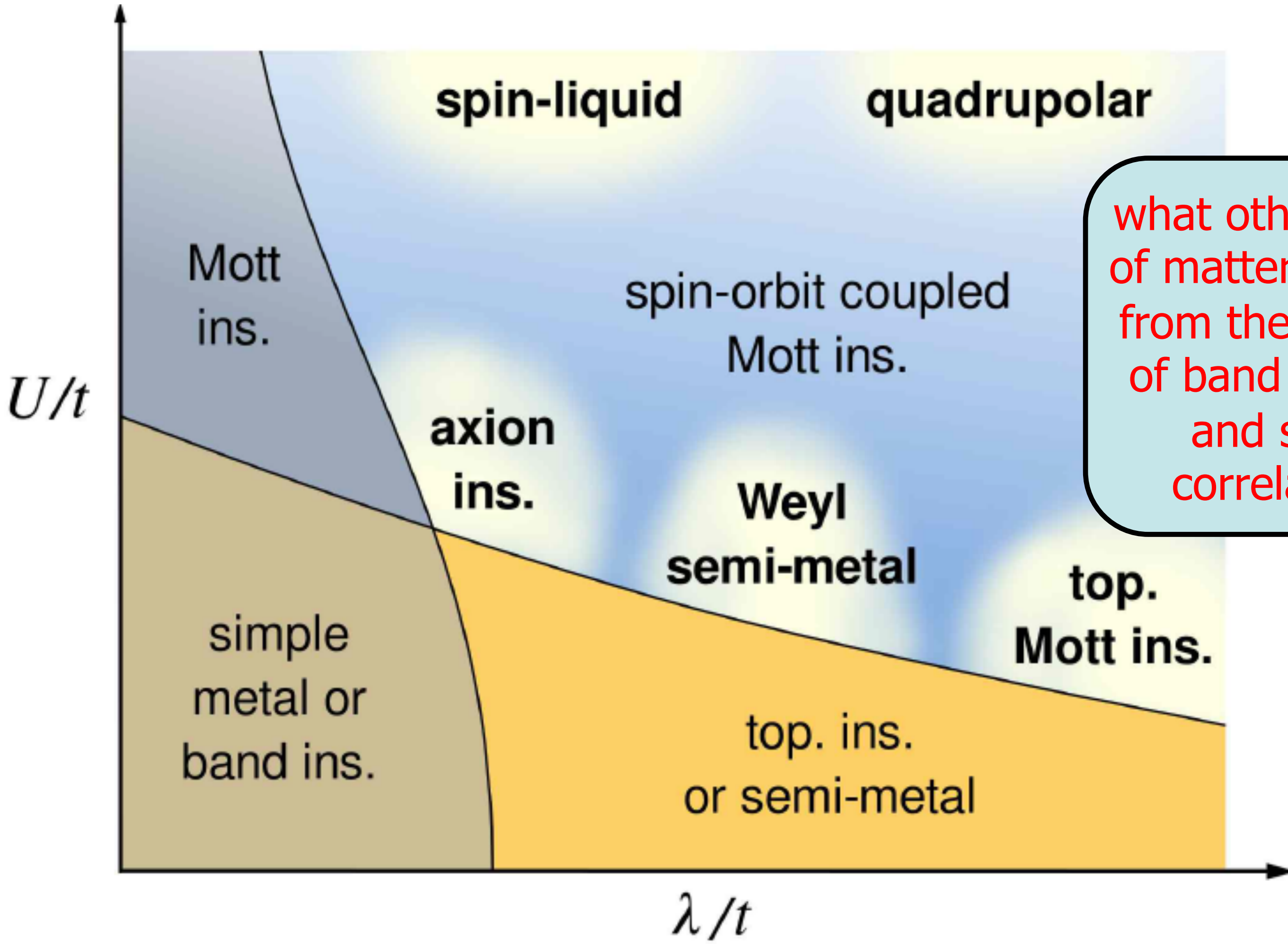


what other phases of matter can arise from the interplay of band topology and strong correlations?

