Protection of the surface states of a topological insulator: Berry phase perspective

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The surface states of a topological insulator has a dual personality: extrovert and introvert

The topological insulator has a *dual* personality.

bulk → insulating

surface → metallic

topologically protected

Its metallic (gapless) surface state also has a *dual* personality.

introvert: staying on the surface; not penetrating into the bulk

extrovert: inducing a fictitious magnetic field on *curved* surfaces; solenoid or magnetic monopole type ↔ (spin) Berry phase

What are the “defining properties” of the topological insulator?

- Existence of a *protected gapless* surface state

cf. (a pair of) Dirac cones in graphene

- The *bulk-surface correspondence*

Bulk: Existence of a non-trivial topological invariant characterizing the *bulk* state

one-to-one correspondence

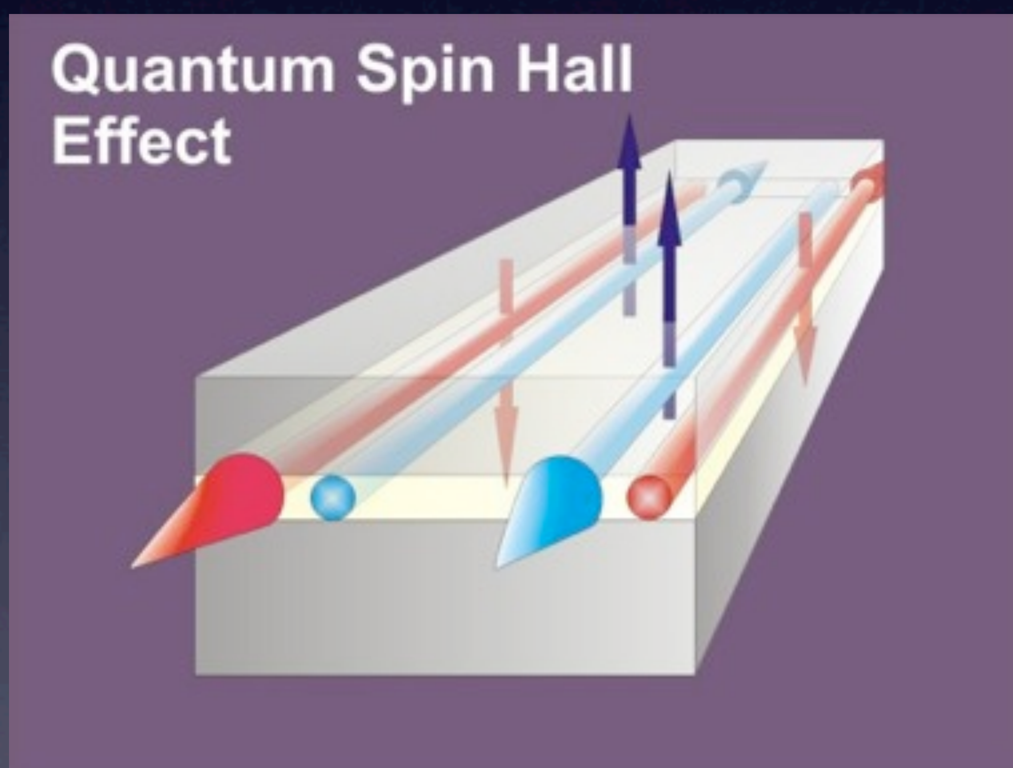
Surface: Emergence of the *protected gapless* state on its *surface*

Introduction to

topological insulators

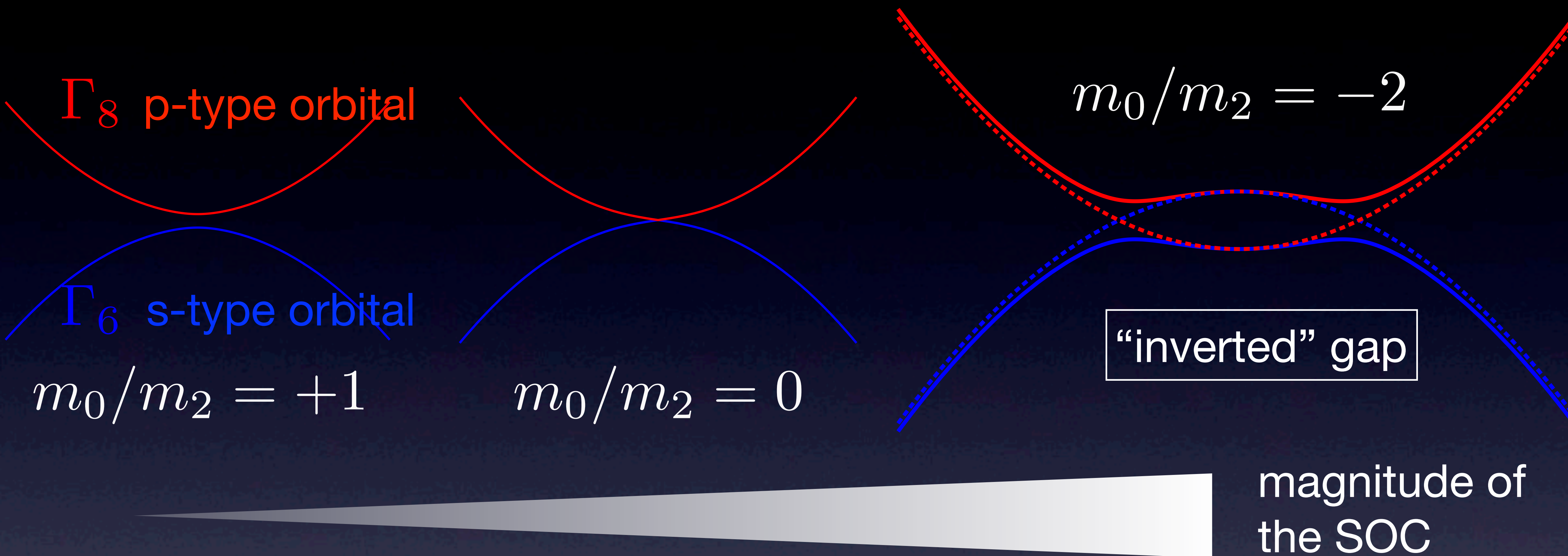
A chronological viewpoint:

spin Hall effect → topological insulator



spatial dimension: **2D** → **3D**
(of the bulk)

Band “inversion” due to spin-orbit coupling



can be modeled by a Wilson-Dirac type effective (bulk) Hamiltonian

gap → mass

$$m \to m(\boldsymbol{p}) = \underbrace{m_0}_{\text{Einstein}} + \underbrace{m_2 \boldsymbol{p}^2}_{\text{Newtonian}}$$

The two bands get inverted by changing the *sign* of a mass term.

Bernevig et al., Science ’07

2D example: how to characterize the bulk

$$H = p_x\sigma_x + p_y\sigma_y + m(\boldsymbol{p})\sigma_z$$
$$= P_\mu(\boldsymbol{p})\sigma_\mu$$

$$m(\boldsymbol{p}) = m_0 + m_2\boldsymbol{p}^2$$

$$n_\mu(\boldsymbol{p}) = \frac{P_\mu(\boldsymbol{p})}{\sqrt{P_\mu P_\mu}}$$

- The winding number (Chern number)

$$N_2 = -\frac{1}{8\pi}\int d^2p\ \epsilon_{\mu\nu}\ \boldsymbol{n}\cdot[\partial_{p_\mu}\boldsymbol{n}\times\partial_{p_\nu}n],$$

mapping: $\boldsymbol{p}\to n_\mu(\boldsymbol{p})$ $\quad \mathbb{R}^2\to\mathbb{S}^2$

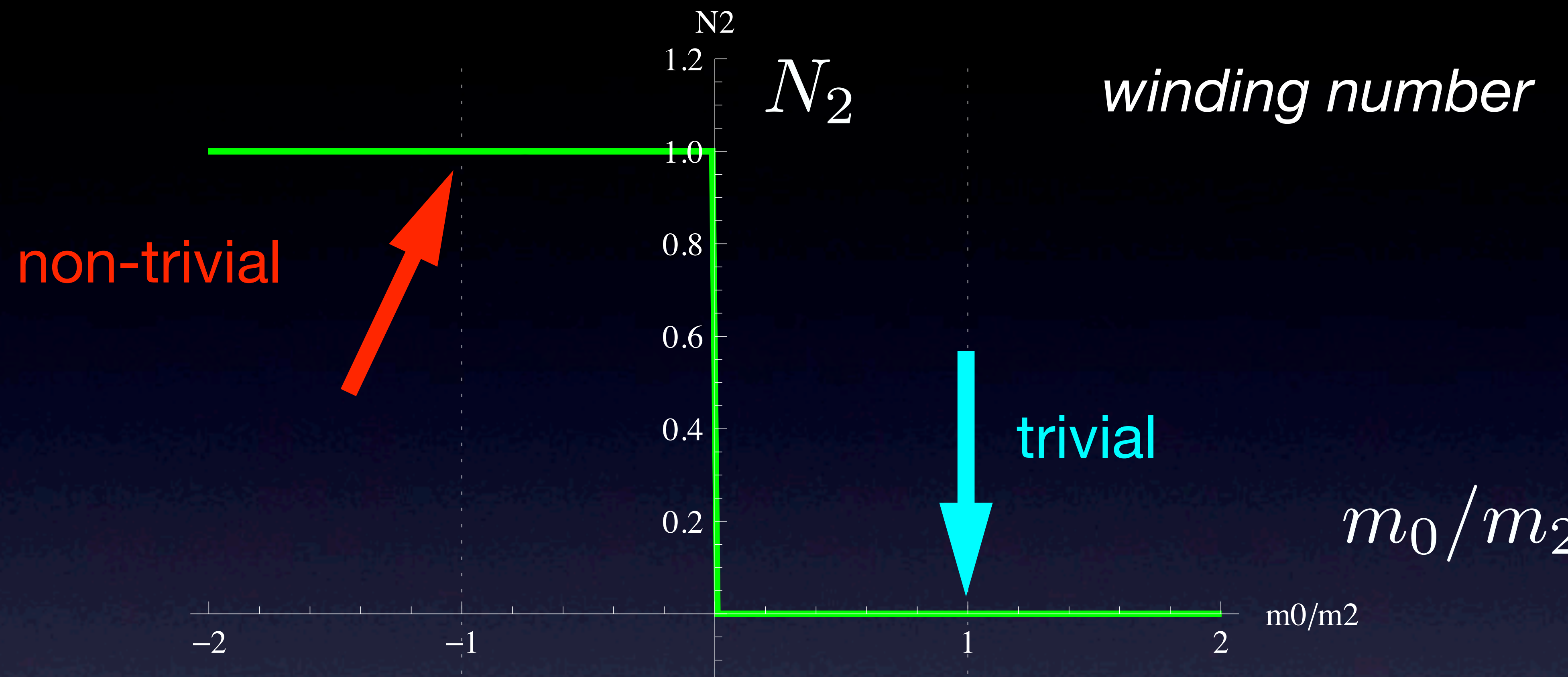
$$p = |\boldsymbol{p}|\to\infty \qquad\qquad p = 0$$
$$\boldsymbol{n}(\boldsymbol{p})\to(0,0,\mathrm{sgn}(m_2)) \qquad \boldsymbol{n}(\boldsymbol{p})\to(0,0,\mathrm{sgn}(m_0))$$

stereographic projection ➜ $\mathbb{S}^2\to\mathbb{S}^2$ $\quad \pi_2(\mathbb{S}^2) = 0,\pm1,\pm2,\cdots$

