Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Topological Superconductors

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Theory Seminar at ISSP
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Homotopy of quasiparticles in superconductors

ISSP
佐藤昌利

Topological Table (1-band SCs)

Sato (04)
(unpublished)

16

Spin-singlet SC

	space of GS 波動関数の空間	homotopy ホモトピー	top. stable node 位相的に安定なノード
T-invariant 時間反転対称性 (T) 有	$\mathcal{H} = S^1/Z_2$	$\pi_1(\mathcal{H}) = Z_2$ $\pi_2(\mathcal{H}) = 0$	line node
T-breaking T 無	$\mathcal{H} = S^3/U(1)$	$\pi_1(\mathcal{H}) = 0$ $\pi_2(\mathcal{H}) = Z$	point node

high Tc cuprate

d+id SC

Spin-triplet SC

波動関数の空間	ホモトピー	位相的に安定なノード
$\mathcal{H} = S^3$	$\pi_1(\mathcal{H}) = 0$ $\pi_2(\mathcal{H}) = 0$	無
$\mathcal{H} \subset S^7/SU(2)\times U(1)$	$\pi_1(\mathcal{H}) = 0$ $\pi_2(\mathcal{H}) = Z$	point node
$\mathcal{H} \subset S^7/U(1)\times U(1)$	$\pi_1(\mathcal{H}) = 0$ $\pi_2(\mathcal{H}) = Z + Z$	point node

^{3}He-B
$\pi_3(S^3) = Z$

^{3}He-A
Sr_2RuO_4

- Orbifold structure due to TRI (= Kramers degeneracy) [Horava (05), Kane-Mele(05)]
- Altland-Zirnbauer classification (= Disorder effects) [Schnyder et al (08)]

In collaboration with

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Review paper
Y. Tanaka, MS, N. Nagaosa, "Symmetry and Topology in Superconductor"
Journal of Physical Society of Japan, 81 (2012) 011013 (open access)

Outline

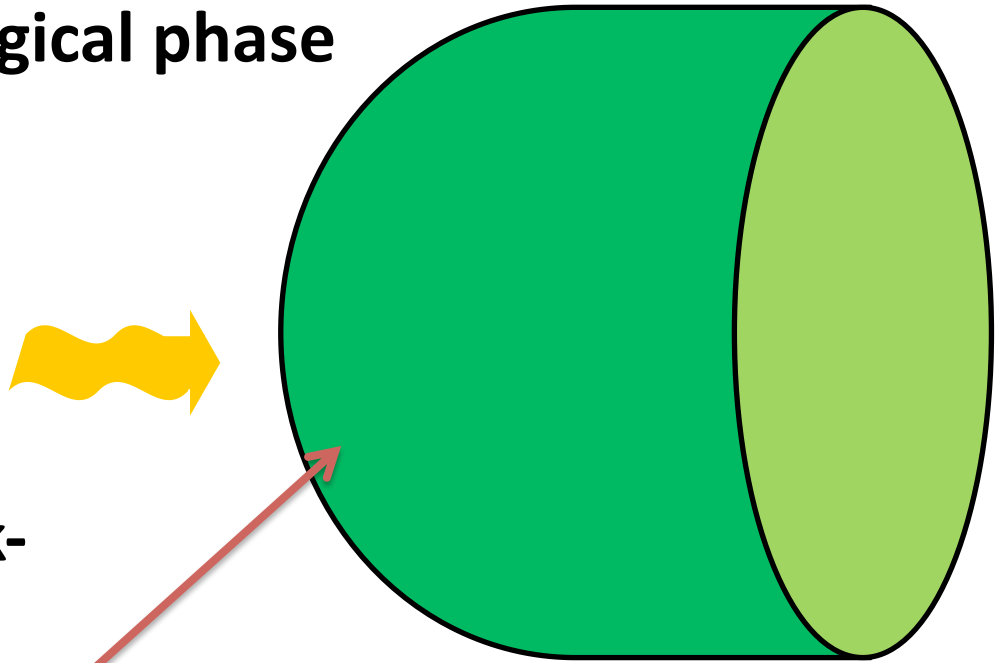
Part 1. What is topological superconductor ?

1. Why topological phase is useful ?
2. Prototype of topological phase – quantum Hall state
3. Topological superconductors
4. Majorana fermions
5. Which system supports Majorana fermions

Part 2. Symmetry Protected Majorana fermions in unconventional SCs

Part 1. What is topological superconductor ?

Topological phase



-Bulk-

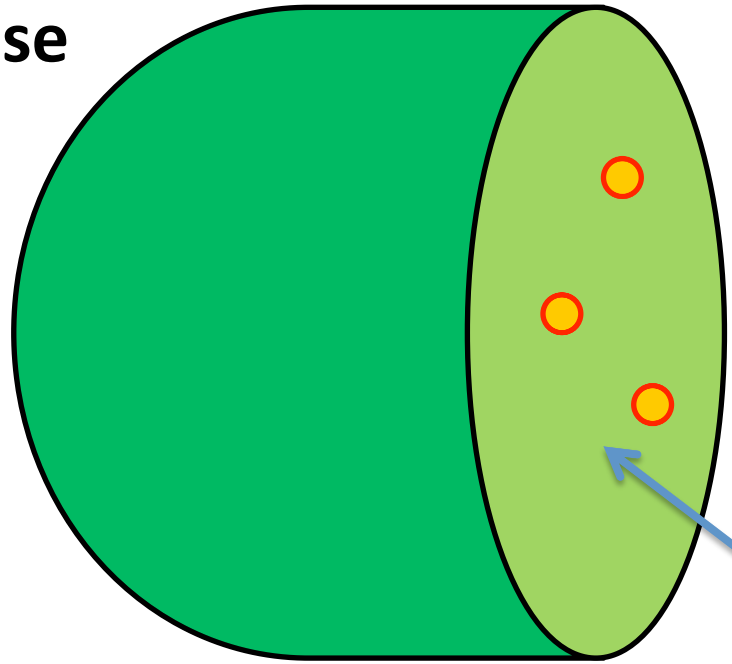
① **Gapped system** such as insulators and superconductors

② **Topological #**

$$\begin{cases} \neq 0 & \textbf{Topological phase} \\ = 0 & \textbf{Non-topological phase} \end{cases}$$

③ Topological # cannot change unless the bulk gap closes

Topological phase



-Boundary-

① **The existence of gapless state on the boundary**

② Mathematically, the existence of gapless states is ensured by the bulk topological #

⇒ **Bulk-edge correspondence**

③ We can operate the gapless state without destroying the bulk state

Why topological phase is useful ?

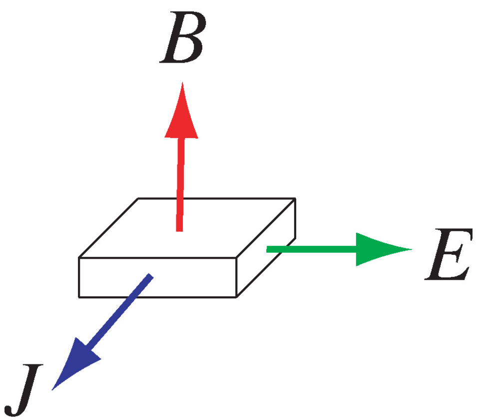
- The bulk state is gapped, so it is stable against local perturbation (i.e. **decoherence free**)

- Nevertheless, it support gapless states on the boundary, so at the same time there exist manageable quantum states (i.e. **qubits**)

Considering these two properties, we can expect that topological phase is an **ideal platform of quantum devices**

(i.e. topological quantum computer)

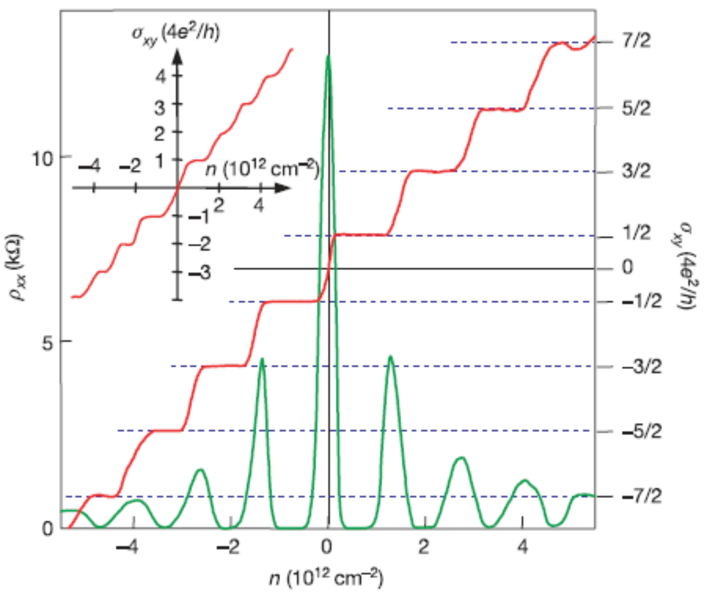
Quantum Hall state: Prototype of topological phase

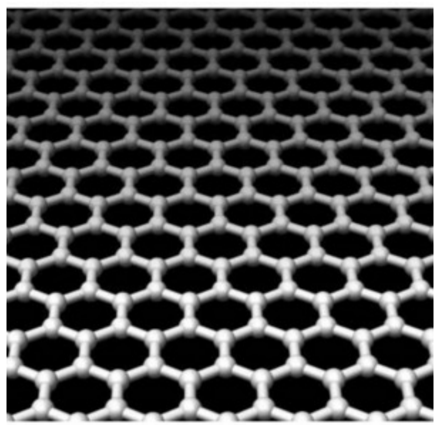


$$\boxed{\sigma_{xy} \equiv \frac{J}{E} = \frac{e^2}{h}\nu}$$

e: electron charge
h: Planck const.

Hall conductance is quantized in the unit of e^2/h





Bulk of QH state

- QH states are gapped in the bulk due to the formation of Landau level.

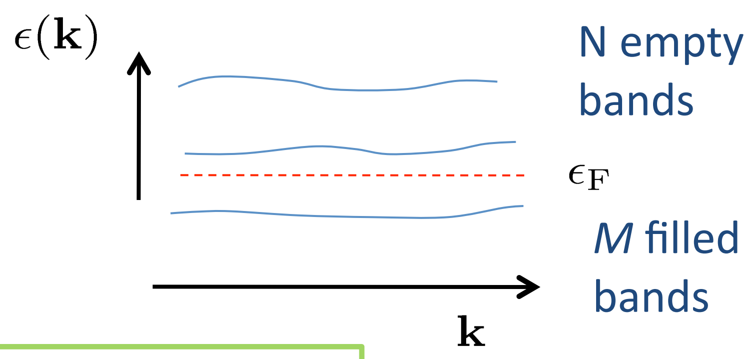
- They have a non-zero topological #.

