

Quantum Criticality and Orbital-dependent
Renormalization of Quasiparticles in
 $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$

–Importance of spatial correlation near a magnetic QCP–

Naoya Arakawa

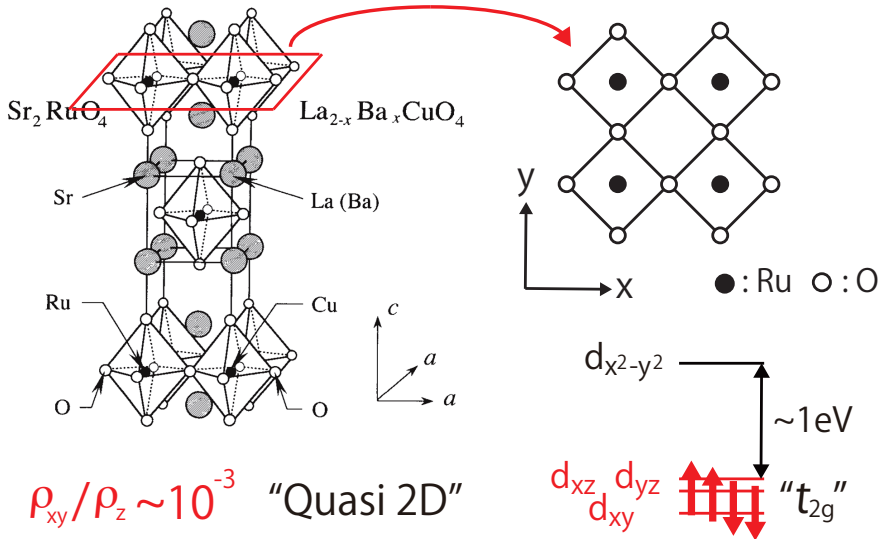
The University of Tokyo

Acknowledgements: Y. Yanase, T. Kariyado,
H. Kontani, S. Onari, Y. Yamakawa, ISSP Super Comp.

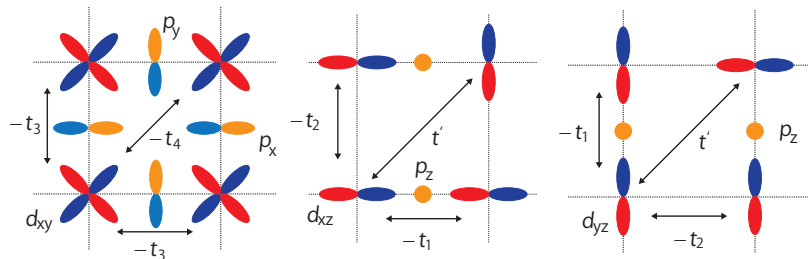
- ▶ Introduction
 - ▶ Electronic structure for Ru oxides; e.g., Sr_2RuO_4
 - ▶ Importance of Ru t_{2g} orbitals and Octahedral distortions
 - ▶ Heavy fermion behavior in $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$ around $x = 0.5$
- ▶ Method
 - ▶ Effective model in the presence of the octahedral rotation
 - ▶ Fluctuation-exchange (FLEX) approximation
- ▶ Results: Mag. prop.s and Renormalization of QPs
 - ▶ Roles of the octahedral rotation
 - ▶ Roles of the van Hove singularity (vHs) for the d_{xy} orbital
- ▶ Summary and Message

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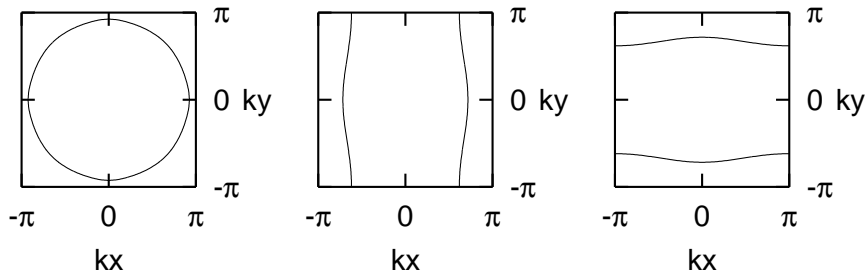
Electronic structure for Ru oxides; e.g., Sr_2RuO_4