$$N_2 = \frac{\mathrm{sgn}(m_2)-\mathrm{sgn}(m_0)}{2}$$

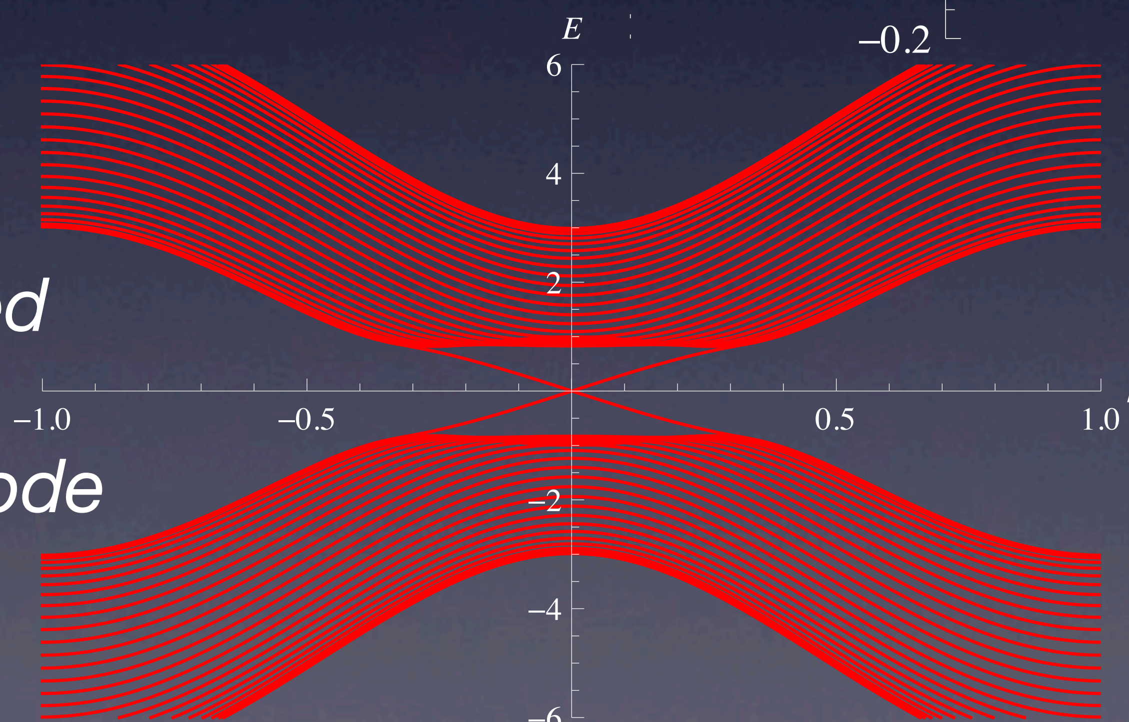
The bulk-edge correspondence

bulk

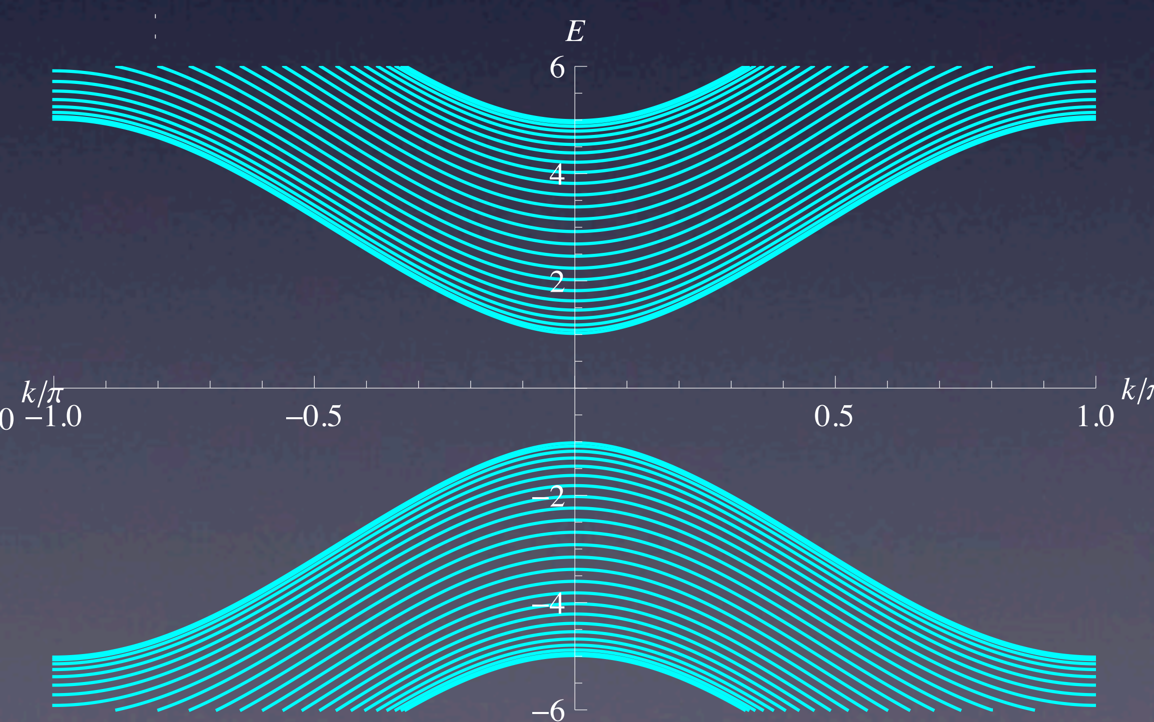


edge

protected gapless edge mode



$$m_0/m_2 = -1$$



$$m_0/m_2 = 1$$

The Wilson-Dirac type (bulk) effective Hamiltonian

Zhang et al., Nature Phys. '09; Liu et al., PRB '10

Dirac equation in 3+1 D $\quad H\psi = E\psi$

$$H = m\gamma_0 + Ap_\mu\gamma_\mu \qquad 4 \times 4 \text{ matrix}$$

$$= m\tau_x + A\tau_y(p_x\sigma_x + p_y\sigma_y + p_z\sigma_z)$$

Time-reversal symmetry preserved

$\Downarrow$

$\mathbb{Z}_2$ topological insulators

- Two types of Pauli matrices: $\begin{cases} \text{spin:} & \sigma_x, \sigma_y, \sigma_z \\ \text{orbital:} & \tau_x, \tau_y, \tau_z \end{cases}$

- gamma matrices: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$

$$\longrightarrow \quad E = E(\boldsymbol{p}) = \pm\sqrt{m^2 + A^2\boldsymbol{p}^2}$$

- Wilson term: a "spice" for the mass term

$$m \to m(\boldsymbol{p}) = m_0 + m_2\boldsymbol{p}^2$$
$$= m_0 + m_2(p_x^2 + p_y^2 + p_z^2)$$

3D generalization

$$\gamma_0 = \tau_x$$
$$\gamma_1 = \tau_y \sigma_x, \quad \gamma_2 = \tau_y \sigma_y, \quad \gamma_3 = \tau_y \sigma_z$$

$$H = P_0 \gamma_0 + P_\mu \gamma_\mu, \qquad P_0 = m(\boldsymbol{k}) = m_0 + 2m_2 \sum_\mu (1 - \cos k_\mu)$$

4×4 matrix

$$P_\mu = P_\mu(\boldsymbol{k}) = \sin k_\mu$$

$$\left(\tilde{H} = \frac{H}{P(\boldsymbol{k})} = \begin{bmatrix} & Q(\boldsymbol{k}) \\ Q^\dagger(\boldsymbol{k}) & \end{bmatrix}, \quad P(\boldsymbol{k}) = \sqrt{P_0^2 + P_\mu P_\mu} \right.$$

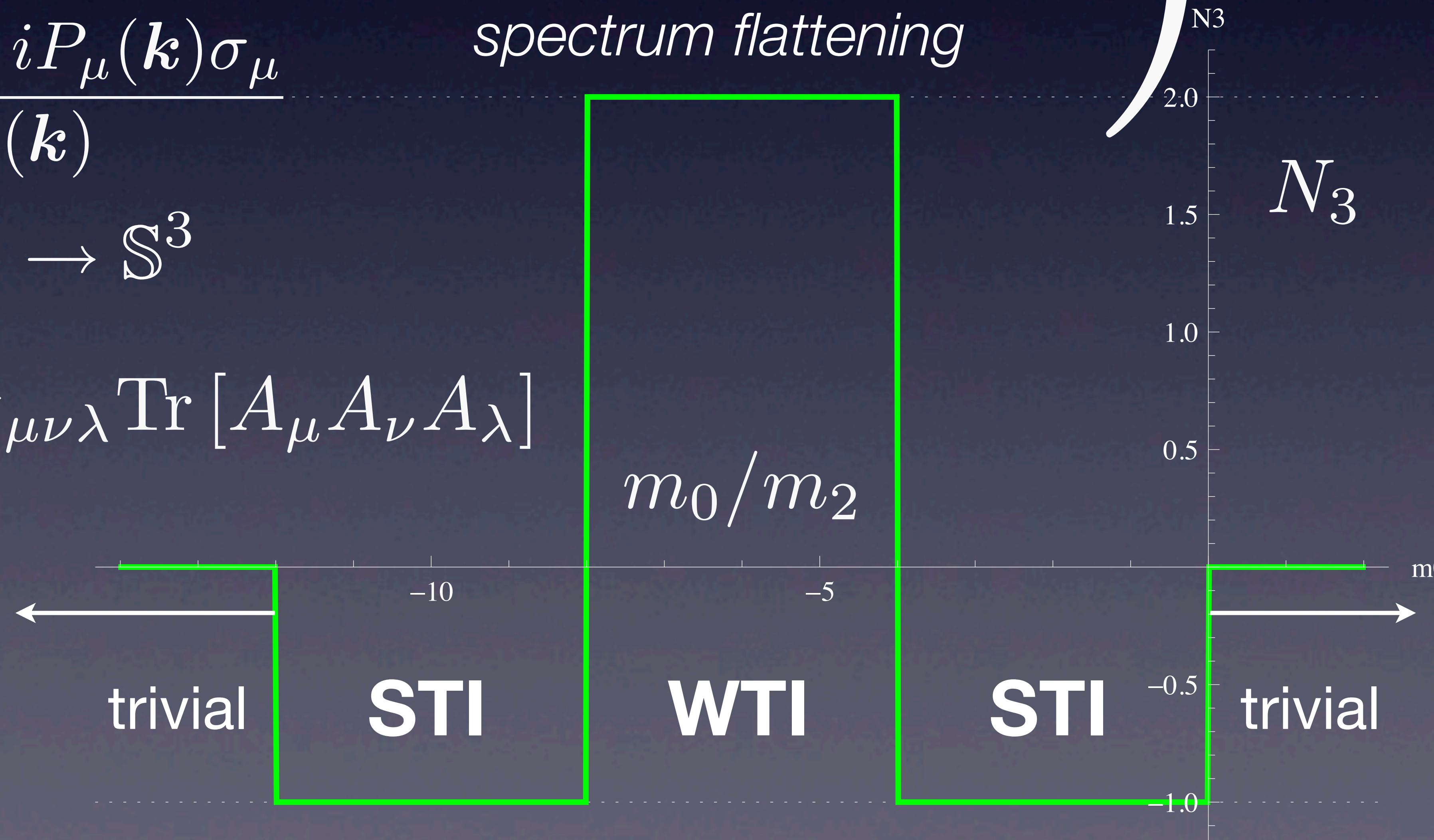
$$\left. Q(\boldsymbol{k}) = \frac{P_0(\boldsymbol{k}) - i P_\mu(\boldsymbol{k}) \sigma_\mu}{P(\boldsymbol{k})} \right)$$

spectrum flattening

winding number $\mathbb{S}^3 \to \mathbb{S}^3$

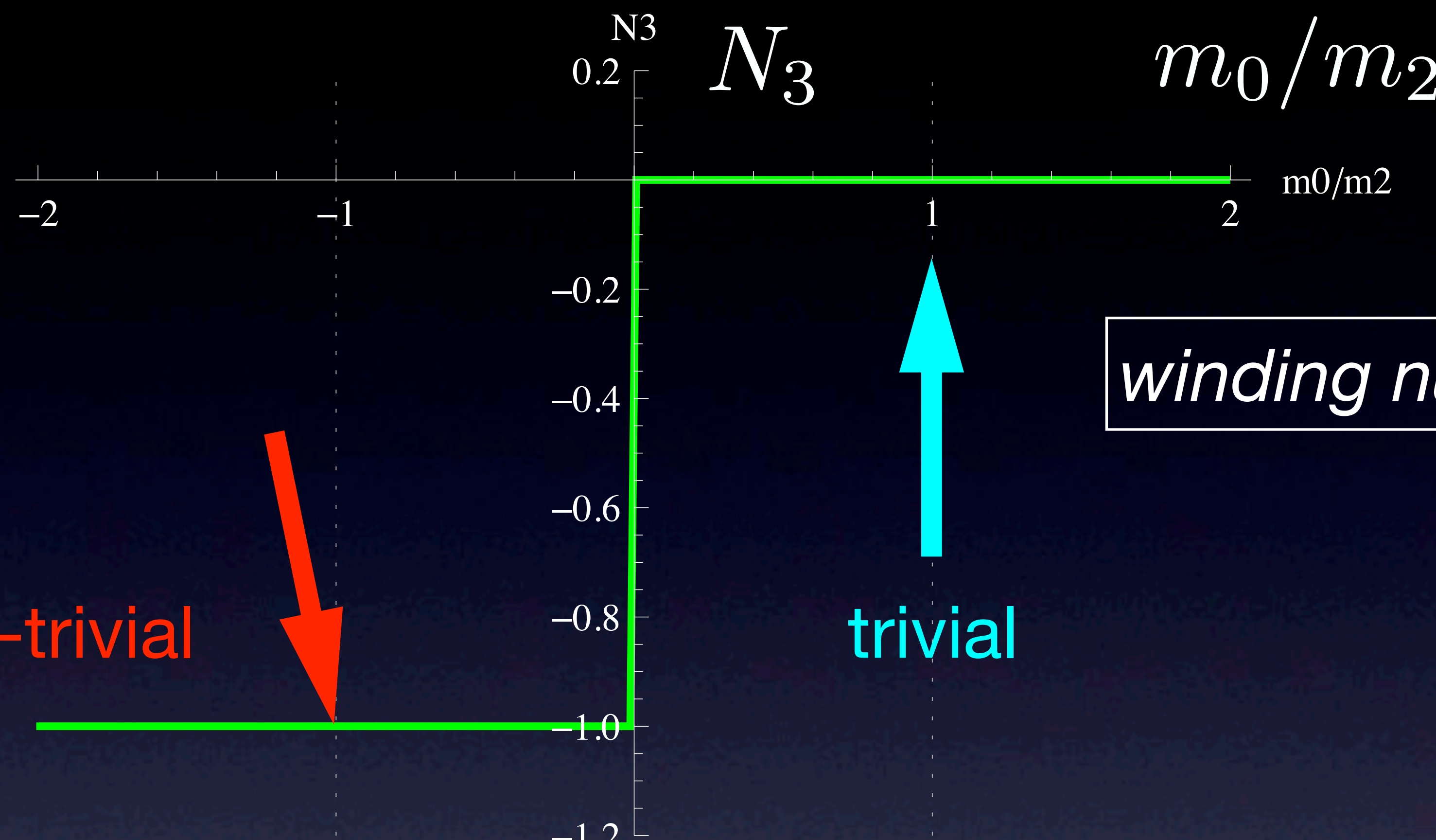
$$N_3 = \frac{1}{24\pi^2} \int d^3p \, \epsilon_{\mu\nu\lambda} \mathrm{Tr}\left[A_\mu A_\nu A_\lambda\right]$$