Thouless-Kohmoto et al. (82)
Kohmoto (85)

$$\mathcal{A}_i(\boldsymbol{k}) = i \sum_{n \in \text{filled}} \langle u_{n,\boldsymbol{k}} | \partial_{k_i} u_{n,\boldsymbol{k}} \rangle$$

Bloch wave fn. of occupied state

Landau level in a crystal filed



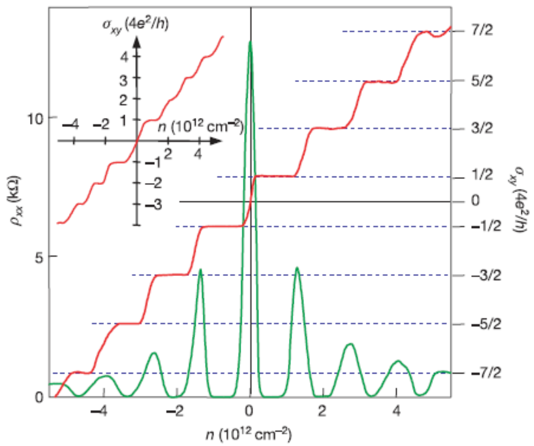
$$\nu_{\mathrm{Ch}} = \frac{1}{2\pi} \int_{\mathrm{BZ}} dk_x dk_y \left[\partial_{k_x} \mathcal{A}_y - \partial_{k_y} \mathcal{A}_x \right]$$

TKNN # (or Chern #)

- The quantization of Hall conductance is explained by the quantization of topological #

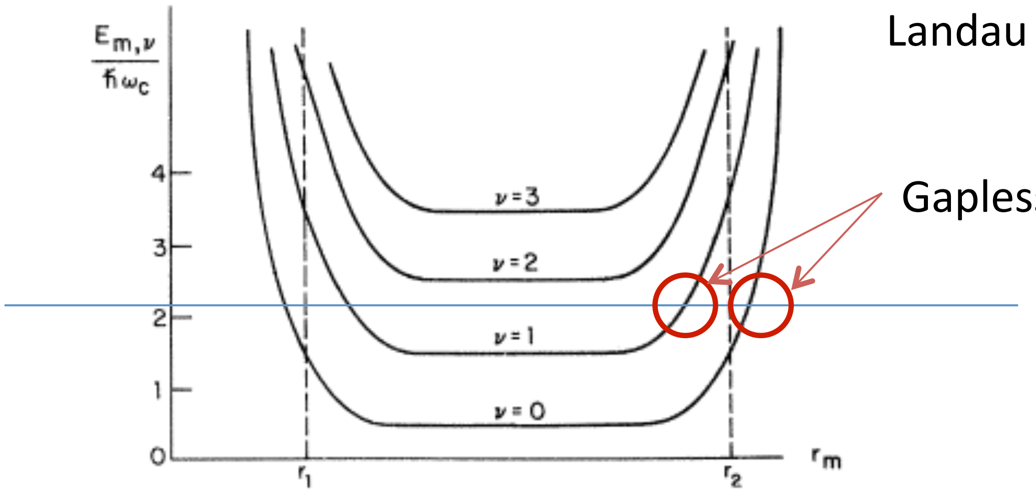
$$\boxed{\sigma_{xy} = \frac{e^2}{h}\nu_{\rm Ch}}$$

$$\nu_{\rm Ch} = \frac{1}{2\pi}\int_{\rm BZ} dk_x dk_y \left[\partial_{k_x}\mathcal{A}_y - \partial_{k_y}\mathcal{A}_x\right]$$



Edge of QH state

To obtain the edge, we introduce confining potential



Landau level + confining potential

Gapless states carry the Hall conductance

$$\sigma_{xy} = \frac{e^2}{h} 2$$

Halperin (81)

• Due to the confining potential, occupied Landau levels cross the Fermi energy near the edge. So there exist gapless states localized on the boundary.

• The quantization of Hall conductance is explained by the quantization of the number of edge states.

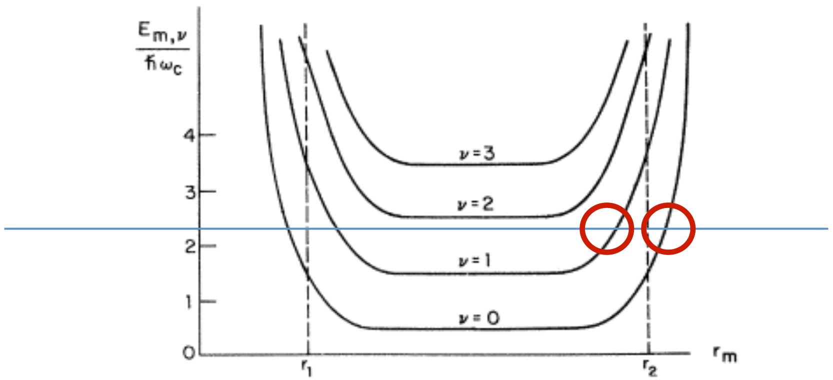
$$\sigma_{xy} = \frac{e^2}{h} \nu_{\text{edge}}$$

We have two different views of Hall conductance

① Bulk

$$\sigma_{xy} = \frac{e^2}{h}\nu_{\mathrm{Ch}} \qquad \nu_{\mathrm{Ch}} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} dk_x dk_y \mathcal{F}(\boldsymbol{k})$$

② Edge



$\nu_{\mathrm{ch}}=\nu_{\mathrm{edge}}$

$$\sigma_{xy} = \frac{e^2}{h}\nu_{\mathrm{edge}}$$

The number of the edge states should be the same as topological #

bulk-edge correspondence

QH states (summary)

- QH states are gapped systems with non-zero topological #.
- QH states support gapless edge states.
- The gapless states are ensured by the bulk topological #.

$\Rightarrow$ **Topological phase**

The Hall current is well-controlled and the quantization is extremely accurate.

Ex.) Hall effect devices

Topological Superconductors

A close similarity between quantum Hall states and SCs

	Integer quantum Hall state	Superconducting state
bulk	gapped (Landau level)	gapped (gap function)
edge	Gapless edge state	Andreev bound state

We can naturally expect that topological phase is possible for superconducting state.

→ **Topological superconductors**

Qi et al.(09) , Schnyder et al (08), MS (09), Roy (08), …

Topologically protected state = Majorana fermion

Majorana Fermion

Dirac fermion with Majorana condition

1. Dirac Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}, \text{ or } \quad \mathcal{H}(k_x) = ck_x$$

2. Majorana condition

$$\Psi = C\Psi^*$$

particle = antiparticle

For the gapless boundary states, their Hamiltonians are naturally given by the Dirac Hamiltonians

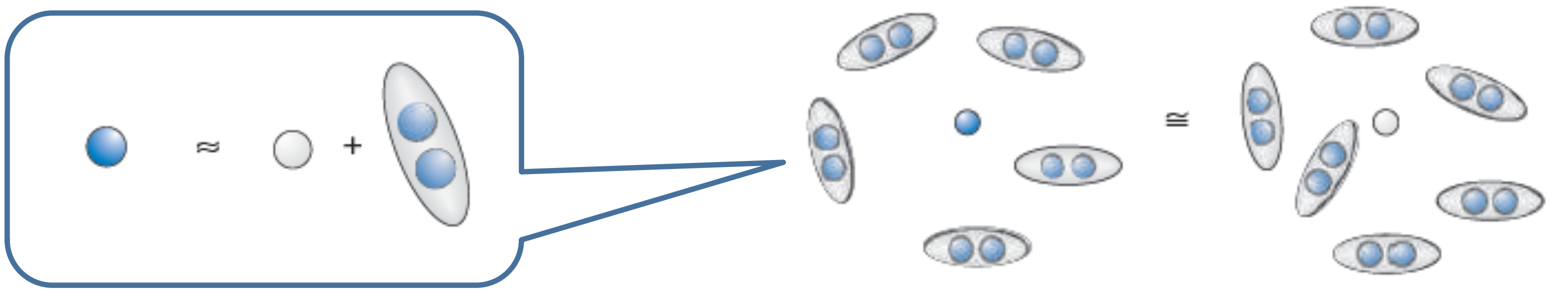
Why the Majorana condition ?

⇒ The Majorana condition is imposed by superconductivity

quasiparticle in Nambu rep. quasiparticle anti-quasiparticle

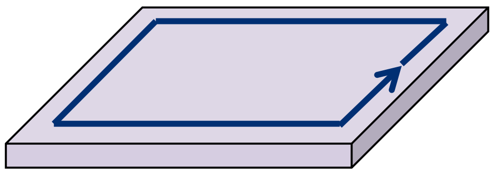
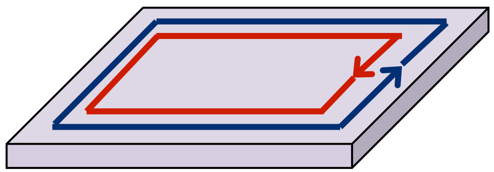
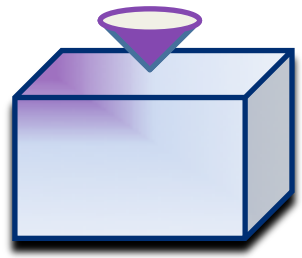
$$\Psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \\ \psi_\uparrow^\dagger(x) \\ \psi_\downarrow^\dagger(x) \end{pmatrix} \Rightarrow \boxed{\Psi(x) = \mathcal{C}\Psi^*(x), \quad \mathcal{C} = \begin{pmatrix} 0 & 1_{2\times 2} \\ 1_{2\times 2} & 0 \end{pmatrix}}$$

Majorana condition



[Wilczek , Nature (09)]

different bulk topological
= different Majorana fermions

2+1D time-reversal breaking SC	2+1D time-reversal invariant SC	3+1D time-reversal invariant SC
1^{st} Chern # (TKNN82, Kohmoto85)	Z_2 number (Kane-Mele 06, Qi et al (08))	3D winding # (Schnyder et al (08))
1+1D **chiral** edge mode 	1+1D **helical** edge mode 	2+1D **helical** surface fermion 
Sr_2RuO_4	Noncentosymmetric SC (MS-Fujimto(09))	^{3}He B

Which system support Majorana fermions ?

