## Topological Phases in Correlated Materials

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#### ISSP, University of Tokyo, June 5, 2013







#### **Topological Phases of Matter**

How can we characterize them?

What are the "Topological Properties"?

Where can we find them?

**Topological Phases of Matter** 

Cannot be fully characterized by a local order parameter such as magnetization in magnets

Cannot be transformed to "simple phases" via local perturbations/operations without going through phase transitions

Often characterized by a variety of "Topological Properties" or "Non-local Properties" Type I Topological Phases (Gapped Phases) **Topological Phases** No "Path" (Local unitary transformations) without closing the bulk gap (fully characterized by local order "simple phases" parameter/information) Quantum Hall States Spin Liquids

(correlated quantum paramagnetic state)

Type-I Topological Phases (Gapped Phases)

Quantum Hall States

Non-trivial ground state degeneracy:

 $\nu=1/3~$  quantum Hall state has "3" degenerate ground states on torus, but "1" on sphere

Non-trivial boundary states:

Edge state is a chiral Luttinger Liquid

Non-trivial topological invariant:  $\sigma_{xy} = \frac{1}{3} \frac{e^2}{h}$ 

Non-trivial excitations:

Fractionally charged e/3 Laughlin quasi-particles

Type-I Topological Phases (Gapped Phases)Spin LiquidsQuantum Paramagnet  $\langle S \rangle = 0$ Correlated insulator with no broken translational symmetry

Resonating Valence Bond state (RVB); Superposition of Valence Bond coverings



P.W.Anderson

**Rokhsar-Kivelson** 



Valence Bond

#### Construction of a Spin Liquid

- BCS superconductor (L x L lattice)
  average number of electrons per site = one (Half-filled)
  g(r r') \Leftrightarrow Cooper pair wave function
  BCS wave function  $|BCS\rangle \propto e^{\sum_{\mathbf{r},\mathbf{r}'} g(\mathbf{r}-\mathbf{r}')c^{\dagger}_{\mathbf{r}\uparrow}c^{\dagger}_{\mathbf{r}'\downarrow}|0\rangle$
- **RVB** wave function  $|RVB\rangle = P_G |BCS\rangle \propto \sum_{vb} A_{vb} |vb\rangle$
- $P_G$  exactly one particle per site; freeze charge fluctuations Hubbard  $U \rightarrow \infty$  in  $Un_{i\uparrow}n_{i\downarrow}$

 $|vb\rangle$  valence bond covering

$$A_{vb} = \prod_{\substack{\text{all valence}\\\text{bond }(\mathbf{r},\mathbf{r}')}} g(\mathbf{r} - \mathbf{r}')$$



No local measurement can distinguish these phases

#### Short 'Coherence Length' Limit

$$|\text{even}\rangle = \frac{1}{2}(|RVB\rangle + |RVB'\rangle) \qquad |\text{odd}\rangle = \frac{1}{2}(|RVB\rangle - |RVB'\rangle)$$

TWO topologically distinct valence bond coverings



Non-trivial ground state degeneracy

### **Elementary Excitations**

- Elementary excitations in superconductors
   Bogoliubov quasiparticles (zero average charge, S=1/2)
- Elementary excitations in the spin liquid state  $P_G (Bogoliubov quasiparticles) = Spinons (Q=0,S=1/2)$
- Fractionalization of electrons !

#### Non-trivial excitations

## Type II "Symmetry-Protected" Topological Phases (Gapped Phases)



"simple phases"

Topological Band Insulator (e.g. time-reversal symmetry)



a Z2 topological invariant

**Trivial Band Insulator** 

**Topological Band Insulator** 



C. L. Kane, E. Mele, L. Fu B. A. Bernevig, T. L. Hughes, X.-L. Qi, S. C. Zhang ....



2D time reversal invariant band structure has a Z2 topological invariant

**Trivial Band Insulator** 

**Topological Band Insulator** 



Spinkarepit value enfinversio Bepfetiger bandsharith lopposite parcity

## **3D** Topological Band Insulator

In 3D there are four  $Z_2$  invariants:  $(\nu; \nu_1 \nu_2 \nu_3)$ characterizing the bulk. These determine how surface states connect.

 $\nu=1$  : Strong Topological Insulator

Fermi surface encloses odd number of Dirac points



 $\nu=0:$  Weak Topological Insulator

Fermi surface encloses even number of Dirac points



L. Fu, C. L. Kane J. E. Moore, L. Balents R. Roy Non-trivial boundary states Non-trivial topological invariant Where can we find them ? especially in correlated materials



5d transition metal (Ir) oxides: New Playground

5d TM 3d TM 4f Ln (Fe,Co,Ni,Cu.) (Ce, Pr, Nd...) (Re, Os, Ir, Pt...) Energy(K) **10**<sup>5</sup> Coulomb U Coulomb U Spin-orbit Coulomb Crystal U field D 104 Crystal Spin-orbit coupling  $\lambda$ field D coupling  $\lambda$ **10**<sup>3</sup> Crystal 5d: U ~ 0.5-1 eV Spin-orbit field D 10<sup>2</sup> coupling  $\lambda$  $\lambda_{SO}$  ~ 0.5 eV

> Traditional playground for correlated electron physics

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# Honeycomb Iridates (A<sub>2</sub>IrO<sub>3</sub>) Type-I Topological Phases ?