$$A_\mu = Q^\dagger \partial_{k_\mu} Q$$



The bulk-surface correspondence: 3D version

bulk



winding number

surface

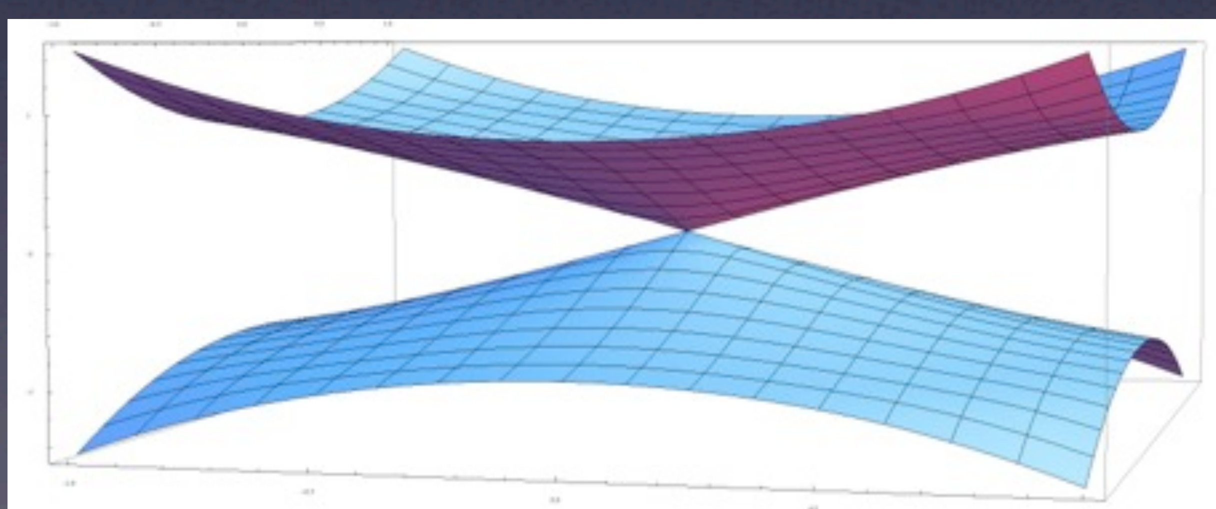
spectrum: gapless

a single helical Dirac cone

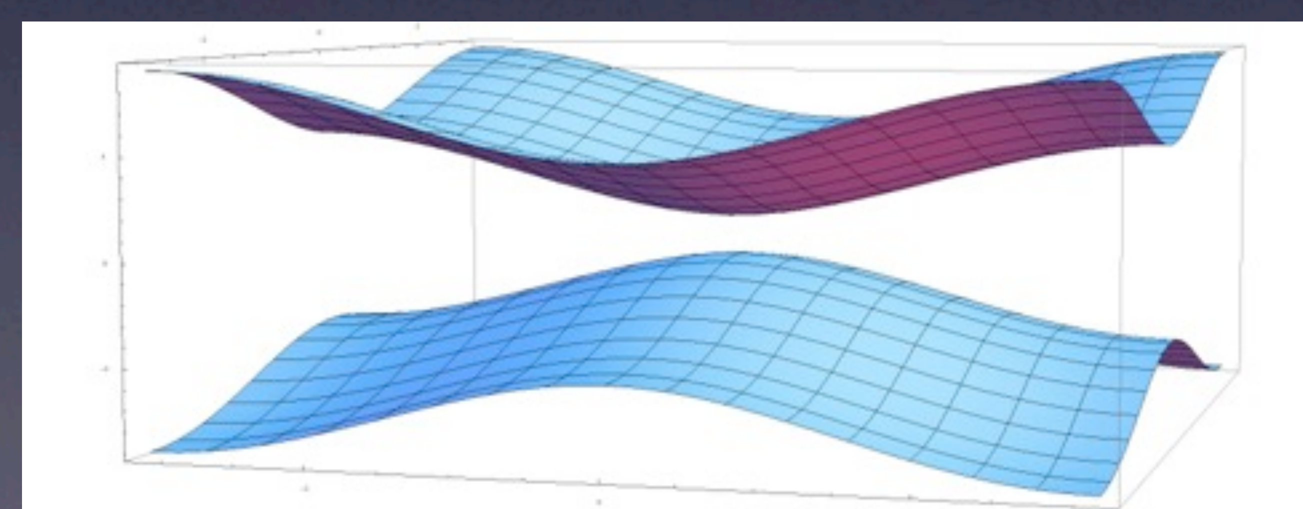
spectrum: gapped

no surface state

slab geometry



$$m_0/m_2 = -1$$



$$m_0/m_2 = 1$$

Bulk
energy gap
(band strcture)

Surface
state

$m_0 m_2 > 0$

normal gap

None

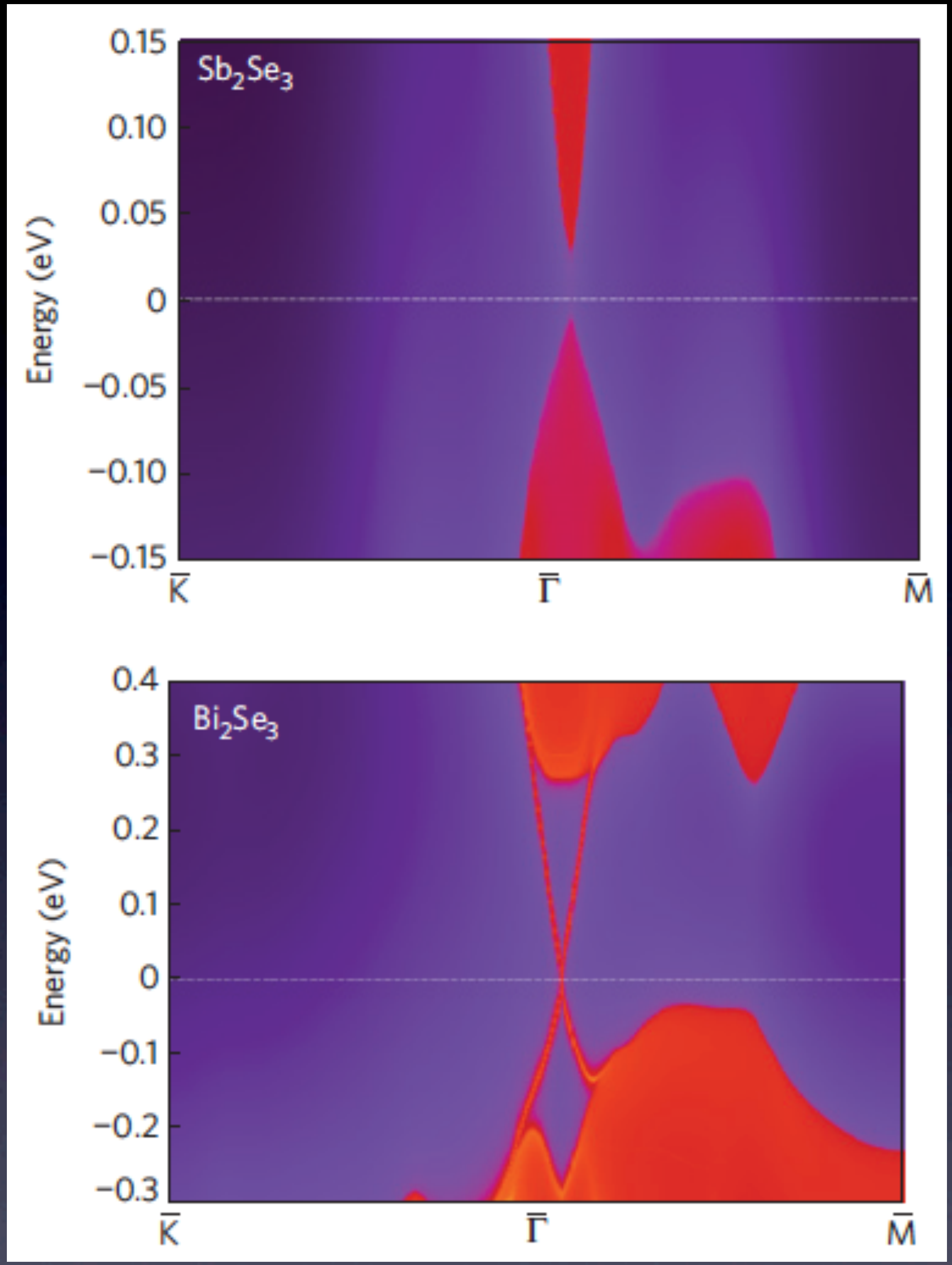
$m_0 m_2 < 0$

inverted gap

Yes

gapless, protected

Bulk-surface correspondence



Zhang et al., Nature Phys. '09

Act I: The extrovert

Inducing a fictitious magnetic filed on *curved* surfaces

→ Finite-size energy gap

- sensitivity to the *geometry* of the sample

spin connection

The topological protection does not help.

<u>cylindrical</u> sample (TI nanowire) → *imaginary* <u>solenoid</u>

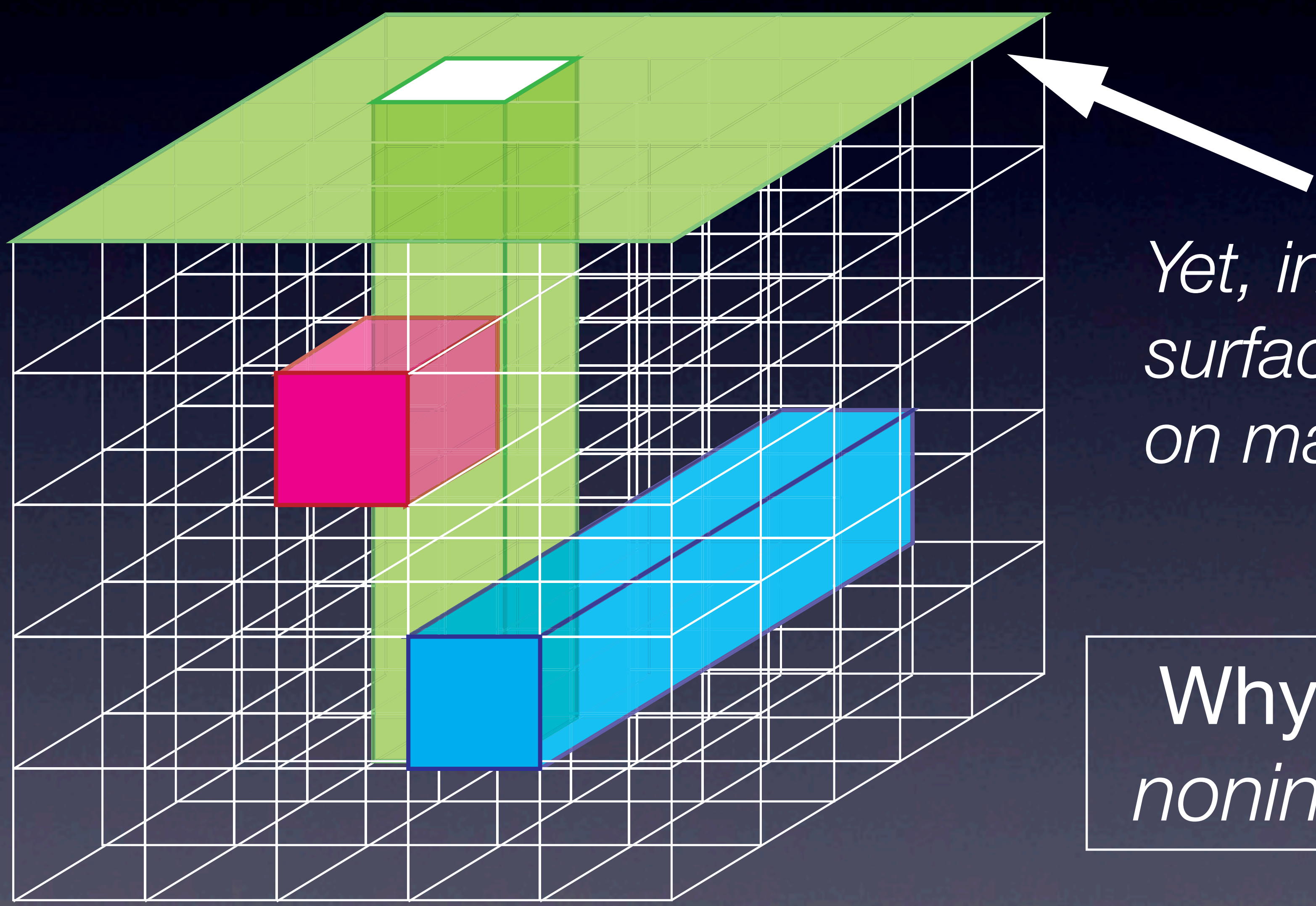
<u>spherical</u> sample (TI nanoparticle) → *effective* <u>monopole</u>

Act II: The *noninvasive* metallic state

The surface stays *introvert* as a consequence of its *extroverted* character.