- Spin-triplet (odd-parity) superconductors

  Volovik (86), Read-Green(00)

- Superconducting states with SO interaction

  MS, Physics Letters B535,126 (03), Fu-Kane (08)

  MS, Takahashi, Fujimoto PRL(09) PRB(10), J.Sau et al (11)

A representative example of topological SC:

spinless chiral p-wave SC in 2+1 dimensions

[Read-Green (00)]

BdG Hamiltonian

spinless chiral p-wave SC

$$
\begin{aligned}
\mathcal{H} &= \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c_{\boldsymbol{k}}^{\dagger} c_{\boldsymbol{k}} + \frac{1}{2} \sum_{\boldsymbol{k}} \left[\Delta(\boldsymbol{k}) c_{\boldsymbol{k}}^{\dagger} c_{-\boldsymbol{k}}^{\dagger} + \mathrm{h.c} \right] \\
&= \frac{1}{2} \sum_{\boldsymbol{k}} \left(c_{\boldsymbol{k}}^{\dagger}, c_{-\boldsymbol{k}} \right) \mathcal{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}} \\ c_{-\boldsymbol{k}}^{\dagger} \end{pmatrix} + \mathrm{const.}
\end{aligned}
$$

with

$$
\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \epsilon(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta(\boldsymbol{k})^{*} & -\epsilon(\boldsymbol{k}) \end{pmatrix}
$$

$$
\epsilon(\boldsymbol{k}) = -2t_x \cos k_x - 2t_y \cos k_y - \mu
$$

$$
\begin{aligned}
\Delta(\boldsymbol{k}) &= d(\sin k_x + i \sin k_y) \\
&\sim d(k_x + i k_y)
\end{aligned}
$$

chiral p-wave

Topological number = 1st Chern number

TKNN (82), Kohmoto(85)

$$A_i(\boldsymbol{k}) = i \sum_{a \in \text{filled}} \langle u_a(\boldsymbol{k}) | \frac{\partial}{\partial k_i} | u_a(\boldsymbol{k}) \rangle$$

$$\begin{aligned} \nu_{\text{Ch}} &= \frac{1}{2\pi} \int d^2k [\partial_{k_x} A_y(\boldsymbol{k}) - \partial_{k_y} A_x(\boldsymbol{k})] \\ &= -\frac{1}{2} \sum_{\Delta(k_0)=0} \text{sgn}\epsilon(k_0) \cdot \text{sgn}[\det(\partial_i R^j(k_0))] \end{aligned}$$

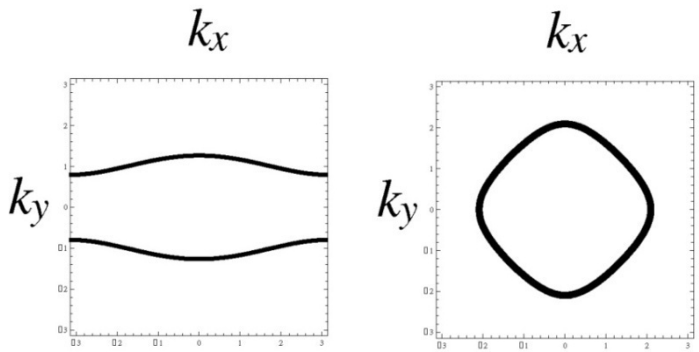
MS (09)

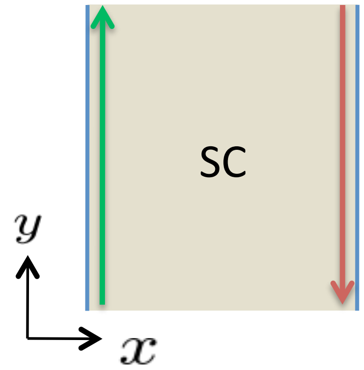
$$(\Delta(\boldsymbol{k}) = R^1(\boldsymbol{k}) + iR^2(\boldsymbol{k}))$$

Edge state

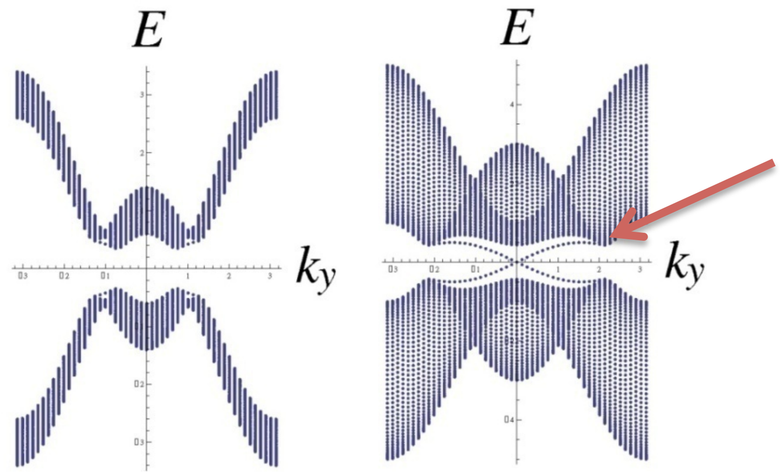
$\mu = -1,\ d = 0.5$

Fermi surface





Spectrum



$t_x = 1,\ t_y = 0.2 \quad t_x = t_y = 1$

$\nu_{\mathrm{Ch}} = 0 \qquad \nu_{\mathrm{Ch}} = 1$

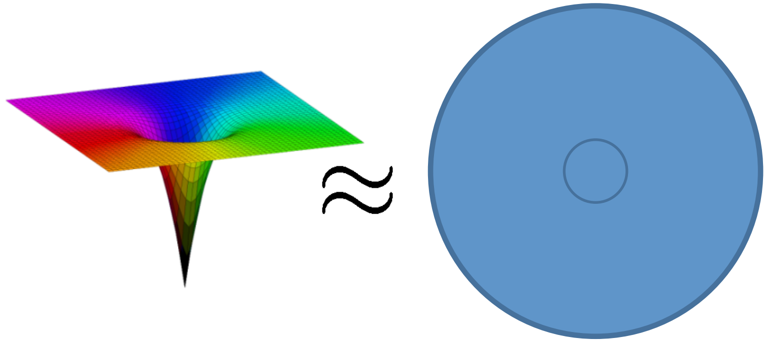
2 gapless edge modes (left-moving , right moving,
on different sides on boundaries)

Majorana fermion

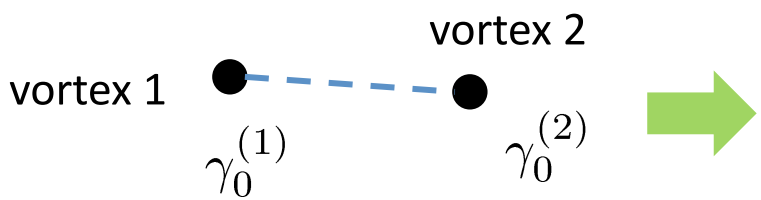
Bulk-edge correspondence

In the second case, there also exist a single Majorana zero mode in a vortex

$$\boxed{\gamma_0^\dagger = \gamma_0}$$



We need a pair of the zero modes to define creation op.



$$\gamma^\dagger = \frac{\gamma_0^{(1)} + i\gamma_0^{(2)}}{\sqrt{2}} \qquad \{\gamma^\dagger, \gamma\} = 1$$

non-Abelian anyon

topological quantum computer

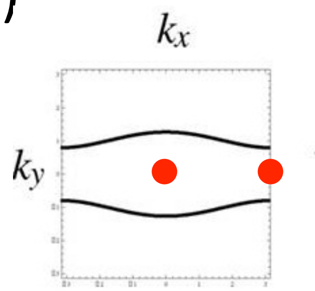
For spin-triplet SCs (or odd parity SCs), there exists a simple criterion for topological phases

> If the number of TRIMs enclosed by the Fermi surface is odd, the spin-triplet SC is (strongly) topological.

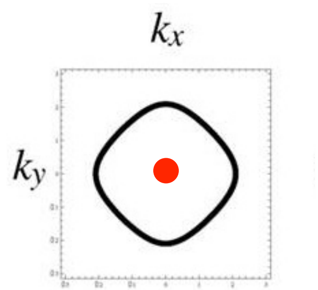
[Sato (09), Sato (10), Fu-Berg (10)]

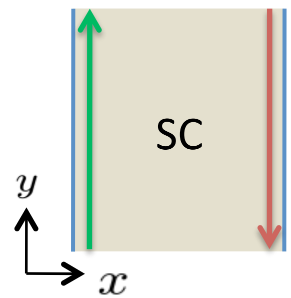
2D spinless SC)

Even



Odd



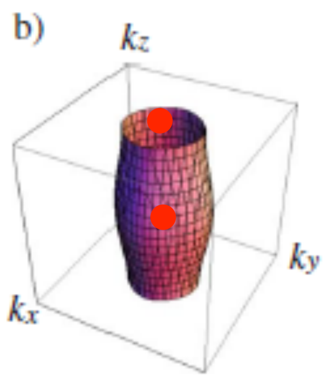


Chiral Majorana mode

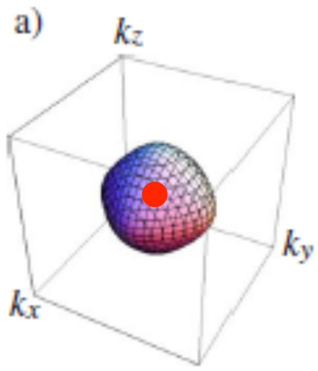
$$\Delta(\boldsymbol{k}) = k_x + ik_y$$

3D time-reversal invariant spin-triplet SC)

Even



Odd



With proper topology of the Fermi surface, spin-triplet SCs (or odd-parity SCs) naturally become topological.

Recently, it has been found that s-wave superconductors also can support Majorana fermion.