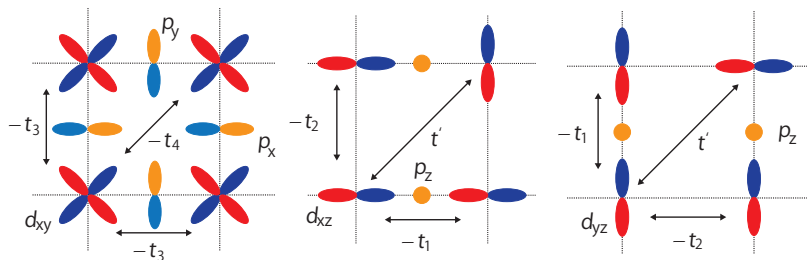
- ▶ Cond. bands: **Antibonding orbitals between Ru t_{2g} and O $2p$**



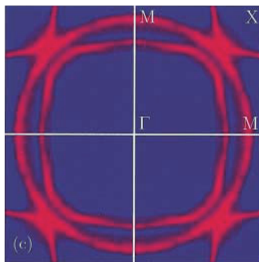
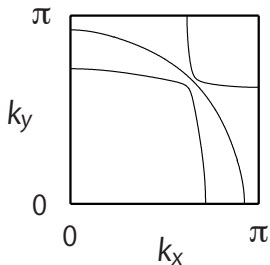
- ▶ quasi 2D γ -FS and quasi 1D α/β -FS:



- ▶ Cond. bands: **Antibonding orbitals between Ru t_{2g} and O 2p**



- ▶ quasi 2dim. γ -FS and quasi 1dim. α/β -FS:



Importance of Ru t_{2g} orbitals and Octahedral distortions

e.g. $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$

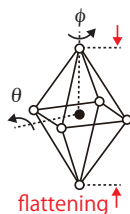
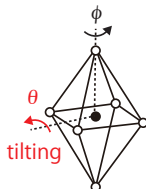
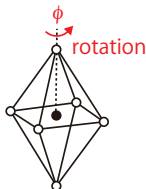
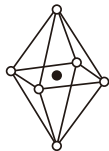
$x = 2$

$1.5 > x \geq 0.5$

$0.5 > x \geq 0.2$

$0.2 > x \geq 0$

RuO_6



Ground states

Spin triplet SC

Paramag. metal
 $x \sim 0.5$: Heavy fermion

Paramag. metal
with Metamag. trans.

AF insulator

e.g., see S. Nakatsuji *et al.*, PRL **90**, 137202 (2003); O. Friedt *et al.*, PRB **63**, 174432 (2001)

Cf. Wrong proposal by μSR : $m_{\text{AF}} \sim 0.25\mu_{\text{B}}$ at $x = 1.5$

→ This should be understood as **short-range order**.

∴ **Elastic neutron**: $m_{\text{AF}} = 0$ at $x = 1.5$

Heavy fermion behavior in $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$ around $x = 0.5$

Q. WHAT is more important in enhancing m^* than the location of the vHs?

e.g. $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$

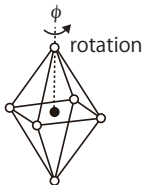
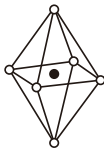
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RuO_6



vHs

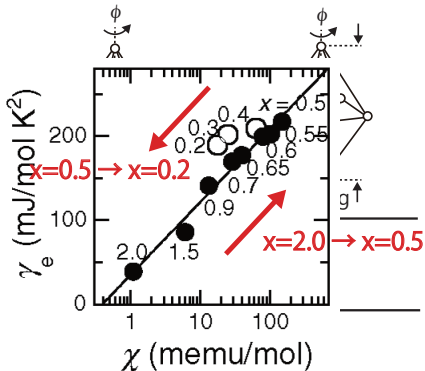
60 meV

-20 meV
@ $x = 0.5$

γ_e

37.5 mJ/mol K²

255 mJ/mol K²
@ $x = 0.5$

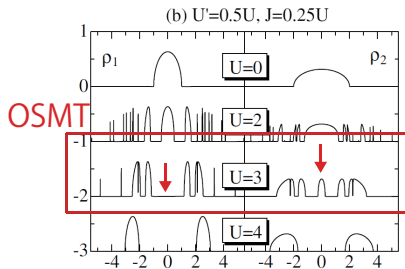


e.g., see S. Nakatsuji *et al.*, PRL **90**, 137202 (2003); O. Friedt *et al.*, PRB **63**, 174432 (2001)