Noninvasiveness protected by the Berry phase

- Consider a lattice (tight-binding) realization of topological insulator
- A lattice model is “sparse”, existing only on sites and links



Yet, in reality protected surface states appear only on macroscopic surfaces.

Why is the surface state *noninvasive* into the bulk?

KI & Takane, arXiv:1211.2088

<u>Which is the genuine surface?</u>

Protection of the surface states in topological insulators: Berry phase perspective

Act I: Extrovert

The property of inducing a fictitious magnetic field on *curved* surfaces

The *cylindrical* topological insulator

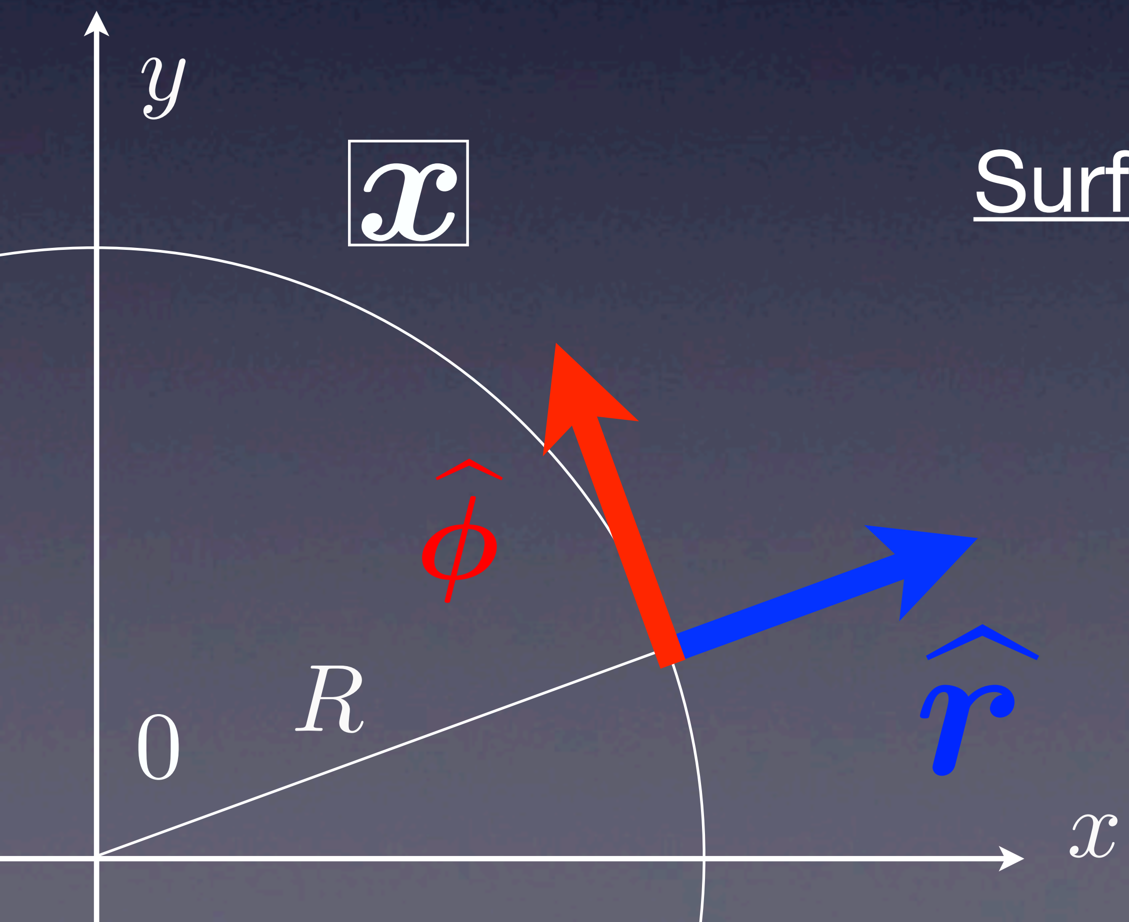
<u>Effective Dirac Hamiltonian on the cylindrical surface:</u>

spin Berry phase

$$H_{\rm surf} = \begin{bmatrix} 0 & -ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) \\ ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) & 0 \end{bmatrix}$$

KI, Takane & Tanaka, Phys. Rev. B ***84****, 195406 (2011)*

bulk effective Hamiltonian: $H_{\rm bulk} = m(\boldsymbol{p})\tau_z + \tau_x(p_x\sigma_x + p_y\sigma_y + p_z\sigma_z)$



<u>Surface eigenstates:</u>

$$H_{\rm surf}\boldsymbol{\alpha} = E\boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_+ \\ \alpha_- \end{bmatrix}$$

$$|\boldsymbol{r}+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ e^{i\phi} \end{bmatrix}$$

$$|\boldsymbol{r}-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -e^{i\phi} \end{bmatrix}$$

$$|\boldsymbol{\alpha}\rangle\rangle = \alpha_+|+\rangle\rangle + \alpha_-|-\rangle\rangle$$

Possible interpretation

If one chooses the base double-valued,

$$|\pm\rangle\rangle = e^{-i\phi/2}|\widetilde{\pm}\rangle\rangle$$

$$|\tilde{\boldsymbol{r}}+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{bmatrix}$$

$$|\tilde{\boldsymbol{r}}-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\phi/2} \\ -e^{i\phi/2} \end{bmatrix}$$

$$\boxed{H_{\text{surf}} = p_\phi \sigma_x + p_z \sigma_y}$$

$$H_{\text{surf}}\boldsymbol{\alpha} = E\boldsymbol{\alpha}$$

$$\boldsymbol{\alpha}(\phi, z) = \begin{bmatrix} \alpha_+(\phi, z) \\ \alpha_-(\phi, z) \end{bmatrix} = e^{ip_\phi R\phi + ip_z z}\begin{bmatrix} \beta_+ \\ \beta_- \end{bmatrix}$$

$$E = \pm\sqrt{p_\phi^2 + p_z^2}$$

$$|\boldsymbol{\alpha}\rangle\rangle = \alpha_+|\widetilde{+}\rangle\rangle + \alpha_-|\widetilde{-}\rangle\rangle$$

orbital part

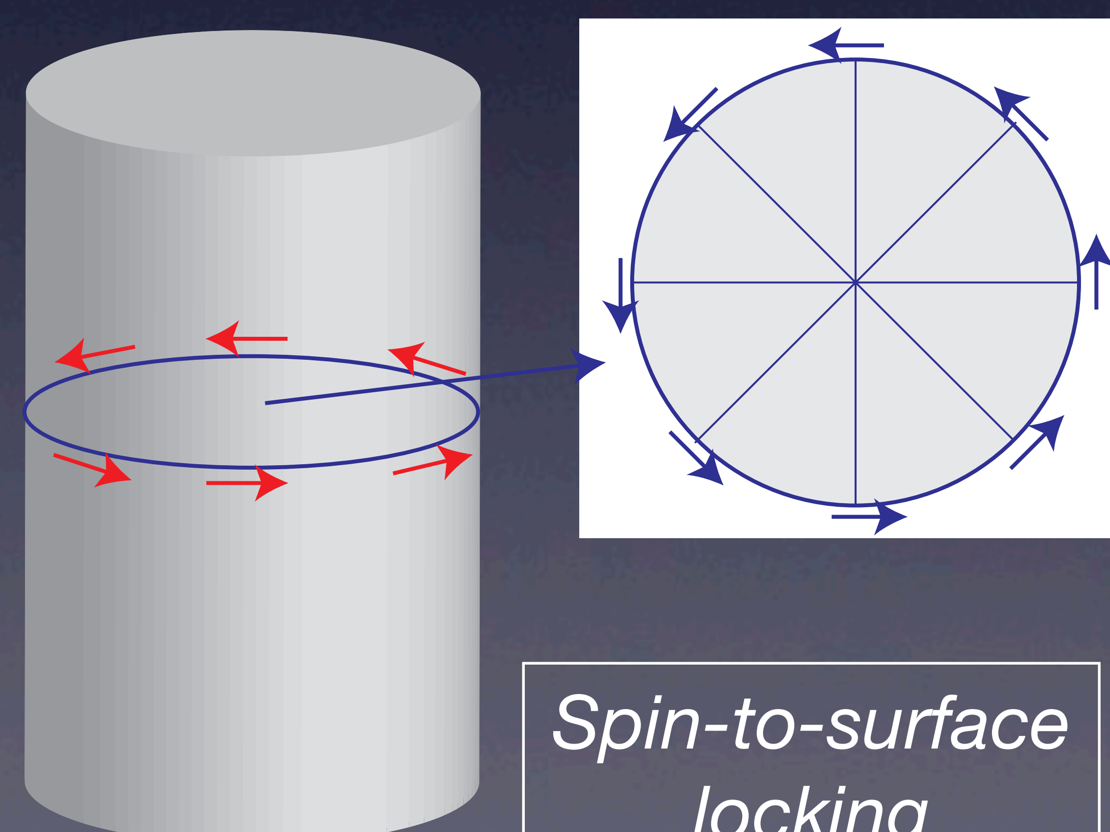
$$L_z = p_\phi R$$

Half-odd integral quantization of the orbital angular momentum

$$L_z = \pm\frac{1}{2}, \pm\frac{3}{2}, \cdots$$

single-valued $\quad |\boldsymbol{\alpha}(\phi + 2\pi)\rangle\rangle = |\boldsymbol{\alpha}(\phi)\rangle\rangle$

anti-periodic $\quad \boldsymbol{\alpha}(\phi + 2\pi) = -\boldsymbol{\alpha}(\phi)$



Spin-to-surface locking

Gapped surface states on the cylinder

- spin Berry phase
- spin-to-surface locking

$\rightarrow$ Half-odd integral quantization $\rightarrow$ Finite-size energy gap

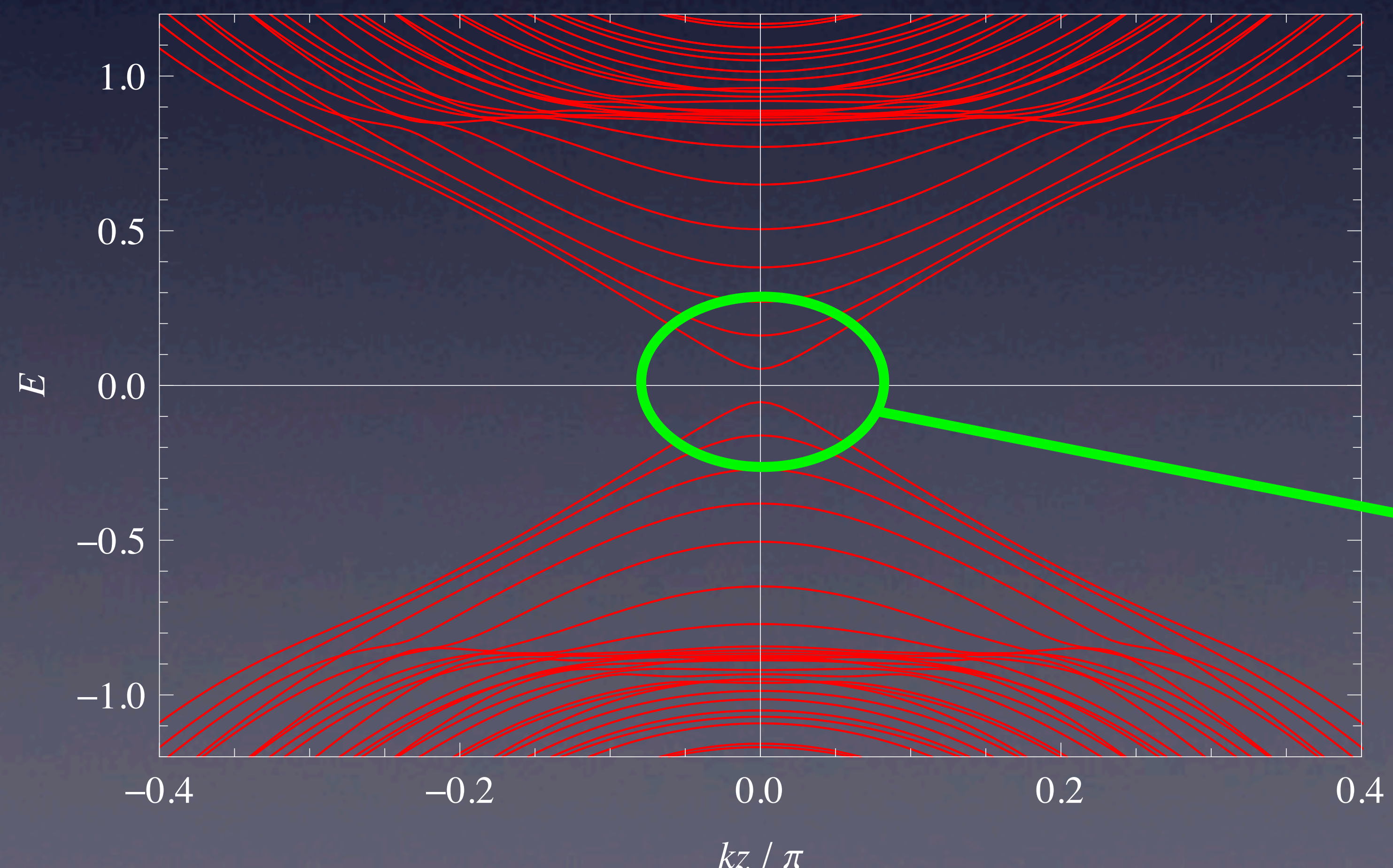
$$e^{iL_z(\phi+2\pi)} \times \underbrace{(-1)}_{\textit{anti-periodic}} = e^{iL_z\phi}$$

$$L_z = \pm\frac{1}{2}, \pm\frac{3}{2}, \cdots$$

$$L_z = p_\phi R$$

$$E = \pm\sqrt{p_\phi^2 + p_z^2}$$

$$\boxed{\Delta E \simeq R^{-1}}$$



Finite-size energy gap decays slowly (only *algebraically*)