A) MS, Physics Letters B535 ,126 (03), Fu-Kane PRL (08)

B) MS-Takahashi-Fujimoto ,Phys. Rev. Lett. 103, 020401 (09) ;
MS-Takahashi-Fujimoto, Phys. Rev. B82, 134521 (10) (Editor's suggestion),
J. Sau et al, PRL (10), J. Alicea PRB (10)

Key point: Spin-orbit interaction

Majorna fermion in spin-singlet SC

MS, Physics Letters B535 ,126 (03)

① 2+1 dim Dirac fermion + s-wave Cooper pair

$$\mathcal{H} = \begin{pmatrix} -i\sigma_i\partial_i & \Phi^* \\ \Phi & -i\sigma_i\partial_i \end{pmatrix} \qquad \Phi = \Phi_0 f(r) e^{i\theta} \quad \text{vortex}$$

⇒

Zero mode in a vortex [Jackiw-Rossi (81), Callan-Harvey(85)]

With Majorana condition, non-Abelian anyon is realized [MS (03)]

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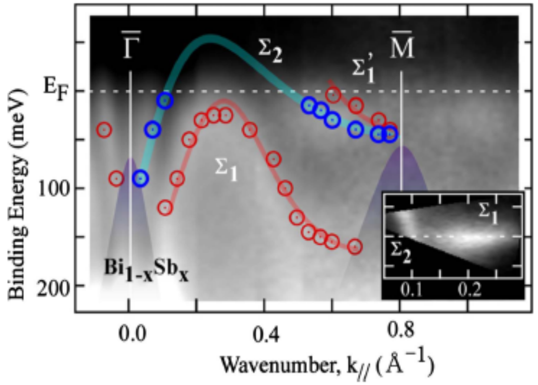
Non-Abelian statistics of axion strings

Masatoshi Sato

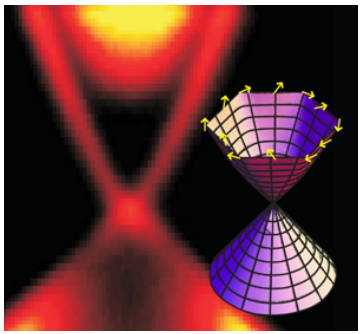
On the surface of topological insulator [Fu-Kane (08)]

$Bi_{1-x}Sb_x$ Hsieh et al., Nature (2008)

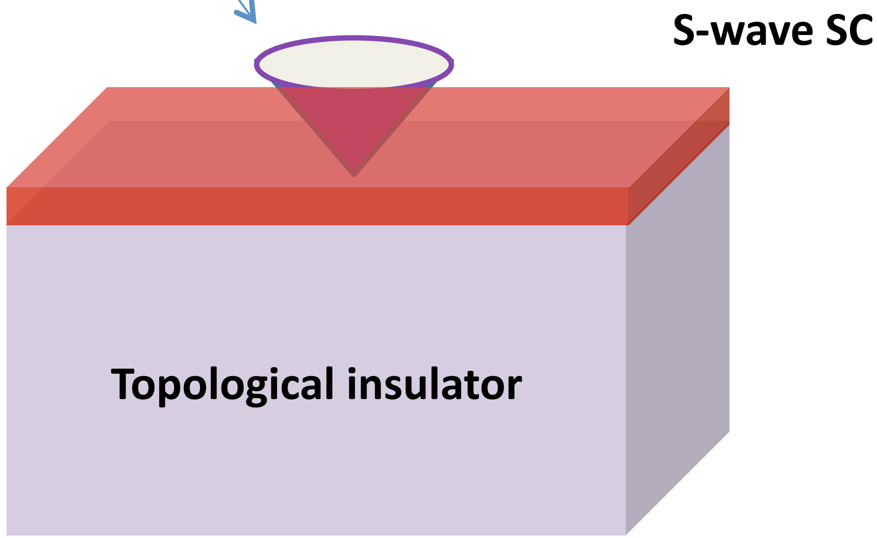


Nishide et al., PRB (2010)

Bi_2Se_3 Hsieh et al., Nature (2009)



Dirac fermion + s-wave SC



Spin-orbit interaction
=> topological insulator

2. S-wave superconductor with Rashba SO interaction

[MS, Takahashi, Fujimoto PRL(09) PRB(10)]

Rashba SO

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F} + \boxed{\boldsymbol{g_k} \cdot \boldsymbol{\sigma}} - \mu_{\rm B} H_z \sigma_z & i\Delta_0 \sigma_y \\ -i\Delta_0 \sigma_y & -\frac{(\hbar \boldsymbol{k})^2}{2m} + E_{\rm F} \boxed{+ \boldsymbol{g_k} \cdot \boldsymbol{\sigma}^*} + \mu_{\rm B} H_z \sigma_z \end{pmatrix}$$

$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = D\mathcal{H}(\boldsymbol{k})D^\dagger, \quad D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i\sigma_y \\ i\sigma_y & 1 \end{pmatrix}$$

$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = \begin{pmatrix} \Delta_0 - \mu_{\rm B} H_z \sigma_z & -i\left[\frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F}\right]\sigma_y - \boxed{i\boldsymbol{g_k} \cdot \boldsymbol{\sigma}\sigma_y} \\ i\left[\frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F}\right]\sigma_y + \boxed{i\boldsymbol{g_k}\sigma_y \cdot \boldsymbol{\sigma}} & -\Delta_0 + \mu_{\rm B} H_z \sigma_z \end{pmatrix}$$

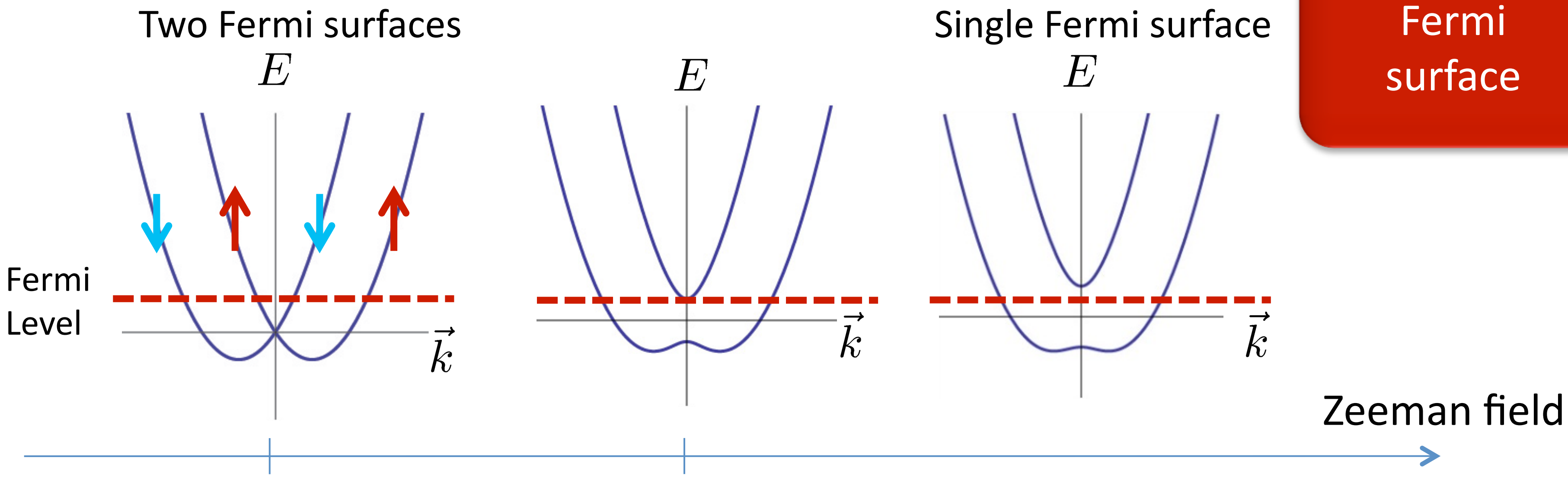
p-wave gap is induced by Rashba SO int.

Topological superconductivity can be obtained if we choose a suitable Fermi surface

Topological Edge state

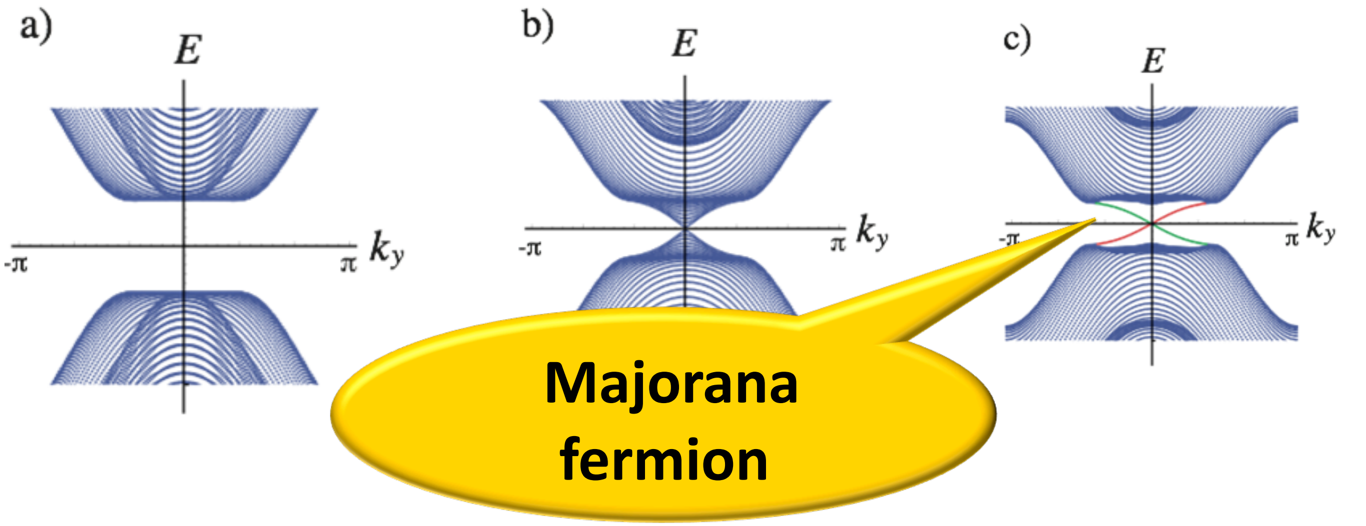
Sato-Takahashi-Fujimoto (09, 10)

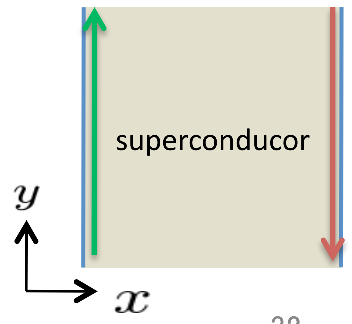
Fermi surface



$$\mu_{\mathrm{B}} H_z = 0 \qquad \mu_{\mathrm{B}} H_z = \sqrt{\Delta_0^2 + E_{\mathrm{F}}^2} \qquad \mu_{\mathrm{B}} H_z > \sqrt{\Delta_0^2 + E_{\mathrm{F}}^2}$$

Edge state





Majorana fermions are realized under a strong magnetic field satisfying $\mu_{\rm B} H_z > \sqrt{\Delta_0^2 + E_{\rm F}^2}$

MS-Takahashi-Fujimoto (09)

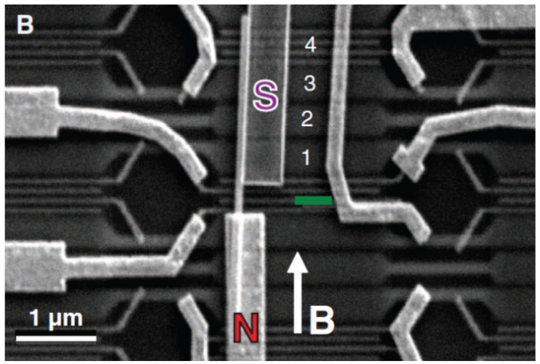
1D Nanowire

MF nanowire

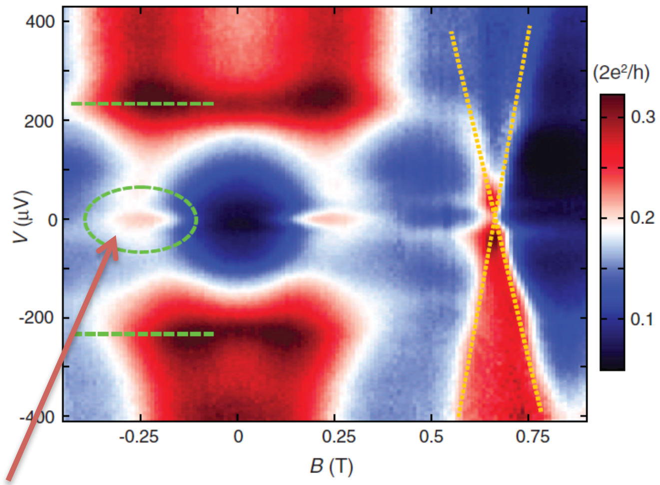
Zeeman field

Lutchyn et al (10), Oreg et al (10)

Mourik *et al.*, Science (2012)



InSb/
NbTiN



Majorana Fermion

Majorana fermions are not merely a theoretical possibility now, but they are what we can realize somehow in experiments.