Cf. Case of La-doped Sr_2RuO_4 : $\gamma_e \approx 50 \text{ mJ/mol K}^2$

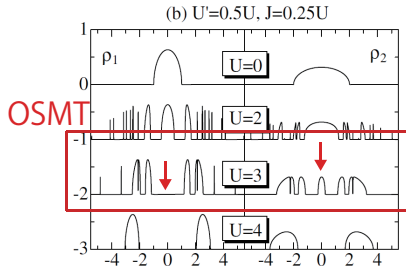
N. Kikugawa *et al.*, PRB **70**, 060508(R) (2004)

► Proposal of Orbital-selective MT for d_{xz}/yz around $x = 0.5$



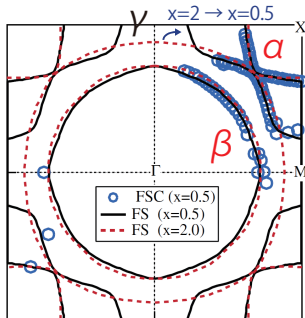
V. I. Anisimov et al., Eur. Phys. J. B (2002); A. Koga et al., PRL (2004)

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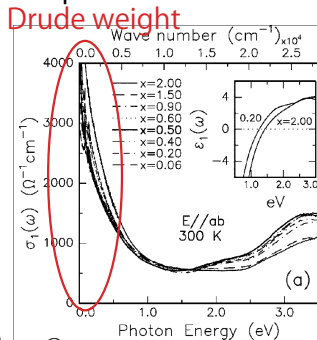


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► Its inconsistency with ARPES and Optical measurements

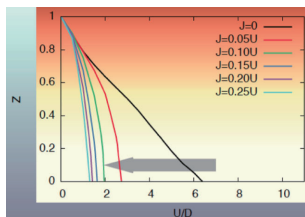


S.-C. Wang *et al.*, *PRL* (2004); J. S. Lee *et al.*, *PRL* (2003)

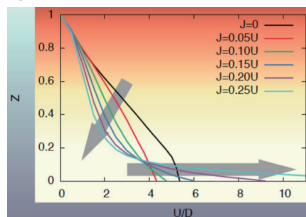


► “Hund’s metal”: Metal, extended by $J \nearrow$, with small Z

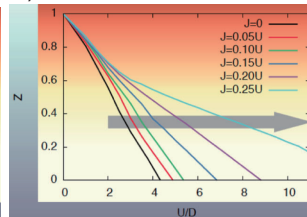
i) $N=3$ in 3 orbitals/site



ii) $N=2$ in 3 orbitals/site



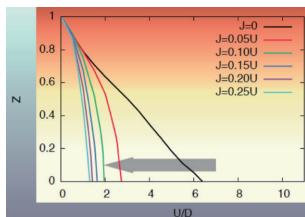
iii) $N=1$ in 3 orbitals/site



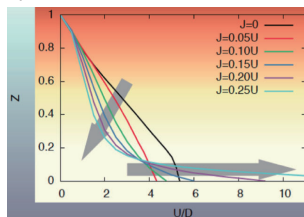
L. de' Medici *et al.*, PRL (2011)

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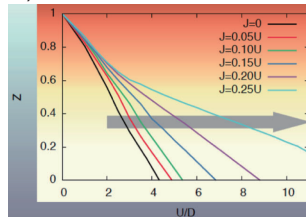
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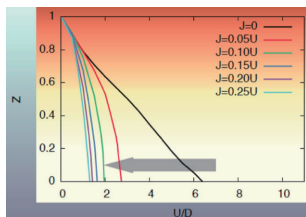
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J [eV]	$m^*/m_{\text{LDA}} _{xy}$	$m^*/m_{\text{LDA}} _{xz}$	T_{xy}^* [K]	T_{xz}^* [K]
0.0, 0.1	1.7	1.7	>1000	>1000
0.2	2.3	2.0	300	800
0.3	3.2	2.4	100	300
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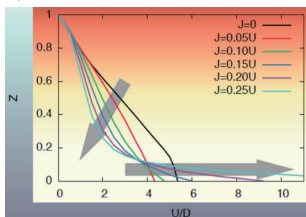
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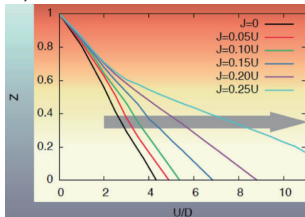
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▶ Its inconsistency with experimental results