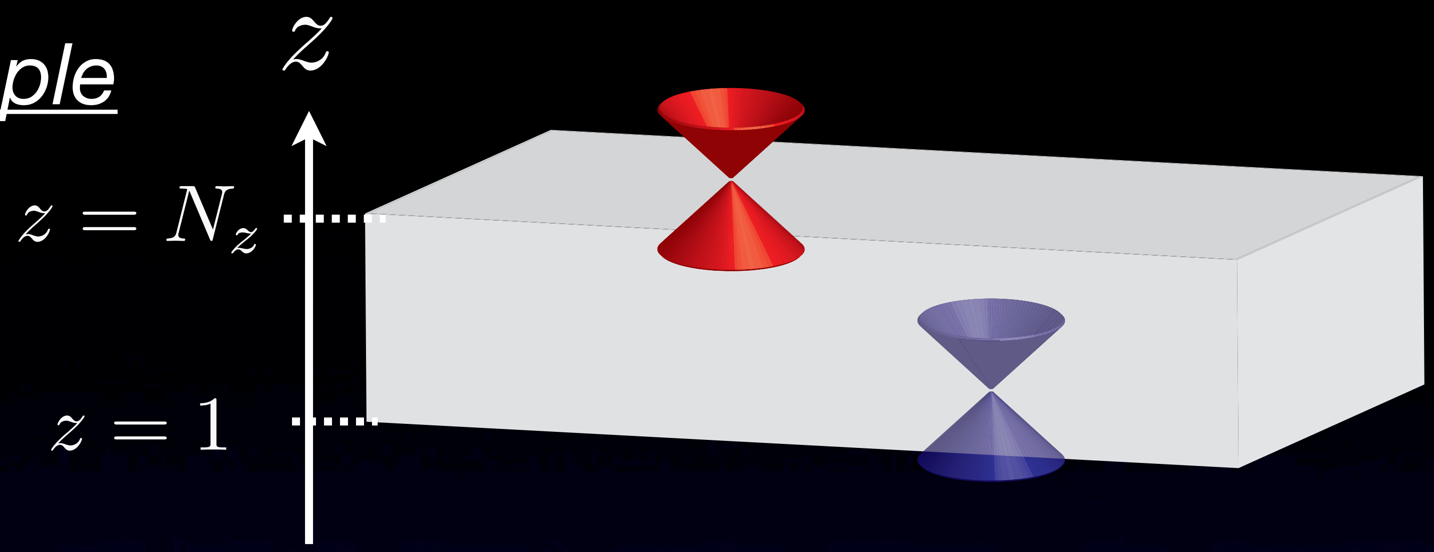
$$\Delta E \propto (N_x + N_y)^{-1}$$

prism: $N_x \times N_y$

***cf.* case of a slab-shaped sample**

- The surfaces are flat.

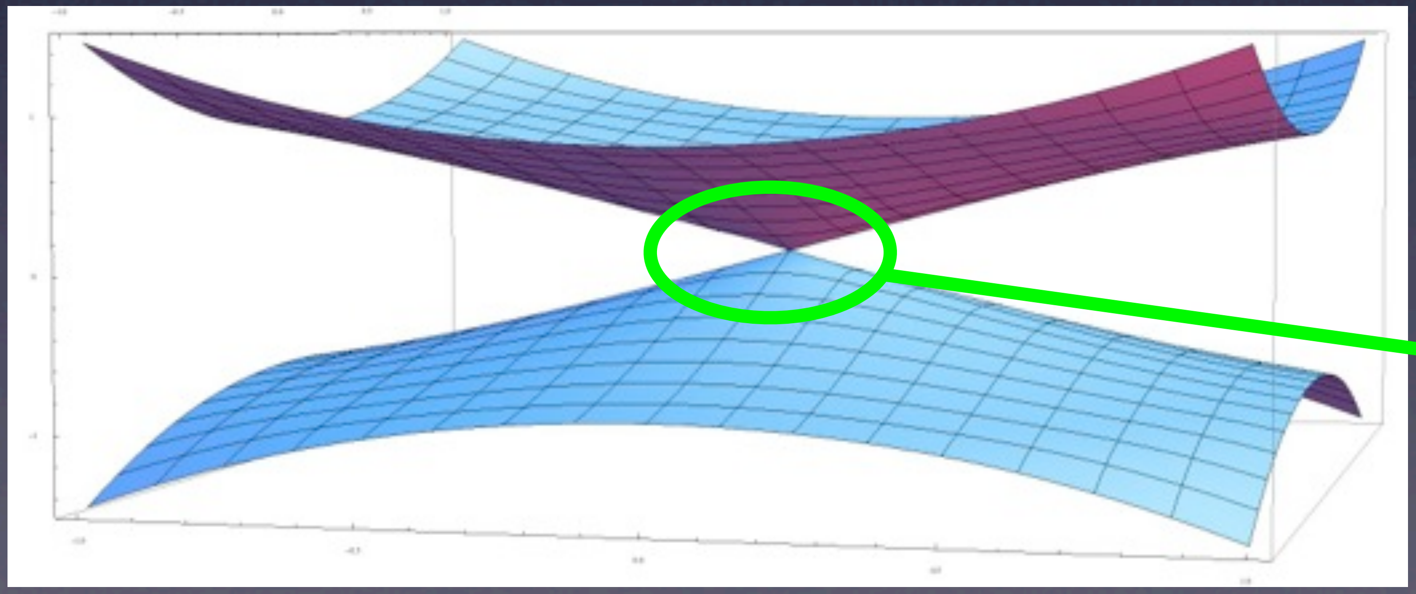
  But there are two of them (two Dirac cones); one on the top and the other at the bottom.



a system of infinitely large slab

- If the thickness is finite, the two Dirac cones *communicate* each other through the bulk

→ Finite-size energy gap



The magnitude of the gap is determined by the *overlap* of the two wave functions.

$$\Delta E \propto e^{-\kappa N_z}$$

exponential decay

Linder, Yokoyama & Sudbo, PRB '09; Lu et al., PRB '10

Physical interpretation of the result

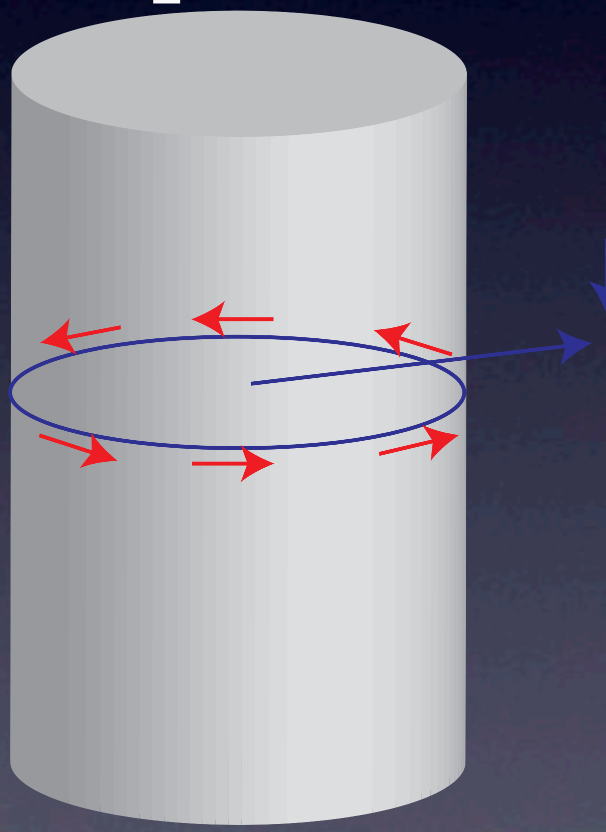
Effective Dirac Hamiltonian on a *cylindrical* surface

spin Berry phase

$$H_{\text{surf}} = \begin{bmatrix} 0 & -ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) \\ ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) & 0 \end{bmatrix}$$

Effects of the Berry phase

An electron on the cylindrical surface behaves as if an *imaginary solenoid* pierces the cylinder.



Electrons on a curved surface of topological insulator exhibits an “active” property reflecting its *geometry*.

Act I-B: The *spherical* topological insulator

*KI, Yoshimura, Takane & Fukui, Phys. Rev. B **86**, 235119 (2012)*

Effective Dirac Hamiltonian on a *spherical* surface

$$H_{\rm surf} = \frac{A}{R}\begin{bmatrix} 0 & -\partial_\theta + \frac{i\partial_\phi}{\sin\theta} - \underline{\frac{\cot\theta}{2}} \\ \partial_\theta + \frac{i\partial_\phi}{\sin\theta} + \underline{\frac{\cot\theta}{2}} & 0 \end{bmatrix}$$

Bulk effective Hamiltonian:

$$H_{\rm bulk} = m(\boldsymbol{p})\tau_x + A\tau_y(p_x\sigma_x + p_y\sigma_y + p_z\sigma_z)$$

Surface states on the sphere:

$$|\boldsymbol{\alpha}\rangle\rangle = \alpha_+|+\rangle\rangle + \alpha_-|-\rangle\rangle$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_+ \\ \alpha_- \end{bmatrix} = \begin{bmatrix} \alpha_+(\theta,\phi) \\ \alpha_-(\theta,\phi) \end{bmatrix} = e^{im\phi}\begin{bmatrix} \alpha_{m+}(\theta) \\ \alpha_{m-}(\theta) \end{bmatrix}$$

$$H_{\rm surf}\boldsymbol{\alpha} = E\boldsymbol{\alpha}$$

Half-odd integral quantization:

$$m = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \cdots$$

$$|\boldsymbol{\alpha}(\theta,\phi+2\pi)\rangle\rangle = |\boldsymbol{\alpha}(\theta,\phi)\rangle\rangle$$

Physical interpretation:

A (fictitious) magnetic monopole is induced at the center of the sphere!

Effective Dirac Hamiltonian in the *single-valued* basis

$$\mathcal{H}_{\rm sv} = \frac{A}{R}\begin{bmatrix} 0 & -\partial_\theta + \frac{i\partial_\phi}{\sin\theta} \underline{- \frac{1}{2}\cot\frac{\theta}{2}} \\ \partial_\theta + \frac{i\partial_\phi}{\sin\theta} \underline{- \frac{1}{2}\tan\frac{\theta}{2}} & 0 \end{bmatrix}$$

induced Berry phase $\simeq$ vector potential associated with a magnetic monopole (+ Dirac string)

The surface Dirac equation:

$$H_{\rm surf}\begin{bmatrix}\alpha_+\\ \alpha_-\end{bmatrix} = E\begin{bmatrix}\alpha_+\\ \alpha_-\end{bmatrix}$$

$$\begin{cases} g = -2\pi & \text{for } \alpha_+ \\ g = 2\pi & \text{for } \alpha_- \end{cases}$$

$$\boldsymbol{A}_I = \frac{g}{4\pi r}\tan\frac{\theta}{2}\,\hat{\boldsymbol{\phi}}$$