#### Honeycomb Lattice of Ir<sup>4+</sup> Na<sub>2</sub> rO<sub>3</sub> 5d transition metal oxides lr Edge-Sharing **Ir** Oxygen Octahedra Na-+ × Na

Plane perpendicular to the [111] direction

#### 5d orbitals of Ir<sup>4+</sup>: large spin-orbit coupling

 $Ir^{4+} = [Xe] 4f^{14} 5d^5$ 



B.J.Kim, C. Kim, J.H.Park, T.W.Noh, G.Cao et al., PRL 101, 076402 (2008)

B.J.Kim, H.Takagi, et al, Science 323, 1329 (2009)

Crystal Field

Spin-Orbit Coupling

$$|\uparrow_{j}\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_{s}\rangle + |yz,\downarrow_{s}\rangle + |xy,\uparrow_{s}\rangle)$$
$$|\downarrow_{j}\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_{s}\rangle - |yz,\uparrow_{s}\rangle + |xy,\downarrow_{s}\rangle)$$

$$\mathcal{P}_{t2g}\mathbf{L}_{\ell=2}\mathcal{P}_{t2g} = -\mathbf{L}_{\ell=1}^{\text{eff}}$$

#### Strong Coupling Limit Considering only the Kitaev Model ? Super-exchange processes



Oxygen Octahedra

Isotropic Heisenberg Exchange suppressed

## Strong Coupling Limit the Kitaev Model ?

Including Hund's coupling and projecting to J<sub>eff</sub>=1/2 manifold



Isotropic Heisenberg Exchange suppressed G. Jackeli and G. Khaliullin, PRL 102, 256403 (2009)

#### Strong Coupling Limit the Kitaev Model ?

Including Hund's coupling and projecting to J<sub>eff</sub>=1/2 manifold

$$\mathcal{H}_{ij}^{(\gamma)} = -JS_i^{\gamma}S_j^{\gamma} \qquad \gamma = x, y, z$$

Exactly Solvable

Quantum Spin Liquid Ground State

Spin-1/2 Spinons are Majorana Fermions (anti-particles to themselves)



G. Jackeli and G. Khaliullin, PRL 102, 256403 (2009)



# Pyrochlore Iridates (A<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>) Type-II Topological Phases ?

#### Pyrochlore Iridates A<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>





A= Y, Ho, Dy, Tb, Gd, Eu, Sm, Nd, Pr

A=Y, Ln and Ir reside on the inter-penetrating two pyrochlore lattices (cubic, FCC Bravais lattice)

D. Yanagishima and Y. Maeno, JPSJ 70, 2880 (2001)K. Matsuhira et al JPSJ 76, 043706 (2007)

#### A<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub> Metal to Insulator Transition ?



K. Matsuhira et al JPSJ 76, 043706 (2007)

Also Earlier Data from Y. Maeno's Group (2001)

#### A<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub> Metal to Insulator Transition ?



#### 5d orbitals of Ir<sup>4+</sup>: large spin-orbit coupling

 $Ir^{4+} = [Xe] 4f^{14} 5d^5$ 



Construct tight binding model for J<sub>eff</sub>=1/2 + Hubbard U

Crystal Field

Spin-Orbit Coupling

$$|\uparrow_{j}\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_{s}\rangle + |yz,\downarrow_{s}\rangle + |xy,\uparrow_{s}\rangle)$$
$$|\downarrow_{j}\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_{s}\rangle - |yz,\uparrow_{s}\rangle + |xy,\downarrow_{s}\rangle)$$

 $\mathcal{P}_{t2g}\mathbf{L}_{\ell=2}\mathcal{P}_{t2g} = -\mathbf{L}_{\ell=1}^{\text{eff}}$ 

#### Generic Phase Diagram



W. Witczak-Krempa, Y. B. Kim (2012)

#### WSM = Weyl Semi-Metal

Semi-metal with 3D Dirac points in the bulk

#### Effect of Interaction: Hartree-Fock



Magnetic Insulator

Weyl Semi-Metal

Semi-Metal

W. Witczak-Krempa, Y. B. Kim, PRB 85, 045124 (2012)

A pair of Weyl fermion points related by inversion; carry opposite chirality

$$\mathcal{H} = \sum_{i=1}^{3} \mathbf{v}_i \cdot \mathbf{k} \ \sigma_i \qquad c = \operatorname{sign}(\mathbf{v}_1 \cdot \mathbf{v}_2 \times \mathbf{v}_3) = \pm 1$$