▶ Exp. for $\text{Ca}_{2-x}\text{Sr}_x\text{RuO}_4$ ($x < 2$): $(m_{xy}^*/m_{xy}) \gg (m_{xz/yz}^*/m_{xz/yz})$

e.g., J. S. Lee et al., PRL (2003)

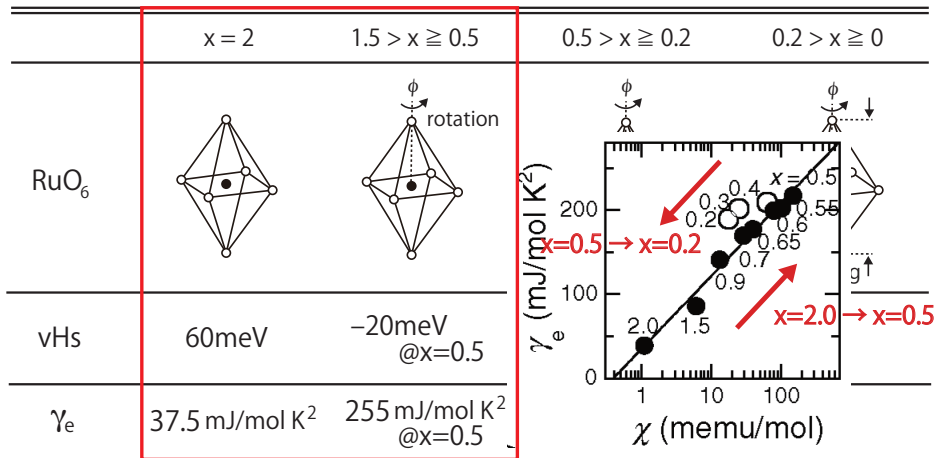
▶ Gutzwiller analysis for $x = 0.5$: $(m_{xy}^*/m_{xy}) < (m_{xz/yz}^*/m_{xz/yz})$

Gutzwiller analysis: N. Arakawa and M. Ogata, PRB **86**, 125126 (2012)

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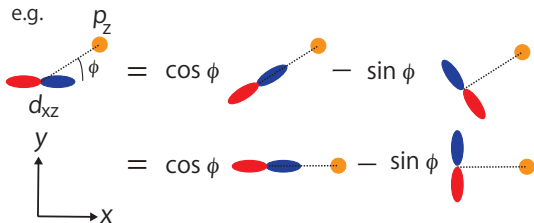
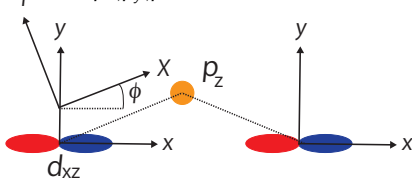
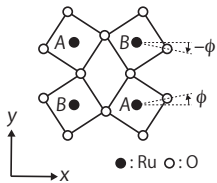
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Effective model in the presence of the octahedral rotation

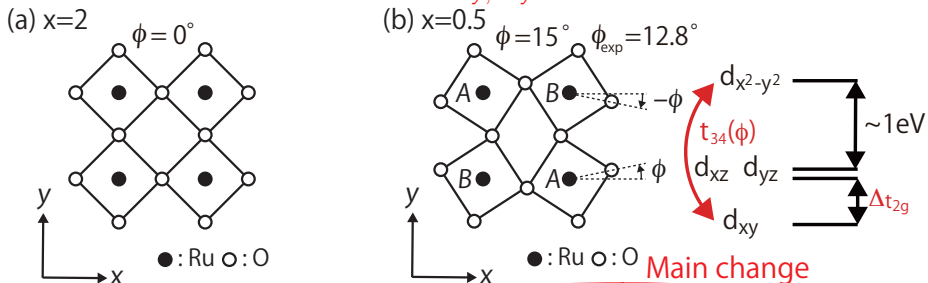
$$\hat{H} = \sum_{\mathbf{k}} \sum_{a,b=1}^3 \sum_{l,l'=A,B} \epsilon_{ab}^{ll'}(\mathbf{k}, \phi) \hat{c}_{\mathbf{k}al}^\dagger \hat{c}_{\mathbf{k}bl'} + U \sum_{i,a} \hat{n}_{ia\uparrow} \hat{n}_{ia\downarrow} + U' \sum_i \sum_{a>b} \hat{n}_{ia} \hat{n}_{ib}$$