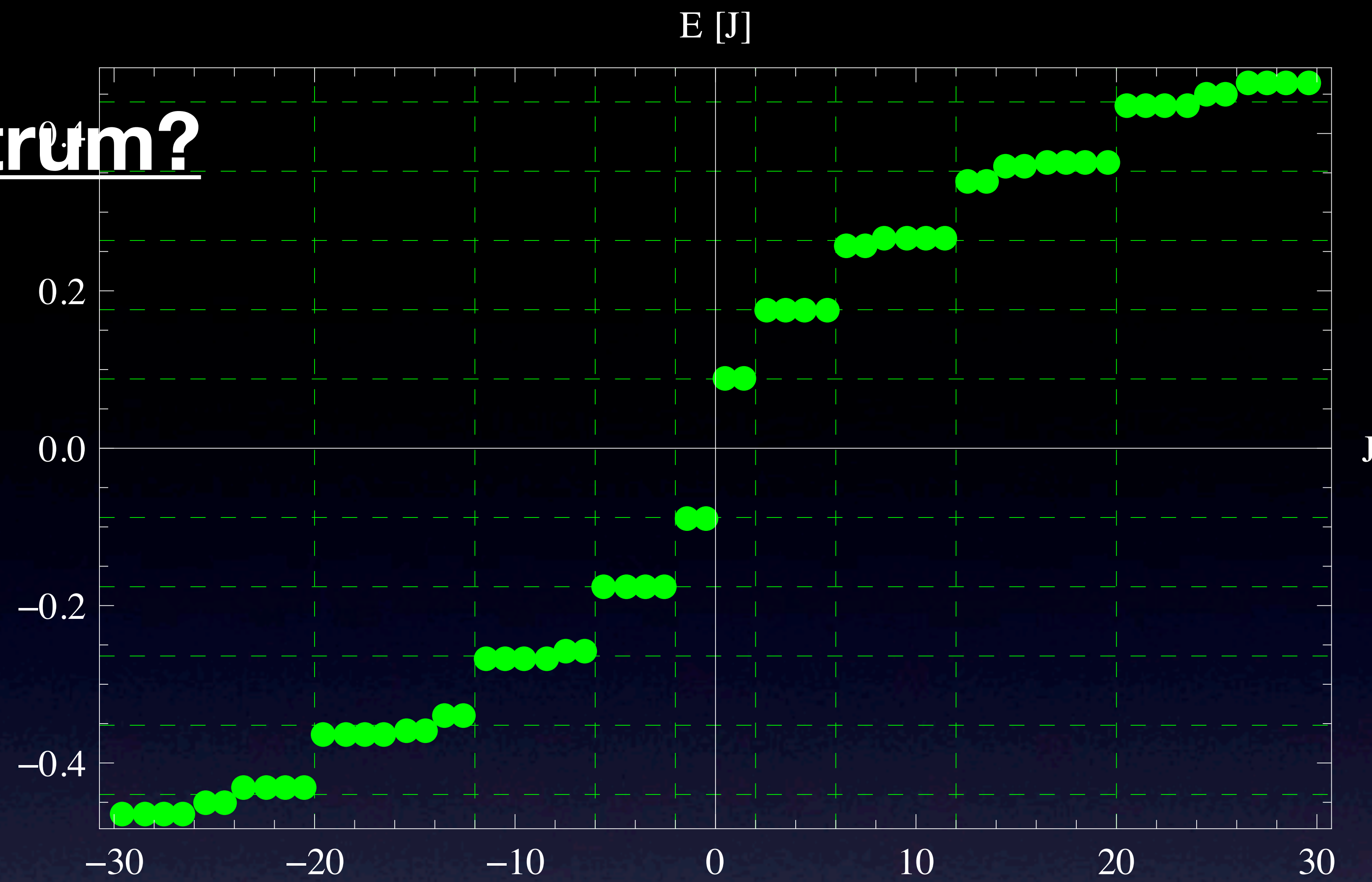
Dirac string on $-\hat{\boldsymbol{z}}$

$$\boldsymbol{A}_{II} = \frac{-g}{4\pi r}\cot\frac{\theta}{2}\,\hat{\boldsymbol{\phi}}$$

Dirac string on $+\hat{\boldsymbol{z}}$

A pair of magnetic monopole with an opposite magnetic charge: $g = \pm 2\pi$ is induced at the origin

How about the spectrum?



$$E = \pm \frac{A}{R}\left(n + |m| + \frac{1}{2}\right)$$

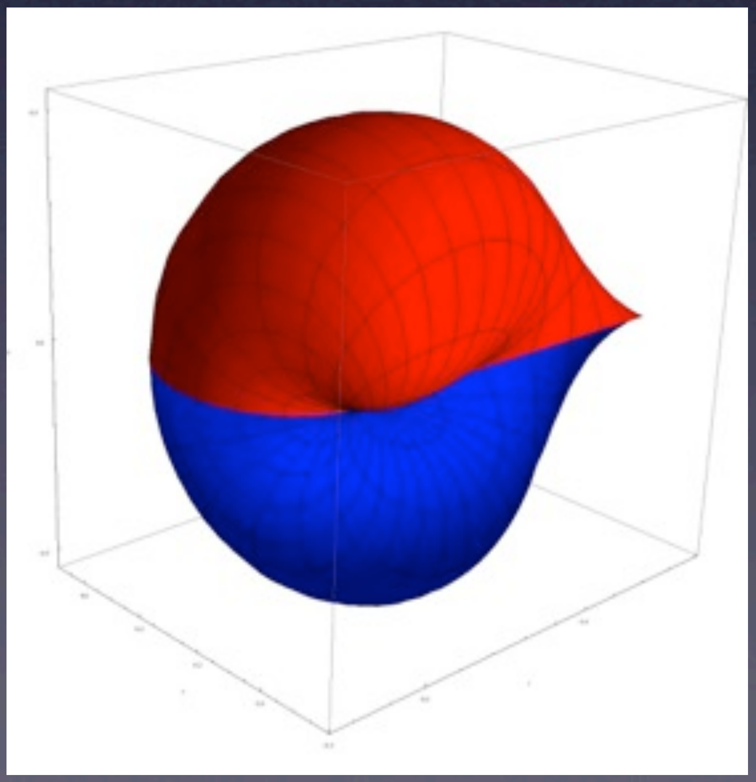
$$n = 0, 1, 2, 3, \cdots$$

reminiscent of Landau levels

- comparison with the tight-binding spectrum

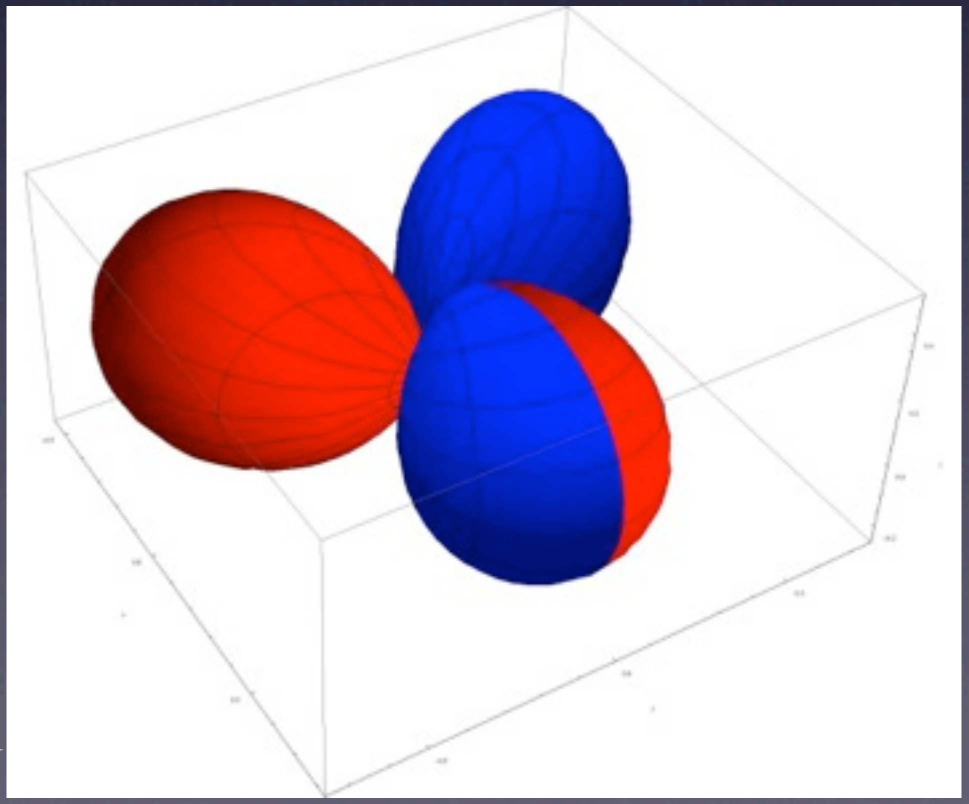
The wave function:

monopole harmonics



$$\boldsymbol{\alpha}_{0\frac{1}{2}} = \frac{1}{\sqrt{4\pi}} \begin{bmatrix} \cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \end{bmatrix}$$

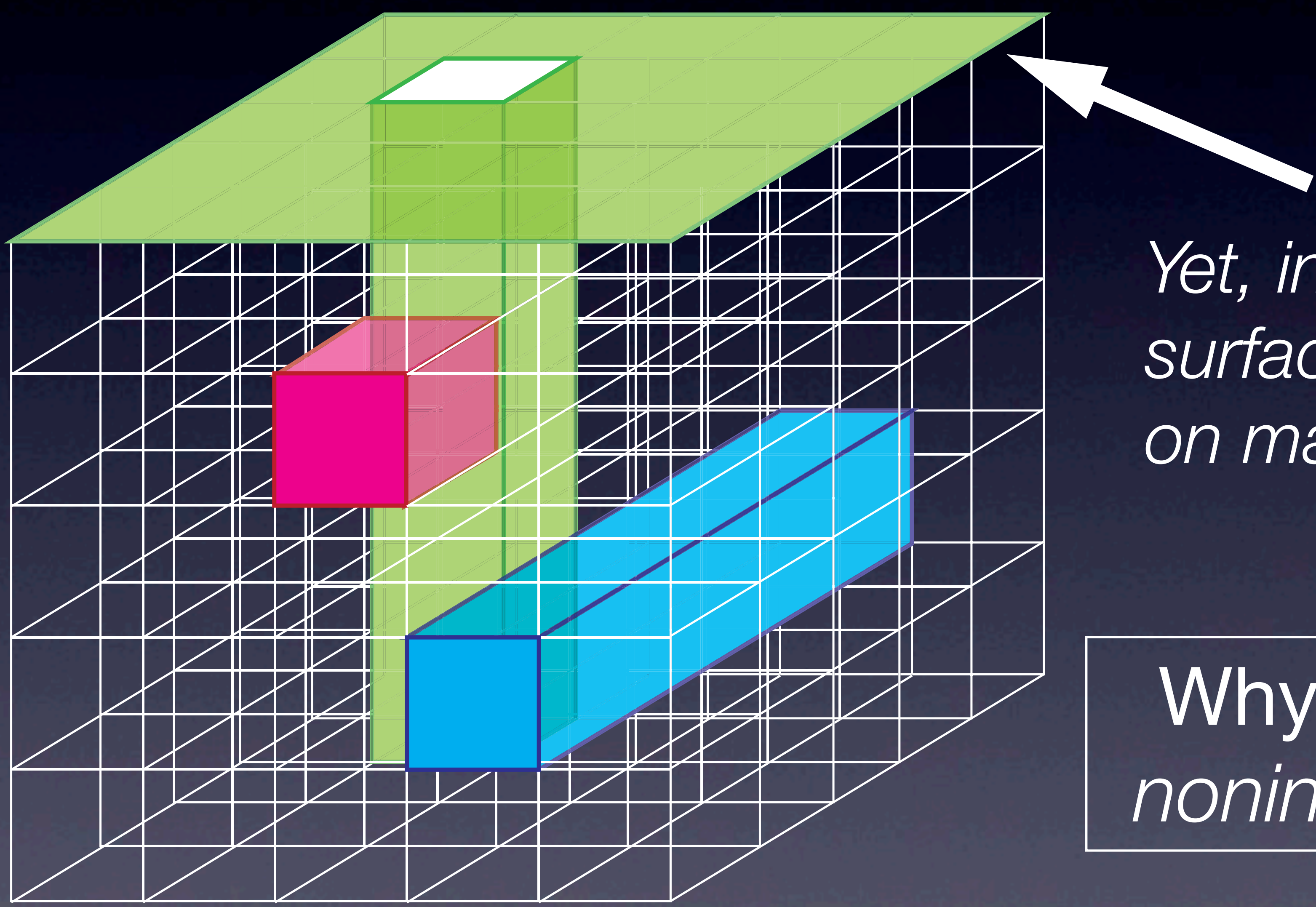
$$\boldsymbol{\alpha}_{0\frac{3}{2}} = \sqrt{\frac{3}{2\pi}} \begin{bmatrix} \sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \\ -\sin^2\frac{\theta}{2}\cos\frac{\theta}{2} \end{bmatrix}$$



Act II: Unification of the two aspects: the extrovert and the introvert

"The *noninvasive* metallic state"

- Consider a lattice (tight-binding) realization of topological insulator
- A lattice model is "sparse", existing only on sites and links



Yet, in reality protected surface states appear only on macroscopic surfaces.

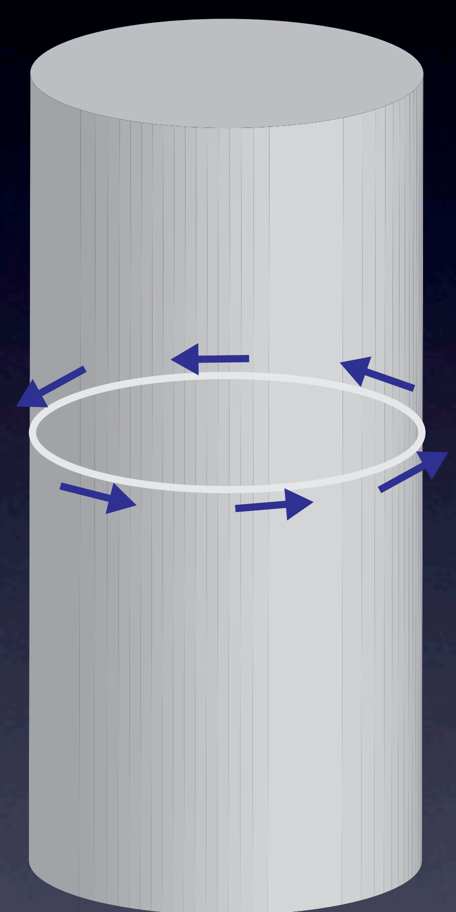
Why is the surface state *noninvasive* into the bulk?