Witczak-Krempa, Chen, Kim, Balents, arXiv:1305.2193

Outline

- **Introduction**
- **Correlation effects in the 2D Chern insulator**

  JM and A. Rüegg, arXiv:1305.1290

- **Correlation effects in the 3D topological insulator**

  JM, V. Chua, and G. A. Fiete, to appear

- **Conclusion**

2D Chern insulator

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

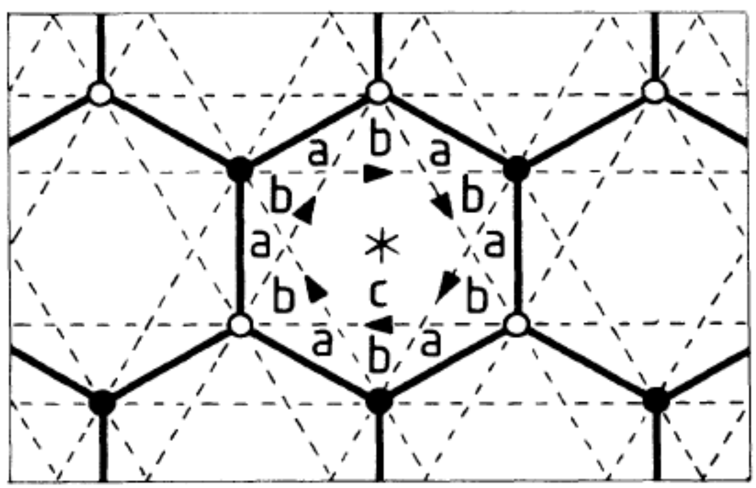
Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

Department of Physics, University of California, San Diego, La Jolla, California 92093

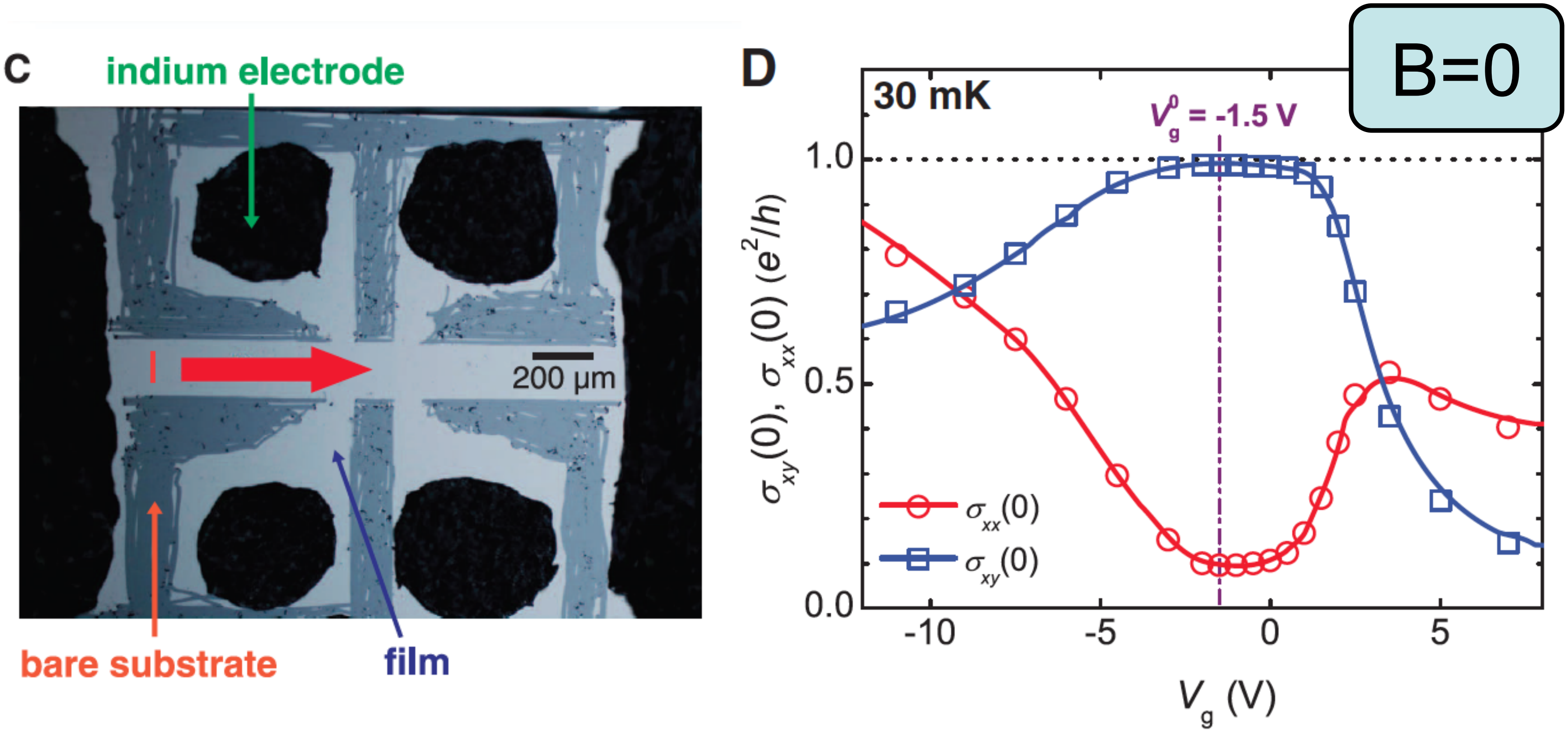
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.