$$- J_H \sum_i \sum_{a>b} (2\hat{s}_{ia} \cdot \hat{s}_{ib} + \frac{1}{2} \hat{n}_{ia} \hat{n}_{ib}) + J' \sum_i \sum_{a>b} \hat{c}_{ia\uparrow}^\dagger \hat{c}_{ia\downarrow}^\dagger \hat{c}_{ib\downarrow} \hat{c}_{ib\uparrow}$$

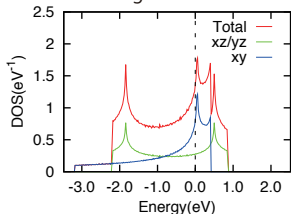
► Change of $V_{ab}(\phi)$; $t_{aa'}^{AB}(\phi) = \sum_{b=p_x, p_y, p_z} \frac{V_{ab}(\phi) V_{ba'}(-\phi)}{E_a(0^\circ) - E_b(\phi)}$



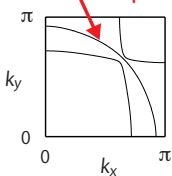
- ▶ Main changes induced by the octahedral rotation:
 - ▶ Reduction of the NN hopping int.s (mainly for d_{xy})
 - ▶ Downwards shift for d_{xy} due to $t_{xy,x^2-y^2}^{AB}(\phi)$



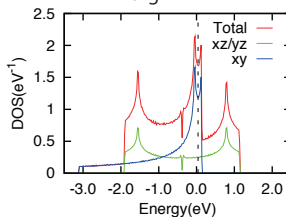
$\Delta t_{2g} = 0 \text{ eV}$



electron γ -FS

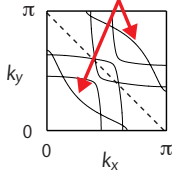


$\Delta t_{2g} = 0.42 \text{ eV}$



Main change

hole γ -FS



Cf. Difference with LSDA: Small FS $\mathbf{k} \approx (0, 0)$ due to $\epsilon_{xy,x^2-y^2}^{AB}(\mathbf{k}, \phi)$

Treatment of electron correlation: FLEX approximation

$$\hat{H} = \sum_{\mathbf{k}} \sum_{a,b=1}^3 \sum_{l,l'=A,B} \epsilon_{ab}^{ll'}(\mathbf{k}, \phi) \hat{c}_{\mathbf{k}al}^\dagger \hat{c}_{\mathbf{k}bl'} + U \sum_{i,a} \hat{n}_{ia\uparrow} \hat{n}_{ia\downarrow} + U' \sum_i \sum_{a>b} \hat{n}_{ia} \hat{n}_{ib}$$
$$- J_H \sum_i \sum_{a>b} (2\hat{\mathbf{s}}_{ia} \cdot \hat{\mathbf{s}}_{ib} + \frac{1}{2} \hat{n}_{ia} \hat{n}_{ib}) + J' \sum_i \sum_{a>b} \hat{c}_{ia\uparrow}^\dagger \hat{c}_{ia\downarrow}^\dagger \hat{c}_{ib\downarrow} \hat{c}_{ib\uparrow}$$

- ▶ Merits of the FLEX approx.:
 - ▶ To partially take account of the mode-mode coupling
 - ▶ To satisfy several conservation laws automatically
- Possible to discuss properties at low T near a QCP!