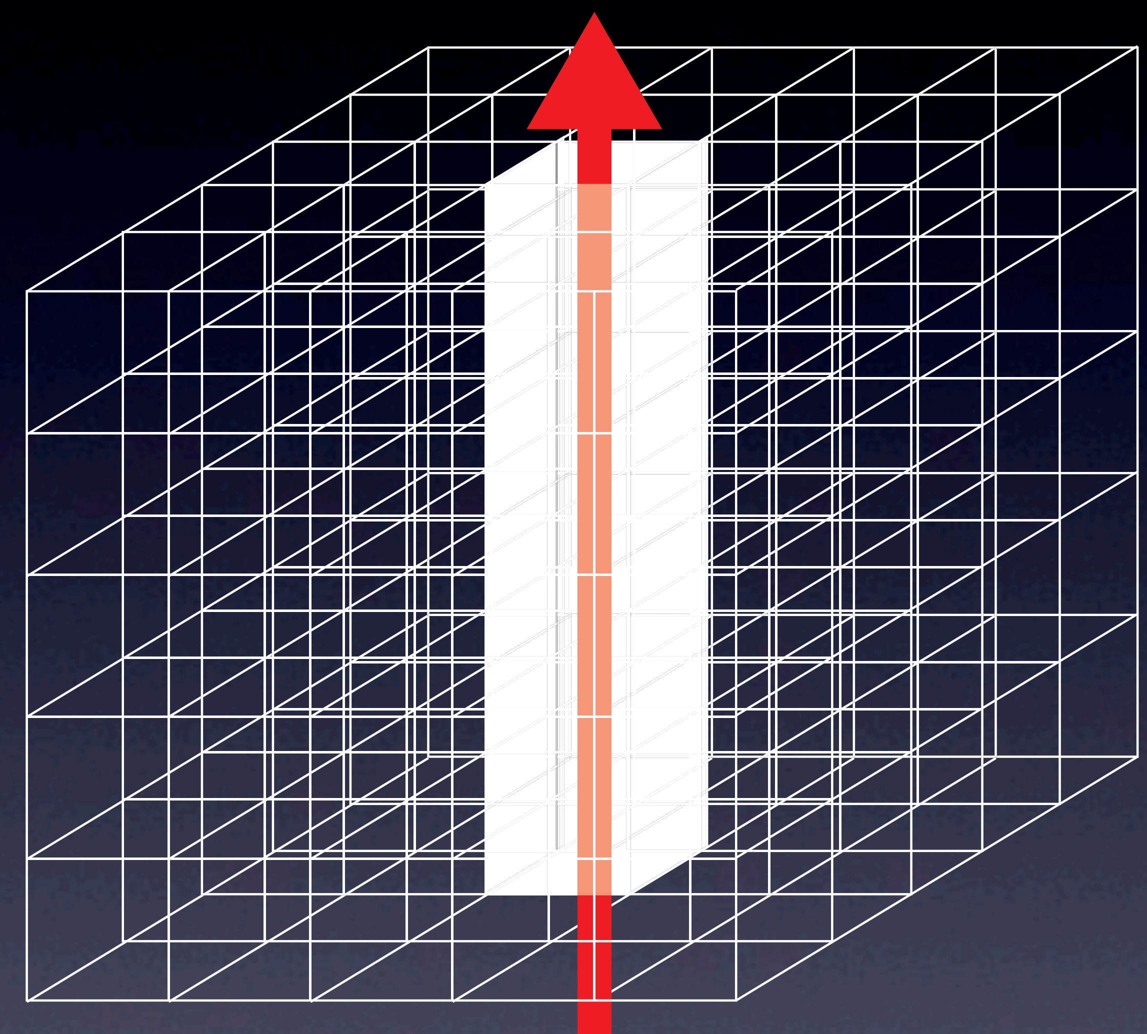
KI & Takane, arXiv:1211.2088

<u>*Where is the genuine surface?*</u>

Why is the surface state not penetrating into the bulk?

Answer: *because of the Berry phase* π



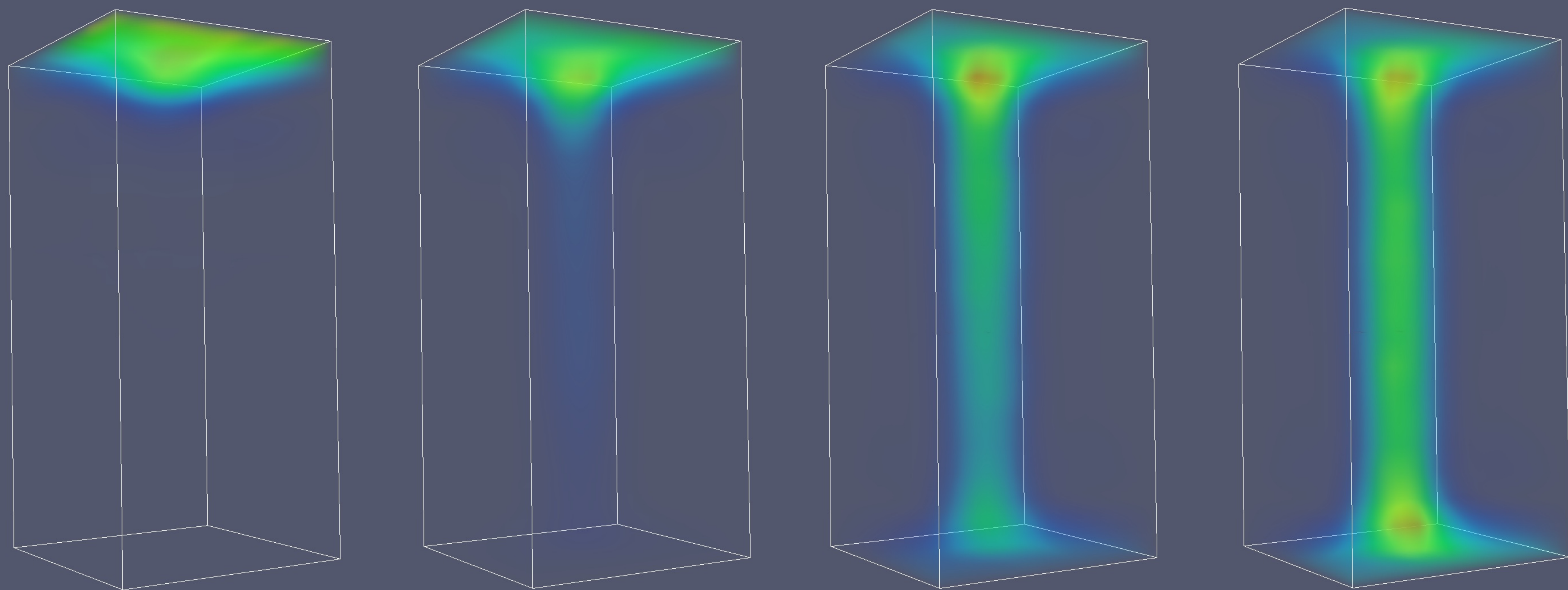


Recall that the *cylindrical* surface state is gapped by the spin-to-surface locking.

A flux tube penetrating the system may *awaken* the invasive nature of surface state.

Invasion of the metallic surface state

deeply into the bulk along a flux tube

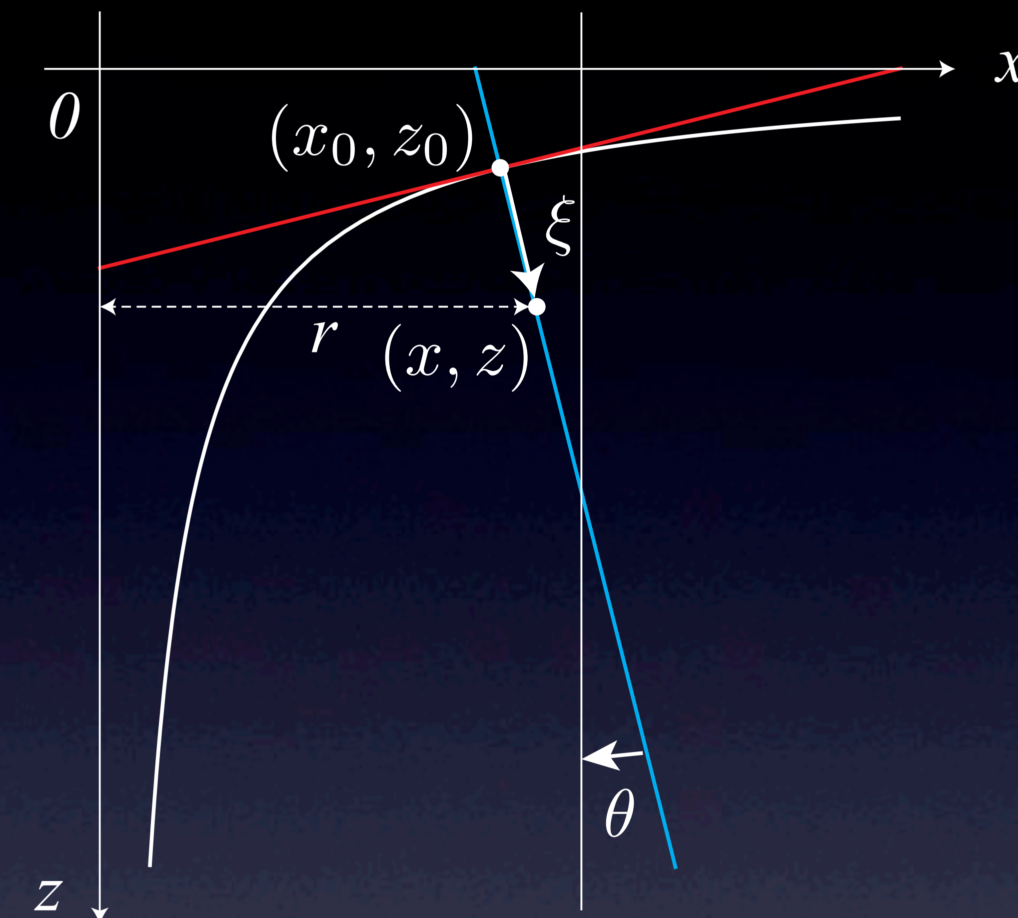


$\Phi = 0.7\pi$ $\Phi = 0.9\pi$ $\Phi = 0.95\pi$ $\Phi = \pi$

invasiveness of the surface state

KI & Takane, arXiv:1211.2088

Analytic treatments: hyperbolic “drain” geometry



The derivatives are rewritten as

$$\nabla = \boldsymbol{e}_\xi \partial_\xi - \frac{1}{\eta(\theta) - \xi} \boldsymbol{e}_\theta \partial_\theta + \frac{1}{r(\xi, \theta)} \boldsymbol{e}_\phi \partial_\phi$$

$$\begin{cases} x = r \cos\phi \\ y = r \sin\phi \\ z = \xi \cos\theta + R\sqrt{\tan\theta} \end{cases}$$

$$\begin{cases} r = r(\xi, \theta) = \xi \sin\theta + a + R\sqrt{\cot\theta} \\ \eta(\theta) = \sqrt{|\partial_\theta \boldsymbol{r}_0|^2} = \dfrac{R}{2} \dfrac{1}{\sqrt{\sin^3\theta \cos^3\theta}} \end{cases}$$

$$\begin{cases} \boldsymbol{e}_\xi = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ \boldsymbol{e}_\theta = (\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta) \\ \boldsymbol{e}_\phi = (-\sin\phi, \cos\phi, 0) \end{cases}$$

Derivation of the surface effective Hamiltonian

The zero-energy (degenerate) basis eigenstates:

$$|\pm\rangle = \frac{1}{\sqrt{c(\theta)}}\left(e^{-\kappa_1\xi} - e^{-\kappa_2\xi}\right)|\pm\rangle\rangle$$

$$|+\rangle\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{bmatrix} \otimes \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$|-\rangle\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} \sin(\theta/2) \\ -e^{i\phi}\cos(\theta/2) \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

The surface effective Hamiltonian:

$$H_{\text{surf}} = \begin{bmatrix} \langle +|H_{\parallel}|+\rangle & \langle -|H_{\parallel}|+\rangle \\ \langle +|H_{\parallel}|-\rangle & \langle -|H_{\parallel}|-\rangle \end{bmatrix} = \begin{bmatrix} 0 & D_- \\ D_+ & 0 \end{bmatrix}$$

where $$D_{\pm} = \pm A_\theta \partial_\theta \pm \frac{\partial_\theta A_\theta}{2} + A_\phi\left(-i\partial_\phi + \underline{\frac{1}{2}}\right)$$

The velocities are renormalized by the curvature

$\boxed{\text{spin Berry phase}}$

$$\begin{cases} A_\theta = \dfrac{\langle r\rangle}{\langle r(\eta-\xi)\rangle}\left(A + \dfrac{\left\langle \frac{r}{\eta-\xi}\right\rangle}{\langle r\rangle} m_2\right) \equiv \dfrac{\langle r\rangle}{\langle r(\eta-\xi)\rangle}\tilde{A}_\theta \\[2ex] A_\phi = \dfrac{\langle \eta-\xi\rangle}{\langle r(\eta-\xi)\rangle}\left(A - \dfrac{\left\langle \frac{\eta-\xi}{r}\right\rangle \sin\theta}{\langle \eta-\xi\rangle} m_2\right) \end{cases}$$