2D Chern insulator

- 2D CI predicted in magnetically doped semiconductors Bi_2Te_3, Bi_2Se_3, and Sb_2Te_3 (Yu et al., Science 2010)
- Experimentally observed recently in Cr-doped thin films of $Bi_xSb_{1-x}Te_3$



Chang et al., Science, 12 April 2013

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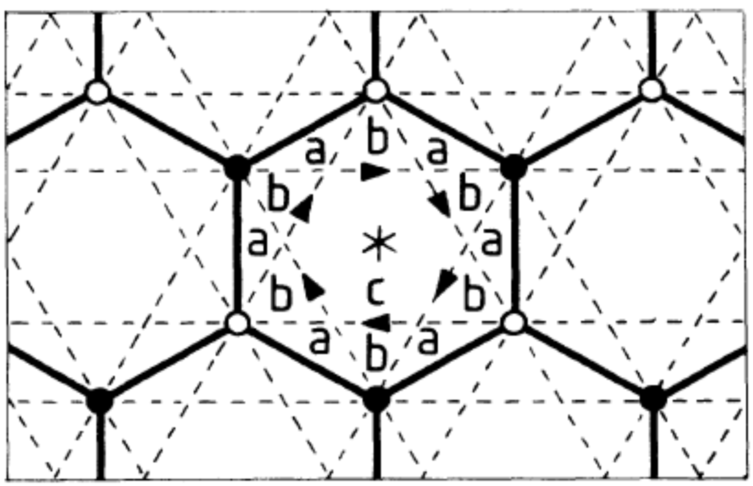
Interaction effects in the 2D CI

- Interactions in a spinless CI at fractional fillings ν=1/3, 1/5, … of the valence band can give a fractional CI = lattice FQH state (Tang, Mei, Wen, PRL 2011; Sun et al., PRL 2011; Neupert et al., PRL 2011; Sheng et al., Nature Commun. 2011; Regnault and Bernevig, PRX 2011)

- Here we focus on a spinful CI with <u>completely filled</u> valence band (C=2 state)



$$H = \sum_{rr'} \sum_{\alpha} t_{rr'} c^\dagger_{r\alpha} c_{r'\alpha} + U \sum_r \left(\sum_\alpha n_{r\alpha} - 1 \right)^2$$

$$\alpha = \uparrow, \downarrow$$

Interaction effects in the 2D CI

- U(1) slave-rotor mean-field theory (He et al., PRB 2011, PRB 2012) and small-cluster ED on the equivalent spin model at U→∞ (Nielsen, Sierra, Cirac, arXiv 2013) predict that the ground state at large U is the **Kalmeyer-Laughlin chiral spin liquid state** (bosonic ν=1/2 Laughlin state)

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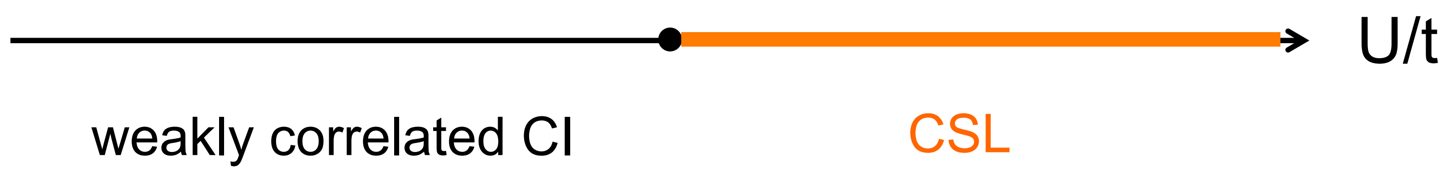