Surface State: Fermi arcs at the surface BZ

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Integer Quantum Hall state: C=I

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#### Effect of Interaction: Hartree-Fock



Magnetic Insulator

Weyl Semi-Metal

Semi-Metal

W. Witczak-Krempa, Y. B. Kim, PRB 85, 045124 (2012)

#### Minimal Hamiltonian: Luttinger Model

 $\vec{k}\cdot\vec{p}~$  expansion near  $\Gamma$ 

$$\mathcal{H}_0(k) = \alpha_1 k^2 + \alpha_2 (\vec{k} \cdot \vec{J})^2 + \alpha_3 (k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)$$
$$= \frac{k^2}{2\tilde{M}_0} + \frac{(\frac{5}{4}k^2 - \vec{k} \cdot \vec{J})^2}{2m} - \frac{(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)}{2M_c}$$

$$ec{J}=(J_x,J_y,J_z)$$
 are J=3/2,4 x 4 matrices

Quadratic band touching of two B. J. Yang, Y. B. Kim doubly-degenerate bands (2010)

$$\begin{aligned} & \text{Time-Reversal Symmetry Breaking} \\ H_{\text{field}} &= C_1 (H_x J_x + H_y J_y + H_z J_z) + C_2 (H_x J_x^3 + H_y J_y^3 + H_z J_z^3) \\ & = U_1 (\{k_x, k_y\} V_z + \{k_y, k_z\} V_y + \{k_z, k_x\} V_y) \\ & + D_2 (J_x J_y J_z + J_z J_y J_x) \\ & V_x &\equiv \{(J_y^2 - J_z^2) J_x\}, V_y &\equiv \{(J_z^2 - J_x^2) J_y\}, V_z &\equiv \{(J_x^2 - J_y^2) J_z\} \end{aligned}$$

T. Hsieh, L. Fu (2012) E. G. Moon, C. Xu, Y. B. Kim, L. Balents (2012)

Both types of perturbations can generate Weyl fermions



#### Time-Reversal Symmetry Breaking and Strain

$$\mathcal{H}' = -\delta(J_z^2 - \frac{5}{4}) - H(\cos(\theta)J_z + \sin(\theta)J_z^3),$$



E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

#### Long-range Coulomb interaction

Relevant in RG sense near the non-interacting limit

 $\varepsilon = 4 - d$  expansion leads to non-trivial interacting (isotropic) fixed point

Non-Fermi Liquid scaling in physical quantities

$$z \approx 1.8 \qquad C_v \sim T^{d/z} \approx T^{1.7} \qquad \chi \sim a + bT^{0.5}$$
  
$$\chi_3 = \left. \frac{\partial^3 M}{\partial H^3} \right|_{H=0} \sim T^{-1.7} \qquad \sigma(\omega, T) \sim T^{1/z} \mathcal{F}(\omega/T) \qquad \sigma_{xy} \sim M^{0.51}$$

E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

#### $Pr_2Ir_2O_7$



Large anomalous Hall effect with "small" magnetization

Large  $\chi_3$ 

he suscep-F scheme 0.77 K) to mot be as Beyond Single-Particle Picture Cluster Dynamical Mean-Field Theory (CDMFT)

#### Interaction Effect in CDMFT

$$H = \sum_{\langle \mathbf{R}i, \mathbf{R}'i' \rangle, \sigma\sigma'} ([T_o]_{\sigma\sigma'}^{ii'} + [T_d]_{\sigma\sigma'}^{ii'}) c_{\mathbf{R}i\sigma}^{\dagger} c_{\mathbf{R}'i'\sigma'} - \mu \sum_{\mathbf{R}i, \sigma} c_{\mathbf{R}i\sigma}^{\dagger} c_{\mathbf{R}i\sigma} + U \sum_{\mathbf{R}i} n_{\mathbf{R}i\uparrow} n_{\mathbf{R}i\downarrow}$$



## Cluster (Tetrahedron) with N<sub>c</sub>=4

#### Bath Sites with Nb=8

A. Go, W. Witczak-Krempa, G.S.Jeon, K. Park, Y.B.Kim, PRL (2012)

## Green's function criteria for inversion correlated topological phases symmetry

$$\hat{G}^{-1}(\omega, \Gamma_i) |\alpha(\omega, \Gamma_i)\rangle = \mu_{\alpha}(\omega, \Gamma_i) |\alpha(\omega, \Gamma_i)\rangle$$

$$\hat{P} |\alpha(\omega, \Gamma_i)\rangle = \eta_{\alpha} |\alpha(\omega, \Gamma_i)\rangle$$

$$\hat{\Gamma}_i : \text{TRIM (time reversative invariant momentum)}$$

$$\hat{P} |\alpha(\omega, \Gamma_i)\rangle = \eta_{\alpha} |\alpha(\omega, \Gamma_i)\rangle$$

