1 Introduction

1.1 Quantum Magnets in the Magnetic Field

1. Magnetization Plateau

Field induced spin gap ⇒ Disordered Phase

2. Field Induced Antiferromagnetic Phase

( a ) Quasi-One-Dimensional System

Magnetic Field > Spin Gap ⇒ Tomonaga-Luttinger Liquid

+ Weak interchain interaction

Antiferromagnetic Long Range Order
Disorder effects on the quantum spin systems in the magnetic field

- Revived spins $\Rightarrow$ magnetic long range order
- Localized singlet pairs = plateau formation $\Rightarrow$ Suppresss the magnetic order

Competition

$\Downarrow$

Field induced reentrant transition
2 Model

Quasi-1-dimensional Random $S = 1/2$ Heisenberg model with bond alternation

$$H = \sum_j \left\{ \sum_{i=1}^{N/2} J_S S_{2i-1,j} S_{2i,j} + \sum_{i=1}^{N/2} J_{ij} S_{2i,j} S_{2i+1,j} \right\}$$

intrachain

$$+ \sum_{i=1}^{N} \sum_{<j,j'>} J_{\text{int}} S_{i,j} S_{i,j'} \quad \text{interchain}$$

- Intrachain Interaction:

  $$J = 1 \quad J_{i,j} = \begin{cases} J_S & \text{probability } p \\ J_W & \text{probability } 1 - p \end{cases}$$

  $$J_S > J > J_W > 0$$

Method DMRG: Keeping 60 $\sim$ 160 states.
[Bond Configuration]

\( p=1 \) \hspace{1cm} \cdots \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \cdots \\

\hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \hspace{0.5cm} J_S \hspace{0.5cm} J \\

\( p=0 \) \hspace{1cm} \cdots \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \cdots \\

\hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \hspace{0.5cm} J_W \hspace{0.5cm} J \\

\( 0 < p < 1 \) \hspace{1cm} \cdots \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \bullet \hspace{0.5cm} \cdots \\

\hspace{0.5cm} J \hspace{1cm} J_W \hspace{0.5cm} J \hspace{1cm} J_S \hspace{0.5cm} J \hspace{1cm} J_S \hspace{0.5cm} J \hspace{1cm} J_S \hspace{0.5cm} J \hspace{1cm} \cdots J \hspace{1cm} J_S \hspace{1cm} J \hspace{1cm} J_W \hspace{0.5cm} J
3 Magnetization Curve of an Isolated Chain at $T = 0$.

$J_S = 2 \quad J_W = 0.1 \quad J = 1$

1. Uniform Chain - DMRG
Averaged over 64 samples with $N = 120$. 

almost free spins
4  Effect of Interchain Interaction

Mean Field Approximation for the Interchain Interaction

\[ \langle S^x_{i,j} \rangle = \begin{cases} 
(−1)^i m & J_{\text{int}} < 0 \quad \text{Interchain ferromagnetic interaction} \\
(−1)^i P_j m & J_{\text{int}} > 0 \quad \text{Interchain antiferromagnetic interaction}
\end{cases} \]

for \( J_{\text{int}} > 0 \)

\[ P_j = +1 \quad j \in \text{A-sublattice} \]

\[ P_j = -1 \quad j \in \text{B-sublattice} \]

Interchain mean field Hamiltonian

\[
H^{\text{IMF}} = \sum_{i=1}^{N} J S_{2i-1} S_{2i} + \sum_{i=1}^{N} J_i S_{2i} S_{2i+1} - H_{\text{st}} \sum_{i=1}^{N} (-1)^i S^x_i
\]
\[ H_{st} = \lambda m(H_{st}) \quad \text{Self-consistent equation} \quad \lambda \equiv \frac{z}{|J_{int}|} \]

\[ \lambda_c = \lim_{H_{st} \to 0} \frac{H_{st}/m(H_{st})}{H_{st}} \]

\( \lambda_c \): critical interchain interaction \( \lambda \)

Multiple reentrant transition

\[ p = 0.2 \quad H_{st} = 0.0005 \quad p = 0.8 \]

\( N = 120 \). Averaged over 512 samples.
• Fine peak structure

\[ \lambda_c(H_{st}) \]

\[ H_{st}=0.0001 \quad H_{st}=0.002 \]

\[ N = 240, 480 \] Averaged over 256 samples (middle 240 sites).

\[ \lambda_c = 0 \] only for discrete points?

• \( H_c \)-dependence of \( \lambda_c \)

\[ 10^{-1.5} \quad 10^{-2} \]

\[ N=240 \quad 480 \]
5 Intrachain spin-spin correlation function

Averaged over 512 samples for $N = 240$

Exponential decay even for $\lambda_c = 0$

Non plateau state: Dense excited states near the ground state

Staggered mean field mixes the excited state into the ground state

⇓

Divergence of $\chi_{st}$

⇓

Long range order with weak interchain interaction
6 Summary

1. Quasi-one-dimensional random alternating bond $S = 1/2$ Heisenberg model exhibits multiple reentrant transitions in the magnetic field.
   
   **Method:** DMRG + Interchain mean field approximation

2. In the absence of interchain coupling, the spin-spin correlation function decays exponentially even in the non-plateau regime. Mix up the low energy excited states by interchain interaction.
   
   $\Rightarrow$ Long range order

3. Bose glass phase (Nohadani et al) is not found
   
   Limitation of interchain mean field approximation

4. • Reentrant transition in 3-D random dimer system
   
   Random destruction of singlet dimer $\Rightarrow$ **Local moment**

• Quasi-One-dimensional random alternating bond system
   
   Competition of 2 types of dimer patterns $\Rightarrow$ **local moment**

   More complicated features = **Multiple reentrant transition**
5. Speculated finite temperature phase diagram

6. Possibility of experimental observation: random substitution of anions on the superexchange path