Topological superconductor is not a fancy way to rewrite the existing theory of the Andreev bound state, but it can be a useful way to predict novel properties in SCs.

Symmetry Protected MFs in Superconductors

Y. Ueno, A. Yamakage, Y. Tanaka, MS, arXiv:1303.0202

MS, A. Yamakage, T. Mizushima, arXiv:1305.7469

Chui, Yao, Ryu, arXiv: 1303.1843
Zhang, Kane, Mele, arXiv:1303.4144

Majorana Fermions

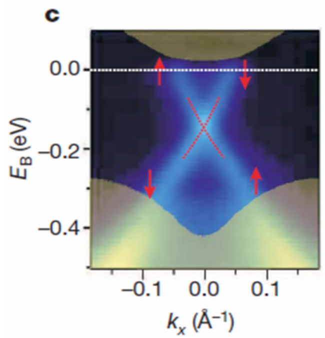
- Majorana fermions are naturally realized in spinless SCs

Spinless chiral p-wave SC Read- Green (00)

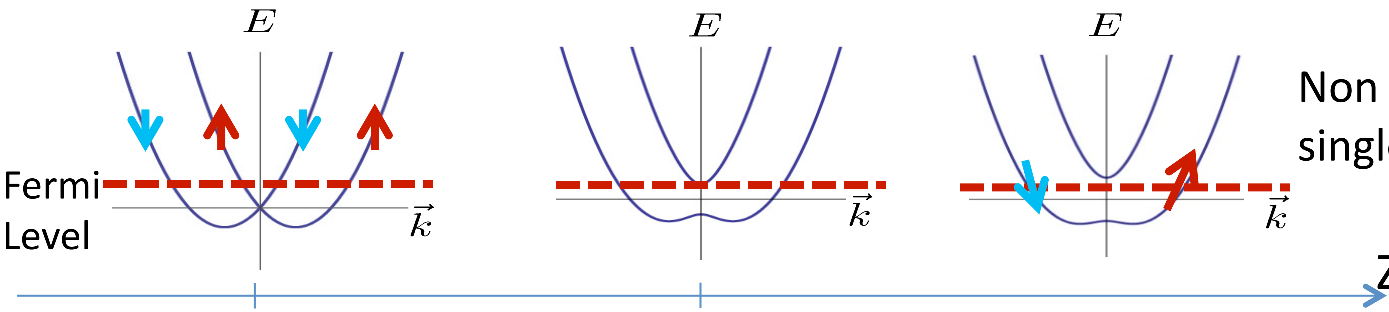
Dirac fermion + s-wave condensate MS(03), Fu-Kane (08)

S-wave superconducting state with Rashba SO + Zeeman field

MS-Takahashi-Fujimoto (09), J. Sau et al (10)



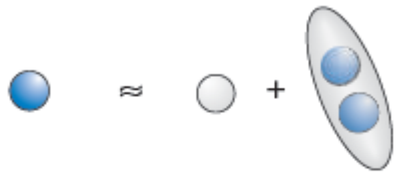
Hsieh et al



Non spin-degenerate
single Fermi surface

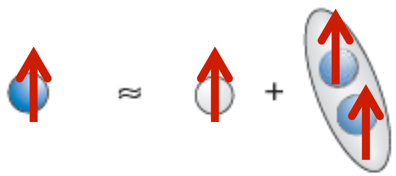
Why Majorana Fermions favor spinless SCs ?

For spinless SCs, we have the Majorana condition naturally.

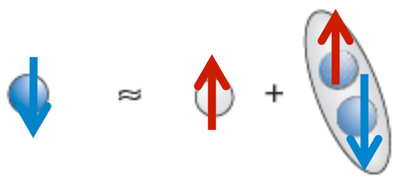


However, the spin degrees of freedoms obscure the Majorana condition

Nitta's talk at JPSJ meeting (12)

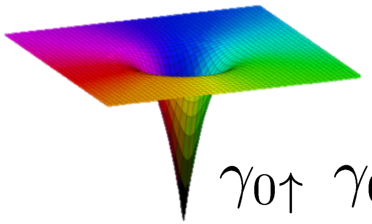


Majorana condition



~~Majorana condition~~

Moreover, spinful SCs support MFs in pairs because of the spin degeneracy.



$\gamma_{0\uparrow} \quad \gamma_{0\downarrow}$

$$\gamma_{0\uparrow}^{\dagger} = \gamma_{0\uparrow} \qquad \gamma_{0\downarrow}^{\dagger} = \gamma_{0\downarrow}$$

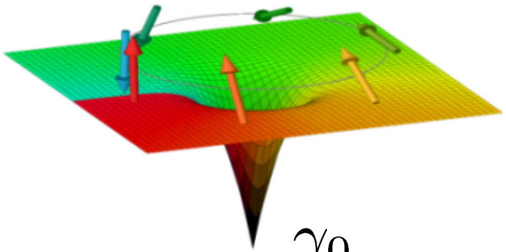
They can be considered as Dirac fermions as well as MFs

$$\psi = \gamma_{0\uparrow} + i\gamma_{0\downarrow}$$

The Dirac fermions are easily gapped away by the Dirac mass term $m\psi^{\dagger}\psi$

No topologically stable MFs

To avoid these difficulties, a half-quantum vortex has been considered. Ivanov (01)



$$\boldsymbol{d}(\boldsymbol{k},\boldsymbol{r}) = i\Delta_0 e^{i\theta/2}\underline{(\sin\theta/2, \cos\theta/2, 0)}$$

Twist in spin space

$$\hat{\Delta}(\boldsymbol{k},\boldsymbol{r}) = i\boldsymbol{d}(\boldsymbol{k},\boldsymbol{r})\cdot\boldsymbol{\sigma}\sigma_y = \begin{pmatrix} -\Delta_0 & 0 \\ 0 & \Delta_0 e^{i\theta} \end{pmatrix}$$

Only downspin sector $\hat{\Delta}_{\downarrow\downarrow}$ supports a vortex

= A vortex in spinless SCs

Topologically stabbe MF

However, the twist in spin space makes the configuration unstable, so the experimental realization is challenging.

Chung-Bluhm-Kim(07)

Question

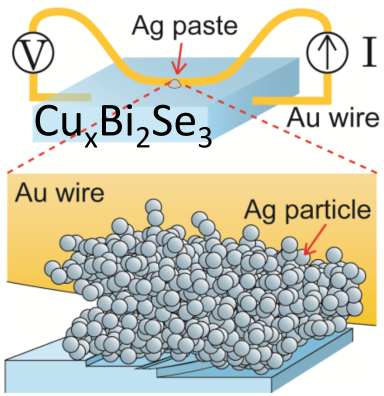
Is there another way to realize Majorana fermions in spinful SCs ?

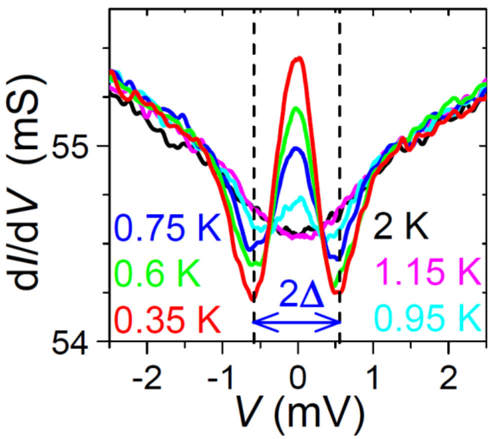
Key observation

If there is an additional symmetry such as time-reversal symmetry, Majorana fermions can be realized in spinful SCs

$Cu_xBi_2Se_3$

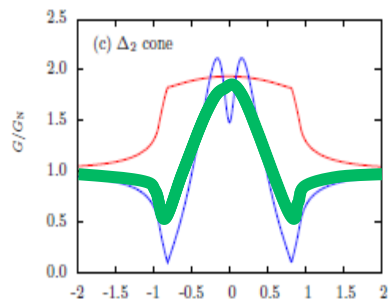
Fu-Berg (10)



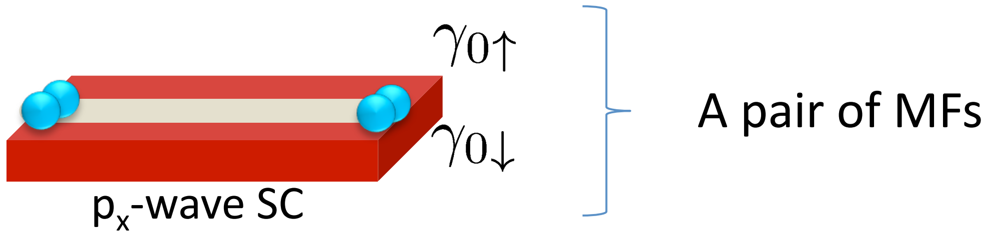


Sasaki-Kriener-Segawa-Yada-Tanaka-MS -Ando(11)

Yamakage--Yada-MS-Tanaka(12)



Ex.) 1D spinful p_x-wave superconductor



Kramers theorem

⇒ No scattering between $\gamma_{0\uparrow}$ and $\gamma_{0\downarrow}$

Thus, they naturally can be considered as two independent particles, not as a single Dirac fermion. $\cancel{\psi = \gamma_{0\uparrow} + i\gamma_{0\downarrow}}$

Actually, the Dirac mass term is forbidden by the time-reversal symmetry.

$$\cancel{m\psi^\dagger\psi = 2im\gamma_{0\uparrow}\gamma_{0\downarrow}}$$

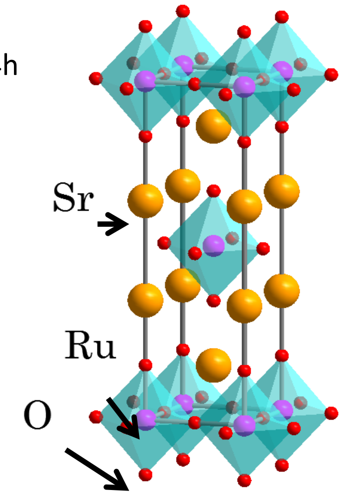
Topologically stable MF

Our idea

To obtain topologically stable MFs in spinful SCs, we use symmetry specific to material structures.