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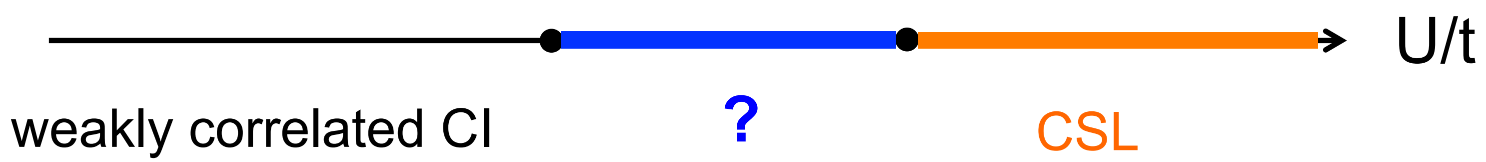


- At large U, charge degrees of freedom are frozen
- Can there be interesting novel phases at intermediate U?

Z_2 slave-spin theory

- The Haldane-Hubbard model can be mapped to a (2+1)D Z_2 gauge theory with bosonic and fermionic matter using the Z_2 slave-spin representation (Huber and Rüegg, PRL 2009; Rüegg, Huber, Sigrist, PRB 2010; Nandkishore, Metlitski, Senthil, PRB 2012)

τ_r^z = +1 -1 -1 +1

$$c^{(\dagger)}_{r\alpha} = f^{(\dagger)}_{r\alpha}\tau^x_r$$

$$H = \sum_{rr'}\sum_{\alpha} t_{rr'} c^\dagger_{r\alpha} c_{r'\alpha} + U\sum_r \left(\sum_\alpha n_{r\alpha} - 1\right)^2$$

$$\Downarrow$$

$$H = \sum_{rr'}\sum_{\alpha} t_{rr'} \tau^x_r \tau^x_{r'} f^\dagger_{r\alpha} f_{r'\alpha} + \frac{U}{2}\sum_r (\tau^z_r + 1)$$

Effective Z_2 gauge theory

- Projection to physical Hilbert space introduces a Z_2 gauge field $\sigma_{ij}=\pm 1$ (Senthil and Fisher, PRB 2000)

$$Z = \int D\bar{f}_{i\alpha} D f_{i\alpha} \sum_{\{\tau_i^x\}} \sum_{\{\sigma_{ij}\}} e^{-S[\bar{f},f,\tau^x,\sigma]}$$

$$S_{\tau^x} = -\kappa \sum_{ij} \tau_i^x \sigma_{ij} \tau_j^x,$$

$$S_f = -\sum_{ij}\sum_{\alpha} t_{ij} \bar{f}_{i\alpha} \sigma_{ij} f_{j\alpha},$$

$$e^{-S_B} = \prod_{i,j=i-\hat{\tau}} \sigma_{ij}$$

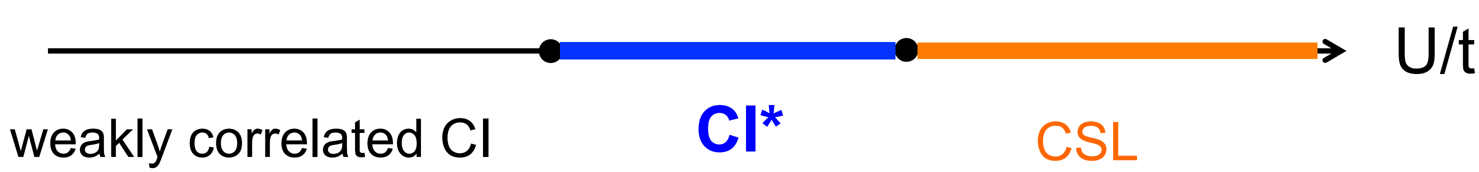
$$\kappa = \frac{1}{2} \ln \coth\left(\frac{\epsilon U}{2}\right)$$

Effective Z_2 gauge theory

$$\kappa = \frac{1}{2} \ln \coth\left(\frac{\epsilon U}{2}\right)$$

- Small U: large κ, slave-spins τ^x condense → weakly correlated CI
- Infinite U: κ=0, slave-spins can be trivially integrated out, effective Hamiltonian has 4-slave-fermion terms $f^\dagger f^\dagger f f$ with the constraint of one slave-fermion per site → effective spin model → possible CSL

- Large but finite U: integrating out slave-spins gives "kinetic" term for Z_2 gauge field
- Focus on **deconfined phase** of the resulting Z_2 gauge theory (Wegner, 1971) which we call CI*



From Z_2 to U(1)