Invasive vs. noninvasive surface state

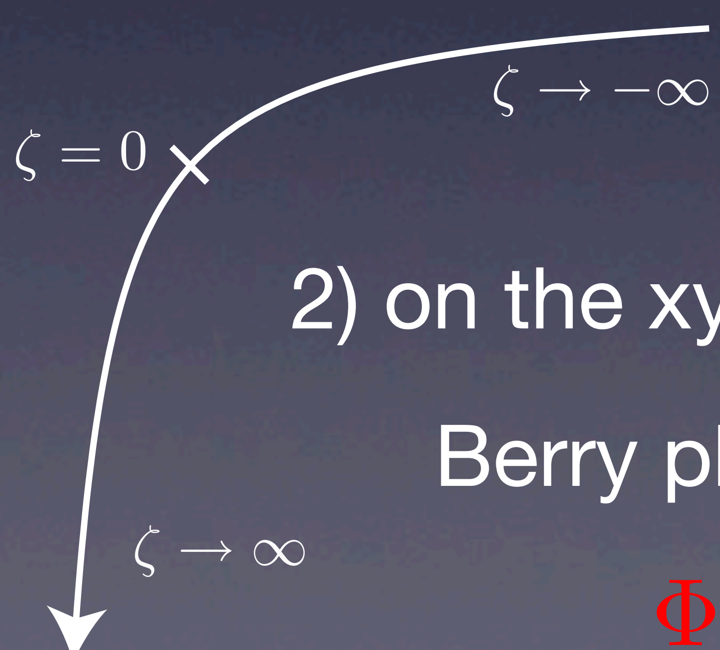
Is there a surface solution?
$$H_{\rm surf}\begin{bmatrix}\psi_+\\ \psi_-\end{bmatrix}=E\begin{bmatrix}\psi_+\\ \psi_-\end{bmatrix}$$
where
$$|\psi\rangle=\psi_+|+\rangle+\psi_-|-\rangle$$

Yes!
$$Z_\pm(\zeta)=\frac{1}{\sqrt{\tilde{A}_\theta(\zeta)}}\exp\left[\mp\tilde{L}_\pm\int_0^\zeta d\zeta'\,\frac{A_\phi(\zeta')}{\tilde{A}_\theta(\zeta')}\right]$$

where
$$\psi^{(+)}_{E=0}=e^{iL_+\phi}Z_+(\zeta)\begin{bmatrix}1\\0\end{bmatrix} \quad \& \quad \psi^{(-)}_{E=0}=e^{iL_-\phi}Z_-(\zeta)\begin{bmatrix}0\\1\end{bmatrix}$$

- contrasting behaviors in the <u>two asymptotic regimes</u>

$$\zeta(\theta)=\int_{\pi/4}^{\theta}d\theta'\,\frac{\langle r(\xi,\theta')(\eta(\theta')-\xi)\rangle}{\langle r(\xi,\theta')\rangle}$$



1) deep in the aperture: exponential decay

$\zeta\to\infty$ $\quad \tilde{L}_\pm=L_\pm+1/2$ constant

2) on the xy-plane, far away from the aperture $\zeta\to-\infty$

Berry phase *inactivated* ⟶ extended E=0 state appears

$\Phi=\pi$ $\quad J_{-1/2}(|E|r/A)$ $\quad J_0(|E|r/A)$

Conclusions

(After a long period of slavery to the bulk) the surface state is now freed.

- The introvert surface state sometimes wants to show off: e.g., on cylindrical and spherical surfaces.

spin Berry phase

role of the system's geometry: slab vs. cylinder, sphere

flat vs. curved surfaces

- The "gaplessness" is not immune to finite-site size corrections associated with the spin connection.

Such a finite-size correction does not decay exponentially.

- The noninvasiveness of the surface state is protected by the Berry phase π
- (Reclassified as a *noninvasive metallic state*) the surface state is shown to be, by itself, topologically protected.

Other perspectives

So far, we have considered only "strong" topological insulators.

= isotropic

Latest tendency of the field:

headed for seek of the diversity?

= anisotropic

A possible direction: *exploring the nature of weak topological insulators (WTI)*

- Specific features associated with the anisotropic character

*KI, Okamoto, Yoshimura, Takane & Ohtsuki, Phys. Rev. B **86**, 245436 (2012)*

- WTI is actually not that weak !

*KI, Takane & Tanaka, Phys. Rev. B **84**, 035443 (2011)*

In WTI, topological non-triviality is *not weak but only hidden.*

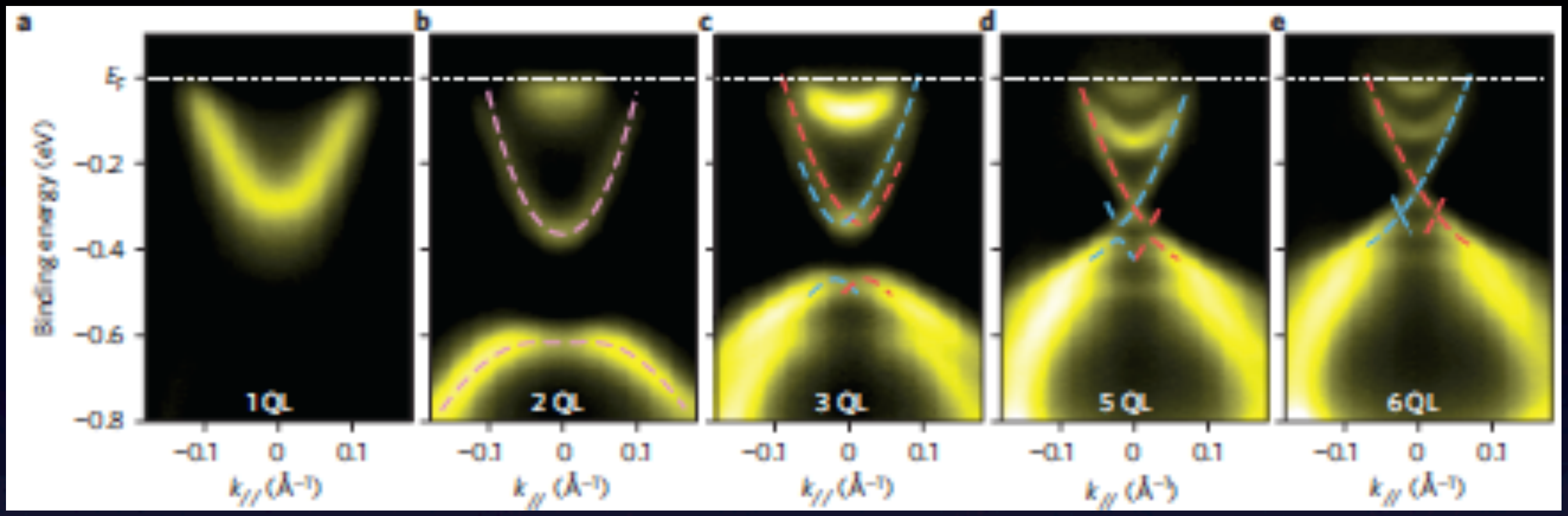
*Ringel, Kraus & Stern, Phys. Rev. B **86**, 045112 (2012).*

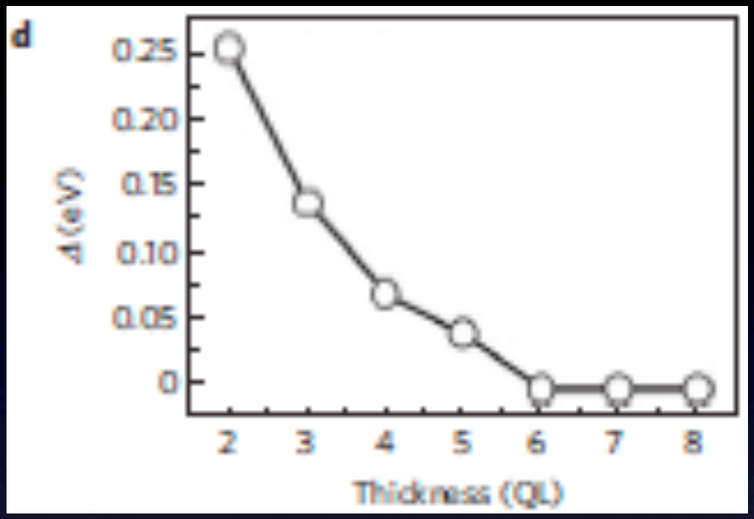
Stability of WTI against disorder: the global phase diagram

Kobayashi, Ohtsuki & KI, arXiv:1210.4656, PRL in press.

Experiments on finite-size effects

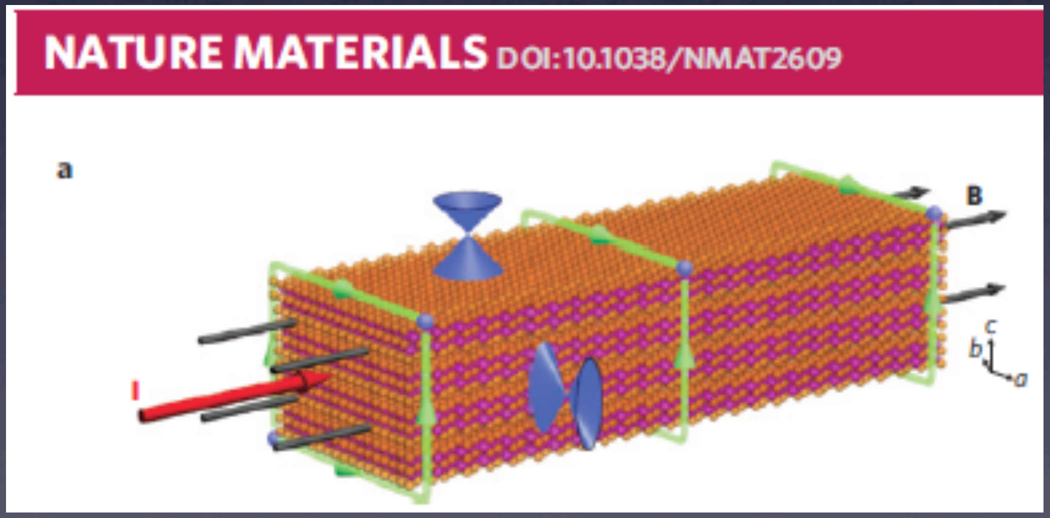
1) Slab (this film) geometry

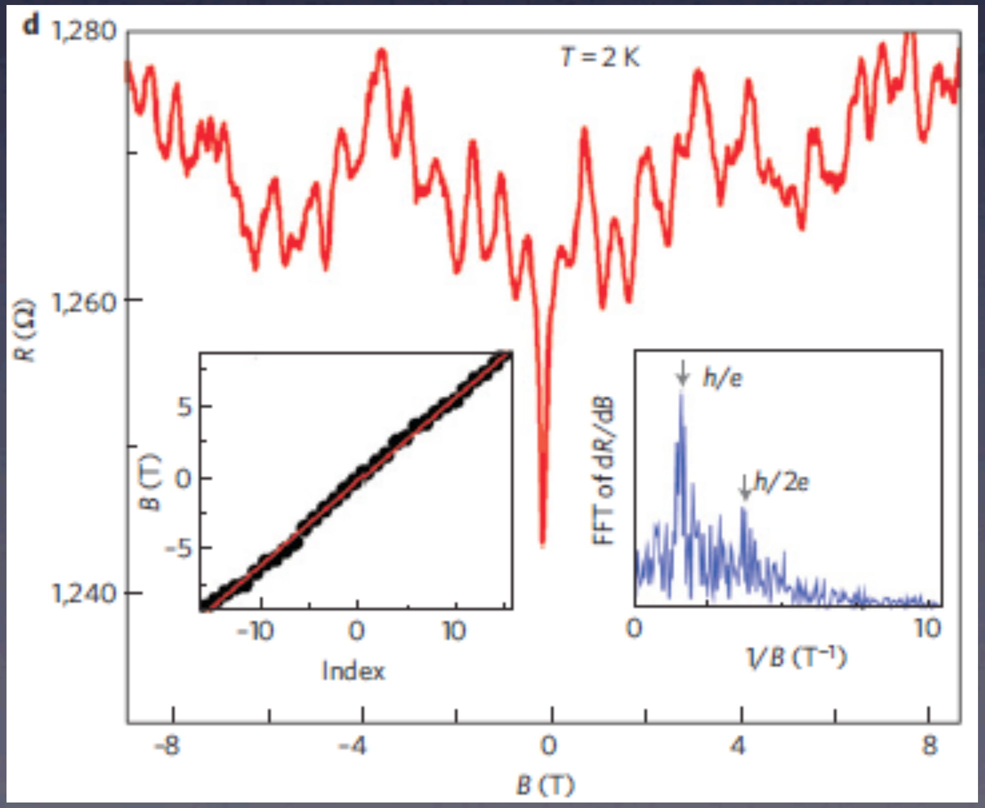




Zhang et al., Nature. Phys. (2010)

2) Prism (nanowire) geometry





Peng et al., Nature. Mat. (2009)