 $\eta_{\alpha} = \pm 1$  Parity Eigenvalue

$$(-1)^{\Delta} = \prod_{\text{R-zero}} \eta_{\alpha}^{1/2}$$

R-zero  $\Leftrightarrow \mu_{\alpha}(0, \mathbf{k}) > 0$ 

 $\Delta=0,1$ 

 $\Delta = 1$  : Correlated Topological Insulator

Z. Wang, X.-L.Qi, and S.-C. Zhang, arXiv: 1201.6431 (2012)

## Green's function criteria for inversion correlated topological phases symmetry

$$(-1)^{\Delta} = \prod_{\text{R-zero}} \eta_{\alpha}^{1/2}$$

R-zero  $\Leftrightarrow \mu_{\alpha}(0, \mathbf{k}) > 0$ 

$$\begin{array}{ll} \text{Magneto-Electric} & \mathbf{P} = \theta \frac{\alpha}{(2\pi)^2} \mathbf{B} & \mathcal{L}_{\theta} = \theta \frac{\alpha}{(2\pi)^2} \mathbf{E} \cdot \mathbf{B} \\ & \text{effect} & \end{array}$$

For TI, 
$$\Delta=1$$
  $heta=\pi$ 

Z. Wang, X.-L.Qi, and S.-C. Zhang, arXiv: 1201.6431 (2012)



A. Go, W. Witczak-Krempa, G.S.Jeon, K. Park, Y.B.Kim, PRL 109, 066401 (2012)

TWS: Topological Weyl Semimetal

AI: Axion Insulator  $\Delta = 1$   $\theta = \pi$  Magnetic Order  $\mathbf{P} = \theta \frac{\alpha}{(2\pi)^2} \mathbf{B}$   $\mathcal{L}_{\theta} = \theta \frac{\alpha}{(2\pi)^2} \mathbf{E} \cdot \mathbf{B}$  Inversion Symmetry



TWS: Topological Weyl Semimetal

AI: Axion Insulator  $\Delta = 1$   $\theta = \pi$  Magnetic Order Gapped surface state Magneto-Electric Effect

possibility discussed in X. Wan, et al, PRB 83, 205101 (2011)



TWS: Topological Weyl Semimetal

AI: Axion Insulator  $\Delta = 1$   $\theta = \pi$  Magnetic Order Correlation in the cluster important; non-existent in HF

possibility discussed in X. Wan, et al, PRB 83, 205101 (2011)

Connection to Experiments

#### muSR: Magnetic Order ?



S. Zhao, D. E. MacLaughlin, S. Nakatsuji et al, (2011)

#### $Eu_2Ir_2O_7$

Eu<sup>3+</sup> is non-magnetic Magnetic moments from  $|r^{4+}|$  $T_M = 120K$ 

Neutron (elastic/inelastic) Non-magnetic muSR A-site  $Eu_2Ir_2O_7$ Yes (< 120K) ? (single crystal) S. Zhao, D. MacLaughlin, Nakatsuji et al, (2011)  $< 0.5 \mu_B$  $Y_2Ir_2O_7$ Yes ( < 150K) M.C. Shapiro, I. R. Fisher et al, (2012) S. M. Disseler et al, (2012) S. M. Disseler et al, (2012) Magnetic A-site Nd<sup>3+</sup> ordered ? K. Tomiyasu, K. Yamada et al, (2011) Damped (< 8K)  $Nd_2Ir_2O_7$ Not detected S. M. Disseler et al, (2012) J=9/2 S. M. Disseler et al, (2012)

**Yb**<sub>2</sub>**I**r<sub>2</sub>**O**<sub>7</sub> J=7/2

**Yes ( < 130K)** 5. M. Disseler et al, (2012)

 $< 0.5 \mu_B$  S. M. Disseler et al, (2012)

#### Elastic Resonant X-ray Scattering

H. Sagayama, D. Uematsu, T. Arima, K. Sugimoto, J. J. Ishikawa, E. O'Farrell, S. Nakatsuji (2013)

#### $Eu_2Ir_2O_7$

q=0 magnetic order confirmed

But need inelastic resonant X-ray scattering to directly prove the all-in/all-out order

#### Finite Temperature (Hartree-Fock)



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#### High Pressure Experiment



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Eu_2Ir_2O_7
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F. F. Tafti,
J. J. Ishikawa,
A. McCollam,
S. Nakatsuji,
S. R. Julian,
2012
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## Spin-Orbit + Interactions + Frustration Exotic Phases of Matter !