- Z_2 gauge theory can be written as U(1) gauge theory coupled to conserved charge-2 "Higgs" link variable (Ukawa, Windey, Guth, PRD 1980)

p=2

$$\prod_{\langle ij\rangle}\sum_{\sigma_{ij}=\pm1} e^{-S[\sigma_{ij}]} = \prod_{\langle ij\rangle}\int_{-\pi}^{\pi} da_{ij} \sum_{n_{ij}=-\infty}^{\infty} e^{ip\sum_{\langle ij\rangle} n_{ij}a_{ij}} \exp\left(-S[\sigma_{ij}=e^{ia_{ij}}]\right)$$

$\Delta_\mu n_{i,i+\mu} = 0$

- Deconfined phase: U(1) gauge field is weakly coupled and we can take the continuum limit

$$\partial_\mu n_\mu = 0 \;\Rightarrow\; n_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu b_\lambda$$

$$S_n = -ip\sum_{ij} n_{ij}a_{ij} \;\Rightarrow\; \frac{p}{2\pi}\int d^3x\, \epsilon^{\mu\nu\lambda} b_\mu \partial_\nu a_\lambda$$

level-p (2+1)D BF term

TQFT of CI* phase

- CI* phase is a chiral topological phase, with TQFT of the BF + Chern-Simons type

BF term describes Z_2 topological order of Z_2 gauge sector (Hansson, Oganesyan, Sondhi, Ann. Phys. 2004)

$$\mathcal{L}_{\mathrm{CI}^*} = \frac{1}{\pi}\epsilon^{\mu\nu\lambda} b_\mu \partial_\nu a_\lambda + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu (a_\lambda^{\uparrow} + a_\lambda^{\downarrow})$$
$$+ \sum_{\sigma=\uparrow,\downarrow}\left(\frac{1}{4\pi}\epsilon^{\mu\nu\lambda} a_\mu^\sigma \partial_\nu a_\lambda^\sigma + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda} a_\mu^\sigma \partial_\nu a_\lambda\right)$$

CS term obtained by integrating out gapped slave-fermions, encodes Chern number C=2 of electron bandstructure

Properties of CI* phase

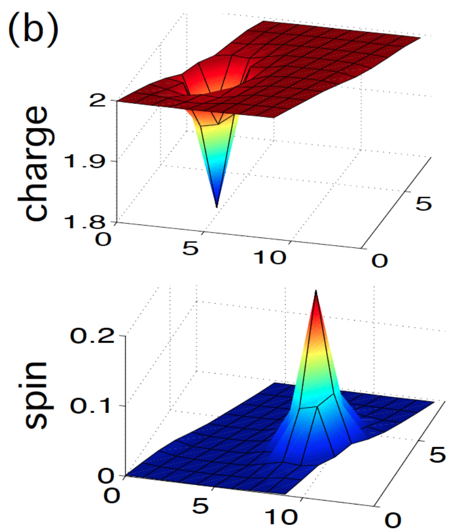
- Hall conductance and quasiparticle charge are **integer**, but statistics are **fractional** (semionic) and ground state on T^2 is 4-fold degenerate