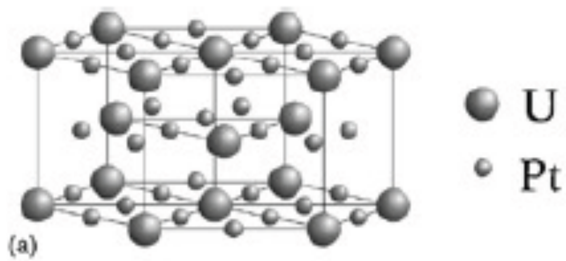
Ex.) point group symmetry

D_{4h}



Sr_2RuO_4

D_{6h}

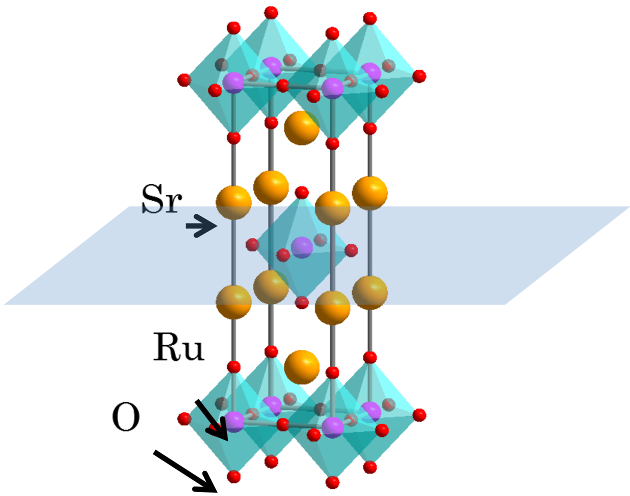


UPt_3

c.f.) Topological crystalline insulators

Fu (11)

In particular, we consider the mirror reflection symmetry



coordinate

$$(x, y, z) \to (x, y, -z)$$

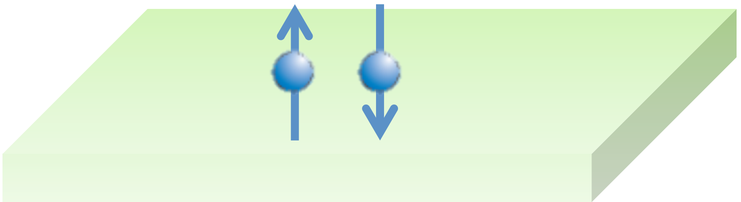
spin

$$(\sigma_x, \sigma_y, \sigma_z) \to (-\sigma_x, -\sigma_y, \sigma_z)$$

Now consider how the mirror reflection symmetry protect MFs

Our system

2dim spinful SC



BdG Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \mathcal{E}(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\mathcal{E}^T(-\boldsymbol{k}) \end{pmatrix}$$

Mirror symmetry $\tilde{\mathcal{M}}_{xy}$ with respect to xy-plane

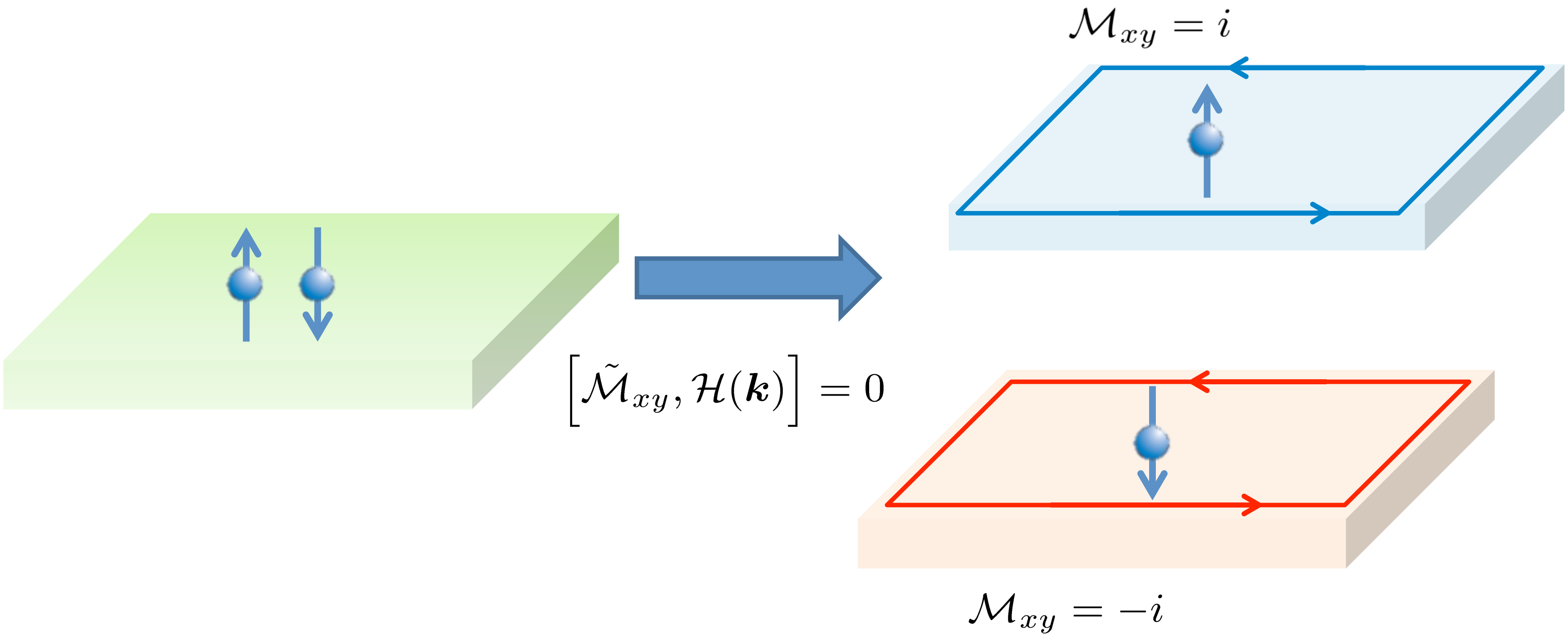
$$k_x \to k_x \quad k_y \to k_y$$

$$\sigma_x \to -\sigma_x \; \sigma_y \to -\sigma_y \quad \sigma_z \to \sigma_z \qquad (\tilde{\mathcal{M}}_{xy} \sim i\sigma_z)$$

$$\left[\tilde{\mathcal{M}}_{xy}, \mathcal{H}(\boldsymbol{k})\right] = 0$$

Basic idea

Using the eigen value of mirror operator, spinful SC can be separated into a pair of spinless SCs

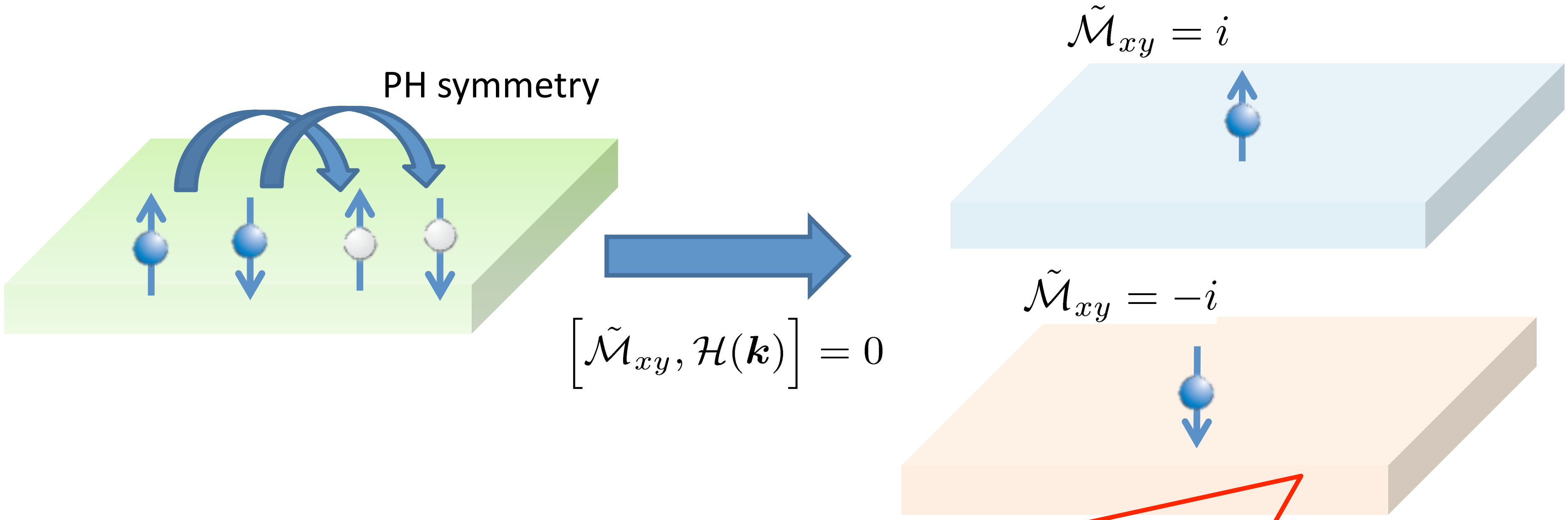


For each spinless SCs, the mirror Chern number can be defined like topological crystalline insulators (TCI).

However, there is an important difference between TCIs and SCs

Particle-hole symmetry = Majorana condition

$$\mathcal{C}\mathcal{H}(\boldsymbol{k})\mathcal{C}^{\dagger} = -\mathcal{H}^{*}(-\boldsymbol{k}) \qquad \Psi = \mathcal{C}\Psi^{\dagger}$$



The problem is how the particle-hole symmetry is realized in the spinless SCs.