Treatment of electron correlation: FLEX approximation

$$\hat{H} = \sum_{\mathbf{k}} \sum_{a,b=1}^3 \sum_{l,l'=A,B} \epsilon_{ab}^{ll'}(\mathbf{k}, \phi) \hat{c}_{\mathbf{k}al}^\dagger \hat{c}_{\mathbf{k}bl'} + U \sum_{i,a} \hat{n}_{ia\uparrow} \hat{n}_{ia\downarrow} + U' \sum_i \sum_{a>b} \hat{n}_{ia} \hat{n}_{ib} - J_H \sum_i \sum_{a>b} (2\hat{s}_{ia} \cdot \hat{s}_{ib} + \frac{1}{2} \hat{n}_{ia} \hat{n}_{ib}) + J' \sum_i \sum_{a>b} \hat{c}_{ia\uparrow}^\dagger \hat{c}_{ia\downarrow}^\dagger \hat{c}_{ib\downarrow} \hat{c}_{ib\uparrow}$$

▶ Merits of the FLEX approx.:

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→ Possible to discuss properties at low T near a QCP!

▶ FLEX approx. for $\Phi_{\text{LW}}[\hat{G}]$ consisting of el-h bubbles and ladders

$$1. \hat{G}(k) = \hat{G}^0(k) + \hat{G}^0(k) \hat{\Sigma}(k) \hat{G}(k)$$

$$2. \hat{\chi}(q) = -\frac{T}{N} \sum \hat{G}(k) \hat{G}(k+q)$$

$$3. \hat{\chi}^S(q) = (\hat{1} - \hat{\chi}(q) \hat{\Gamma}^S)^{-1} \hat{\chi}(q), \quad \hat{\chi}^C(q) = (\hat{1} - \hat{\chi}(q) \hat{\Gamma}^C)^{-1} \hat{\chi}(q)$$

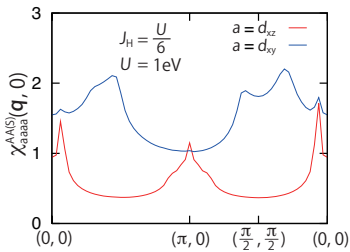
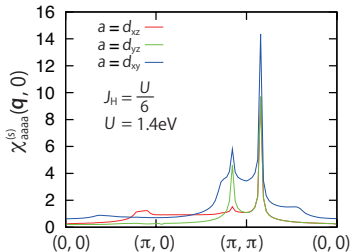
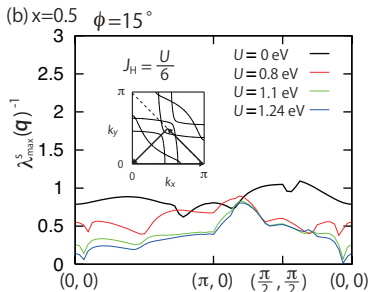
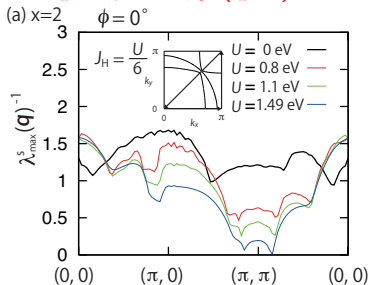
$$4. \hat{V}(q) = \frac{3}{2} \hat{\Gamma}^S \hat{\chi}^S(q) \hat{\Gamma}^S + \frac{1}{2} \hat{\Gamma}^C \hat{\chi}^C(q) \hat{\Gamma}^C + \left(\frac{3}{2} \hat{\Gamma}^S + \frac{1}{2} \hat{\Gamma}^C \right) - \hat{\Gamma}^{\uparrow\downarrow} \hat{\chi}(q) \hat{\Gamma}^{\uparrow\downarrow}$$

$$5. \hat{\Sigma}(k) = \frac{T}{N} \sum_q \hat{V}(q) \hat{G}(k-q)$$

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Roles of the rotation in Mag. prop.s

- ▶ IC FM spin fluc.s ↗, related with two dominant fluc.s for $x = 2$
- ▶ Flat q dep. of $\hat{\chi}^S(\mathbf{q}, 0)$