$$K = \begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

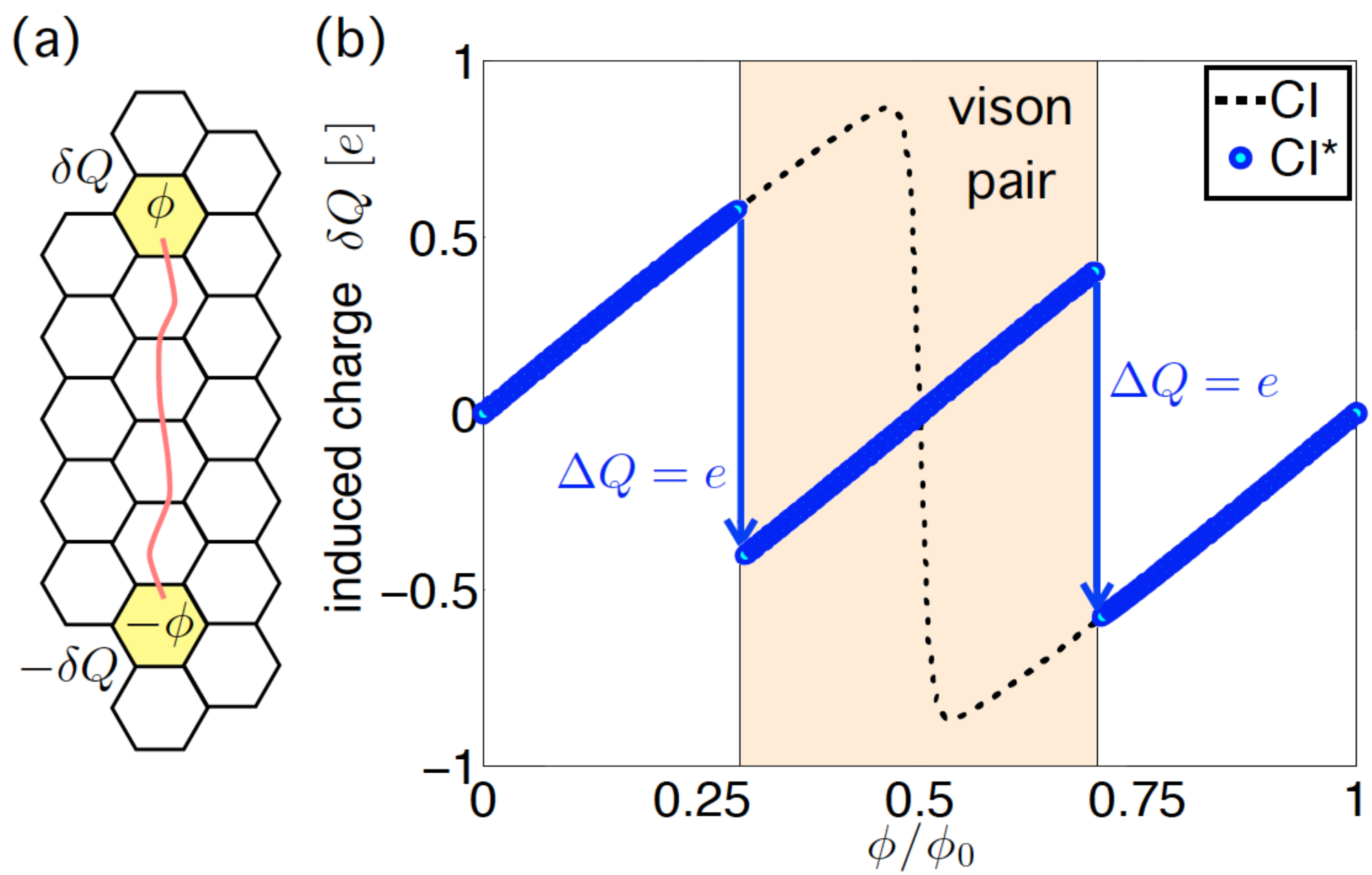
$$\sigma_{xy} = \frac{e^2}{h} t^T K^{-1} t = \frac{2e^2}{h}$$

$$|\det K| = 4$$

quasiparticle	Q	S_z	θ_{ll}	$\theta_{ll'}$
$l_1 = (1,0,0,0)$	$-e$	0	$\pi/2$	$\theta_{13} = \pi$
$l_2 = (0,-1,0,0)$	e	0	$-\pi/2$	$\theta_{23} = -\pi$
$l_3 = (1,-1,0,0)$	0	0	0	$\theta_{31} = -\theta_{32} = \pi$
$l_4 = (0,0,1,0)$	e	$1/2$	π	
$l_5 = (0,0,0,1)$	e	$-1/2$	π	



Linear vs nonlinear response



- In CI* phase, spontaneously created Z_2 vortices can screen external fluxes and modify Hall response in nonlinear regime

Quasiparticle excitations of CI* phase

- Excitations of Z_2 gauge field are Z_2 vortices (visons) = π fluxes
- A π flux in a $\nu = 1$ QH state traps a fermionic mode with charge e/2, and the resulting bound state is an anyon with $\theta = \pi/4$
(Weeks et al., Nature Phys. 2007)
- A π flux in a spinful (C = 2) CI state traps a fermionic mode with charge e: bound state is a semion ($\theta = \pi/2$)

Band topology + interactions → novel states of matter

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Conclusion

- The interplay of band topology and electron correlations can lead to qualitatively new quantum phenomena
- Hubbard interaction in a spinful (C=2) Chern insulator can lead to a “hybrid” topological phase with integer Hall conductance and quasiparticle charge but nontrivial GSD and fractional statistics
- Hubbard interaction in a 3D topological insulator can likewise lead to a gapped fractionalized phase, the TI*, and possibly also a condensed matter realization of oblique confinement
- The search for exotic phases in 4d and 5d transition metal compounds is a promising direction for the experimental realization of novel topological quantum phenomena