Key point

Two different mirror symmetries are possible in SCs.

a) $\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t}=\Delta(\boldsymbol{k})$

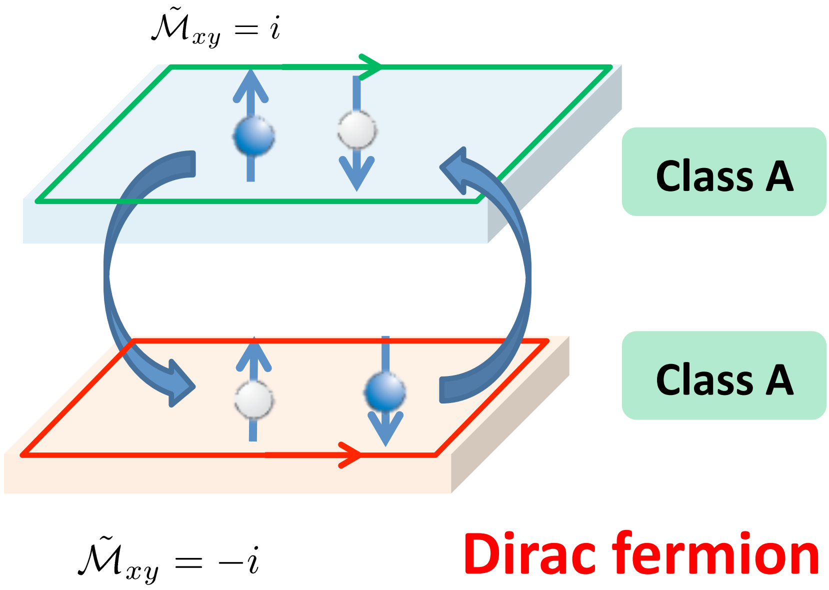
S-wave SC	$\Delta(\boldsymbol{k})=i\psi\sigma_y$	
Spin-triplet SC with $\boldsymbol{d}\parallel\boldsymbol{z}$	$\Delta(\boldsymbol{k})=i\boldsymbol{d}(\boldsymbol{k})\boldsymbol{\sigma}\sigma_y$	$(\tilde{\mathcal{M}}_{xy}\sim i\sigma_z)$

b) $\tilde{\mathcal{M}}_{xy}\Delta(\boldsymbol{k})\tilde{\mathcal{M}}_{xy}^{t}=-\Delta(\boldsymbol{k})$ $\xrightarrow{\text{U(1) gauge sym}}$ $\Delta(\boldsymbol{k})$

Spin-triplet SC with $\boldsymbol{d}\perp\boldsymbol{z}$	$\Delta(\boldsymbol{k})=i\boldsymbol{d}(\boldsymbol{k})\boldsymbol{\sigma}\sigma_y$

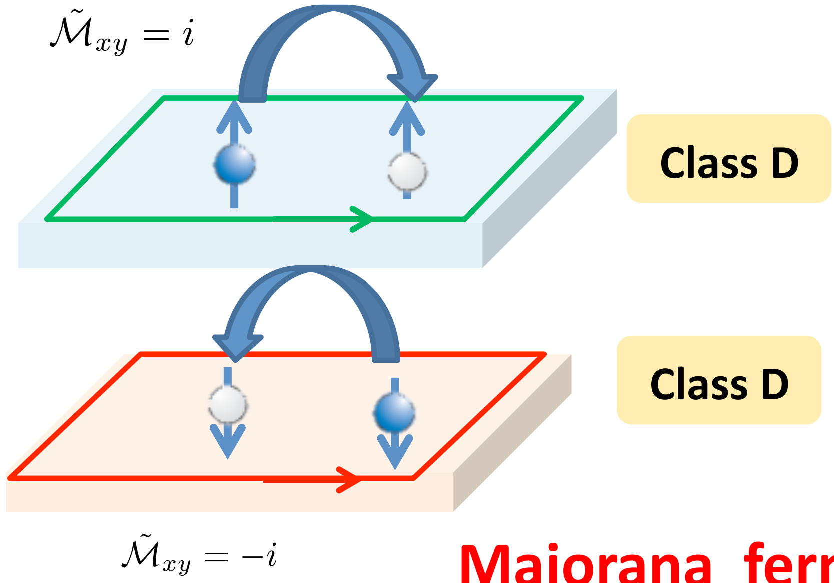
$$\boxed{\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = \Delta(\boldsymbol{k})}$$

Even



- Mirror subsector does not support its own particle-hole symmetry.
- Mirror subsector is topologically the same as quantum Hall states.
- Symmetry protected topological states are Dirac fermions.

$$\boxed{\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = -\Delta(\boldsymbol{k})}$$ **Odd**



Majorana fermion

- Mirror subsector supports its own particle-hole symmetry .
- Mirror subsector is topologically the same as spinless SCs.
- Symmetry protected topological states are Majorana fermions
- Majorana zero mode can exit in a vortex or in a dislocation

Schnyder et al (08)
Teo-Kane (10)

	1D	2D	3D
class D	Z_2	Z	-

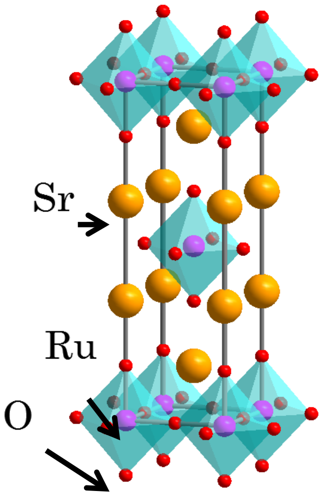
$$\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = \Delta(\boldsymbol{k})$$

- **No PH symmetry** in mirror subsector
- Mirror subsector is **class A**
- Only **Dirac fermions** are possible

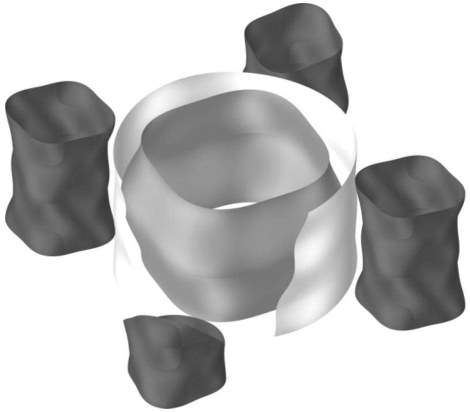
$$\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = -\Delta(\boldsymbol{k})$$

- **PH symmetry** in mirror subsector
- Mirror subsector is **class D**
- **Majorana fermions** are possible in each mirror sector

Application to Sr_2RuO_4



Fermi surface

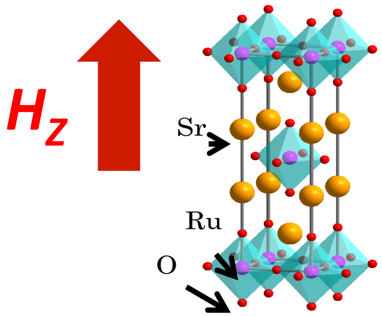


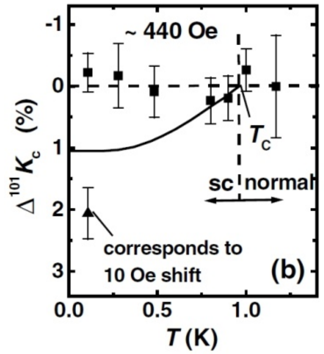
C. Bergemann

- (quasi) 2-dimensional time-reversal breaking spin-triplet SC

Maeno, Kittaka, Nomura, Yonezawa, Ishida, JPSJ (12)

- Interestingly, the NMR measurements have suggested that **d-vector is normal to the z-direction** in the presence of magnetic fields along the z-direction.





$$\boxed{\mathcal{M}_{xy}\Delta(\boldsymbol{k})\mathcal{M}_{xy}^{t} = -\Delta(\boldsymbol{k})}$$

Odd

We can expect Majorana Fermions !!

Our model

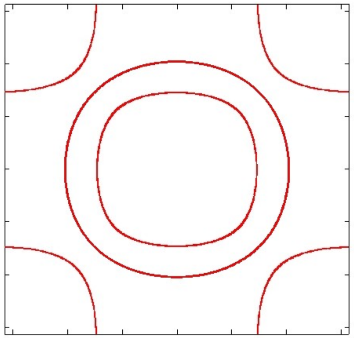
All three bands and relevant SO int. are taken into account.

$$\mathcal{H}_{\text{kin}} = \sum_{\boldsymbol{k},s} (c^\dagger_{\boldsymbol{k}s1}, c^\dagger_{\boldsymbol{k}s2}, c^\dagger_{\boldsymbol{k}s3}) \begin{pmatrix} \epsilon_{\boldsymbol{k}1} & g_{\boldsymbol{k}} & 0 \\ g_{\boldsymbol{k}} & \epsilon_{\boldsymbol{k}2} & 0 \\ 0 & 0 & \epsilon_{\boldsymbol{k}3} \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}s1} \\ c_{\boldsymbol{k}s2} \\ c_{\boldsymbol{k}s3} \end{pmatrix}$$

$$\mathcal{H}_{\text{SO}} = i\lambda \sum_{lmn} \epsilon_{lmn} \sum_{\boldsymbol{k}ss'} c^\dagger_{\boldsymbol{k}sl} c_{\boldsymbol{k}s'm} \sigma^n_{ss'}$$

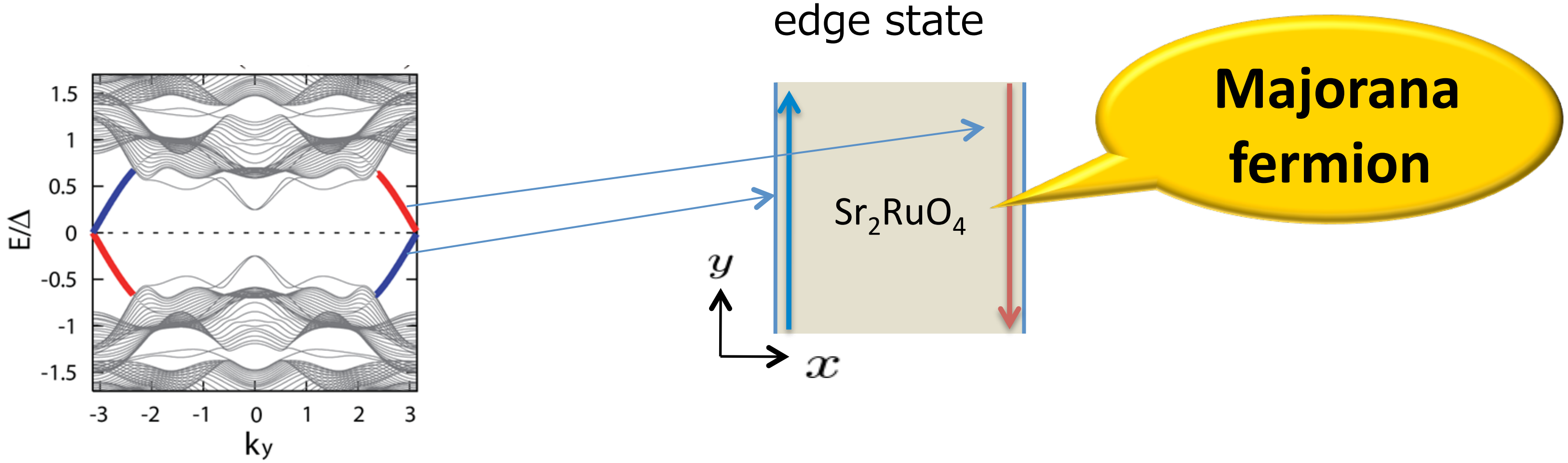
$$\mathcal{H}_{\text{pair}} = \frac{i}{2} \sum_{\boldsymbol{k}lss'} \Delta^l \boldsymbol{d}(\boldsymbol{k}) (\boldsymbol{\sigma}\sigma_y)_{ss'} c^\dagger_{\boldsymbol{k}sl} c^\dagger_{-\boldsymbol{k}s'l} + \text{h.c.}$$