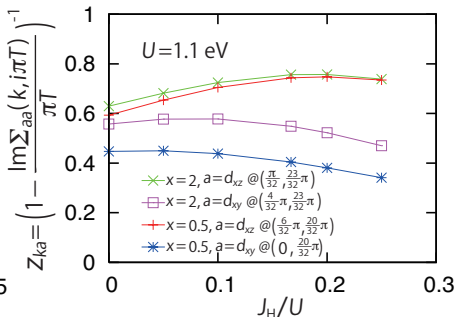
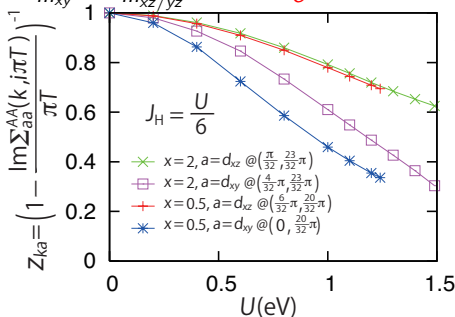
BZ = 64×64 , 1024 Matsubara freq., $T = 0.007 \text{ eV}$

Roles of the rotation in Renormalization of QPs

▶ Enhancement of $\frac{m_{xy}^*}{m_{xy}} (\sim Z_{kd_{xy}}^{-1})$ and $\frac{m_{xz/yz}^*}{m_{xz/yz}} (\sim Z_{kd_{xz/yz}}^{-1})$

▶ Enhancement of $\frac{m_{xy}^*}{m_{xz/yz}^*}$

▶ $\frac{m_{xy}^*}{m_{xy}} > \frac{m_{xz/yz}^*}{m_{xz/yz}}$ for all $\frac{J_H}{U}$



Cf. DMFT for $x = 2$ with $U = 2.1\text{eV}$;

J [eV]	$m^*/m_{\text{LDA}} _{xy}$	$m^*/m_{\text{LDA}} _{xz}$	T_{xy}^* [K]	T_{xz}^* [K]
0.0, 0.1	1.7	1.7	>1000	>1000
0.2	2.3	2.0	300	800
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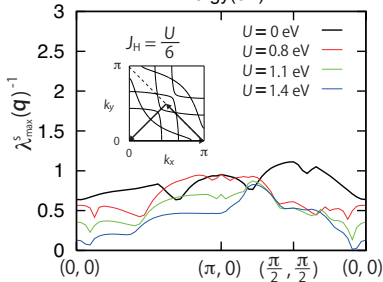
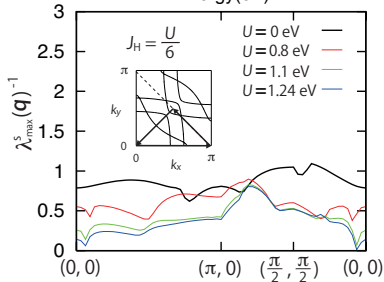
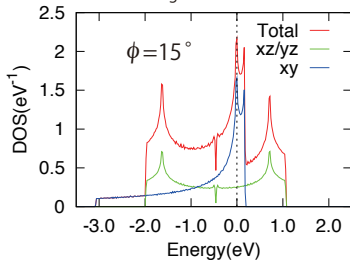
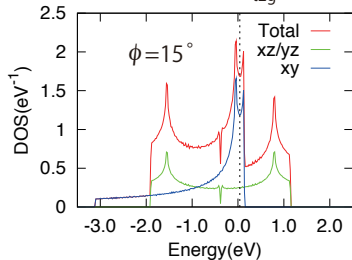
Roles of the vHs in Mag. prop.s

- ▶ IC FM spin fluc.s: dominant

- ▶ Location of the vHs \leftrightarrow Flat q dep. of $\hat{\chi}^S(\mathbf{q}, 0)$

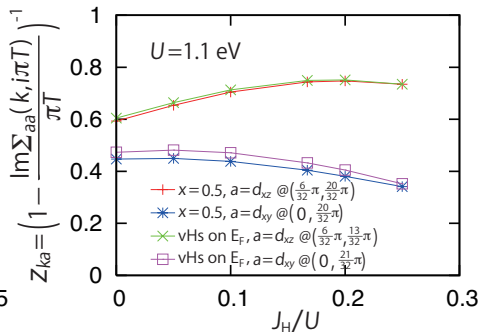
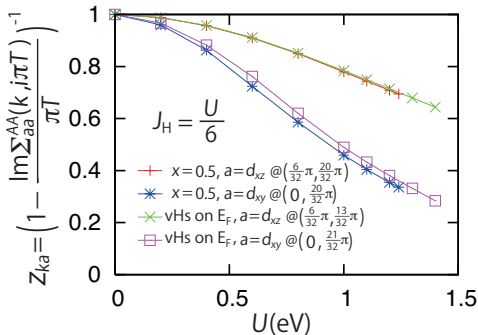
ii) $x = 0.5$ (vHs below E_F) $\Delta_{t2g} = 0.39$ eV

iii) vHs on E_F $\Delta_{t2g} = 0.27$ eV



Roles of the vHs in Renormalization of QPs

- ▶ Slight deviation of vHs \rightarrow Enhancement of $\frac{m_{xy}^*}{m_{xy}}$
- ▶ $\frac{m_{xy}^*}{m_{xy}} > \frac{m_{xz/yz}^*}{m_{xz/yz}}$ for all $\frac{J_H}{U}$



Q. WHAT is more important in enhancing m^* than the location of the vHs?

A. Flat \mathbf{q} dep. of $\hat{\chi}^S(\mathbf{q}, 0)$.

Summary and Message

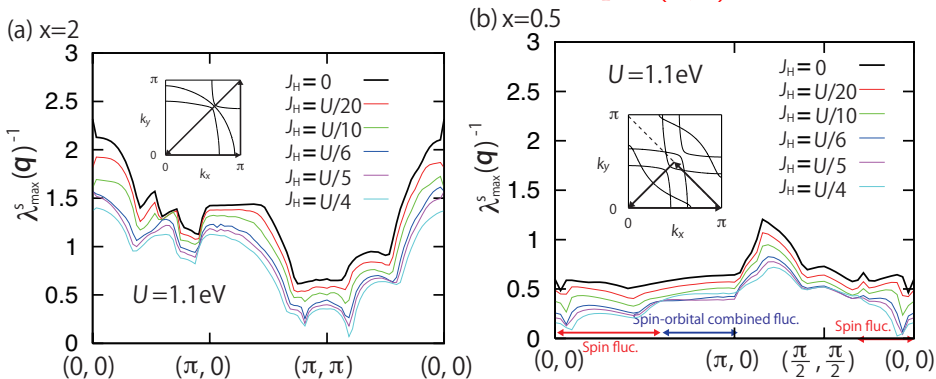
- ▶ Roles of the octahedral rotation:
 - ▶ IC FM spin fluc.s ↗, related with dominant fluc.s for $x = 2$
 - ▶ Flat \mathbf{q} dep. of $\hat{\chi}^S(\mathbf{q}, 0)$
 - ▶ Enhancement of $\frac{m_{xy}^*}{m_{xy}}$ and $\frac{m_{xz/yz}^*}{m_{xz/yz}}$
 - ▶ Enhancement of $\frac{m_{xy}^*}{m_{xz/yz}^*}$
 - ▶ $\frac{m_{xy}^*}{m_{xy}} > \frac{m_{xz/yz}^*}{m_{xz/yz}}$ for all $\frac{J_H}{U}$
- ▶ Roles of the vHs for the d_{xy} orbital:
 - ▶ IC FM spin fluc.s: dominant
 - ▶ Flat \mathbf{q} dep. of $\hat{\chi}^S(\mathbf{q}, 0)$ due to the slight deviation of vHs
 - ▶ $\frac{m_{xy}^*}{m_{xy}} \nearrow$ due to the slight deviation of vHs
 - ▶ $\frac{m_{xy}^*}{m_{xy}} > \frac{m_{xz/yz}^*}{m_{xz/yz}}$ for all $\frac{J_H}{U}$

Q. WHAT is more important in enhancing m^* than the location of the vHs?

A. Flat \mathbf{q} dep. of $\hat{\chi}^S(\mathbf{q}, 0)$.

Effects of J_H on Mag. prop.s

- ▶ $x = 2$: Monotonic enhancement, i.e. enhancement of spin fluc.s
- ▶ $x = 0.5$: Monotonic enhancement around $\mathbf{q} = (0, 0)$ and **Nonmonotonic enhancement around $\mathbf{q} = (\pi, 0)$**



Cf. For spin-orbital combined fluc., see Y. Yamashita and K. Ueda, PRB (2003).