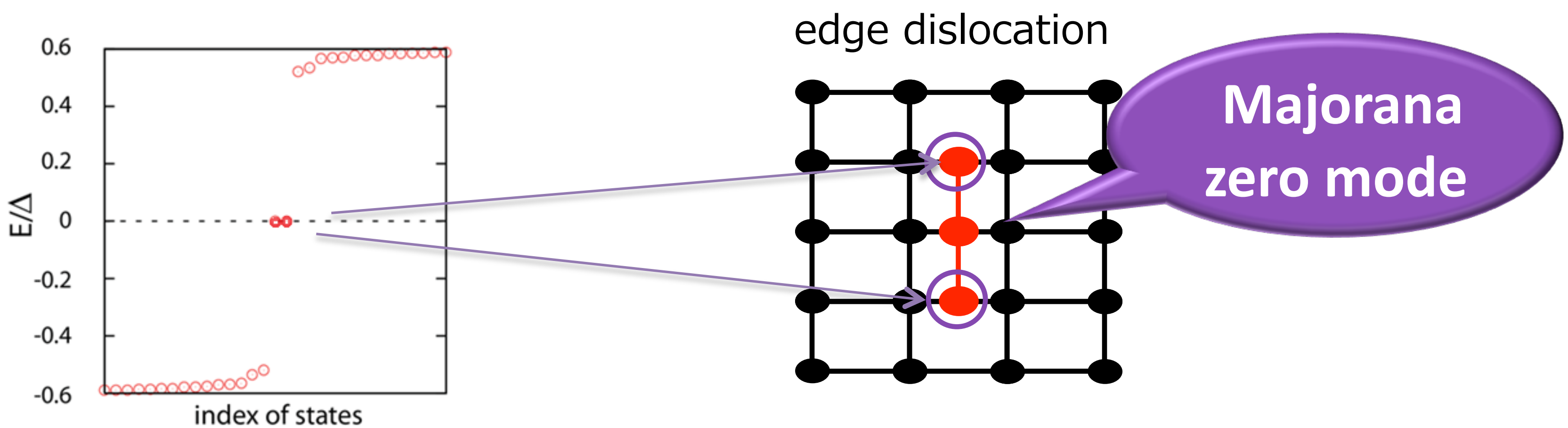
Fermi surface



Γ	$\boldsymbol{d}$-vector	$\mathcal{M}_{xy}$	subsector	$[\nu(\lambda), \nu_x(\lambda), \nu_y(\lambda)]$
A_u	$\hat{\boldsymbol{x}} \sin k_x + \hat{\boldsymbol{y}} \sin k_y$	odd	class D	$[\pm 1, 1, 1]_{\lambda=\pm i}$
	$\hat{\boldsymbol{x}} \sin k_y - \hat{\boldsymbol{y}} \sin k_x$			
B_u	$\hat{\boldsymbol{x}} \sin k_x - \hat{\boldsymbol{y}} \sin k_y$	odd	class D	$[\mp 1, 1, 1]_{\lambda=\pm i}$
	$\hat{\boldsymbol{x}} \sin k_y + \hat{\boldsymbol{y}} \sin k_x$			
E_u	$\hat{\boldsymbol{z}}(\sin k_x + i \sin k_y)$	even	class A	$[1, -, -]_{\lambda=\pm i}$

Topological state $(\mathcal{M}_{xy} = i \text{ sector})$ $\boldsymbol{d} = \hat{\boldsymbol{x}} \sin k_y - \hat{\boldsymbol{y}} \sin k_x$



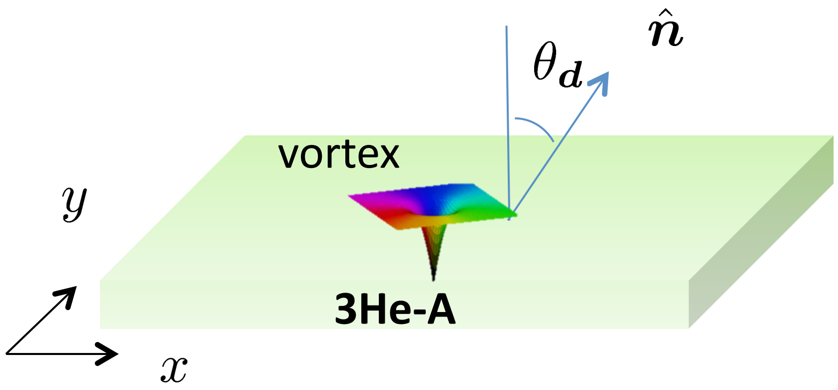


The other three mirror odd gap functions also show qualitatively the same results.

Our arguments also work for other unconventional SCs/SFs

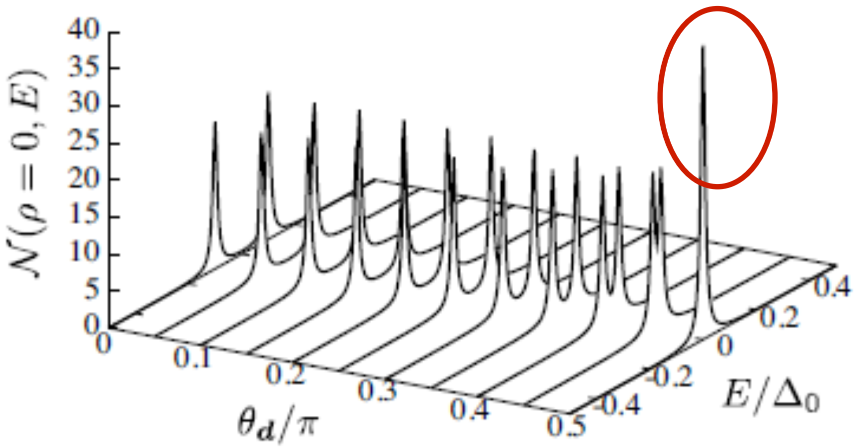
Thin film of ^{3}He-A

MS, Yamakage, Mizushima (13)



$$\boldsymbol{d} = \hat{\boldsymbol{n}}(k_x + ik_y)$$

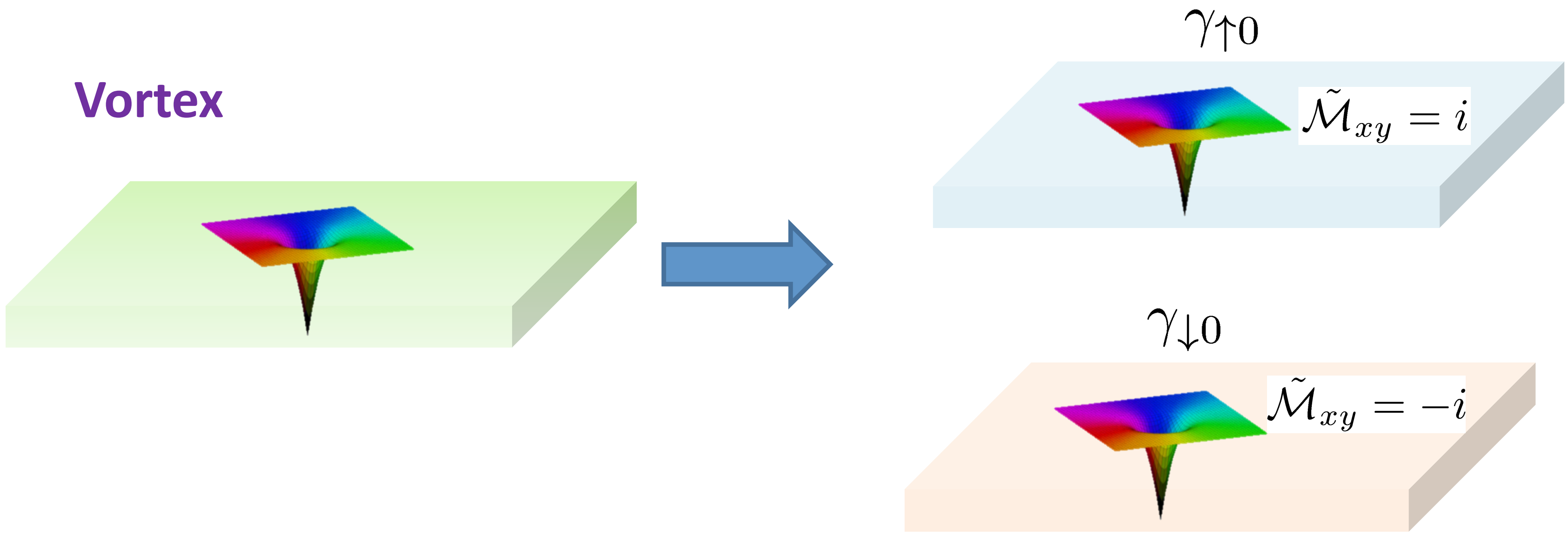
LDOS at core of integer vortex



Majorana zero modes exist in integer vortex when $\boldsymbol{d} \perp \boldsymbol{z}$ $(\theta_{\boldsymbol{d}} = \pi/2)$

Remark

These symmetry protected MFs show non-trivial phenomena like usual MF if the mirror symmetry is preserved.



Due to the existence of MF, a vortex in each mirror subsector obeys non-Abelian statisitics . Therefore, **a vortex in original spinful SC also obeys non-Abelian statistics**

Summary

1. We have revealed the condition necessary to obtain Majorana fermions protected by the mirror symmetry.

2. With the mirror symmetry, unconventinal spinful SCs can host Majorana fermions

   In particular, Sr_2RuO_4 can hosts Majorana zero mode in a vortex and a dislocation without considering half-quantum vortex

3. Our arguments are general and applicable to 3 dimensional spinful SCs.